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Distinguishability of some ununified models

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We calculate the left-right asymmetry parameters A^-, B^-, and $C^-_{L,R}$ in the polarized e–d and e–p deep-inelastic scattering processes for two recently proposed ununified models, in which the quarks and leptons transform nontrivially under different electroweak gauge groups. We then compare the various asymmetry parameters in these ununified models with those of the standard electroweak model. None of the ununified models can be discriminated from the standard model through measurements of the parameters A^-, B^-, and C^-_L. Both of these models are markedly distinguishable through measurement of C^-_R, which is particularly sensitive in the range $0.3 \le y \le 0.5$.

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I. INTRODUCTION

The remarkable success of the standard electroweak model at low energies has encouraged formulation of viable alternative theories with expanded gauge groups [1] and motivated measurements of parameters [2] which can discriminate these theories from the standard model. Cahn and Gilman [3] have shown that some of the extensions of the standard model can be distinguished through measurements of asymmetry parameters in the deep-inelastic scattering of polarized electrons by unpolarized protons at $Q^2 = 1\ \mathrm{GeV}^2/c^2$. It is this point of view which has motivated us to calculate the asymmetry parameters in two recently proposed models [4, 5] in order to establish their distinguishability from the standard model.

Recently Georgi, Jenkins, and Simmons [4] have proposed an extension of the standard model (hereafter referred to as the partially ununified model) based on the gauge group $\mathrm{SU}(2)_q \otimes \mathrm{SU}(2)_l \otimes \mathrm{U}(1)_Y$ which partially ununifies the standard model in the sense that the left-handed quarks and leptons couple to different SU(2) gauge groups and right-handed fermions transform as singlets under both. Several authors [6–8] have undertaken some phenomenological studies such as the hadronic decays of W's and Z's, forward-backward asymmetries in $e^+e^- \longrightarrow \mu^+\mu^-, b\bar{b}$ processes, B^0-$\bar{B}0$ mixing and the decay widths $\Gamma(Z \longrightarrow l^+l^-)$ and $\Gamma(Z \longrightarrow \text{hadrons})$ in the context of this partially ununified model and obtained some constraints for the model parameters.

Another fully ununified electroweak model [5] has also been recently proposed which ununifies completely the quarks and the leptons based on the group $G_{qL} \otimes G_{lL}$ where $G_{qL} = \mathrm{SU}(2)_{qL} \otimes \mathrm{U}(1)_{Yq}$ and $G_{lL} = \mathrm{SU}(2)_{lL} \otimes \mathrm{U}(1)_{Yl}$. The quarks (leptons) transform under G_{qL} (G_{lL}) exactly as they do under the standard electroweak model. In this model the charged-gauge-boson sector remains the same as in the partially ununified model, but the neutral-gauge-boson sector has been enlarged due to the inclusion of another Z, of course, the lightest of which is the one corresponding to the standard model.

The ununified models were originally proposed as yet another possible extension of the standard model. But afterwards it was found that in some grand unified theories such as SU(15) or SU(16), where baryon number is a gauge symmetry, one can embed the ununified models naturally. In these theories it is possible to suppress proton decay and one can have unification at low energies. These theories require ununified models to be present at the TeV scale [9]. Thus, observing a signature of the ununified model in various asymmetry experiments can imply low-energy grand unification and very rich and interesting new phenomenology in next-generation experiments.

In the present work we focus our attention on the mixing of weak neutral gauge bosons in the partially ununified and fully ununified model which leads to the deviations from the standard-model prediction in the weak-neutral-current sector and we propose to calculate the left-right asymmetry parameters $A^-, B^-, C^-_{L,R}$, in deep-inelastic e^--d and e^--p scattering in the context of both the models and compare them with the standard model.

Our analysis shows that none of the ununified models can be discriminated from the standard model through measurements of the parameters A^-, B^-, and C^-_L in the deep-inelastic e-d and e-p scattering processes. However, both the models can be distinguished from the standard model if the parameter B^- is measured for values of y in the range $0 \le y \le 0.1$. Interestingly the parameter C^-_R in e-d and e-p scattering will unambiguously discriminate the partially ununified as well as the fully ununified model from the standard electroweak model.

The structure of the paper is as follows. In Sec. II we describe the models and the new parameters entering in them. Section III contains our calculation of the various asymmetry parameters in these models. The results are presented in Sec. IV. We summarize our analysis in Sec. V.

II. THE MODELS

In this section we briefly summarize the essential features of the two models. The first one [4] ununifies the SU(2) group while the second one [5] ununifies both the SU(2) and the U(1) groups of the standard $SU(2)_L \otimes U(1)_Y$ electroweak model. As a result, in the first there will be one extra neutral gauge boson, whose mixing with the fermions will affect the asymmetry parameters, while there will be two extra neutral gauge bosons in the second model, both of which couple to the fermions.

A. Partially ununified model

The representation contents of quarks and leptons in the model [4] based on the electroweak gauge group $SU(2)_q \otimes SU(2)_l \otimes U(1)_Y$ are as follows:

$$q_L \equiv (2,1,\tfrac{1}{6}) \quad l_L \equiv (1,2,-\tfrac{1}{2})$$
$$q_R \equiv (1,1,Q) \quad l_R \equiv (1,1,Q)\ .$$

The gauge-covariant derivative is given by

$$D^\mu = \partial^\mu + ig_q T_q^a W_{qa}^\mu + ig_l T_l^a W_{la}^\mu + ig' Y B^\mu\ , \quad (1)$$

where T_q^a and T_l^a, $a = 1,2,3$, denote the SU(2) generators and Y is the usual hypercharge generator. $W_{qa}^\mu, W_{la}^\mu, B^\mu$ denote the weak gauge boson eigenstates and corresponding coupling constants are g_q, g_l, and g'. Two scalar fields $\Sigma(2,2,0)$ and $\phi(1,2,\frac{1}{2})$ are necessary for the spontaneous breakdown of the electroweak symmetry to $U(1)_{\rm em}$ when they acquire the vacuum expectation values (VEV's)

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}\ ,$$
$$\langle\Sigma\rangle = \begin{pmatrix} u & 0 \\ 0 & u \end{pmatrix}\ . \quad (2)$$

The electric charge Q is defined in terms of the generators of the partially ununified model as

$$Q = T_{3q} + T_{3l} + Y\ . \quad (3)$$

The neutral gauge boson eigenstates can be expressed as

$$A_\mu = s_w s_\phi W_{3q}^\mu + s_w c_\phi W_{3l}^\mu + c_w B^\mu\ ,$$
$$Z_1^\mu = c_w(s_\phi W_{3q}^\mu + c_\phi W_{3l}^\mu) - s_w B^\mu\ , \quad (4)$$
$$Z_2^\mu = c_\phi W_{3q}^\mu - s_\phi W_{3l}^\mu\ ,$$

where A_μ is the massless photon and Z_1^μ and Z_2^μ are the two neutral massive weak gauge bosons. The squared-mass-matrix (M_Z^2) for these weak neutral gauge bosons can be written as

$$M_Z^2 = M_W^2 \begin{pmatrix} \frac{1}{c_w^2} & -\frac{s_\phi}{c_\phi c_w} \\ -\frac{s_\phi}{c_\phi c_w} & \frac{x}{s_\phi^2 c_\phi^2} + \frac{s_\phi^2}{c_\phi^2} \end{pmatrix} \quad (5)$$

where $s_\phi = \sin\phi$, $c_\phi = \cos\phi$, $s_w = \sin\theta_w$, $c_w = \cos\theta_w$, $x = \frac{u^2}{v^2}$, and M_W is the tree-level charged-weak-gauge-boson mass in the standard model. Here θ_w is the usual Weinberg angle and the additional parameters of the model are a new mixing angle ϕ and the ratio x. Now the couplings can be expressed in terms of the two angles θ_w and ϕ as

$$g_q = \frac{g}{s_\phi},\ g_l = \frac{g}{c_\phi},\ g = \frac{e}{s_w},\ \text{and}\ g' = \frac{e}{c_w}. \quad (6)$$

The gauge-covariant derivative for the neutral sector is given by

$$D^\mu = \partial^\mu + \frac{e}{s_w c_w}(T_{3q} + T_{3l} - \sin^2\theta\, Q)Z_1^\mu + ig\left(\frac{c_\phi}{s_\phi}T_{3q} - \frac{s_\phi}{c_\phi}T_{3l}\right)Z_2^\mu + ieQA^\mu\ . \quad (7)$$

In the approximation $\frac{x}{s_\phi^2} \gg 1$ the masses for the Z's are given by

$$M_{Z_L} \approx M_Z\left(1 - \frac{s_\phi^4}{2x}\right)\ ,$$
$$M_{Z_H} \approx \frac{M_W\sqrt{x}}{s_\phi c_\phi}\left(1 + \frac{s_\phi^4}{2x}\right)\ . \quad (8)$$

In the large-(x/s_ϕ^2) approximation, the corresponding mass eigenstates Z_L, Z_H are given by

$$Z_L \approx Z_1 + \frac{s_\phi^3 c_\phi}{x c_w} Z_2\ ,$$
$$Z_H \approx Z_2 - \frac{s_\phi^3 c_\phi}{x c_w} Z_1\ . \quad (9)$$

The lighter one of these (Z_L), which is dominantly Z_1, corresponds to the neutral weak gauge boson of the standard model. We intend to calculate the effect of the mixing of Z_2 with Z_1 in the various left-right asymmetry parameters.

B. Fully ununified model

The other model [5] is based on the gauge group $G_q \otimes G_l$, where $G_q = SU(2)_{qL} \otimes U(1)_{Y_q}$ and similarly for G_l. The quarks (leptons) transform under G_q (G_l) exactly the same way as they do in the standard model and are also singlets under G_l (G_q). The representation content of quarks and leptons in this model are as follows:

$$q_L \equiv (2,\tfrac{1}{6},1,0)\ ,$$
$$l_L \equiv (1,0,2,-\tfrac{1}{2})\ ,$$
$$q_R \equiv (1,Q,1,0)\ ,$$
$$l_R \equiv (1,0,1,Q)\ .$$

The corresponding gauge-covariant derivative is

$$D^\mu = \partial^\mu - i(g_q T_q^a W_{qa}^\mu + g_l T_l^a W_{la}^\mu + g_q' Y_q B_q^\mu + g_l' Y_l B_l^\mu)\ , \quad (10)$$

where T^a's are the SU(2) generators and Y_q and Y_l are the two hypercharge generators. $W^{\mu}_{qa}, W^{\mu}_{la}, B^{\mu}_{q}$, and B^{μ}_{l} denote the weak-boson eigenstates and g_q, g_l, g'_q, and g'_l are the respective couplings. Now the minimal Higgs field chosen in the model are $\phi_q\,(2,\frac{1}{2},1,0)$, $\phi_l\,(1,0,2,\frac{1}{2})$, $\Sigma\,(1,\frac{1}{6},1,-\frac{1}{6})$, and $H\,(2,0,2,0)$. H is a real field and is responsible for the breaking of $\mathrm{SU}(2)_{qL}\otimes\mathrm{SU}(2)_{lL}$ to diagonal $\mathrm{SU}(2)_L$. A nonzero VEV of Σ breaks $\mathrm{U}(1)_{Y_q}\otimes\mathrm{U}(1)_{Y_l}\longrightarrow\mathrm{U}(1)_Y$. ϕ_q and ϕ_l are used to break the standard electroweak symmetry and give masses to the quarks and leptons respectively. The choice of the VEV's of the Higgs fields are as follows :

$$\langle H\rangle=\begin{pmatrix}u&0\\0&u\end{pmatrix},\quad\langle\Sigma\rangle=\sigma,\quad\langle\phi_q\rangle=\begin{pmatrix}0\\v_q\end{pmatrix}, \tag{11}$$

$$\langle\phi_l\rangle=(0\quad v_l)\ . \tag{12}$$

The electric charge in the model is defined in terms of the fully-ununified-model generators as

$$Q=T_{3q}+T_{3l}+Y_q+Y_l. \tag{13}$$

The neutral-gauge-boson eigenstates can be defined through the orthogonal transformations:

$$\begin{pmatrix}Z_l\\Z_n\\Z_d\\A\end{pmatrix}^{\mu}=\begin{pmatrix}0&0&c_\theta&-s_\theta\\c_\phi&-s_\phi&0&0\\c_w s_\phi&c_w c_\phi&-s_w s_\theta&-s_w c_\theta\\s_w s_\phi&s_w c_\phi&c_w s_\theta&c_w c_\theta\end{pmatrix}\begin{pmatrix}W_q^3\\W_l^3\\B_q\\B_l\end{pmatrix}^{\mu} \tag{14}$$

with $c_i=\cos i$, $s_i=\sin i$, $t_i=\tan i$, $i=\theta_w,\phi,\theta$. In these bases the gauge-covariant derivative for the neutral current is

$$D^\mu=\partial^\mu+gt_w(t_\theta^{-1}Y_q-t_\theta Y_l)Z_l^\mu+g(t_\phi^{-1}T_q^3-t_\phi T_l^3)Z_n^\mu+gc_w^{-1}(T_d^3-s_w^2Q)Z_d^\mu+eQA^\mu. \tag{15}$$

Here $T_d^i=T_q^i+T_l^i$. The mass eigenstates of the neutral gauge bosons are

$$\begin{pmatrix}Z_S\\Z_H\\Z_L\end{pmatrix}=\begin{pmatrix}1&0&0\\0&c_{nd}&-s_{nd}\\0&s_{nd}&c_{nd}\end{pmatrix}\begin{pmatrix}c_{ld}&0&-s_{ld}\\0&1&0\\s_{ld}&0&c_{ld}\end{pmatrix}\begin{pmatrix}c_{nl}&-s_{nl}&0\\s_{nl}&c_{nl}&0\\0&0&1\end{pmatrix}\begin{pmatrix}Z_l\\Z_n\\Z_d\end{pmatrix}, \tag{16}$$

where

$$\tan\theta_{nd}\approx\frac{q_nq_d-l_nl_d}{m_n^2+l_n^2+q_n^2-l_d^2-q_d^2},$$
$$\tan\theta_{ld}\approx\frac{l_ll_d-q_lq_d}{m_l^2+l_l^2+q_l^2-l_d^2-q_d^2},$$
$$\tan\theta_{nl}\approx-\frac{q_lq_n+l_ll_n}{m_l^2+l_l^2+q_l^2-m_n^2-l_n^2-q_n^2}.$$

The mass eigenvalues of the neutral gauge bosons are given by

$$\begin{aligned}M^2(Z_S)&\approx\frac{1}{2}g^2(m_l^2+l_l^2+q_l^2+\delta_{ld}+\delta_{nl}),\\M^2(Z_H)&\approx\frac{1}{2}g^2(m_n^2+l_n^2+q_n^2+\delta_{nd}-\delta_{nl}),\\M^2(Z_L)&\approx\frac{1}{2}g^2(l_d^2+q_d^2-\delta_{ld}-\delta_{nd}),\end{aligned} \tag{17}$$

where

$$\delta_{nd}\approx(q_nq_d-l_nl_d)\tan\theta_{nd},$$
$$\delta_{ld}\approx(l_ll_d-q_lq_d)\tan\theta_{ld},$$
$$\delta_{nl}\approx-(q_nq_l+l_nl_l)\tan\theta_{nl},$$

$$m_l^2=\frac{4a^2\sigma^2}{c_\theta^2s_\theta^2}\ \text{with}\ a=\tfrac{1}{6},\quad m_n^2=\frac{2u^2}{c_\phi^2s_\phi^2},$$
$$l_l=v_lt_wt_\theta,\quad q_l=v_qt_wt_\theta^{-1},$$
$$l_n=v_lt_\phi,\quad q_n=v_qt_\phi^{-1},$$
$$l_d=v_lc_w^{-1},\quad q_d=v_qc_w^{-1}.$$

The lightest of these bosons, Z_L, which is dominantly Z_d, is the neutral gauge boson of the standard model. We shall next see how the mixing of these neutral gauge bosons affect the various left-right asymmetry parameters.

III. CALCULATION OF ASYMMETRY PARAMETERS

Following the notations of Refs. [2] and [3] the asymmetry parameters in e-p and e-d scattering are defined as follows:

$$A^-=\frac{d\sigma(e_R^-)-d\sigma(e_L^-)}{d\sigma(e_R^-)+d\sigma(e_L^-)}, \tag{18}$$

$$B^-=\frac{d\sigma(e_R^-)-d\sigma(e_L^+)}{d\sigma(e_R^-)+d\sigma(e_L^+)}, \tag{19}$$

$$C_{L,R}^-=\frac{d\sigma(e_{L,R}^+)-d\sigma(e_{L,R}^-)}{d\sigma(e_{L,R}^+)+d\sigma(e_{L,R}^-)}, \tag{20}$$

where $d\sigma_R=\sum(d\sigma_{RL}^i+d\sigma_{RR}^i)q_i(x)$ and $d\sigma_L=\sum(d\sigma_{LL}^i+d\sigma_{LR}^i)q_i(x)$, where the summation is over all quarks and antiquarks. If the Dirac structure of photon-and-Z–fermion vertex is

$$[\tfrac{1}{2}\gamma_\mu(1-\gamma_5)Q_{Lf}^{\gamma,Z(\alpha)}+\tfrac{1}{2}\gamma_\mu(1+\gamma_5)Q_{Rf}^{\gamma,Z(\alpha)}][A^\mu,Z^{(\alpha)\mu}], \tag{21}$$

where $Q_{Lf}^{\gamma,Z}$ and $Q_{Rf}^{\gamma,Z}$ are the strength by which the photon or $Z^{(\alpha)}$ couples with left- and right-handed fermions, respectively, obviously, one then obtains the following cross sections,

$$d\sigma_{RR}^i\propto\left|\frac{Q_{Re}^\gamma Q_{Ri}^\gamma}{Q^2}+\sum\frac{Q_{Re}^{Z(\alpha)}Q_{Ri}^{Z(\alpha)}}{Q^2+M_Z^2(\alpha)}\right|^2, \tag{22}$$

$$d\sigma^i_{RL} \propto \left| \frac{Q^\gamma_{Re} Q^\gamma_{Li}}{Q^2} + \sum \frac{Q^{Z(\alpha)}_{Re} Q^{Z(\alpha)}_{Li}}{Q^2 + M^2_Z(\alpha)} \right|^2 (1-y)^2 , \quad (23)$$

$$d\sigma^i_{LL} \propto \left| \frac{Q^\gamma_{Le} Q^\gamma_{Li}}{Q^2} + \sum \frac{Q^{Z(\alpha)}_{Le} Q^{Z(\alpha)}_{Li}}{Q^2 + M^2_Z(\alpha)} \right|^2 , \quad (24)$$

and

$$d\sigma^i_{LR} \propto \left| \frac{Q^\gamma_{Le} Q^\gamma_{Ri}}{Q^2} + \sum \frac{Q^{Z(\alpha)}_{Le} Q^{Z(\alpha)}_{Ri}}{Q^2 + M^2_Z(\alpha)} \right|^2 (1-y)^2 , \quad (25)$$

where $d\sigma^i_{L(R)L(R)}(e^-)$ is the cross section for the scattering of left- (right-) handed electrons from left- (right-) handed quarks of type i. In order to get the total contribution one has to multiply each $d\sigma^i$ by corresponding weight factors given by the quark-antiquark distribution function obtainable from deep-inelastic scattering.

Since the electromagnetic current in every gauge model remains unchanged we have $Q^\gamma_{Le} = Q^\gamma_{Re} = -e$, $Q^\gamma_{Lu} = Q^\gamma_{Ru} = \frac{2}{3}e$, and $Q^\gamma_{Ld} = Q^\gamma_{Rd} = -\frac{1}{3}e$. The electromagnetic coupling e and the weak Fermi coupling G_F are related as follows,

$$e = g \sin\theta_w ,$$
$$\frac{G_F}{2\sqrt{2}\pi\alpha} = \frac{1}{4 \sin^2\theta_w \cos^2\theta_w M_Z^2} . \quad (26)$$

The weak charges of the fermions are given by

$$Q^Z_L = \frac{e}{s_w c_w}(T_{3L} - Q^\gamma_{Lf} \sin^2\theta_w) , \quad (27)$$
$$Q^Z_R = \frac{e}{s_w c_w}(T_{3R} - Q^\gamma_{Rf} \sin^2\theta_w) . \quad (28)$$

Here T_{3L} and T_{3R} are third components of weak isospin for left- and right-handed fermions, respectively. Similar expressions for the weak charges $Q^{Z(\alpha)}$ etc. for the other models can be easily computed by considering the relations $Q^{Z(\alpha)}_L = V^{Z(\alpha)} - A^{Z(\alpha)}$ and $Q^{Z(\alpha)}_R = V^{Z(\alpha)} + A^{Z(\alpha)}$ where $V^{Z(\alpha)}$ and $A^{Z(\alpha)}$ are the appropriate weak vector and axial-vector charges for the respective $Z^{(\alpha)}$'s.

A. Partially ununified model

In the partially ununified model we have two neutral weak bosons with mass eigenstates Z_L and Z_H and the corresponding weak charges are

$$Q^{Z_L}_{Le} = \frac{e}{s_w c_w}\left(\frac{s^4_\phi}{2x} + s^2_w - \frac{1}{2}\right) ,$$
$$Q^{Z_L}_{Re} = e\, t_w$$
$$Q^{Z_L}_{Lu} = \frac{e}{s_w c_w}\left(\frac{s^2_\phi c^2_\phi}{2x} - \frac{2}{3}s^2_w + \frac{1}{2}\right) ,$$
$$Q^{Z_L}_{Ru} = -\frac{e}{s_w c_w}\left(\frac{2}{3}s^2_w\right) ,$$
$$Q^{Z_L}_{Ld} = \frac{e}{s_w c_w}\left(-\frac{s^2_\phi c^2_\phi}{2x} + \frac{1}{3}s^2_w - \frac{1}{2}\right) ,$$
$$Q^{Z_L}_{Rd} = \frac{e}{3}\, t_w$$

and

$$Q^{Z_H}_{Le} = \frac{e}{s_w c_w}\left(\frac{s^3_\phi c_\phi}{2x c_w} - \frac{s^3_\phi c_\phi s^2_w}{x c_w} + \frac{s_\phi c_w}{2c_\phi}\right) ,$$
$$Q^{Z_H}_{Re} = \frac{e}{s_w c_w}\left(-\frac{s^3_\phi c_\phi s^2_w}{x c_w}\right) ,$$
$$Q^{Z_H}_{Lu} = \frac{e}{s_w c_w}\left(-\frac{s^3_\phi c_\phi}{2x c_w} + \frac{2 s^3_\phi c_\phi s^2_w}{3x c_w} + \frac{c_\phi c_w}{2 s_\phi}\right) ,$$
$$Q^{Z_H}_{Ru} = \frac{e}{s_w c_w}\left(\frac{2 s^3_\phi c_\phi s^2_w}{3x c_w}\right) ,$$
$$Q^{Z_H}_{Ld} = \frac{e}{s_w c_w}\left(\frac{s^3_\phi c_\phi}{2x c_w} - \frac{s^3_\phi c_\phi s^2_w}{3x c_w} - \frac{c_\phi c_w}{2 s_\phi}\right) ,$$
$$Q^{Z_H}_{Rd} = \frac{e}{s_w c_w}\left(-\frac{s^3_\phi c_\phi s^2_w}{3x c_w}\right) ,$$

where ϕ is the additional mixing angle arising from the mixing of the neutral bosons associated with the splitting of the two SU(2)'s. The parameter s_ϕ plays a crucial role in this model. When s_ϕ approaches zero one expects $SU(2)_q \otimes SU(2)_l \otimes U(1)_Y \longrightarrow SU(2)_L \otimes U(1)_Y$.

B. Fully ununified model

In the second model,

$$Q^{Z_S}_{Le} = \frac{e}{s_w}\left(\frac{1}{2}P_1 t_\theta t_w - \frac{1}{2}U t_\phi - \frac{1}{2}s_{ld} O \frac{1}{c_w}\right) ,$$
$$Q^{Z_S}_{Re} = \frac{e}{s_w}(P_1 t_\theta t_w - s_{ld} t_w s_w) ,$$
$$Q^{Z_S}_{Lu} = \frac{e}{s_w}\left(P_1 \frac{t_w}{6t_\theta} - \frac{U}{2t_\phi} - K\frac{s_{ld}}{2c_w}\right) ,$$
$$Q^{Z_S}_{Ru} = \frac{e}{s_w}\left(P_1 \frac{2t_w}{3t_\theta} + \frac{2}{3}s_{ld} t_w s_w\right) ,$$
$$Q^{Z_S}_{Ld} = \frac{e}{s_w}\left(P_1 \frac{t_w}{6t_\theta} + \frac{U}{2t_\phi} - K'\frac{s_{ld}}{2c_w}\right) ,$$
$$Q^{Z_S}_{Rd} = \frac{e}{s_w}\left(-P_1 \frac{t_w}{3t_\theta} - \frac{1}{3}s_{ld} t_w s_w\right) ,$$
$$Q^{Z_H}_{Le} = \frac{e}{2s_w}\left(-X t_\theta t_w + Y t_\phi - R_1 O \frac{1}{c_w}\right) ,$$
$$Q^{Z_H}_{Re} = -\frac{e}{s_w}(X t_\theta t_w - R_1 t_w s_w) ,$$
$$Q^{Z_H}_{Lu} = \frac{e}{2s_w}\left(-X\frac{t_w}{3t_\theta} + \frac{Y}{t_\phi} - R_1 K \frac{1}{c_w}\right) ,$$
$$Q^{Z_H}_{Ru} = \frac{2e}{3s_w}\left(-X\frac{t_w}{t_\theta} + R_1 t_w s_w\right) ,$$
$$Q^{Z_H}_{Ld} = \frac{e}{2s_w}\left(-X\frac{t_w}{3t_\theta} - \frac{Y}{t_\phi} - R_1 K' \frac{1}{c_w}\right) ,$$
$$Q^{Z_H}_{Rd} = \frac{e}{3s_w}\left(X\frac{t_w}{t_\theta} - R_1 t_w s_w\right)$$

and

$$Q^{Z_L}_{Le} = \frac{e}{2s_w}\left(X' t_\theta t_w - Y' t_\phi + SO\frac{1}{c_w}\right) ,$$
$$Q^{Z_L}_{Re} = \frac{e}{s_w}(X' t_\theta t_w + S t_w s_w) ,$$
$$Q^{Z_L}_{Lu} = \frac{e}{2s_w}\left(X'\frac{t_w}{3t_\theta} - \frac{Y'}{t_\phi} + SK\frac{1}{c_w}\right) ,$$
$$Q^{Z_L}_{Ru} = \frac{2e}{3s_w}\left(X'\frac{t_w}{t_\theta} - S t_w s_w\right) ,$$
$$Q^{Z_L}_{Ld} = \frac{e}{2s_w}\left(X'\frac{t_w}{3t_\theta} + \frac{Y'}{t_\phi} + SK'\frac{1}{c_w}\right) ,$$
$$Q^{Z_L}_{Rd} = \frac{e}{3s_w}\left(-X'\frac{t_w}{t_\theta} + S t_w s_w\right) ,$$

where

$$X = c_{nl}s_{ld}s_{nd} - c_{nd}s_{nl} , \quad Y = s_{nl}s_{ld}s_{nd} + c_{nl}c_{nd} ,$$
$$X' = c_{nl}s_{ld}c_{nd} + s_{nd}s_{nl} , \quad Y' = s_{nl}s_{ld}c_{nd} - c_{nl}s_{nd} ,$$
$$P_1 = c_{nl}c_{ld} , \quad R_1 = c_{ld}s_{nd} ,$$
$$S = c_{ld}c_{nd} , \quad U = s_{nl}c_{ld} ,$$
$$K = (1 - \tfrac{4}{3}s_w^2) , \quad K' = (\tfrac{2}{3}s_w^2 - 1) , \quad O = (2s_w^2 - 1).$$

In this model there are two extra neutral gauge bosons and hence there are two new mixing angles ϕ and θ. Thus we recover the standard model result in the limit $s_{\phi,\theta} \longrightarrow 0$.

IV. RESULTS

We have calculated the y dependence of the ratios $\frac{A^-}{Q^2}$, $\frac{B^-}{Q^2}$, $\frac{C_L^-}{Q^2}$, and $\frac{C_R^-}{Q^2}$ for polarized e-d and e-p deep-inelastic scattering. We have used $q_p^u(x) = 2q_p^d(x)$ for proton target and $q_d^u(x) = q_d^d(x)$ for deuteron target and neglected the QCD correction to the quark distribution function including the contribution of the quark-antiquark sea. We have compared the asymmetry parameters (A^-, $B,^-$ and $C^-_{L,R}$) in both the ununified models under consideration with those in the standard model (A_0^-, B_0^-, and $C^-_{L0,R0}$) and considered the asymmetry ratios for different values of y (fraction of the incident lepton's energy which is transferred to the nucleon).

A. Partially ununified model

The additional parameters in the partially ununified model are the Z_1-Z_2 mixing angle ϕ and a ratio $x = (\frac{u}{v})^2$ of the square of the VEV's. It is to be noted [6] that the prediction for the $e^+e^- \longrightarrow \mu^+\mu^-$ asymmetry at $\sqrt{s} = 35$ GeV, relevant to measurements of the DESY e^+e^- collider PETRA, deviates significantly from the standard-model results only at $s_\phi \geq 0.8$ for $M_{Z_H} \approx 100$ GeV ($s_\phi \geq 0.95$ for $M_{Z_H} = 200$ GeV). The $e^+e^- \longrightarrow b\bar{b}$ asymmetry is essentially independent of s_ϕ for $s_\phi \geq 0.1$ and the deviation from the standard model is $\delta A_{FB} = 0.2(100\ \text{GeV}/M_Z)^2$. Thus for $M_{Z_H} \geq 200$ GeV the asymmetry measurements do not give a restriction on the model. We have chosen M_{Z_L} in the range 91.07 GeV $\leq M_{Z_L} \leq$ 91.27 GeV and $M_{Z_H} > 500$ GeV and $x = 1$, so that the lighter Z mass agrees with the standard model Z mass [10] and the heavier one lies within the phenomenological bound [6]. We thus get the range of ϕ in the limit $0° \leq \phi \leq 9°$. The upper limit on ϕ, for $x = 1$, is obtained from the lower bound of M_{Z_H}. There is no significant variation in the asymmetry parameters A^-, B^-, $C^-_{L,R}$ for variation of ϕ in this range. The model reproduces the phenomenology of the standard model for $u \gg v$ but the situation $u \approx v$ is permitted for a wide range of ϕ and this situation is of interest to us in the present work.

For the purpose of the present analysis, we have calculated the asymmetry parameters for values of y in the range $0.1 \leq y \leq 1.0$ keeping $x = 1$ with $\phi = 5°$. Figure 1(a) shows the variation of the ratio $\frac{A^-}{A_0^-}$ with respect to y for deep-inelastic e-d and e-p scattering. It is evident that the ratio varies from ~ 1.0031 (1.0025) at $y = 0.1$ to 0.99384 (0.99363) at $y = 1$ for e-d (e-p) scattering. The experimental value of the asymmetry parameter A^- of Prescott *et al.* [11] for e-d scattering at $y = 0.20$ agrees within the errors with that predicted from the ununified model. In summary the ratio $\frac{A^-}{A_0^-}$ is close to unity within 0.6% for all values of y for both e-d and e-p scattering. Figure 1(b) depicts the plot of $\frac{B^-}{B_0^-}$ versus y for deep-inelastic e-d and e-p scattering. B^- is markedly different from B_0^- for both e-d and e-p scattering in the range $0 \leq y \leq 0.1$, since at $y = 0$ the value of B_0^- is always zero while B^- has some finite value. The value of the ratio $\frac{B^-}{B_0^-}$ remains almost constant at a value ~ 1.008 in the region $0.1 \leq y \leq 1$. Figure 1(c) exhibits the y dependence of $\frac{C_L^-}{C_{L0}^-}$ for e-d as well as e-p scattering. The ratio appears to be independent of y with the value $\frac{C_L^-}{C_{L0}^-} \approx 1.003$. Figure 1(d) shows the plot of $\frac{C_R^-}{C_{R0}^-}$ as a function of y. The ratio is markedly different from unity for both the scattering pro-

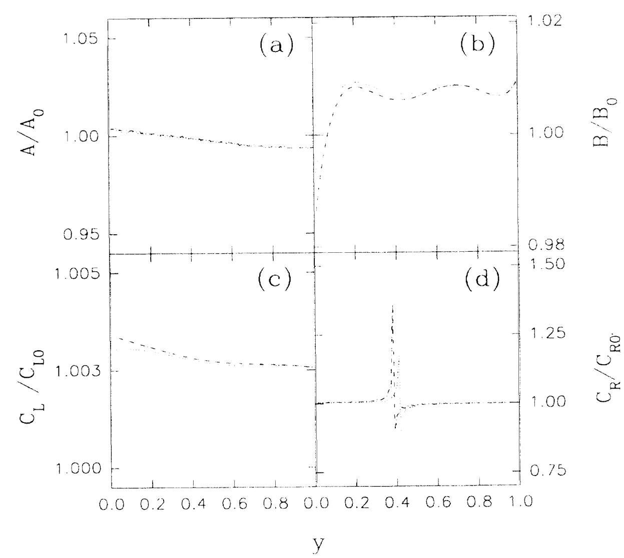

FIG. 1. The ratios (a) $\frac{A^-}{A_0^-}$, (b) $\frac{B^-}{B_0^-}$, (c) $\frac{C_L^-}{C_{L0}^-}$, and (d) $\frac{C_R^-}{C_{R0}^-}$ are plotted against y for e-d scattering (dotted lines) and e-p scattering (dash-dotted lines) for the partially ununified model.

cesses in the range $0.35 \leq y \leq 0.45$ with $\frac{C_R^-}{C_{R0}^-} \approx 1.1818$ at $y = 0.41$ for *e-d* scattering and $\frac{C_R^-}{C_{R0}^-} \approx 1.3636$ at $y = 0.38$ for *e-p* scattering. The ratio turns out to be ~ 1.006 for *e-d* (*e-p*) scattering in the range $0 \leq y \leq 0.3$ while the ratio has the value ~ 0.998 (0.999) in *e-d* (*e-p*) scattering for $y = 0.6$ to 1.0. Therefore, the parameter C_R^- is indistinguishable from C_{R0}^- in the above ranges.

It may be noted that for $x = 5$, ϕ lies in the range $0° \leq \phi \leq 19°$ with the above choice of M_{Z_L} and M_{Z_H}. With $x = 5$ and $\phi = 5°$ the asymmetry parameters except C_R^- agree with those for $x = 1$ within less than one percent. For example, the value of C_R^- for *e-d* (*e-p*) scattering for $x = 1$, $\phi = 5°$ is 1.1818 (1.3636) while that for $x = 5$, $\phi = 5°$ is 1.0455 (1.1818) at $y = 0.41$ (0.38).

B. Fully ununified model

The fully ununified model, in addition to the Weinberg angle θ_w, has two extra angles ϕ and θ which mix the three neutral gauge bosons. We choose the following mass hierarchy for the neutral gauge bosons: 91.07 GeV $\leq M_{Z_L} \leq$ 91.27 GeV, $M_{Z_H} > 500$ GeV and $M_{Z_S} > 700$ GeV. To compare it with the partially ununified model we choose $\theta = 5°$. The asymmetry parameters are insensitive to the variation of ϕ for $\theta = 5°$ and y in the range $0.1 \leq y \leq 1.0$. Hence, for the present analysis, we set $\phi = 45°$.

Figure 2 shows the plots of the ratios $\frac{A^-}{A_0^-}, \frac{B^-}{B_0^-}, \frac{C_L^-}{C_{L0}^-}$, and $\frac{C_R^-}{C_{R0}^-}$ as a function of y $(0.1 \leq y \leq 1.0)$ for $\theta = 5°$ and $\phi = 45°$. It is obvious from Fig. 2(a) that the variation of the parameter A^- with y in the range $0.1 \leq y \leq 1.0$ does not vary over 1%. The plot of $\frac{B^-}{B_0^-}$ versus y as shown in Fig. 2(b) unambiguously demonstrates that the ratio is almost the same for both *e-d* and *e-p* scattering processes and independent of y with $\frac{B^-}{B_0^-} \approx 1.003$. Although the ratio $\frac{C_L^-}{C_{L0}^-}$, in Fig. 2(c), increases with increasing y with $\frac{C_L^-}{C_{L0}^-} \approx 0.995$ (0.993) at $y = 0.1$ and with $\frac{C_L^-}{C_{L0}^-} \approx 0.9995$ (0.9989) at $y = 1.0$ for *e-d* (*e-p*) scattering processes this variation is negligible and the ratio is independent of y. The variation of the ratio $\frac{C_R^-}{C_{R0}^-}$ as a function of y [Fig. 2(d)] is quite spectacular in the range $0.3 \leq y \leq 0.5$ for both scattering processes. The ratio $\frac{C_R^-}{C_{R0}^-}$ first decreases sharply from $y = 0.35$ to 0.41 and then immediately increases to reach its maximum $\sim$ 1.5806 at $y = 0.42$ and then at $y = 0.45$ it becomes 1.1023 for *e-d* scattering. For *e-p* scattering the ratio decreases from 0.878 57 at $y = 0.35$ to $-0.888 89$ at $y = 0.38$ and then increases sharply to 1.5938 at $y = 0.39$. The ratio $\frac{C_R^-}{C_{R0}^-}$ turns out to be ~ 1 for y in the range $0.44 \leq y \leq 1.0$. Furthermore, the ratio becomes almost the same for both *e-d* and *e-p* scattering for y in the range $0.1 \leq y \leq 0.3$ and $0.5 \leq y \leq 1.0$.

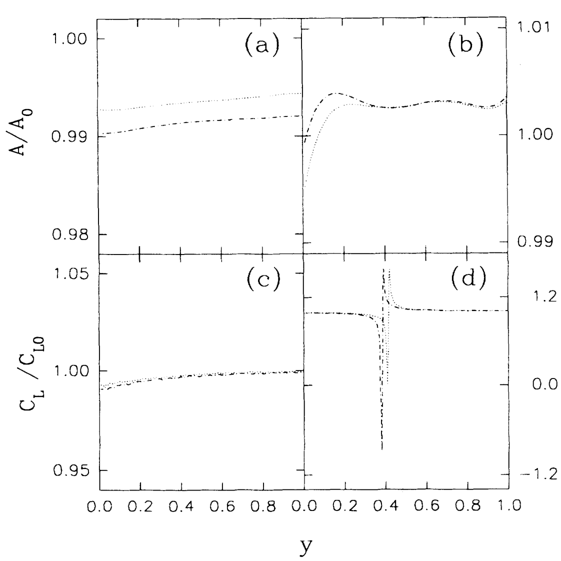

FIG. 2. The ratios (a) $\frac{A^-}{A_0^-}$, (b) $\frac{B^-}{B_0^-}$, (c) $\frac{C_L^-}{C_{L0}^-}$, and (d) $\frac{C_R^-}{C_{R0}^-}$ are plotted against y for *e-d* scattering (dotted lines) and *e-p* scattering (dash-dotted lines) for $\theta = 5°$ and $\phi = 45°$ in the fully ununified model.

V. CONCLUSION

We have calculated the asymmetry parameters A^-, B^-, C_L^-, and C_R^- in the standard, partially ununified and the fully ununified models in polarized *e–d* and *e–p* deep-inelastic scattering processes. Our analysis shows that none of the ununified models can be discriminated at the level of 5 σ effect from the standard model through measurement of the asymmetry parameters A^-, B^-, and C_L^- in both of the processes assuming 2% accuracy in the measured values of the parameters for values of y in the range $0.1 \leq y \leq 1.0$. The partially ununified as well as the fully ununified model can be discriminated from the standard model through measurement of the parameter B^- for values of y in the range $0 < y < 0.1$. However, the measurement of the asymmetry parameter C_R^- in both polarized *e–d* and *e–p* scattering processes can unambiguously distinguish among the standard, partially ununified, and fully ununified models.

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