Electroproduction of the Roper resonance as a hybrid state

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We derive the Q^2 dependence of the helicity amplitudes of the Roper resonance assuming that it is (1) a radially excited q^3 state, and (2) a q^3G hybrid baryon. Our study shows that for a hybrid baryon assignment the magnitude of the transverse helicity amplitude decreases rapidly as Q^2 increases, and the longitudinal helicity amplitude vanishes. This behavior is quite different from the predictions of the q^3 quark potential model, which assumes a radially excited q^3 assignment. Comparison with data shows that the hybrid interpretation of the Roper resonance is favored. Future experiments at the Continuous Electron Beam Accelerator Facility should be able to clearly distinguish between these two possible assignments.

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Hybrid baryons are states presumably dominated by the state of three quarks oscillating against explicitly excited glue field configurations. We denote these states by q^3G. Within the framework of QCD, such configurations are expected to exist in nature. However, because QCD is not well understood in the nonperturbative regime, little is known about the masses of hybrid states, and mass estimates have to rely on the use of models. Estimates within the bag-model approach [1] showed that the gluonic excitation energy should be about 550–700 MeV. This suggests that the masses of the lightest hybrid baryons could be as low as <1500 MeV. Therefore, it is possible that some of the experimentally observed baryons are hybrid states rather than ordinary q^3 states. Recently, it has been suggested [2] that there should be a gluonic partner of the nucleon whose ratio of "proton" and "neutron" magnetic moments and photoproduction helicity amplitudes is $-\frac{3}{2}$. It is possible that the Roper resonance $P_{11}(1440)$ and $P_{33}(1600)$, which, in the quark model, are assigned to radial excitations of the nucleon and $P_{33}(1232)$, respectively, are such hybrid states. The low mass of the Roper resonance as well as of the $P_{33}(1600)$ are difficult to explain within the framework of the quark potential model [3]. Moreover, there are indications of the existence of $P_{31}(1550)$ and $P_{13}(1540)$ states, which are expected to exist as hybrid states, but have no place in the quark potential model.

Three quark states and hybrid states may have the same quantum numbers; a study of the spectroscopic assignments will therefore not be sufficient to discriminate between the q^3 and q^3G states. However, because of their different internal structure (spatial wave function), as well as their different spin-flavor structure, studies of transition properties, especially of the photon transition amplitudes and the electromagnetic transition form factors, may prove very effective in distinguishing between these states. A hybrid state is excited in the spin-flavor space, and has an SU(6) spin-flavor wave function orthogonal to that of the nucleon, whereas the spin-flavor wave function of a radially excited state is identical to that of the nucleon. The difference between the spin-flavor excitation and the radial excitation of the Roper resonance will have important phenomenological consequences for the electromagnetic transition, in particular for the Q^2 dependence of the electroproduction helicity amplitudes. The purpose of this paper is to investigate the Q^2 dependence of the electroproduction helicity amplitudes of the Roper resonance assuming alternative assignments as a q^3 or a q^3G state in order to provide possible experimental signatures of hybrid baryons.

The wave function of the nucleon with gluonic degrees of freedom can be written in lowest nontrivial order as [2]

$$|N\rangle=\frac{1}{\sqrt{1+2\delta^2}}[|N_0\rangle-\delta(|^4N_g\rangle+|^2N_g\rangle)] , \qquad (1)$$

where $|N_0\rangle$ represents the wave function for a three-quark system, which transforms as a **56** under SU(6) for nucleons [4], $|^4N_g\rangle$ and $|^2N_g\rangle$ are the wave functions for a q^3G system with the total quark spin $\frac{3}{2}$ and $\frac{1}{2}$, respectively [5], which transform as a **70** under SU(6), and the parameter δ is determined by the quark-gluon interaction. The wave function given by Eq. (1) preserves the success of the q^3 quark model, especially the ratio of magnetic moments between the proton and the neutron. The corresponding state orthogonal to the nucleon in spin-flavor space is

$$|N',J=\tfrac{1}{2}\rangle=\left[\frac{2}{1+2\delta^2}\right]^{1/2}[\delta|N_0\rangle+\tfrac{1}{2}(|^2N_g\rangle+|^4N_g\rangle)] \qquad (2)$$

whose ratio of magnetic moments and photoproduction amplitudes for "proton" and "neutron" states is also $-\frac{3}{2}$.

The transverse helicity amplitude is given by

$$A_\lambda = 3\sqrt{\pi k}\,\mu_0 \left\langle N,J,\lambda \left| e_3 \left[\sigma_3^+ + 2i \left[\sigma_3 \times \frac{\mathbf{J}_3^{\rm TE}}{\omega_g} \right]^+ \right] e^{(ikz_3)} \right| N, J=\tfrac{1}{2}, \lambda-1 \right\rangle , \tag{3}$$

where the first term comes from the normal quark-photon vertex and the second from the $\gamma q \to qG$ process (see Ref. [2] for details). Assuming that the spatial wave function in Eq. (2) has a Gaussian form, we find an explicit expression for $A_{1/2}^p$:

$$A_{1/2}^p = \frac{4\sqrt{2}}{3}\delta\sqrt{\pi k}\,\mu_0 e^{-k^2/6\alpha_h^2} , \tag{4}$$

where the parameter α_h is determined by the harmonic-oscillator potential for a hybrid state. A study [2] in the real photon limit shows that the choices $\delta=-0.35$ and $\alpha_h=0.25$ GeV give a good systematic description of the helicity amplitudes for the Roper resonance, as well as for $P_{31}(1550)$, $P_{13}(1540)$, and $P_{33}(1600)$, which are all candidates for hybrid states.

Following the procedure of Foster and Hughes [6] (without introducing an *ad hoc* form factor), the Q^2 dependence of Eq. (4) is

$$A_{1/2}^p(Q^2) = \frac{4\sqrt{2}}{3}\delta\sqrt{\pi/k_0}\,\mu_0 \frac{k}{\gamma^2} e^{-k^2/6\alpha_h^2\gamma^2} , \tag{5}$$

where the Lorentz boost factor γ can be written as

$$\gamma = \left[1 + \frac{k^2}{(M_r+M_p)^2} \right]^{1/2} \tag{6}$$

in the equal velocity frame, and

$$k^2 = \frac{(M_r^2-M_p^2)^2}{4M_rM_p} + \frac{Q^2(M_r+M_p)^2}{4M_rM_p} . \tag{7}$$

The equal velocity frame has been chosen to minimize relativistic corrections. The Q^2 dependence of the transverse helicity amplitude for a radially excited state in the quark model [7] is

$$A_{1/2}^p(Q^2) = -\frac{1}{3\sqrt{3}}\sqrt{\pi/k_0}\,\mu_0 \frac{k}{\gamma^2}\frac{k^2}{\alpha^2\gamma^2} e^{-k^2/6\alpha^2\gamma^2} . \tag{8}$$

In Fig. 1 we show the Q^2 dependence of $A_{1/2}^p$ for the Roper as a q^3 or a q^3G state. The calculations are also compared with experimental data. The expressions for $A_{1/2}^p(Q^2)$ corresponding to the hybrid state in Eq. (5) and to the radial excitation state in Eq. (8) lead to very different Q^2 dependences. In particular, $A_{1/2}^p$ for a hybrid state decreases much faster with increasing Q^2 than for a state with radial excitation. Similar behavior is expected for the $P_{33}(1600)$ resonance. The corresponding Q^2 dependence of $A_{1/2}^p$ is [2,7]

$$A_{1/2}^p = \begin{cases} -\dfrac{2\sqrt{2}}{9}\delta\sqrt{\pi/k_0}\,\mu_0\dfrac{k}{\gamma^2}e^{-k^2/6\alpha_h^2\gamma^2} & \text{for a hybrid state ,} \\ -\dfrac{\sqrt{2}}{9\sqrt{3}}\sqrt{\pi/k_0}\,\mu_0\dfrac{k}{\gamma^2}\dfrac{k^2}{\alpha^2\gamma^2}e^{-k^2/6\alpha^2\gamma^2} & \text{for a three-quark state .} \end{cases} \tag{9}$$

The corresponding dependence of $A_{1/2}^p$ on Q^2 is shown in Fig. 2.

The difference in the electroproduction of a hybrid state and a radially excited state becomes even more apparent in the longitudinal helicity amplitudes for these states, which are defined as

$$S_{1/2} = \frac{k_z}{\sqrt{Q^2}} \langle N,J,\lambda=\tfrac{1}{2} | H_{e,m}^L | N=0, J=\tfrac{1}{2}, \lambda=\tfrac{1}{2} \rangle$$
$$= \langle N,J,\lambda=\tfrac{1}{2} | J_0 | N=0, J=\tfrac{1}{2}, \lambda=\tfrac{1}{2} \rangle , \tag{10}$$

where, to order $(v/c)^2$ (see Ref. [8]),

$$J_0 = 2\pi/k_0 \left\{ \sum_j \left[e_j + \frac{ie_j}{4m_j^2}\mathbf{k}\cdot(\boldsymbol{\sigma}_j\times\mathbf{p}_j) \right] e^{i\mathbf{k}\cdot\mathbf{r}_j} - \sum_{j<l}\frac{i}{4M_T}\left[\frac{\boldsymbol{\sigma}_j}{m_j} - \frac{\boldsymbol{\sigma}_l}{m_l} \right] \times [e_j\mathbf{k}\times\mathbf{p}_l e^{i\mathbf{k}\cdot\mathbf{r}_j} - e_l\mathbf{k}\times\mathbf{p}_j e^{i\mathbf{k}\cdot\mathbf{r}_l}] \right\} . \tag{11}$$

In this approach, current conversation for the longitudinal transition operator is satisfied automatically without

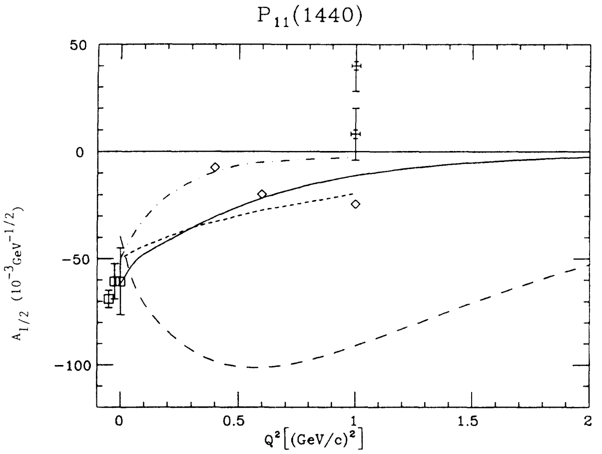

FIG. 1. $A_{1/2}^{p}(Q^2)$ for the transition $\gamma_v p \rightarrow P_{11}$ if the Roper is a q^3 state [5] (long-dashed line), or a q^3G state (solid line), respectively. The short-dashed and the dash-dotted lines represent different results of the analysis by Gerhardt. Note that the analysis was constrained by fixing the amplitude at the photoproduction point. The data points indicate the results of fits at fixed Q^2. Gerhardt, diamond symbols; Boden and Krösen, cross symbols.

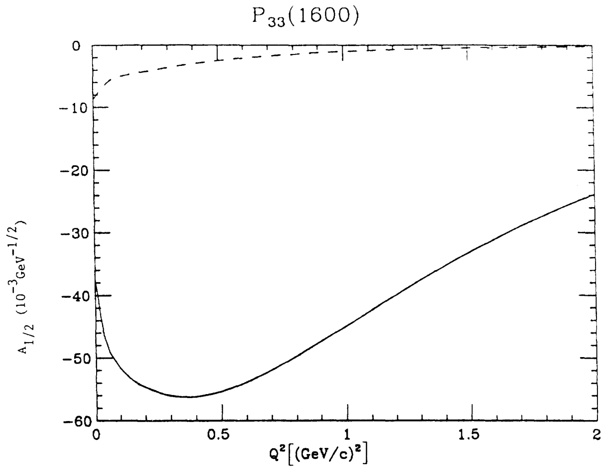

FIG. 2. $A_{1/2}^{p}(Q^2)$ for the transition $\gamma_v p \rightarrow P_{33}(1600)$. The dashed line represents the hybrid prediction, and the solid line is the result of the radially excited state of $P_{33}(1232)$. Note that, in both cases, $A_{1/2}^{p}(Q^2)=(1/\sqrt{3})A_{3/2}^{p}(Q^2)$.

a subtraction procedure [9] in the electromagnetic interaction.

Note that the leading term of Eq. (11) is proportional to the charge operator in the nonrelativistic limit, so it probes only the relative charge distribution of the nucleon and the excited states. The second and third terms of Eq. (11) are the spin-orbit and nonadditive terms [8], respectively. The latter represent the $(v/c)^2$ corrections to the leading term which require that both spin and orbital angular momentum flip by one unit. Thus, the $(v/c)^2$ terms do not contribute to the nucleon-Roper transition since $L=0$ for both states; this is also true for the nucleon-hybrid state transition. Therefore, the longitudinal helicity amplitude for a radially excited state is the Fourier transformation of the product of the spatial wave functions of the nucleon and the q^3 Roper state. In the harmonic-oscillator basis this gives

$$S_{1/2}^{p}(q^3)=-\frac{1}{3\sqrt{3}}\sqrt{2\pi/k_0}\,m_q\mu_p\frac{k^2}{\alpha^2}e^{-k^2/6\alpha^2}\,. \quad (12)$$

For a hybrid state, only the spin-orbit and nonadditive terms contribute to the transition operator for the process $\gamma_L q \rightarrow qG$ (see Ref. [2]):

$$\begin{aligned}\mathcal{O}^L&=\frac{1}{\omega_g}[h_{\mathrm{QCD}}^{\mathrm{TE}},J_0]\\&=\sqrt{2\pi/k_0}\,g_s\left\{\sum_j\frac{ie_j\lambda_j^a}{8m_j^3}\boldsymbol{\sigma}_j\cdot\left[\frac{\mathbf{B}_g}{\omega_g}\times(\mathbf{k}\times\mathbf{p}_j)\right]\right.\\&\qquad\left.-\sum_{j<l}\frac{i}{8M_T}\left[\frac{\lambda_j^a\boldsymbol{\sigma}_j}{m_j^2}-\frac{\lambda_l^a\boldsymbol{\sigma}_l}{m_l^2}\right]\left[e_j\frac{\mathbf{B}_g}{\omega_g}\times(\mathbf{k}\times\mathbf{p}_l)e^{i\mathbf{k}\cdot\mathbf{r}_j}-e_l\frac{\mathbf{B}_g}{\omega_g}\times(\mathbf{k}\times\mathbf{p}_j)e^{i\mathbf{k}\cdot\mathbf{r}_l}\right]\right\},\end{aligned} \quad (13)$$

where $\mathbf{B}_g$ is the gluonic field, and g_s and λ_j^a are the strong coupling parameter and SU(3) matrix elements, respectively. Thus, the operator $\mathcal{O}^L$ vanishes for the nucleon-hybrid transition as well. Therefore, to $\mathcal{O}(v^2/c^2)$ we have

$$\begin{aligned}S_{1/2}^{p}(q^3G)&=6\sqrt{2\pi/k_0}\,m_q\mu_p\frac{\sqrt{2}\delta}{1+2\delta^2}\{\langle N_0|q_3|N_0\rangle-\tfrac{1}{2}[\langle{}^2N_g|q_3|{}^2N_g\rangle+\langle{}^4N_g|q_3|{}^4N_g\rangle]\}\langle\phi(q^3G)|e^{i\mathbf{k}\cdot\mathbf{r}_j}|\phi(q^3)\rangle\\&=0\end{aligned} \quad (14)$$

[here $\phi(q^3G)$ and $\phi(q^3)$ are the spatial wave functions for the hybrid state and the nucleon, respectively]. The physical origin of this zero is that the hybrid state is excited in spin-flavor space; therefore, the spin-flavor wave function is orthogonal to the wave function of the nucleon. [It is possible that there is a small D-wave component in the nucleon or hybrid state due to configuration mixing induced by the hyperfine interaction. Consequently, the longitudinal transition could be nonzero; however, just as for the small amount of longitudinal transition in $\gamma_v N \rightarrow P_{33}(1232)$, this contribution is expected to be very small compared to the transverse helicity amplitude].

In Fig. 3 we show the Q^2 dependence of $S_{1/2}^{p}(q^3)$ for a radially excited Roper state together with the experimental data. $S_{1/2}^{p}(q^3)$ is significant at small Q^2 in contrast with the vanishing hybrid $S_{1/2}^{p}$. Similar results for the q^3 radially excited Roper have been obtained in other calculations [10]. This result, combined with the vanishing longitudinal helicity amplitude, implies that the total cross section for a hybrid state should decrease much faster with Q^2 than that for a three-quark baryon. Such a behavior of the hybrid state is consistent with inclusive electron-proton scattering data at low momentum transfer. It has been shown in the quark model [9,11,12] that the strength of a radially excited Roper would become dominant over $S_{11}(1535)$ and $D_{13}(1520)$ as well as $P_{33}(1232)$ as Q^2 increases, while our calculation shows that the hybrid Roper would be further suppressed.

However, it would be premature to draw definite conclusions from the nonrelativistic calculation [7,13] of the radially excited state. The investigation showed that relativistic effects are significant [14], and large configuration mixings are expected for the Roper resonance due to the anharmonicity of the spectrum [15], if it is considered as a three-quark state. A model that gives a better description of the spectrum would be essential for the calculation of the transition properties in this case. This has not been treated consistently in the literature, and more investigations are needed. Other dynamical approaches, such as the calculation by Gavela *et al.* [16] using the quark pair creation (QPC) model, may not be inconsistent with the existing data. However, since the Q^2 dependence predicted by this model is quite different from our prediction, more precise measurements will be able to discriminate between these approaches.

A more definite way to distinguish the electromagnetic transitions between a radial excitation and a hybrid is the ratio between transverse and longitudinal helicity amplitudes. Because uncertainties resulting from the lack of knowledge of the precise radial wave function cancel out in the ratio, the spin-flavor structure of the excited state can be probed directly. Theoretical predictions will be less model dependent than in the case of individual helicity amplitudes, allowing more stringent tests of the correct assignment of the state. In Fig. 4, we show the ratio $R = S_{1/2}^{p}/A_{1/2}^{p}$ for a three-quark configuration. This ratio is significant for a radially excited state, and vanishes for a hybrid state. Therefore, it is possible to distinguish hybrid states from radially excited three-quark states experimentally.

It has been pointed out [17] that, already at small Q^2, indications of the Roper excitation have disappeared from the inclusive cross section. Moreover, a recent analysis [18] of the high-Q^2 behavior of the inclusive cross section finds no indication of the excitation of the Roper, even at the highest Q^2 of 20 GeV^2. This is consistent with the Q^2 dependence of the transverse helicity amplitude for a hybrid state according to perturbative QCD [19]. Detailed analyses of single-pion production in the region of the Roper have been performed by Devenish and Lyth [20], Gerhardt [21], and Boden and Krösen [22]. Devenish and Lyth conclude that $P_{11}(1440)$ drops so fast with Q^2 that it is effectively absent from the fits to electroproduction data. Gerhardt, as well as Boden and Krösen, use more detailed electroproduction data to extract quantitative values for the transition amplitudes to the Roper. Using various data sets and different assumptions in the fit, ranges for $A_{1/2}^{p}(Q^2)$ and $S_{1/2}^{p}(Q^2)$ as shown in Figs. 1 and 2 were obtained. Systematic uncertainties, due to incomplete data sets, and associated with various theoretical assumptions, are significant, and estimated to be no smaller than $\pm 12 \times 10^{-3}$ GeV$^{1/2}$. The results, both for the transverse and for the longitudinal

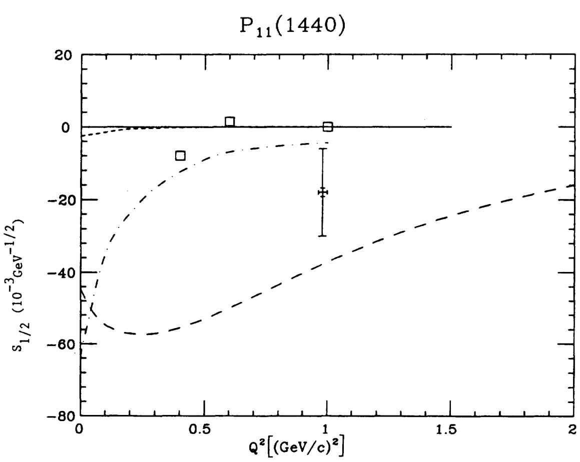

FIG. 3. $S_{1/2}^{p}(Q^2)$ for the transition $\gamma_v p \rightarrow P_{11}(1440)$ if the Roper is a q^3 state (long-dashed line). The short-dashed and dash-dotted lines represent different results of the analysis by Gerhardt. $S_{1/2}^{p}(Q^2)$ is zero for a hybrid state. Data from Gerhardt (squares) and from Boden and Krösen (crosses).

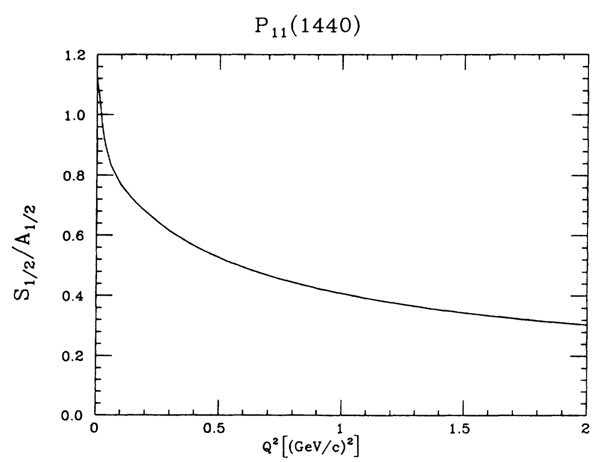

FIG. 4. Ratio $S_{1/2}^{p}(Q^2)/A_{1/2}^{p}(Q^2)$ for the transition $\gamma_v p \rightarrow P_{11}(1440)$ if the Roper is a q^3 state, while this quantity vanishes for a hybrid state.

coupling, clearly favor the interpretation of the Roper as a q^3G hybrid state. However, considering the uncertainties in the data analysis, more precise data for the exclusive process $\gamma_v p \to P_{11}(1440)$ at low and intermediate momentum transfers are needed to firmly establish the nature of the Roper resonance. In particular, pion electroproduction off polarized protons will be very sensitive to the excitation strength of the Roper [23].

For $P_{33}(1600)$, experimental evidence indicates a weak electromagnetic coupling in the real-photon limit, which is consistent with our prediction for a hybrid baryon. At $Q^2 \simeq 0.5-1.0$ GeV^2, the helicity amplitudes for this state would be significant, and should be clearly seen in the data if it were a radial excitation of $P_{33}(1232)$; on the other hand, a vanishing $P_{33}(1600)$ in electroproduction would suggest that it is a hybrid state.

In summary, our investigation shows that the transverse helicity amplitude for a hybrid state decreases much faster than that for a radially excited state; in addition, the hybrid longitudinal helicity amplitude vanishes, in contrast with the large longitudinal helicity amplitude predicted for a radially excited state at small Q^2. Because of these major differences, precise data on the electroproduction of the Roper resonance are crucial in determining its spin-flavor content. Our investigation indicates that the hybrid interpretation of the Roper is consistent with existing data. However, more precise data are needed before hybrid baryon states can be considered as firmly established. Experiments planned [24] at the continuous Electron Beam Accelerator Facility (CEBAF) should give a definite answer about the existence of these states.

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