Quantum black holes

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Static solutions of large-N quantum dilaton gravity in $1+1$ dimensions are analyzed and found to exhibit some unusual behavior. As expected from previous work, infinite-mass solutions are found describing a black hole in equilibrium with a bath of Hawking radiation. Surprisingly, the finite-mass solutions are found to approach zero coupling both at the horizon and spatial infinity, with a "bounce" off of strong coupling in between. Several new zero-mass solutions, candidate quantum vacua, are also described.

I. INTRODUCTION

In his seminal work [1] Hawking argued that the laws of quantum mechanics, when applied to black holes, predict their own demise: a pure state that collapses into a black hole evaporates into a mixed final state. In the intervening fifteen years, progress in verifying or refuting his claims has been stymied by several formidable obstacles. One of these is that regions of Planck-scale curvature and strongly coupled quantum gravity probably arise in four-dimensional gravitational collapse. While string theory provides a model for describing weakly coupled quantum gravity, a description of strongly coupled quantum gravity is well beyond our reach. Another obstacle is the problem of analyzing the back reaction of Hawking radiation on the gravitational field. There are indications that this is qualitatively important in resolving the puzzle of information loss; yet practical methods for describing it have not been forthcoming.

Recently, a strategy for sidestepping the first obstacle and overcoming the second was proposed [2]. A great simplification occurs by considering the problem of black hole formation and/or evaporation in a renormalizable theory of "dilaton" gravity in $1+1$ dimensions coupled to conformal matter. This "toy" problem contains most of the important conceptual issues present in the four-

dimensional case, yet is computationally much more tractable. The region of strongly coupled quantum gravity can be analyzed within the framework of a $1/N$ expansion where N is the central charge of the conformal matter. The problem of black hole formation and/or evaporation, including the gravitational back reaction, is thus formally reduced to a system of second-order partial differential equations. In addition to serving as a two-dimensional model for four-dimensional gravitational collapse, this two-dimensional theory is also directly relevant to four-dimensional physics as the effective theory describing the absorption and/or reemission of incident particles by certain extremal dilatonic black holes in four dimensions [2–4].

The analysis of [2] expanded the theory around the "linear dilaton vacuum" configuration. This is a static solution of the large-N equations of motion for which the dilaton varies linearly across space. Since the dilaton governs the strength of quantum loops, quantum fluctuations are large in half of space (referred to as the "Liouville region") and small in the other half (the "dilaton region"). There is a sharp line dividing these two regions along which the dilaton take the critical value $\phi = \phi_{cr}$. A black hole is potentially formed by sending matter in from the dilaton region to the Liouville region. While the equations describing this process were not solved, it was conjectured in [2] that the collapsing matter loses all its energy via Hawking radiation before the black hole has a chance to form.

This conjecture was shown in [3,5] to be false. In fact something rather different occurs; when the collapsing matter tries to cross ϕ_{cr} from the dilaton to the Liouville region, a singularity appears [3,5]. This singularity is

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quite different in nature from the black hole singularities of the classical theory: it occurs at the finite value $\phi=\phi_{cr}$ (as opposed to the strong-coupling value $\phi=\infty$), and the metric also remains finite at the singularity. Its physical significance is somewhat mysterious.

In an attempt to better understand the nature of the critical line $\phi=\phi_{cr}$ separating the two phases, and the physical implications of the singularities, in this paper we investigate numerically and analytically the static solutions of the large-N equations. We find that there is a rich variety of solutions with some rather unexpected behavior: there is a tendency to "bounce" off of the critical line. We also find vacuum solutions with greater symmetry than the linear dilaton vacuum that lie wholly within the Liouville region, as well as a zero-mass black hole with a singular horizon that lies entirely within the dilaton region.

After a brief summary of the relevant formula in Sec. II, some static solutions are discussed in Sec. III. We also argue there that the end point of evaporation of black holes formed in the linear dilaton vacuum is a truncated linear dilaton vacuum terminated just before a singularity at ϕ_{cr}. In Sec. IV we argue that the singularities at ϕ_{cr} signal a breakdown of the $1/N$ expansion, and are conceivably resolved in the exact quantum theory. The related possibility of defining the large-N theory by boundary conditions at ϕ_{cr} is also discussed.

II. SUMMARY OF RELEVANT PREVIOUS RESULTS

Classical dilaton gravity coupled to N conformal matter fields is described by the action

$$S=\frac{1}{2\pi}\int d^2\sigma\sqrt{-g}\left[e^{-2\phi}[R+4(\nabla\phi)^2+4\lambda^2]\right.$$
$$\left.-\frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right],\qquad(2.1)$$

where g, ϕ, and f_i are the metric, dilaton, and matter fields, respectively, and λ^2 is a cosmological constant. The matter fields can be explicitly integrated out to provide an effective action for the metric and dilaton. In the conformal gauge this action is

$$S_N=\frac{1}{\pi}\int d^2\sigma\left[e^{-2\phi}[\partial_+(2\phi-\rho)\partial_-(2\phi-\rho)-\lambda^2e^{2\rho}]\right.$$
$$\left.+\left[\frac{N}{12}-e^{-2\phi}\right]\partial_+\rho\partial_-\rho\right],\qquad(2.2)$$

where we have chosen the conformal gauge

$$g_{+-}=-\tfrac{1}{2}e^{2\rho},\quad g_{--}=g_{++}=0.\qquad(2.3)$$

The term proportional to N is the Liouville term induced by the matter fields. The equations of motion for ρ and ϕ are

$$T_{+-}=e^{-2\phi}(2\partial_+\partial_-\phi-4\partial_+\phi\partial_-\phi-\lambda^2e^{2\rho})$$
$$-\frac{N}{12}\partial_+\partial_-\rho=0,\qquad(2.4)$$

$$-4\partial_+\partial_-\phi+4\partial_+\phi\partial_-\phi+2\partial_+\partial_-\rho+\lambda^2e^{2\rho}=0.\qquad(2.5)$$

Because the action (2.2) is gauge fixed, these equations of motion should be supplemented by the constraint

$$T_{++}=e^{-2\phi}(4\partial_+\phi\partial_+\rho-2\partial_+^2\phi)$$
$$-\frac{N}{12}[\partial_+\rho\partial_+\rho-\partial_+^2\rho+t_+(\sigma^+)]=0,\qquad(2.6)$$

as well as a similar equation for T_{--}.[1] t_+ is an integration function which must be fixed by boundary conditions. Solving these large-N equations includes the effects of Hawking radiation as well as the gravitational back reaction.

An important solution of these equations is known as the linear dilaton vacuum:

$$\rho=0,\quad\phi=-\frac{\lambda}{2}(\sigma^+-\sigma^-).\qquad(2.7)$$

This vacuum is divided into two regions by the critical line

$$\phi=-\frac{1}{2}\ln\frac{N}{12}\equiv\phi_{cr},\qquad(2.8)$$

across which, as easily seen from (2.2), an eigenvalue of the kinetic operator changes sign. Quantum effects are small in the region $\phi<\phi_{cr}$, which is referred to as the dilaton region. The gravitational dynamics are governed by the Liouville action in the region $\phi>\phi_{cr}$, which is referred to as the Liouville region.

III. STATIC SOLUTIONS

In this section we shall describe some static solutions of Eqs. (2.4)–(2.6). In the static limit these equations become

$$0=T_{+-}=e^{-2\phi}\left[-\frac{1}{2}\phi''+\phi'^2-\lambda^2e^{2\rho}\right]+\frac{N}{48}\rho'',$$

$$0=T_{++}=T_{--}=e^{-2\phi}\left[\phi'\rho'-\frac{1}{2}\phi''\right]-\frac{N}{48}(\rho'^2-\rho''+t),$$
$$(3.1)$$

$$0=\frac{\delta S}{\delta\phi}\propto\phi''-\phi'^2-\frac{1}{2}\rho''+\lambda^2e^{2\rho},$$

where t is a constant and a prime denotes $d/d\sigma$, with $\sigma=\frac{1}{2}(\sigma^+-\sigma^-)$. These equations are of course redundant; the T_{+-} and dilaton equations imply the vanishing of $(T_{++}-T_{+-})'$. The T_{+-} and dilaton equations can also be rewritten in the form

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[1]However, we note that the Bianchi identity implies that any solution of (2.4) and (2.5) is also a solution of the constraints for some choice of $t_\pm$.

$$\left[1-\frac{Ne^{2\phi}}{24}\right]\rho''=\phi'' \ ,$$

$$\frac{1}{2}\left[1-\frac{Ne^{2\phi}}{12}\right]\phi''=\left[1-\frac{Ne^{2\phi}}{24}\right](\phi'^2-\lambda^2e^{2\rho}) \ , \qquad (3.2)$$

which will be convenient for later use.

A. Quantum kinks

Solutions to Eqs. (3.1) may be specified by fixing ρ,ϕ, and their derivatives at infinity. The mass of a solution asymptotic to (2.7) is

$$M=2e^{2\lambda\sigma}(\lambda\delta\rho+\delta\phi') \qquad (3.3)$$

evaluated at infinity where $\delta\rho,\delta\phi$ are the deviations from (2.7). We are in particular interested in solutions asymptotic to the linear dilaton vacuum, with a finite mass and with no incoming or outgoing energy flux at infinity. These are candidates for the final state of black hole evaporation. The asymptotic behavior of such solutions can be determined by linearizing (3.1) around the linear dila-

ton vacuum in the gauge (2.7). The resulting equations can be put in the form

$$2\lambda\delta\phi'+2\lambda^2\delta\rho-\lambda\delta\rho'=\frac{N}{48}e^{-2\lambda\sigma}t \ ,$$

$$\delta\phi''=\delta\rho''\left[1-\frac{N}{24}e^{-2\lambda\sigma}\right] \ . \qquad (3.4)$$

Solving (3.4) for the asymptotic perturbations of the linear dilaton vacuum yields an infinite-mass solution for $t\neq0$. This makes physical sense: $t\neq0$ corresponds to a constant incoming and outgoing energy flux, and the solution has divergent Arnowitt-Deser-Misner (ADM) mass. Such solutions, while perhaps interesting for other reasons, are therefore not candidates for the final state of black hole evaporation.

With $t=0$, the asymptotic solutions are of the form

$$\delta\rho=-\frac{M}{2\lambda}e^{-2\lambda\sigma}+O(e^{-4\lambda\sigma}) \ ,$$

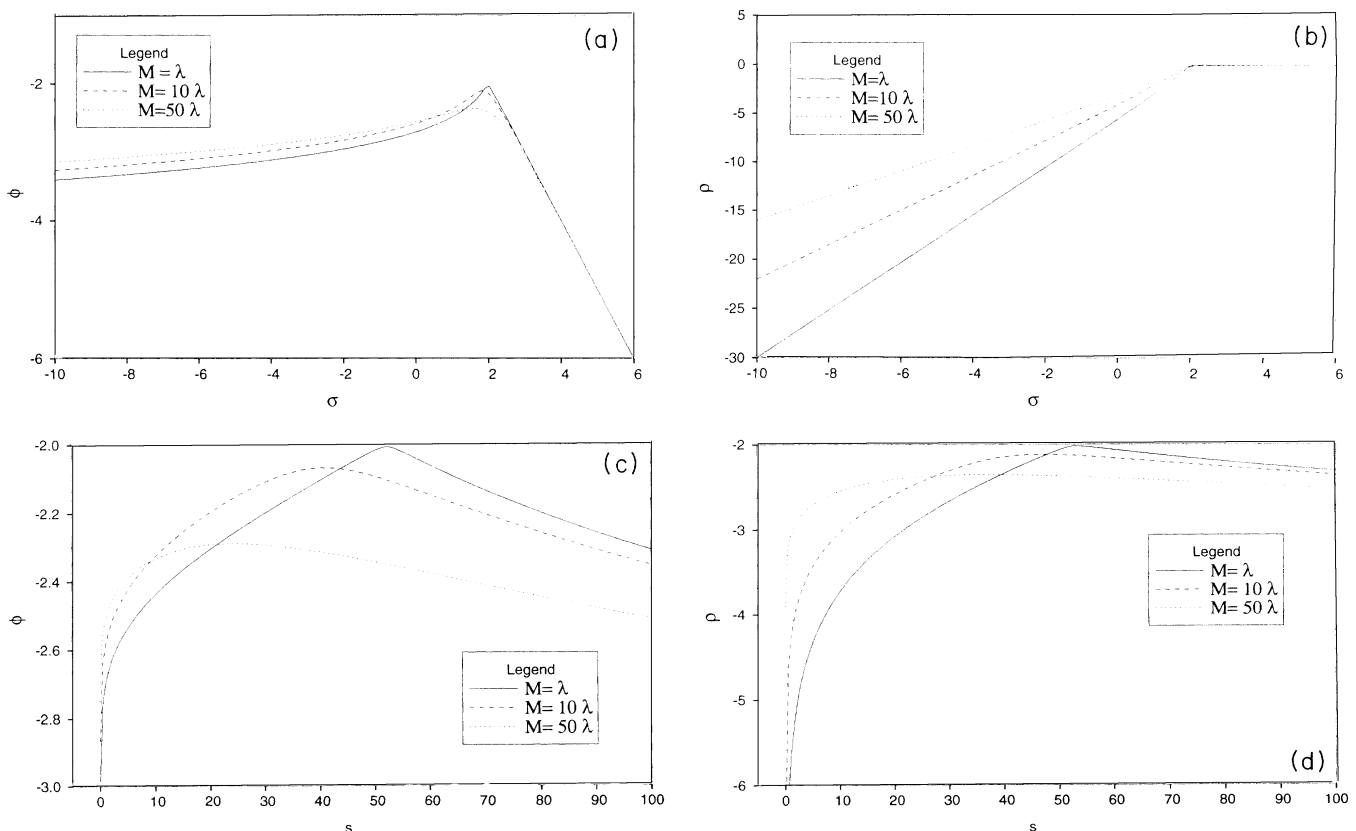$$\delta\phi=-\frac{M}{2\lambda}e^{-2\lambda\sigma}+O(e^{-4\lambda\sigma}) \qquad (3.5)$$

FIG. 1. (a) A plot of ϕ vs σ for quantum kinks of mass $M=\lambda,10\lambda,50\lambda$ for $\phi_{\rm cr}=-2$. Integrating in from infinity, the solutions closely resemble the linear dilaton vacuum until $\phi_{\rm cr}$ is reached, at which point the solutions bounce back towards weak coupling at minus infinity. (b) A plot of ρ vs σ for quantum kinks of mass $M=\lambda,10\lambda,50\lambda$ for $\phi_{\rm cr}=-2$. Integrating in from infinity, ρ is nearly zero until the bounce occurs. It asymptotes to minus infinity at $\sigma=-\infty$ with a constant linear slope plus logarithmic corrections. This implies that $\sigma=-\infty$ is a finite distance away, and that there is a horizon there. (c) A plot of ϕ vs $s=e^{2\lambda\sigma}$ for quantum kinks of mass $M=\lambda,10\lambda,50\lambda$ for $\phi_{\rm cr}=-2$. The horizon is pulled into $s=0$ in these coordinates, and it is evident that the dilaton goes to zero coupling there. (d) A plot of ρ vs $s=e^{2\lambda\sigma}$ for quantum kinks of mass $M=\lambda,10\lambda,50\lambda$ for $\phi_{\rm cr}=-2$. ρ diverges on the horizon.

and have a finite mass M. To find the geometry of these solutions one can integrate Eqs. (3.1) in from infinity. $t = 0$ defines an invariant surface of fixed $T_{++} - T_{+-}$. Different orbits are chosen by initial data on this surface as illustrated in Figs. 1(a)–1(d). These are solutions with distinct masses that asymptotically approach the linear dilaton vacuum.

These solutions may be qualitatively understood as follows. Asymptotically they approach the linear dilaton vacuum plus the perturbations in (3.5). Integrating in from infinity, for a positive mass one finds from (3.2) that $\phi'' < 0$ and so they begin to turn over. ϕ'' becomes very large and negative in the critical region $\phi \simeq \phi_{cr} = -\frac{1}{2}\ln(N/12)$. In this vicinity an approximate solution can be found by setting $\rho = \text{const} = 0$ and $\phi = \phi_{cr} + \varphi$, where $|\varphi| \ll |\phi_{cr}|$. (The consistency of this can then be checked using the full equations.) The resulting equation is

$$-\varphi''\varphi \simeq \tfrac{1}{2}(\varphi'^2 - \lambda^2) , \tag{3.6}$$

with first integral

$$\varphi'^2 - \frac{A}{\varphi} \simeq \lambda^2 , \tag{3.7}$$

where A is an integration constant. For positive mass, $A > 0$ and the equation is that for motion of a particle in a repulsive potential centered at $\varphi = 0$ (note that $\varphi < 0$). Hence ϕ bounces near $\phi = \phi_{cr}$ (ϕ' flips from roughly $-\lambda$ to $+\lambda$) and then begins to decrease towards $\sigma = -\infty$. The sharp turnover in ϕ also forces ρ' to jump, by (3.2). ρ is then found to be asymptotically linear and approaches minus infinity.

The qualitative behavior as $\sigma \to -\infty$ can also be understood, using the fact that $e^{2\rho}$ becomes negligible as $\rho \to -\infty$. Without this term the equations can be integrated to find

$$e^{-2\phi} = -\frac{N}{12}\rho + a\sigma + b ,$$

$$e^{-\phi}\left[e^{-2\phi} - \frac{N}{12} \right]^{1/2} \tag{3.8}$$

$$-\frac{N}{12}\ln\left[\left[e^{-2\phi} - \frac{N}{12} \right]^{1/2} + e^{-\phi} \right] = -a\sigma + c ,$$

where a, b, and c are constants. These have asymptotic solutions as $\sigma \to -\infty$:

$$e^{-2\phi} \sim -a\sigma + \frac{N}{24}\ln(-a\sigma) + \cdots ,$$

$$ds^2 \sim \frac{e^{48a\sigma/N}}{-a\sigma}(1 + \cdots)(-d\tau^2 + d\sigma^2) . \tag{3.9}$$

One sees that $\sigma = -\infty$ is in fact an event horizon at finite distance, and its vicinity is more easily investigated by introducing the new coordinates

$$x^+ = e^{24a\sigma^+/N} , \quad x^- = -e^{-24a\sigma^-/N} . \tag{3.10}$$

From (3.9) one finds infinite curvature at $x^+ = 0$ and $x^- = 0$; the horizon is singular. The singularity occurs at

zero coupling.

As the mass goes to zero, the solution gets closer and closer to ϕ_{cr} before bouncing back to weak coupling. Furthermore, it is very close to the linear dilaton vacuum outside a region whose boundary gets closer to ϕ_{cr}. A configuration can be defined in the zero-mass limit that agrees with the linear dilaton vacuum up to ϕ_{cr}, but then bounces back to weak coupling ($\phi = -\infty$) rather than continuing on to strong coupling ($\phi = +\infty$). The existence of distinct "solutions" that agree up to ϕ_{cr} but then disagree afterwards is due to the fact that the equations themselves degenerate at ϕ_{cr}. To resolve this ambiguity one must go beyond the large-N limit, as will be discussed in the last section.

Is this zero-mass bounce solution a plausible end point for black hole evaporation? We think not. As described in [5,3], the black holes formed by f-wave collapse have a dilaton that increases monotonically up to a singularity at ϕ_{cr}. The static solutions described here are nonsingular at ϕ_{cr}: rather they bounce off of ϕ_{cr} and reach a singularity at $\phi = -\infty$. It is hard to see how the black holes formed in a collapse process could smoothly evolve into such a configuration.

It is tempting to try to instead interpret the zero-mass bounce solution as the true quantum vacuum of the theory. The singularities described in [5,3] might then be viewed as punishment for expanding around the wrong vacuum. Making sense of this idea would require finding some sensible choice of boundary conditions at the horizon, as well as for propagating through the kink at ϕ_{cr}; we have done neither.

B. Quantum black holes

The conditions for solutions with regular horizons are most easily investigated by introducing a new spatial coordinate $s = -x^+x^-$, so that the horizon is at $s = 0$. In terms of this coordinate the static equations become

$$-2\phi' - 2s\phi'' + 4s\phi'^2 - \lambda^2 e^{2\rho}$$

$$= -\frac{N}{12}e^{2\phi}(s\rho'' + \rho') \quad (+-) ,$$

$$4\phi'\rho' - 2\phi'' = \frac{N}{12}e^{2\phi}\left[\rho'^2 - \rho'' + \frac{\hat{t}}{s^2} \right] \quad (++, --) ,$$

$$\tag{3.11}$$

$$4\phi' + 4s\phi'' - 4s\phi'^2 - 2\rho' - 2s\rho'' + \lambda^2 e^{2\rho} = 0 \quad (\text{dilaton}) .$$

The conditions for the solution to be regular at the horizon then follows from the finiteness of ϕ'' and ρ'' or, equivalently, the vanishing of $s\phi''$ and $s\rho''$ at $s = 0$. One finds

$$\rho'(0) = -\frac{\lambda^2}{2}\frac{e^{2\rho(0)}}{1 - (N/12)e^{2\phi(0)}} ,$$

$$\phi'(0) = -\frac{\lambda^2}{2}e^{2\rho(0)}\frac{1 - (N/24)e^{2\phi(0)}}{1 - (N/12)e^{2\phi(0)}} , \tag{3.12}$$

$$\hat{t} = 0 .$$

When translated into x coordinates the last condition implies that $t_{\pm} = -\lambda^2/4 \neq 0$. In the s coordinates $t = 0$ generically implies nonzero Hawking flux. This means that the solutions with regular horizons only remain static when supported by an incoming flux that matches the outgoing Hawking flux, as expected. This nonzero radiation density extending out to infinity implies that these solutions have an infinite mass. Numerical plots of these solutions can be found in Figs. 2(a) and 2(b). They can be continued across the horizon, and a singularity appears at $\phi = \phi_{cr}$. The solutions therefore have causal structures identical to the classical black hole. Although they are not candidates for the final state, they approximate a slowly evaporating black hole before it reaches zero mass.

C. The end point of black hole evaporation

To summarize the results up to this point, we have found static solutions of two types.

(1) Solutions with $0 < M < \infty$. These have horizons with weak-coupling singularities.

(2) Black holes with regular horizons. These are solutions with constant incoming radiation to balance the outgoing Hawking radiation, and thus have infinite mass.

Neither of these two types of solutions are good candidates for the final state. The only remaining candidate is at $M = 0$: the linear dilaton vacuum. However, because of the singularity at $\phi = \phi_{cr}$, one should terminate the linear dilaton vacuum just to the right of ϕ_{cr}. One therefore presumes that in the far future the solution formed from infalling matter settles down to the linear dilaton vacuum for ϕ in the range less than $\phi_{cr} - \epsilon$, for some small ϵ that approaches zero in the infinite future. Thus the best candidate for the end point of black hole evaporation may be pictured as the linear dilaton vacuum truncated just before the end of the Universe at ϕ_{cr}.

D. The Liouville region

In the preceding, static solutions that approached weak coupling at spatial infinity were discussed. It is also of interest to consider solutions that remain wholly within the Liouville region. Since the main problems are associated with the crossover between the two regions, perturbation theory around such solutions may well be better defined. Furthermore, as argued in [3,4], such solutions are physically relevant to the description of four-dimensional extremal black holes for which the asymptotic value ϕ_0 of the dilaton obeys $\phi_0 > \phi_{cr}$.

The simplest such solution is the vacuum configuration

$$e^{-2\phi} = 0, \quad \rho = 0 . \tag{3.13}$$

Perturbation theory about this vacuum can be defined in terms of the variable[2]

$$\psi \equiv e^{-\phi} , \tag{3.14}$$

in terms of which the large-N gravitational action becomes

$$S = \frac{1}{\pi} \int d^2\sigma \left[4\partial_+ \psi \partial_- \psi + 4\psi \partial_+ \psi \partial_- \rho \right.$$

$$\left. -\lambda^2 \psi^2 e^{2\rho} + \frac{N}{12} \partial_+ \rho \partial_- \rho \right] . \tag{3.15}$$

Excitations about this vacuum are characterized by the conserved energy

$$E = -\frac{1}{2} \int d\sigma \left[(2\partial_+ \psi + \psi \partial_+ \rho)^2 + (2\partial_- \psi + \psi \partial_- \rho)^2 \right.$$

$$+ 2\lambda^2 \psi^2 e^{2\rho}$$

$$+ \left[\frac{N}{12} - \psi^2 \right] [(\partial_+ \rho)^2 + (\partial_- \rho)^2]$$

$$\left. + \frac{N}{12} (t_+ + t_-) \right] . \tag{3.16}$$

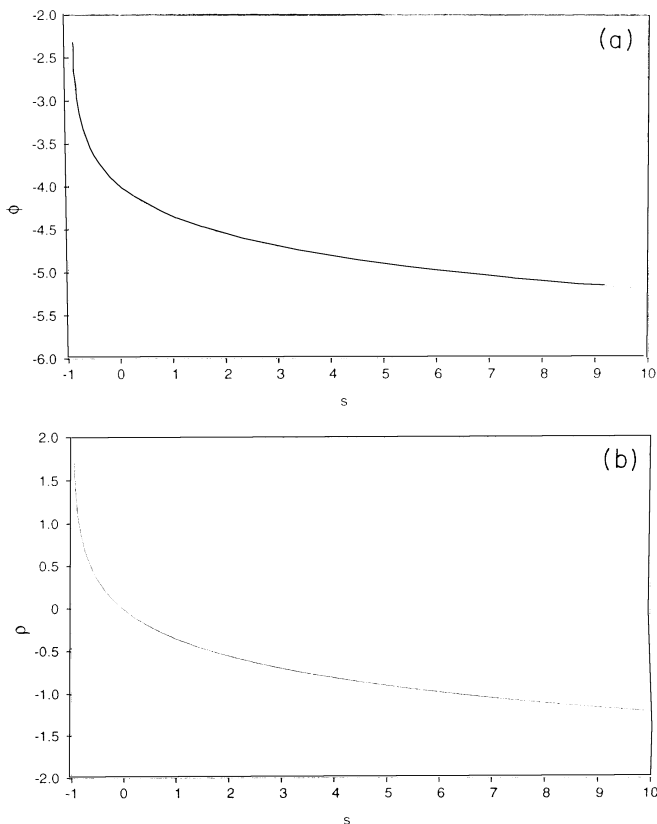E may also be expressed as a surface integral:

FIG. 2. (a) A plot of ϕ vs $s = e^{2\lambda\sigma}$ for a black hole in equilibrium with Hawking radiation. The initial conditions are chosen so as to ensure regularity at the horizon ($s = 0$). Inside the horizon, where $s < 0$ is a timelike coordinate, ϕ rapidly increases and reaches a singularity at $\phi_{cr} = -2$ in finite time. (b) A plot of ρ vs s for a black hole in equilibrium with Hawking radiation.

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[2]We require ψ, but not ϕ, to be real.

$$E = -\frac{1}{2}\left[\frac{N}{12}(\partial_+ - \partial_-)\rho + 2\psi(\partial_+ - \partial_-)\psi\right]_{-\infty}^{+\infty}, \quad (3.17)$$

from which it is evident that $E=0$ for configurations asymptotic to (3.13). A nonzero conserved quantity

$$\hat{E} = \frac{1}{2}\int d\sigma \left[(2\partial_+\psi + \psi\partial_+\rho)^2 + (2\partial_-\psi + \psi\partial_-\rho)^2 \right.$$
$$\left. + 2\lambda^2\psi^2 e^{2\rho} + \left[\frac{N}{12} - \psi^2\right][(\partial_+\rho)^2 + (\partial_-\rho)^2]\right]$$

$$(3.18)$$

can also be defined by virtue of the fact that $t_\pm$ are functions only of $x^\pm$. Note that for $\psi^2 < N/12$, which defines the Liouville region, $\hat{E}$ is positive semidefinite, which constrains possible choices of $t_\pm$.

There is also a static solution corresponding to anti–de Sitter space with a constant dilaton. It is easy to see directly from (2.4) and (2.5) that the field configuration

$$e^{-2\phi} = \frac{N}{24}, \quad R = 8e^{-2\rho}\partial_+\partial_-\rho = -4\lambda^2 \quad (3.19)$$

is a solution. In static coordinates ρ is given by

$$\rho = -\ln(\sqrt{2}\lambda\sigma). \quad (3.20)$$

IV. DISCUSSION

In [5,3] it was argued in the quantum theory that a small perturbation of the linear dilaton vacuum produces a black hole. The black hole then evaporates leaving in its place a configuration close to the linear dilaton vacuum until very near $\phi = \phi_{cr}$, at which point there is a singularity. Thus the linear dilaton vacuum is unstable under small perturbations, and is in this sense not the true vacuum of the theory. A candidate for the true vacuum is the zero-mass configuration with a singularity at $\phi = \phi_{cr}$. One should endeavor to understand this configuration.

Can we really reliably conclude that there is a singularity at $\phi = \phi_{cr}$? The answer to this is no, because the $1/N$ expansion breaks down before the singularity is reached, and the equations used to find the singularity are not a good approximation to the quantum theory described by (2.1). In terms of Feynman diagrams for perturbation theory about the linear dilaton vacuum, the large-N action (2.2) describes graviton-dilaton tree diagrams plus one-loop matter. Graviton-dilaton loops are suppressed as long as the propagator is of order $1/N$. Equivalently, the determinant of the matrix K governing small fluctuations of ρ, ϕ should be of order N^2. That determinant is given by

$$\det K = e^{-2\phi}\left[e^{-2\phi} - \frac{N}{12}\right]. \quad (4.1)$$

This is indeed of order N^2 in both the Liouville and dilaton regions for $e^{-2\phi}$ of order N, except when $e^{-2\phi}$ is near $N/12$, which demarcates the two regions of the linear dilaton vacuum. Since $\phi = -\lambda\sigma$, the $1/N$ expansion breaks

down in a region of width of order λ^{-1} containing the critical line or, equivalently wherever ϕ differs from ϕ_{cr} by an amount less than one. Large N fails to suppress quantum fluctuations of the dilaton and metric within this region.

In a sense this brings us back to where we started from; the interesting physics occurs in a region outside the reach of perturbation theory. Large N has failed to fully tame strongly coupled quantum gravity. However, some small progress has nevertheless been made in the following sense. The breakdown of large-N perturbation theory is limited to a small region of width λ^{-1} (in contrast with ordinary perturbation theory, which is bad in half of spacetime). If we restrict our attention to processes on scales large compared to λ^{-1},[3] this region is effectively a one-dimensional line.

Progress might then be made if one *assumes* that the exact quantum theory has a well-defined evolution. (Of course we have no evidence in favor of this; it is important and difficult to find out if this is indeed the case.) This exact quantum theory will then imply some effective boundary conditions along the critical line in the effective theory at scales larger than λ^{-1}. Constraints on these boundary conditions can be derived from consistency of the low-energy theory.[4] In the following we consider two possible types of boundary conditions.

A. Bouncing off the singularity

In this picture the Universe ends at $\phi = \phi_{cr}$ (or just before there, so that the large-N equations are valid), and boundary conditions must be imposed there. In the vacuum, this line is timelike (if it is drawn just before ϕ_{cr}), so it might appear sensible to apply Dirichlet or Neumann boundary conditions there. One might hope that, rather than leaving the Universe, information can be reflected off of the boundary line. However, there appears to be a severe problem with this: the effect of throwing matter at the line $\phi = \phi_{cr}$ is to change the trajectory of the boundary from a timelike one to a spacelike one [5]. No local boundary condition can alter this conclusion. It does not seem possible to define sensible, unitary dynamics for a system with spacelike boundaries.

B. Sailing through the singularity

Another possibility is that the boundary conditions give a prescription for continuing through the singularity. It is well known that there are certain types of mild "shock-wave" singularities in general relativity that do

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[3]This is in any case necessary if the two-dimensional theory is viewed as an effective theory for four-dimensional black holes.

[4]This is reminiscent of the problem of fermion-monopole scattering analyzed by Callan and Rubakov [6].

not prevent a unique and consistent evolution. The present class of singularities are in fact quite mild — neither the dilaton nor metric diverge, while their first derivatives typically blow up like $t^{-1/3}$ as the singularity is approached. There is a substantial literature on the problem of continuing through singularities arising in partial differential equations of this general type. It in fact appears likely that it is possible to find a rule for continuing through the singularity — the difficulty is in keeping all the fields real while doing so. Work on this issue is in progress.

Noted added in proof. The work of Hawking has appeared in Caltech Report No. CALT-68-1774 (unpublished) and hepth@xxx/9203052, and the work of Susskind and Thorlacius has appeared in Stanford Report No. SU-ITP-92-12 (unpublished) and hepth@xxx/9203054.

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