# Combined fit to $R(e^+e^-\rightarrow$hadrons) and data from the CERN $e^+e^-$ collider LEP

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A combined fit to $R(e^+e^-\rightarrow$hadrons) in the range 5–61.4 GeV and to the data from the CERN $e^+e^-$ collider LEP is presented to obtain a global estimate of the strong-interaction scale parameter $\Lambda^{(5)}_{\overline{\rm MS}}$ free of uncertainties due to hadronization and choice of the momentum scale, where $\overline{\rm MS}$ denotes the modified minimal subtraction scheme. Our fitted values $\Lambda^{(5)}_{\overline{\rm MS}}=545^{+221}_{-182}$ MeV, in the range 5–94 GeV, and $\Lambda^{(5)}_{\overline{\rm MS}}=961^{+383}_{-314}$ MeV, in the range 14–94 GeV, do not agree, within two standard deviations, with the low-energy determinations from $\Upsilon$ branching ratios and deep-inelastic scattering.

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## I. INTRODUCTION

The process $e^+e^-\rightarrow$hadrons represents the most fundamental test of perturbative QCD. Indeed, the theoretical prediction is very clean and completely insensitive to hadronization effects. At the same time, experimental data are available in a very wide range of $Q^2$ so that, within the same process, it should be possible to detect in an unambiguous way the predicted "running" of $\alpha_s$.

In this paper we have refined the analysis of the data from SLAC, DESY, and KEK $e^+e^-$ collider PEP, PETRA, and TRISTAN presented in Ref. [1]; our sample now includes (1) the new data collected by the Crystal Ball Collaboration [2] in the range $5<\sqrt{s}<7.4$ GeV, (2) additional low-energy data in the continuum just below and just above the $Y$ threshold collected by the Crystal Ball [3], CLEO [4], CUSB [5], DHHM [6], and LENA [7], Collaborations (the data are treated homogeneously as presented in Ref. [8]), (3) the recent determination by the Mark II Collaboration [9] at $\sqrt{s}=29$ GeV, (4) the complete PETRA-PEP set, collected by the PLUTO [10], TASSO [11], HRS [12], MAC [13], Mark J [14], JADE [15], and CELLO [16] Collaborations (the data are treated homogeneously as presented in Ref. [16]), (5) the full TRISTAN sample as collected by the AMY [17], TOPAZ [18], and VENUS [19] Collaborations, and (6) finally, also, the most recent data from the CERN $e^+e^-$ collider LEP as published by the ALEPH [20], DELPHI [21], L3 [22], and OPAL [23] Collaborations, included for a global fit as explained in Sec. III. As such, we are presenting here the most complete analysis available today.

Our aim is to investigate the energy dependence of the strong-interaction correction to the quark-parton model and the correlations in the fit among the strong-interaction effects and electroweak parameters $M_Z$ and $\sin^2\theta_W$.

In Sec. II we introduce the formalism we have been using for our fit in the range 5–61 GeV. In Sec. III we show how to include the LEP data for a global fit. In Sec. IV we present our results for the range 5–61 GeV, and in Sec. V the LEP data are included in the analysis. Finally, Sec. VI contains our conclusions. For the convenience of the reader, the full sample of data for the range $5<\sqrt{s}<61$ GeV is reported in Table I. For the LEP data, on the other hand, we shall address the interested reader to Refs. [20–23].

## II. GENERAL FORMALISM FOR $R(e^+e^-\rightarrow$hadrons) IN THE RANGE 5–61 GeV

The values of $R$ measured by the various collaborations are determined by the expression

$$R=\frac{N_h-N_{\rm bg}}{\sigma_{\mu\mu}(s)\mathcal{L}\varepsilon(1+\delta)}\,, \tag{1}$$

where $N_h$ is the number of hadronic events detected, $N_{\rm bg}$ is the number of estimated background events, $\sigma_{\mu\mu}(s)$ $(=4\pi\alpha^2/3s)$ is the lowest-order QED cross section for $e^+e^-\rightarrow\mu^+\mu^-$, $\mathcal{L}$ is the integrated luminosity, $\varepsilon$ the detection efficiency, and $1+\delta$ the radiative correction factor. The factor $1+\delta$ is treated differently by the various collaborations; namely, for data in the ranges 5–10.5 GeV [2–7] and 14–46.6 GeV [9–16], only purely electromagnetic corrections have been taken into account, while at TRISTAN [17–19] (50–61.4 GeV) full electroweak corrections have been implemented.

The theoretical expression for the ratio $R$ we have been using to fit the data, calculated in perturbative QCD, is

$$R=3\sum_i [\tfrac{1}{2}\beta_i(3-\beta_i^2)R_i^V(1+C^V)+\beta_i^3 R_i^A(1+C^A)] , \quad (2)$$

where the sum runs over the quark flavors which can be produced at the relevant center-of-mass energy. Let us now briefly explain the right-hand side of Eq. (2).

The quantities $R_i^V$ and $R_i^A$, coming from the vector and axial-vector couplings, are, respectively,

$$\begin{aligned} R_i^V &= e_i^2-8e_i v_e v_i \mathrm{Re}(\chi)+16(v_e^2+a_e^2)v_i^2|\chi|^2 , \\ R_i^A &= 16(v_e^2+a_e^2)a_i^2|\chi|^2 , \end{aligned} \quad (3)$$

with $a_i=\frac{1}{2}\ (-\frac{1}{2})$, $v_i=\frac{1}{2}-\frac{4}{3}y\ (-\frac{1}{2}+\frac{2}{3}y)$, $e_i=\frac{2}{3}\ (-\frac{1}{3})$, for up and down quarks, respectively; $a_e=-\frac{1}{2}$, $v_e=-\frac{1}{2}+2y$; $\beta_i$ is the quark's velocity in the center-of-mass system.

Because of the different way of treating the radiative corrections, the neutral-current strength $\chi(s)$ and the quantity $y$ have different expressions for the three energy ranges 5–10.5, 14–46.6, and 50–61.4 GeV. On the one hand, the TRISTAN data have to be described in terms of the Born-approximation expressions

$$\begin{aligned} \chi(s) &= \frac{1}{16\sin^2\theta_W\cos^2\theta_W}\,\frac{s}{s-M_Z^2+iM_Z\Gamma_Z} , \\ y &= \sin^2\theta_W , \end{aligned} \quad (4)$$

where $\sin^2\theta_W=1-M_W^2/M_Z^2$.

Equations (4), corresponding to the "on-shell" renormalization, make essential use of the $W$ mass, which, differently from the $Z$ mass, is not yet known to the required level of accuracy. In this situation it is more convenient to express $M_W$ in terms of the precisely determined quantities $\alpha=\alpha(0)=1/137.0359\ldots$, the Fermi constant $G_F=1.166\,38\ldots 10^{-5}$ GeV$^{-2}$, and of the $Z$ mass from LEP [20–23]:

$$M_Z=91.176\pm\underline{0.020}\ \mathrm{GeV} \quad (5)$$

(the underlined error is essentially due to the uncertainty in the energy scale) through the relations [24] ($\mu^2=\pi\alpha/G_F\sqrt{2}$)

$$M_W^2=M_Z^2\cos^2\theta_W \quad (6)$$

and

$$\begin{aligned} \sin^2\theta_W &= 1-\cos^2\theta_W \\ &= \frac{1}{2}\left[1-\left[1-\frac{4\mu^2}{M_Z^2}\,\frac{1}{1-\Delta r}\right]^{1/2}\right] . \end{aligned} \quad (7)$$

In the standard model (SM), the radiative correction $\Delta r$ is a function of the $Z$ mass itself and of the unknown Higgs-boson and $t$-quark masses. Numerical tables for $\Delta r$ can be found in Ref. [25].

On the other hand, for the PEP-PETRA data, one has to employ an "improved" Born approximation [26–29] which takes into account the neglected relevant higher-order weak effects. Therefore, in this case, we have to use the expressions

$$\begin{aligned} \chi(s) &= \frac{1}{16}\,\frac{M_Z^2}{\mu^2}\,\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}(1+\Delta\rho) , \\ y &= \sin^2\bar{\theta} , \end{aligned} \quad (8)$$

where the effective value of the weak mixing angle $\sin^2\bar{\theta}$, appearing in the couplings of the $Z$ with quarks and leptons, can be expressed [26,28,29] in terms of $\sin^2\theta_W$ through the relation

$$\sin^2\bar{\theta}\sim\sin^2\theta_W+\Delta\rho\cos^2\theta_W . \quad (9)$$

In the minimal SM, $\Delta\rho$ depends mainly on the top-quark $m_t$ [30] and Higgs-boson [31] masses and one finds the leading behavior for large $m_t$ and very large $m_H$ ($m_H \gg M_Z$):

$$\Delta\rho\simeq\frac{3G_F}{8\pi^2\sqrt{2}}m_t^2-3\frac{G_F M_Z^2\sin^2\bar{\theta}}{4\pi^2\sqrt{2}}\ln\left[\frac{m_H}{M_Z}\right] . \quad (10)$$

As for $\sin^2\theta_W$, $\sin^2\bar{\theta}$ can also be expressed in terms of $M_Z$, $\alpha$, and $G_F$ analogously to Eq. (7):

$$\sin^2\bar{\theta}=\frac{1}{2}\left[1-\left[1-\frac{4\mu^2}{M_Z^2}\,\frac{1}{1-\overline{\Delta r}}\right]^{1/2}\right] , \quad (11)$$

and $\overline{\Delta r}$ is also tabulated in Ref. [25].

At the same time, since vector-boson vertex parts and box diagrams are not included in the analysis of the PETRA-PEP data, it is sensitive to neglect nonleading effects in Eqs. (7) and (11) and, to a level of accuracy $\sim 5\times 10^{-3}$, express both $\sin^2\theta_W$ and $\sin^2\bar{\theta}$ through the simpler relations [32] [$\rho=1/(1-\Delta\rho)$]

$$\begin{aligned} 1-\sin^2\theta_W &= \cos^2\theta_W \\ &= \frac{\rho}{2}\left[1+\left[1-\frac{4\mu^2}{\rho M_Z^2}\,\frac{\alpha(M_Z)}{\alpha}\right]^{1/2}\right] , \end{aligned} \quad (12)$$

$$\sin^2\bar{\theta}=\frac{1}{2}\left[1-\left[1-\frac{4\mu^2}{\rho M_Z^2}\,\frac{\alpha(M_Z)}{\alpha}\right]^{1/2}\right] . \quad (13)$$

Equation (12), as shown in Ref. [32], reproduces the correct $M_W-M_Z$ interdependence, including terms $(\Delta\rho)^2$, and Eq. (13) follows from Eq. (9).

In the following we shall use the above accurate parametrization in Eqs. (12) and (13), where the purely electroweak effects can be expressed in terms of the global quantity $\rho$, of the $Z$ mass $M_Z^2$, and the precisely known quantities $\alpha$ and $G_F$. At the same time, Eqs. (12) and (13) represent a more general parametrization than in Eqs. (7) and (11) since they allow for deviations from the standard-model minimal structure associated with $\rho\neq 1$ at the tree level.

Finally, for the 5–10.5-GeV data, the effects due to the $Z$ are negligible; then $\chi(s)$ in this range is substantially zero.

The QCD correction factors $C^V$ and $C^A$ in Eq. (2), up to the third order in $\alpha_s$, have the form [33,34]

$$C^V(A)=C_1^{V(A)}(\alpha_s/\pi)+C_2^{V(A)}(\alpha_s/\pi)^2+C_3^{V(A)}(\alpha_s/\pi)^3\ , \tag{14}$$

with

$$C_1^V=\frac{4\pi}{3}\left[\frac{\pi}{2\beta_i}-\frac{3+\beta_i}{4}\left[\frac{\pi}{2}-\frac{3}{4\pi}\right]\right],$$

$$C_1^A=\frac{4\pi}{3}\left[\frac{\pi}{2\beta_i}-\left[\frac{19}{10}-\frac{22\beta_i}{5}+\frac{7\beta_i^2}{2}\right]\left[\frac{\pi}{2}-\frac{3}{4\pi}\right]\right], \tag{15}$$

$$C_2^V=C_2^A=1.986-0.115N_f$$

and $N_f$ is the number of quark flavors. Finally, from Ref. [34], one obtains

$$C_3^V=C_3^A=-12.805,\quad N_f=5\ , \tag{16}$$

$$C_3^V=C_3^A=-11.682,\quad N_f=4\ . \tag{17}$$

Mass terms other than those prescribing to $O(\alpha_s)$, the $c$- and $b$-quark threshold effects, have been neglected in the QCD corrections.

The three-loop coupling constant is related to the QCD scale parameter $\Lambda_{\overline{\rm MS}}^{(N_f)}$ where $\overline{\rm MS}$ denotes the modified minimal-subtraction scheme, through the expression [35]

$$\alpha_s(s)=\frac{4\pi}{\beta_0 L}\left\{1-\frac{\beta_1\ln L}{\beta_0^2 L}+\left[\frac{\beta_1}{\beta_0^2 L}\right]^2\left[\left[\ln L-\frac{1}{2}\right]^2+\frac{\beta_2\beta_0}{\beta_1^2}-\frac{5}{4}\right]\right\}, \tag{18}$$

where

$$L=\ln\frac{s}{\Lambda_{\overline{\rm MS}}^2}\ ,$$

$$\beta_0=11-\tfrac{2}{3}N_f\ ,$$

$$\beta_1=102-\tfrac{38}{3}N_f\ ,$$

$$\beta_2=\tfrac{2857}{2}-\tfrac{5033}{18}N_f+\tfrac{325}{54}N_f^2\ .$$

In the range of energy below the threshold of the $\Upsilon$ production, the QCD scale parameter is $\Lambda_{\overline{\rm MS}}^{(4)}$, which can be expressed in terms of $\Lambda_{\overline{\rm MS}}^{(5)}$ [36]. In the following we shall always refer to $\Lambda_{\overline{\rm MS}}^{(5)}$.

Finally, to make an alternative fit, we can describe, as suggested in Ref. [16], the strong-interaction correction in a model-independent way by replacing both factors $(1+C^V)$ and $(1+C^A)$ in Eq. (2) with the linearized form $(E=\sqrt{s}\,)$

$$R=3[A(E_0)+B(E-E_0)]\times\sum_f[\tfrac{1}{2}\beta_f(3-\beta_f^2)R_f^V+\beta_f^3R_f^A]\ . \tag{19}$$

## III. INCLUSION OF THE LEP DATA

The experimental data from LEP can be easily included in a global fit to sharpen the determination of both the strong-interaction scale parameter $\Lambda_{\overline{\rm MS}}^{(5)}$ and electroweak parameters. We shall use the model-independent analysis of the $Z$ line shape introduced in Ref. [37]. However, since the theoretical prediction for the $e^+e^-\to$hadrons data in the range 5–61 GeV can be described in terms of $\Lambda_{\overline{\rm MS}}^{(5)}$, $\rho$, and $M_Z^2$ (and the precisely determined quantities $\alpha$ and $G_F$), we have to introduce a suitable modification of Ref. [37] to be employed in the common fit. To this end, in the spirit of an improved Born approximation which takes into account the most important electroweak corrections and which is accurate to $\sim 5\times10^{-3}$, we can use the results of Refs. [26,29] by parametrizing the leptonic widths as ($l=e,\mu,\tau$)

$$\Gamma(Z\to l^+l^-)=\Gamma_l=\rho\frac{G_FM_Z^3}{24\pi\sqrt{2}}[1+(1-4\sin^2\bar\theta)^2]\left[1+\frac{3}{4}\frac{\alpha}{\pi}\right], \tag{20}$$

$$\Gamma(Z\to\nu\bar\nu)=\rho\frac{G_FM_Z^3}{12\pi\sqrt{2}}\ , \tag{21}$$

and $\sin^2\bar\theta$ is defined in Eq. (13).

At the same time, in the standard model, independently of the top-quark and Higgs-boson masses, the ratio between the hadronic and leptonic widths is severely constrained. In the quark-parton model, one gets [38]

$$\frac{\Gamma_h^{(0)}}{\Gamma_l}=R^{(0)}=19.97\pm0.03\ ; \tag{22}$$

therefore, by expressing the hadronic width as

$$\Gamma_h=R^{(0)}\Gamma_lA_{\rm QCD}\ , \tag{23}$$

with

$$A_{\rm QCD}=1+(\alpha_s/\pi)+1.411(\alpha_s/\pi)^2-12.805(\alpha_s/\pi)^3\ , \tag{24}$$

and using Eqs. (20) and (21), we can perform a three-parameter fit $(\rho,M_Z,\Lambda_{\overline{\rm MS}}^{(5)})$ to the LEP data, just as in the case of the $e^+e^-\to$hadrons data in the range 5–61 GeV.

Analogously to Eq. (19), the LEP data can be easily included in a model-independent fit by replacing the perturbative predictions for $A_{\rm QCD}$ in Eq. (24) with the linearized form

$$A_{\rm QCD}=A(E_0)+B(M_Z-E_0)\ .$$

## IV. RESULTS IN THE RANGE 5–61 GeV

The sample of data we have been using is reported for convenience of the reader in Table I. The center-of-mass energy runs from 5 to 61.4 GeV (total sample of data: 126). Systematic errors are treated by introducing in the $\chi^2$ a penalty function which takes into account the normalization of the various experiments.

The simplest choice to have an idea of the various effects is to start with the very-low-energy data in the range 5–10.5 GeV. Indeed, in this case, the analysis

TABLE I. Total sample of experimental values of $R$ we have been using, with the relative center-of-mass energy $\sqrt{s}$, the statistical $\sigma_{\text{stat}}$ and systematic $\sigma_{\text{syst}}$, errors. The low-energy data (5–7.4 GeV) of the Crystal Ball Collaboration (SPEAR) are taken from Ref. [2]. The additional Crystal Ball, LENA, DHHM, CUSB, and CLEO data are reported as quoted in Ref. [8] (energy points which lie closer than a width from the $\Upsilon$ system are not included in our fit). The Mark II value is taken from Ref. [9]. The PEP-PETRA data (14–46.6 GeV data) of Refs. [10–16] are reported as quoted in Ref. [16]. The statistical and point-to-point errors appearing in Ref. [16] have been combined in quadrature, as indicated in Ref. [16], to obtain our $\sigma_{\text{stat}}$, just like $\sigma_{\text{norm1}}$ and $\sigma_{\text{norm2}}$ of Ref. [16] to obtain our $\sigma_{\text{syst}}$. The TRISTAN data (50–61.4 GeV) are taken from Refs. [17–19].

| Experiment | $\sqrt{s}$ (GeV) | $R$ | $\sigma_{\text{stat}}$ (%) | $\sigma_{\text{syst}}$ (%) |
|---|---|---|---|---|
| C Ball | 5.20 | 3.44 | 4.42 | 5.2 |
| | 6.00 | 3.44 | 4.59 | 5.2 |
| | 6.50 | 3.62 | 5.59 | 5.2 |
| | 7.00 | 3.71 | 5.18 | 5.2 |
| | 5.00 | 3.42 | 4.96 | 5.2 |
| | 5.25 | 3.57 | 3.19 | 5.2 |
| | 5.50 | 3.41 | 3.42 | 5.2 |
| | 5.75 | 3.44 | 3.25 | 5.2 |
| | 6.00 | 3.50 | 3.66 | 5.2 |
| | 6.25 | 3.31 | 3.15 | 5.2 |
| | 6.50 | 3.37 | 3.10 | 5.2 |
| | 6.75 | 3.42 | 3.16 | 5.2 |
| | 7.00 | 3.35 | 3.21 | 5.2 |
| | 7.25 | 3.57 | 3.65 | 5.2 |
| | 7.40 | 3.35 | 5.14 | 5.2 |
| LENA | 7.44 | 3.37 | 3.90 | 6.7 |
| | 8.91 | 3.42 | 2.90 | 6.7 |
| | 9.28 | 3.31 | 2.70 | 6.7 |
| C Ball | 9.39 | 3.48 | 1.10 | 4.6 |
| DHHM | 9.40 | 3.80 | 7.0 | 11.0 |
| CUSB | 10.40 | 3.54 | 1.40 | 11.3 |
| CLEO | 10.49 | 3.77 | 1.60 | 6.4 |
| Mark II | 29.00 | 3.92 | 1.20 | 2.2 |
| HRS | 29.00 | 4.20 | 0.80 | 7.0 |
| MAC | 29.00 | 4.00 | 0.80 | 2.1 |
| JADE | 14.04 | 3.94 | 3.60 | 2.4 |
| | 22.00 | 4.11 | 3.20 | 2.4 |
| | 25.01 | 4.24 | 6.80 | 2.4 |
| | 27.66 | 3.85 | 12.50 | 2.4 |
| | 29.93 | 3.55 | 11.30 | 2.4 |
| | 30.38 | 3.85 | 4.90 | 2.4 |
| | 31.29 | 3.84 | 7.30 | 2.4 |
| | 33.89 | 4.17 | 2.40 | 2.4 |
| | 34.50 | 3.94 | 5.10 | 2.4 |
| | 35.01 | 3.94 | 2.50 | 2.4 |
| | 35.45 | 3.94 | 4.60 | 2.4 |
| | 36.38 | 3.72 | 5.70 | 2.4 |
| | 40.32 | 4.07 | 4.70 | 2.6 |
| | 41.18 | 4.24 | 5.20 | 2.6 |
| | 42.55 | 4.24 | 5.20 | 2.6 |
| | 43.53 | 4.05 | 5.00 | 2.6 |
| | 44.41 | 4.04 | 5.00 | 2.6 |
| | 45.59 | 4.47 | 5.00 | 2.6 |
| | 46.47 | 4.11 | 5.90 | 2.6 |
| PLUTO | 17.00 | 3.60 | 10.28 | 6.0 |
| | 22.00 | 3.47 | 17.29 | 6.0 |
| | 27.60 | 4.07 | 7.10 | 6.0 |
| | 30.80 | 4.11 | 3.20 | 6.0 |

| Experiment | $\sqrt{s}$ (GeV) | $R$ | $\sigma_{\text{stat}}$ (%) | $\sigma_{\text{syst}}$ (%) |
|---|---|---|---|---|
| CELLO | 14.04 | 4.10 | 3.41 | 1.7 |
| | 22.00 | 3.86 | 3.66 | 1.7 |
| | 33.80 | 3.74 | 3.22 | 1.7 |
| | 38.28 | 3.89 | 3.10 | 1.7 |
| | 41.50 | 4.03 | 4.48 | 1.7 |
| | 43.60 | 3.97 | 2.44 | 1.7 |
| | 44.20 | 4.01 | 2.77 | 1.7 |
| | 46.00 | 4.09 | 5.44 | 1.7 |
| | 46.60 | 4.20 | 8.67 | 1.7 |
| Mark J | 22.00 | 3.66 | 3.72 | 2.1 |
| | 25.00 | 3.89 | 6.18 | 2.1 |
| | 30.60 | 4.09 | 4.53 | 2.1 |
| | 33.82 | 3.71 | 3.40 | 2.1 |
| | 34.63 | 3.74 | 3.10 | 2.1 |
| | 35.11 | 3.85 | 3.40 | 2.1 |
| | 36.36 | 3.78 | 5.00 | 2.1 |
| | 37.40 | 3.97 | 9.77 | 2.1 |
| | 38.30 | 4.16 | 3.72 | 2.1 |
| | 40.36 | 3.75 | 5.00 | 2.1 |
| | 42.50 | 4.32 | 5.49 | 2.1 |
| | 42.50 | 3.85 | 6.00 | 2.1 |
| | 43.58 | 3.91 | 3.35 | 2.1 |
| | 44.23 | 4.14 | 3.55 | 2.1 |
| | 45.48 | 4.17 | 5.66 | 2.1 |
| | 46.47 | 4.35 | 4.92 | 2.1 |
| TASSO | 14.00 | 4.14 | 7.30 | 4.0 |
| | 22.00 | 3.89 | 4.40 | 4.0 |
| | 25.00 | 3.72 | 10.20 | 4.0 |
| | 27.50 | 3.91 | 8.20 | 4.0 |
| | 30.10 | 3.94 | 4.60 | 4.0 |
| | 31.10 | 3.67 | 4.90 | 4.0 |
| | 33.00 | 3.74 | 7.20 | 4.0 |
| | 33.20 | 4.49 | 6.30 | 4.0 |
| | 34.00 | 4.14 | 3.10 | 4.0 |
| | 34.00 | 4.10 | 4.90 | 4.0 |
| | 35.00 | 4.23 | 2.10 | 4.0 |
| | 35.00 | 4.04 | 4.20 | 4.0 |
| | 36.10 | 3.94 | 4.30 | 4.0 |
| | 41.50 | 4.11 | 2.90 | 4.6 |
| | 44.20 | 4.28 | 3.80 | 3.0 |
| AMY | 50.00 | 4.54 | 11.01 | 3.2 |
| | 52.00 | 4.33 | 5.08 | 3.2 |
| | 54.00 | 4.72 | 13.35 | 3.2 |
| | 55.00 | 4.67 | 5.35 | 3.2 |
| | 56.00 | 5.24 | 3.81 | 3.2 |
| | 56.50 | 5.37 | 9.31 | 3.2 |
| | 57.00 | 4.95 | 4.65 | 3.2 |
| | 58.50 | 5.34 | 11.05 | 3.2 |
| | 59.00 | 5.44 | 11.58 | 3.2 |
| | 59.05 | 6.61 | 12.57 | 3.2 |

TABLE I. (*Continued*).

| Experiment | $\sqrt{s}$ (GeV) | $R$ | $\sigma_{\rm stat}$ (%) | $\sigma_{\rm syst}$ (%) |
|---|---|---|---|---|
| | 60.00 | 5.83 | 5.14 | 3.2 |
| | 60.80 | 5.56 | 5.39 | 3.2 |
| | 61.40 | 5.38 | 5.02 | 3.2 |
| TOPAZ | 50.00 | 4.53 | 13.02 | 5.5 |
| | 52.00 | 4.53 | 4.64 | 5.5 |
| | 54.00 | 4.98 | 11.45 | 5.5 |
| | 55.00 | 4.64 | 5.38 | 5.5 |
| | 56.00 | 5.07 | 4.34 | 5.5 |
| | 56.50 | 5.11 | 9.39 | 5.5 |
| | 57.00 | 5.15 | 4.85 | 5.5 |
| | 58.29 | 5.34 | 8.24 | 5.5 |
| | 59.06 | 5.74 | 7.49 | 5.5 |
| | 60.00 | 5.31 | 5.46 | 5.5 |
| | 60.80 | 5.66 | 4.95 | 5.5 |

| Experiment | $\sqrt{s}$ (GeV) | $R$ | $\sigma_{\rm stat}$ (%) | $\sigma_{\rm syst}$ (%) |
|---|---|---|---|---|
| | 61.40 | 5.86 | 5.29 | 5.5 |
| VENUS | 50.00 | 4.40 | 11.36 | 3.8 |
| | 52.00 | 4.70 | 6.38 | 3.8 |
| | 54.00 | 4.69 | 9.38 | 3.8 |
| | 55.00 | 4.32 | 6.94 | 3.8 |
| | 56.00 | 4.66 | 3.86 | 3.8 |
| | 56.50 | 3.94 | 10.41 | 3.8 |
| | 57.00 | 4.99 | 4.41 | 3.8 |
| | 58.50 | 4.92 | 8.74 | 3.8 |
| | 59.00 | 4.86 | 9.46 | 3.8 |
| | 59.05 | 6.07 | 10.71 | 3.8 |
| | 60.00 | 5.29 | 4.73 | 3.8 |
| | 60.80 | 5.70 | 4.21 | 3.8 |
| | 61.40 | 5.01 | 4.19 | 3.8 |

reduces to a one-parameter fit since the electroweak parameters play no role here. We find in this range a 90% C.L. ($\Delta\chi^2 = +2.7$) upper limit

$$\Lambda_{\overline{\rm MS}}^{(5)} < 277 \text{ MeV} ,$$

which is equivalent to

$$\alpha_s(34 \text{ GeV}) < 0.146, \quad 90\% \text{ C.L.} , \tag{25}$$

in very good agreement with the result of Ref. [2], $\alpha_s$(34 GeV) $< 0.14$ at 90% C.L.

One should also note that this determination of $\Lambda_{\overline{\rm MS}}^{(5)}$ is less precise than (but in good agreement with) the informations from $\Upsilon$ branching ratios [39]

$$\Lambda_{\overline{\rm MS}}^{(5)} = 120 \pm 50 \text{ MeV} \tag{26}$$

and deep-inelastic scattering [39]

$$\Lambda_{\overline{\rm MS}}^{(5)} = 140 \pm 40 \text{ MeV} , \tag{27}$$

in the same range of $Q^2$. With our statistics, however, we are unable to obtain any meaningful lower bound, as in Ref. [2].

Let us now include the PETRA-PEP-TRISTAN data. In this case one can adopt different strategies. We shall closely follow Ref. [8].

When performing the first kind of fit in perturbative QCD, we leave $\Lambda_{\overline{\rm MS}}^{(5)}$ as a free parameter, fixing the $Z$ mass $M_Z$ to the precisely known value from LEP [20-23] reported in Eq. (5) and the quantity $\Delta\rho$, which takes into account the difference between $\sin^2\theta_W$ and the effective value $\sin^2\bar{\theta}$, fixed to the value obtained from Eq. (10) in correspondence with a top-quark mass $m_t = 120$ GeV and a Higgs-boson mass $m_H = 300$ GeV. We have considered the two ranges 5–61.4 and 14–61.4 GeV. The results are reported in Table II.

It should be noted that when we analyze the restricted sample of data (14–61.4 GeV), the value of $\Lambda_{\overline{\rm MS}}^{(5)}$ increases considerably with respect to the determinations corresponding to the larger range. However, even in the latter case, the value obtained is large when compared with the determinations coming from $\Upsilon$ branching ratios and deep-inelastic scattering.

Let us now investigate the interdependence between the electroweak parameters and the QCD scale parameter $\Lambda_{\overline{\rm MS}}^{(5)}$ by performing a set of two-parameter fits. In this case, by restricting ourselves to the range allowed by the other experiments, say, 100 MeV $< \Lambda_{\overline{\rm MS}}^{(5)} <$ 250 MeV, we obtain a very poor estimate of the $Z$ mass. The results in this case are shown in Table III.

Finally, when performing a three-parameter fit in which $\Lambda_{\overline{\rm MS}}^{(5)}$, $\rho$, and $M_Z$ are left out as free parameters, we obtain the results shown in Table IV for the two ranges 5–61 and 14–61 GeV. Again, a substantial difference is obtained in the fitted value of $\Lambda_{\overline{\rm MS}}^{(5)}$ when including the low-energy data, and the smaller value implies a worse determination of $M_Z$.

TABLE II. Results of two one-parameter fits to the data in the ranges 5–61.4 and 14–61.4 GeV. The $Z$ mass is fixed to its value in Eq. (5), while $\rho$ has been determined from Eq. (10) by assuming $m_t = 120$ GeV and $m_H = 300$ GeV.

| | 5–61.4 GeV | 14–61.4 GeV |
|---|---|---|
| $\Lambda_{\overline{\rm MS}}^{(5)} =$ | $397^{+256}_{-193}$ MeV | $990^{+608}_{-485}$ MeV |
| $\alpha_s$(34 GeV) $=$ | $0.157 \pm 0.018$ | $0.193 \pm 0.026$ |
| $\chi^2/N_{\rm DF} =$ | 104/125 | 87.2/103 |

TABLE III. Results of three two-parameter fits for several values of $\Lambda_{\overline{\rm MS}}^{(5)}$.

| $\Lambda_{\overline{\rm MS}}^{(5)}$ MeV | 100 | 150 | 250 |
|---|---|---|---|
| $\rho =$ | $0.929^{+0.051}_{-0.073}$ | $0.938^{+0.047}_{-0.074}$ | $0.951^{+0.041}_{-0.069}$ |
| $M_Z =$ | $88.5^{+1.1}_{-1.0}$ GeV | $88.7^{+1.1}_{-1.0}$ GeV | $88.9^{+1.1}_{-1.0}$ GeV |
| $\chi^2/N_{\rm DF} =$ | 99.6/124 | 99.5/124 | 99.6/124 |

TABLE IV. Results of two three-parameter fits in the ranges 5–61.4 and 14–61.4 GeV.

| | 5–61.4 GeV | 14–61.4 GeV |
|---|---|---|
| $\Lambda_{\overline{\rm MS}}^{(5)}=$ | $169^{+225}_{-124}$ MeV | $732^{+702}_{-477}$ MeV |
| $\rho=$ | $0.941^{+0.048}_{-0.078}$ | $0.978^{+0.032}_{-0.060}$ |
| $M_Z=$ | $88.7^{+1.2}_{-1.1}$ GeV | $89.8^{+1.5}_{-1.4}$ GeV |
| $\chi^2/N_{\rm DF}=$ | 99.5/123 | 86.3/101 |

It is clear from the above results that a strong-interaction correction compatible with the low-energy experiments does *not* reproduce the correct value of the $Z$ mass in the higher-energy data. At the same time, as noted in Ref. [35], the TRISTAN data are not in disagreement with the highest PETRA data, and therefore, if one assumes a systematic effect and lowers all TRISTAN data (by about 4%), the $\chi^2$ gets considerably worse.

In order to investigate, in a model-independent way, the energy dependence of the strong-interaction correction, we have used Eq. (19), choosing $E_0=34$ GeV. By fixing $M_Z$ to its value from LEP in Eq. (5) and leaving out $\rho$, $A$, and $B$ as free parameters, we obtain

5–61.4 GeV :

$$\begin{aligned} &A(34\ \text{GeV})=1.066\pm0.009\ , \\ &B=(6\pm55)\times10^{-5}\ \text{GeV}^{-1}\ , \\ &\rho=0.994\pm0.027\ , \\ &\chi^2/N_{\rm DF}=99.4/124\ . \end{aligned} \tag{28}$$

Note that the model-independent strong-interaction correction at $\sqrt{s}=34$ GeV in Eq. (28) is equivalent to a rather large value in the three-loop correction, $\alpha_s(34\ \text{GeV})\sim0.199\pm0.023$, in excellent agreement with the value reported in Table II for the restricted range 14–61 GeV, but not with the result from the fit in the range 5–61 GeV, since the low-energy data in the range 5–10.5 GeV do not tolerate values of $\Lambda_{\overline{\rm MS}}^{(5)}$ larger than 300 MeV [see Eq. (25)]. At the same time, there is a very good agreement between our value of $A(34\ \text{GeV})$ and the result of Ref. [16], $A(34\ \text{GeV})=1.062\pm0.011$, obtained in the restricted range 14–47 GeV (i.e., without the TRISTAN data).

In Ref. [1], by analyzing the restricted sample from 22 to 61.4 GeV, it was shown that the strong-interaction correction can be described fairly well by a constant factor. The inclusion of the new lower-energy data, apparently, supports that previous result. Indeed, the value of $B$ obtained in Eq. (28) shows that there is no evidence for an energy dependence in the strong-interaction correction in the range 5–61.4 GeV, despite $Q^2$ changes by more than two orders of magnitude.

## V. ANALYSIS IN THE RANGE 5–94 GeV

Let us first consider the LEP data alone as deduced from Refs. [20–23]. By performing a fully model-independent fit [37] to the hadronic and leptonic line shapes, we obtain (the errors include uncertainties from luminosity and acceptance as published by each experimental collaboration)

$$\begin{aligned} &M_Z=91.176\pm\underline{0.020}\ \text{GeV}\ , \\ &\Gamma_l=83.1\pm0.4\ \text{MeV}\ , \\ &\Gamma_{\rm inv}=495\pm8\ \text{MeV}\ , \\ &\Gamma_h=1741\pm9\ \text{MeV}\ , \\ &\Gamma_Z=2486\pm9\ \text{MeV}\ , \\ &\chi^2/N_{\rm DF}=142/(180-4)\ . \end{aligned}$$

By using Eqs. (20), (21) and (23), we have repeated the fit with the result

$$\begin{aligned} &M_Z=91.176\pm\underline{0.020}\ \text{GeV}\ , \\ &A_{\rm QCD}=1.049\pm0.006\ , \end{aligned} \tag{29}$$

$$\begin{aligned} &\rho=0.996\pm0.004\ , \\ &\chi^2/N_{\rm DF}=142/(180-3)\ . \end{aligned} \tag{30}$$

Our result for $\rho$ in Eq. (30) is in good agreement with the estimate presented in Ref. [40] for the effective vector and axial-vector coupling of the $Z$ to the charged leptons. Indeed, by writing Eq. (20) in the alternative form [40]

$$\Gamma(Z\to l^+l^-)=\Gamma_l=\frac{G_F M_Z^3}{6\pi\sqrt{2}}(g_V^2+g_A^2)\ ,$$

one can perform a simultaneous fit to the $Z$ line shape and to the leptonic forward-backward asymmetries (which are sensitive to the product $g_V^2 g_A^2$). The relation between our notation in terms of $(\rho,\sin^2\bar\theta)$ and $(g_V^2,g_A^2)$ is

$$g_V^2=\frac{\rho}{4}(1-4\sin^2\bar\theta)^2\left[1+\frac{3}{4}\frac{\alpha}{\pi}\right]\ ,$$

$$g_A^2=\frac{\rho}{4}\left[1+\frac{3}{4}\frac{\alpha}{\pi}\right]\ .$$

The result of Ref. [40] is

$$g_V^2=0.0012\pm0.0003\ ,$$

$$g_A^2=0.2492\pm0.0012\ ,$$

or, in our notation,

$$\rho=0.995\pm0.005\ ,$$

$$\sin^2\bar\theta=0.2327\pm0.0021\ ,$$

in excellent agreement with our fit $\rho=0.996\pm0.004$ quoted in Eq. (30) and the corresponding value $\sin^2\bar\theta=0.2328\pm0.0019$ obtained by using Eq. (13).

Our fitted value of $A_{\rm QCD}$ in Eq. (29), by assuming the validity of Eqs. (22) and (24), amounts to

$$\alpha_s(M_Z)=0.148\pm0.019(\text{expt})\pm0.005(\text{theor})\ ,$$

to compare with the prediction from low-energy experiments,

$$\alpha_s(M_Z)=0.11\pm0.01\ ,$$

corresponding to the range $100<\Lambda_{\overline{\rm MS}}^{(5)}<250$ MeV.

We shall now combine LEP with the $e^+e^-\to$hadrons data in the range 5–61 GeV considered in Sec. IV. In this case, because of the very high statistics from LEP, the $Z$ mass and $\rho$ are totally constrained; therefore the determination of $\Lambda_{\overline{\rm MS}}^{(5)}$ becomes more precise. Two three-parameter fits in the ranges 5–94 and 14–94 GeV are shown in Table V.

In order to have a better understanding of the region 14–94 GeV, we have divided it into two pieces 14–47 GeV (i.e., the PETRA-PEP range) and 50–94 GeV (i.e., the TRISTAN-LEP data) to compare the various determinations. The results are shown in Table VI, where for the range 14–47 GeV we have used the same values of $\rho$ and $M_Z$ obtained from the LEP data.

We note that the "prediction" for $\alpha_s$ (34 GeV) obtained from the analysis of the range 50–94 GeV is in excellent agreement with its "measured" value in the range 14–47 GeV. On the other hand, both from Tables V and VI, we find evidence for values of the strong-interaction scale parameter not compatible with the low-energy determinations from $\Upsilon$ branching ratios and deep-inelastic scattering and with the prediction in Eq. (25) within more than two standard deviations.

It is interesting that the inclusion of the LEP data provides clear evidence for an energy dependence of the strong-interaction correction to the quark-parton model. By using Eq. (19) consistently in the range 5–94 GeV and choosing again $E_0=34$ GeV, we have performed a four-parameter fit with the results

5–94 GeV :

$$\begin{aligned}&A(34\ \text{GeV})=1.065\pm0.008\ ,\\&B=(-26\pm17)\times10^{-5}\ \text{GeV}^{-1}\ ,\\&M_Z=91.176\pm\underline{0.020}\ \text{GeV}\ ,\\&\rho=0.996\pm0.004\ ,\\&\chi^2/N_{\rm DF}=242/(306-4)\ .\end{aligned}\tag{31}$$

Note that, even though $B$ is negative, still the same value of $A$(34 GeV) as in Eq. (28) and, therefore, the same large $\alpha_s$(34 GeV) are obtained. As a consequence, the observed energy dependence cannot be explained with a strong-interaction scale parameter consistent with the low-energy experiments.

It is interesting to perform a similar fit where only the very-low-energy data in the range 5–10.5 GeV and the LEP data are included. This procedure is sensible since according to perturbative QCD the maximal difference in the strong-interaction correction should be found in connection with the extreme sets of data. By still retaining $E_0=34$ GeV, we find

(5–10.5)+(88–94) GeV :

$$\begin{aligned}&A(34\ \text{GeV})=1.045\pm0.019\ ,\\&B=(+8\pm34)10^{-5}\ \text{GeV}^{-1}\ ,\\&M_Z=91.176\pm\underline{0.020}\ \text{GeV}\ ,\\&\rho=0.996\pm0.004\ ,\\&\chi^2/N_{\rm DF}=153/(202-4)\ .\end{aligned}\tag{32}$$

In this case the energy dependence is not the one expected and the "predicted" value of $A$(34 GeV) is considerably lower than the one obtained from the full fit. Therefore the observed decreasing energy dependence of the fit in the full region 5–94 GeV is only due to the fact that the PETRA-PEP-TRISTAN data are *higher* than what one would expect since the LEP data are not lower than the very-low-energy ones. In this situation, we believe, the final conclusions have to be very cautious.

TABLE V. Three-parameter fits including the LEP data.

| | 5–94 GeV | 14–94 GeV |
|---|---|---|
| $\Lambda_{\overline{\rm MS}}^{(5)}=$ | $545^{+221}_{-182}$ MeV | $961^{+383}_{-314}$ MeV |
| $\alpha_s$(34 GeV)= | 0.168±0.014 | 0.192±0.018 |
| $\rho=$ | 0.998±0.003 | 0.996±0.003 |
| $M_Z=$ | 91.176±$\underline{0.020}$ GeV | 91.176±$\underline{0.020}$ GeV |
| $\chi^2/N_{\rm DF}=$ | 246/(306−3) | 229/(284−3) |

TABLE VI. Two separate fits in the ranges 14–47 and 50–94 GeV.

| | 14–47 GeV | 50–94 GeV |
|---|---|---|
| $\Lambda_{\overline{\rm MS}}^{(5)}=$ | $960^{+598}_{-459}$ MeV | $962^{+542}_{-402}$ MeV |
| $\alpha_s$(34 GeV)= | 0.192±0.027 | 0.192±0.023 |
| $\rho=$ | 0.996=fixed | 0.996±0.004 |
| $M_Z=$ | 91.176 =fixed | 91.176±$\underline{0.020}$ GeV |
| $\chi^2/N_{\rm DF}=$ | 54.8/(66−1) | 174.6/(218−3) |

## VI. COMPARISON WITH PREVIOUS ANALYSIS AND CONCLUSIONS

In this paper we have presented an analysis of the experimental data for $R(e^+e^-\to$hadrons), restricting ourselves to energy ranges which are free of resonances. It covers a very wide range

$$25\le s\le8836\ \text{GeV}^2$$

and includes very recent experimental data, namely, the Crystal Ball [2], Mark II [9], and LEP [20–23] results. Previous analyses [8,41] did not include all the TRISTAN data and, obviously, could not take into account the more recent experimental result considered here.

Let us compare our results with those of Ref. [8], where the two-loop result (see their Table 2, p. 165, left-hand side) $\alpha_s$(34 GeV)=0.170±0.025, in the range 14–57 GeV, has been obtained with a value of the $Z$ mass $M_Z=89.3$ GeV. We have checked that, by replacing our value in Eq. (5) with $M_Z=89.3$ GeV, our fitted value of $\alpha_s$(34 GeV) in the range 14–61 GeV decreases by about 5%. Therefore, by using Eq. (5), their estimate of $\alpha_s$(34 GeV) would increase by about 5% to compensate the decrease of the resonating contribution in the higher-energy data. At the same time, the use of the three-loop correction in $R$ introduces an additional increase by about 5% because of the new negative $O(\alpha_s^3)$ term. Altogether, we expect their value at 34 GeV to be modified into $\alpha_s$(34 GeV)$=0.187\pm0.025$, which is slightly lower than (but well consistent with) our value $0.193\pm0.026$ reported in Table II.

The comparison of the results of Ref. [8] with ours for the full range 5–61 GeV, on the other hand, is very difficult to understand. Indeed, we obtain exactly their result in the range 7–57 GeV after including new data at 5 GeV, changing the $Z$ mass, and switching to the three-loop calculation. Evidently, there is a compensation among the various effects, and we have no particular explanation. Also, as remarked in Sec. IV, our three-loop result in Table II for the full range 5–61 GeV is ambiguous since it would produce an effective correction $A$(34 GeV)$=1.054\pm0.007$, considerably lower than the model-independent result in Eq. (28) and similar to the value $A$(34 GeV)$=1.056\pm0.008$ obtained in Ref. [8] from the QCD fit in the range 7–57 GeV.

In the more recent analysis performed by De Boer [35], the fit to the sample of data from 14 to 61.4 GeV (PEPPETRA and TRISTAN) at the two-loop level, gives (with $M_Z=91.11$ GeV and $\sin^2\theta_W=0.2293$ fixed)

$$\Lambda_{\overline{\rm MS}}^{(5)}=650^{+450}_{-340}\ \text{MeV}$$

and

$$\alpha_s(34\ \text{GeV})=0.172\pm0.024\ .$$

Again, by employing the three-loop correction, the value of Ref. [35] should increase up to $0.181\pm0.024$, somewhat lower but consistent with our value in Table II. Concerning his result in the range 7–65 GeV, at the two-loop level $\alpha_s$(34 GeV)$=0.159\pm0.019$, we note again the asymmetric effect of the three-loop correction when including the very-low-energy data.

Finally, we cannot compare with the three-loop estimate of $\alpha_s$(34 GeV) in Eq. (14) of Ref. [34(a)] since the set of data and energy range are not specified.

Summarizing, the results obtained from a three-parameter fit $(\rho, M_Z, \Lambda_{\overline{\rm MS}}^{(5)})$ to the $e^+e^-\to$hadrons and LEP data in the range 5–94 and 14–94 GeV, reported in Table V, show that the strong-interaction correction to the quark-parton model, as calculated in perturbative QCD, requires a very large value of the scale parameter which is not consistent, within two standard deviations, with the low-energy determinations from $\Upsilon$ branching ratios and deep-inelastic scattering. As a consequence, it is not clear the meaning of the observed energy dependence in Eq. (31) especially noticing that the very-low-energy data in the range 5–10.5 GeV, apparently, require a lower strong-interaction correction than at LEP [see Eq. (32)].


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