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Dynamics of extremal black holes

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Particle scattering and radiation by a magnetically charged, dilatonic black hole is investigated near the extremal limit at which the mass is a constant times the charge. Near this limit a neighborhood of the horizon of the black hole is closely approximated by a trivial product of a two-dimensional black hole with a sphere. This is shown to imply that the scattering of long-wavelength particles can be described by a (previously analyzed) two-dimensional effective field theory, and is related to the formation and/or evaporation of two-dimensional black holes. The scattering proceeds via particle capture followed by Hawking reemission, and naively appears to violate unitarity. However this conclusion can be altered when the effects of back reaction are included. Particle-hole scattering is discussed in the light of a recent analysis of the two-dimensional back-reaction problem. It is argued that the quantum-mechanical possibility of scattering off of extremal black holes implies the potential existence of additional quantum numbers, referred to as "quantum whiskers," characterizing the black hole.

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I. INTRODUCTION

Consider the theory consisting of Einstein gravity coupled to electromagnetism. This theory contains charged black hole solutions for which the mass M equals or exceeds, in Planck units, the charge Q. For $M<Q$, a naked singularity appears. According to Hawking [1], a quantum-mechanical black hole of mass $M>Q$ will evaporate incoherently until it reaches the extremal value $M=Q$, at which point the Hawking temperature vanishes and the evaporation ceases. Thus the extremal solutions are expected to be the end points of Hawking evaporation and correspond to stable quantum ground states.

Let us now consider throwing a long-wavelength particle into the extremal black hole. This results in a nonextremal black hole with a nonzero Hawking temperature. It should therefore decay back to (one of) its extremal ground state(s) via Hawking emission.[1] This raises the following interesting question: how does one describe the scattering of long-wavelength particles in the presence of an extremal black hole? A naive application of Hawking's calculation suggests that it cannot be described by a unitary S matrix, but rather should follow from a nonfactorizable (but probability-conserving) "$\mathcal{S}$ matrix" [2] mapping density matrices to density matrices. On the other hand, the stability of such objects suggests that their scattering might be similar to that of an elementary particle; indeed many have speculated that in a strong sense extremal black holes are equivalent to elementary particles.

As emphasized by Preskill *et al.* [3] the semiclassical methods used by Hawking to estimate decay rates of highly nonextremal black holes break down when applied to this problem. The reason for this is that the back reaction of the emitted radiation on the black hole inevitably becomes very large near the extremal limit. Consequently new approximation methods must be found to describe particle-hole scattering.

In this paper we investigate some basic features of particle capture by charged dilatonic black holes (such as are found in string theory) followed by reemission. It is also true in this context that near extremality a discussion of the Hawking process is incomplete without including the effects of the back reaction of the emitted particles on the black hole.[2] Several features of the dilatonic black holes render them more amenable to analysis. First, as stressed in [4], the extremal, magnetically charged, dilatonic black holes are (unlike their Reissner-Nordström cousins) completely nonsingular. The upper bound on the curvature can be made arbitrarily small by increasing the charge. Thus there is no reason to believe that short-distance physics plays any role in low-energy particle-hole scattering. This strongly suggests that the *scattering of low-energy particles off of extremal black holes is essentially a problem in low-energy quantum gravity, and is indepen-*

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[1]Although for $Q>1$ it might also decay to several extremal black holes. We shall ignore this possibility in the following.

[2]This is, however, for slightly different reasons than those given in [3]. Unlike the Reissner-Nordström case, the temperature of a charged dilatonic black hole does not rapidly vary near extremality. The back reaction is nevertheless important because energy conservation implies a large change in the geometry during a typical emission process.

dent of the cutoff.[3] Therefore the role played by string theory in the present developments is *not* to provide a consistent cutoff for quantum gravity, but rather to suggest modifications of the low-energy theory which render the computations more tractable.

Second, we shall see that the low-energy dynamics of near-extremal dilatonic black holes reduces in a simple way to a two-dimensional problem. This allows us to introduce a two-dimensional effective theory to summarize the essential physics. Progress in understanding this two-dimensional problem, and in particular in treating the back reaction, was recently made in [5], and will be applied to higher-dimensional physics in the present work.

Our understanding of the two-dimensional problem is unfortunately insufficient to answer the basic question of whether or not there is loss of coherence in scattering from black holes. The discussion of [5], translated into the present higher-dimensional context, suggested the possibility that the extremal black hole behaves like an ordinary quantum system with a possibly infinite ground-state degeneracy.[4] The existence of a unitary S matrix then follows if the number of ground states is finite. However if the ground state degeneracy is infinite, scattering *may* still lead in practice to quantum incoherence because the quantum state of the black hole is not observable. However, it is argued that even in this case there can be superselection sectors leading to coherent scattering. This must be so if the accessible black hole entropy is finite, as indicated by Hawking's area law. The superselection sectors are labeled by conserved quantum numbers which are examples of quantities we refer to as "quantum whiskers." These are a potentially infinite set of new parameters which are determined by direct scattering off of the black hole ground state

This paper is organized as follows. Section II contains a description of the near-extremal geometry of four- and five-dimensional dilatonic black holes, reviewing and extending results of [8,9,4,10,11]. In Sec. III we explain how coupling to the dilaton imparts an effective mass to most modes in a large region surrounding the black hole. Section IV contains a discussion of semiclassical Hawking radiation in a regime for which the back reaction can be ignored, and a brief comment on stringy black holes at or above the Hagedorn temperature. In Sec. V a two-dimensional effective field theory is derived for the description of low-lying excitations of the extremal black holes. Section VI addresses the issue of quantum coherence of particle-hole scattering. The implications of the two-dimensional analysis of [5] are discussed, and the notion of quantum whiskers is explained. Brief concluding comments are made in Sec. VII.

Although there are occasional references to issues in string theory throughout the paper, we believe that the implications of our work extend beyond string theory and hope to have written this paper in a manner accessible to nonstring theorists.

II. APPROACHING THE EXTREMAL LIMIT

We begin by describing the peculiar behavior of dilatonic black holes near the extremal limit where the mass M equals a constant times the charge Q. As will be seen, the geometry greatly simplifies in this limit. We first consider the case of large Q (in Planck units) and restrict attention to the region near or outside the horizon. In that case the curvature is everywhere weak and σ model perturbation theory should be valid.

The four-dimensional black hole solutions of string theory described in [8,4] have higher-dimensional generalizations as found in [10]. In particular, the five-dimensional member of this family of solutions (studied in [9,12,11]) has a simple and interesting structure which makes it a natural laboratory, along with the four-dimensional solutions, for studying black hole dynamics. We begin with this five-dimensional case.

A. The five-dimensional case

The five-dimensional black hole can be derived as a solution of the low-energy effective action for string compactification down to five dimensions:

$$S_5=\int d^5x\sqrt{-g}\,e^{-2\phi}[R+4(\nabla\phi)^2-\tfrac{1}{3}H^2]\ , \tag{2.1}$$

where $H=dB$ is an antisymmetric tensor field strength, fields which do not enter into the solution are omitted from (2.1), and we have set $\alpha'=1$. The solution can be expressed as [10][5]

$$\begin{aligned} ds^2 &= -Q\tanh^2\sigma\,d\tau^2 \\ &\quad +(\sqrt{Q^2+\Delta_5^2}+\Delta_5\cosh 2\sigma)(d\sigma^2+d\Omega_3^2)\ , \\ e^{2(\phi-\phi_0)} &= \frac{\sqrt{Q^2+\Delta_5^2}+\Delta_5\cosh 2\sigma}{2\Delta_5\cosh^2\sigma}\ , \\ H &= Q\epsilon_3\ , \end{aligned} \tag{2.2}$$

where the quantity Δ_5 is related to the mass[6] M (in the σ-model metric g) by

$$\Delta_5=M-\left[\frac{M^2}{4}+\frac{Q^2}{3}\right]^{1/2}\ ; \tag{2.3}$$

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[3]This is less evident, but we suspect nevertheless true, in the Reissner-Nordström case because the extremal solutions are singular. We also note that this conclusion could be affected if there is a tendency for large Q black holes to quantum-mechanically fracture to minimal Q black holes, for which the curvature is large and quantum gravity effects may be important.

[4]However very recent work [6,7] rules out some of the conjectures made in [5], at least in the form given therein. The relevance of [6,7] of the present work will be discussed in Sec. VI.

[5]The following metric is a simple coordinate transformation of the expression found in [10].

[6]Note that the present definition of the mass differs from [11] but agrees with [10].

$d\Omega_3^2$ and ϵ_3 are the line element and volume form on the unit three-sphere; and Q is integrally quantized in units of α'. A black fivebrane solution of ten-dimensional string theory may be obtained from (2.2) by simply tensoring with the flat five-dimensional metric.

The extremal limit is

$$\Delta_5 \to 0 , \tag{2.4}$$

which implies $M = 2Q/3$. Near this limit one can distinguish four regions of the black hole (see Fig. 1):

(i) $\sigma \gg \frac{1}{2}\ln(Q/\Delta_5)$ AF region ,

(ii) $\sigma \sim \frac{1}{2}\ln(Q/\Delta_5)$ mouth

(iii) $\frac{1}{2}\ln(Q/\Delta_5) \gg \sigma \gg 1$ throat

(iv) $\sigma = 0$ horizon .

Region (i) is far from the black hole where the metric is nearly flat and the dilaton nearly constant. Region (ii) is the mouth of the black hole at which the curvature becomes large. At the mouth one enters the long throat region. The proper length of the throat region is

$$D_T \sim \frac{\sqrt{Q}}{2}\ln(Q/\Delta_5) \tag{2.5}$$

which diverges as $\Delta_5 \to 0$. The dilaton varies nearly linearly along the throat, and the radius of the three-spheres of constant σ and τ is nearly constant. The throat then ends at the horizon. The coordinate system (2.2) does not cover the region inside the horizon, where in any case σ model perturbation theory breaks down.

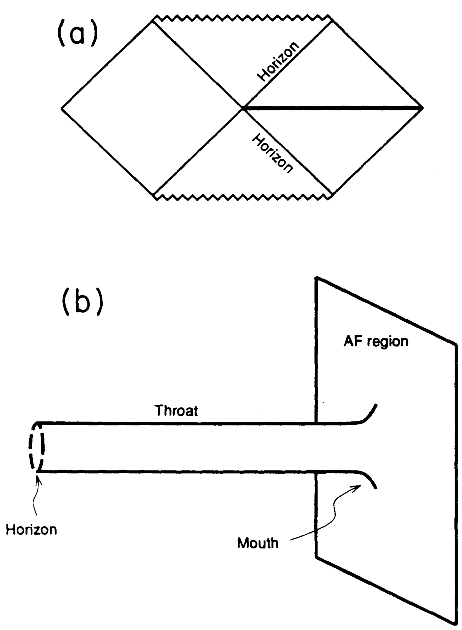

FIG. 1. (a) Pictured is the Penrose diagram for the four- or five-dimensional dilatonic black hole. Also shown is a spatial slice through the geometry. (b) The intrinsic geometry of the spatial slice of (a) is that of an asymptotically flat region with an attached long tube. In the extremal limit the length of the tube becomes infinite.

The distinguishing feature of this geometry, shared by the four-dimensional black hole discussed in the next subsection, is that the distance from the mouth to the horizon diverges in the extremal limit. This will be seen to have interesting physical consequences. It also leads to several inequivalent ways of approaching the extremal limit. In the first approach, one keeps the AF and mouth regions (i) and (ii) fixed, while allowing the horizon to move off to infinity as $\Delta_5 \to 0$. The geometry is then [9]

$$\begin{aligned} ds^2 &= -Q\,d\tau^2 + \left[1+\frac{Q}{y^2}\right](dy^2+y^2 d\Omega_3^2) , \\ e^{2(\phi-\phi_0)} &= 1+\frac{Q}{y^2} , \\ H &= Q\epsilon_3 , \end{aligned} \tag{2.6}$$

where $y=\sqrt{2\Delta_5}\cosh\sigma$. This describes a supersymmetric "black hole" with no horizon or singularity. Rather there is a semi-infinite throat attached to the AF region. The dilaton grows linearly and therefore the action (2.1) becomes strongly coupled far down the throat. As argued in [9], in string theory there is an enhanced (4,4) world-sheet supersymmetry in this limit, which implies a nonrenormalization theorem. Thus (2.6) describes an exact solution to the classical string equations of motion [9].

A second method of approaching the limit [11] is to keep the horizon and $e^{-2\hat{\phi}_0} = 2\Delta_5 e^{-2\phi_0}$ fixed while letting the AF region move off to infinity as $\Delta_5 \to 0$. One then obtains

$$\begin{aligned} ds^2 &= -Q\tanh^2\sigma\,d\tau^2 + Q\,d\sigma^2 + Q\,d\Omega_3^2 , \\ e^{2(\phi-\hat{\phi}_0)} &= Q\cosh^{-2}\sigma , \\ H &= Q\epsilon_3 . \end{aligned} \tag{2.7}$$

which describes the horizon region attached to a semi-infinite throat. The dilaton again grows linearly along the throat, but approaches weak coupling at the end. This limit can also be shown [11] to correspond to an exact classical string solution by constructing the underlying conformal field theory as the tensor product of SU(1,1)/U(1) and SU(2) Wess-Zumino-Witter (WZW) theories. This construction corresponds to a spacetime which includes the region inside the horizon and near the "singularity."

It is also of interest to consider the "throat limit" in which both the horizon and the AF region tend to infinity, leaving an infinite throat described by

$$\begin{aligned} ds^2 &= -Q\,d\tau^2 + Q\,d\sigma^2 + Q\,d\Omega_3^2 , \\ \phi &= -\sigma + \tilde{\phi}_0 , \\ H &= Q\epsilon_3 , \end{aligned} \tag{2.8}$$

with $\tilde{\phi}_0 = \hat{\phi}_0 + \ln(2\sqrt{Q}\,)$. At one end the linear dilaton is strongly coupled, while at the other it is weakly coupled. This limit also corresponds to a string solution [9], given by a level Q, SU(2) WZW model together with a Feigin-Fuks-like theory.

B. The four-dimensional case

The extremal limit of the four-dimensional magnetically charged black hole of [8,4] has behavior similar to that of the five-dimensional black hole of the previous section. It is a solution of the effective action arising in string compactification to four dimensions:

$$S_4=\int d^4x\sqrt{-g}\,e^{-2\phi}[R+4(\nabla\phi)^2-\tfrac{1}{2}F^2]\ . \tag{2.9}$$

where F is the electromagnetic field strength and irrelevant terms are again omitted. The black hole is given by

$$ds^2=-4Q^2\tanh^2\sigma\,d\tau^2+(2M+\Delta_4\sinh^2\sigma)^2(4\,d\sigma^2+d\Omega_2^2),$$
$$e^{2(\phi-\phi_0)}=\frac{2M+\Delta_4\sinh^2\sigma}{\Delta_4\cosh^2\sigma}\ , \tag{2.10}$$
$$F=Q\epsilon_2\ ,$$

where $d\Omega_2^2$ and ϵ_2 are the line element and volume form on the unit two-sphere. $M=e^{-\phi_0}M_{\rm ADM}$ is again the σ-model mass, the magnetic charge Q is quantized, and Δ_4 is given by

$$\Delta_4=2M-\frac{Q^2}{2M}\ . \tag{2.11}$$

As previously, in the extremal limit $M\to Q/2$ there are four regions:

(i) $\sigma\gg\frac{1}{2}\ln(Q/\Delta_4)$ AF region

(ii) $\sigma\sim\frac{1}{2}\ln(Q/\Delta_4)$ mouth

(iii) $\frac{1}{2}\ln(Q/\Delta_4)\gg\sigma\gg1$ throat

(iv) $\sigma=0$ horizon .

In the extremal limit the throat length approaches infinity as before. Therefore at $\Delta_4=0$ there are three distinct solutions. The AF region plus infinite throat is given by

$$ds^2=-4Q^2d\tau^2+\left[1+\frac{Q}{y}\right]^2(dy^2+y^2d\Omega_2^2)\ ,$$
$$e^{2(\phi-\phi_0)}=1+\frac{Q}{y}\ , \tag{2.12}$$
$$F=Q\epsilon_2\ .$$

Unlike the five-dimensional case, this extremal limit is not supersymmetric [4]. The horizon plus infinite throat is

$$ds^2=-4Q^2\tanh^2\sigma\,d\tau^2+4Q^2d\sigma^2+Q^2d\Omega_2^2\ ,$$
$$e^{2(\phi-\hat{\phi}_0)}=\frac{Q}{\cosh^2\sigma}\ , \tag{2.13}$$
$$F=Q\epsilon_2\ .$$

Note that this solution is a product of two two-dimensional solutions, and that again one of them is the low-energy limit of the exact black hole solution of [13]. This means that the extremal four-dimensional black hole should correspond to an exact solution of string theory which is the product of two conformal field theories [11]. The theory describing the angular variables corresponds to the round two-sphere penetrated by uniform magnetic flux. Finally, there is the limit corresponding to the infinite throat with linear dilaton as in (2.8):

$$ds^2=-4Q^2d\tau^2+4Q^2d\sigma^2+Q^2d\Omega_2^2\ ,$$
$$\phi=-\sigma+\tilde{\phi}_0 \tag{2.14}$$
$$F=Q\epsilon_2\ .$$

We note that at the classical level there are also electrically charged [8,4] as well as dyonic solutions [8,14]. However in a theory with massless charged fermions, such as heterotic string theory, the electric charge will be rapidly discharged through Schwinger pair production.

III. PERTURBATIONS ON THE THROAT

We have argued that near the extremal limit the essential features of the geometry are the asymptotically flat region and the attached long throat. For the purpose of studying emission and absorption of particles it is important to elucidate the propagation of fields along this throat. In particular, in the Hawking process particles are emitted in the vicinity of the horizon and must propagate up the throat to escape to infinity. In this section we will argue that the interaction with the linear dilaton results in significant attenuation for most low-energy particles. (This simple conclusion was also independently derived in the more detailed analysis of [15]; there the argument was made in the "canonical" metric, which differs from the metric herein by a dilaton rescaling.)

This propagation is described by considering small perturbations of the fields about the linear-dilaton solution; the equations governing these perturbations are obtained by linearizing the equations arising from the actions (2.1), (2.9), or their generalizations incorporating other fields. Although straightforward, this is somewhat complicated due to the mixing between the perturbations, etc. However, for perturbations moving along the throat, the couplings to the background are quite simple: the metric in the direction along the throat is flat, and there is an exponential coupling to the linear dilaton. Thus when one diagonalizes the kinetic matrix one finds actions of the form

$$S_\psi=-\int d^Dx\sqrt{-g}\,e^{-2b\phi}(\nabla\psi)^2 \tag{3.1}$$

governing the generic perturbation ψ, and the constant b depends on the mode in question. Were it not for the dilaton background this action would describe a free, massless propagation along the throat. However, the linear dilaton modifies the dispersion relation. This can be seen by defining a new (canonically normalized) fluctuation by

$$\tilde{\psi}=e^{-b\phi}\psi\ . \tag{3.2}$$

One then finds

$$S=-\int d^Dx\sqrt{-g}\,[(\nabla\tilde{\psi})^2+b^2(\nabla\phi)^2\tilde{\psi}^2+b\nabla\phi\nabla\tilde{\psi}^2]\ . \tag{3.3}$$

Along the throat the last term is a total derivative and the effect of the linearly varying dilaton is simply to give a mass $m^2=b^2(\nabla\phi)^2$ to $\tilde{\psi}$:

$$S_{\text{throat}}=-\int d^Dx\sqrt{-g}\,[(\nabla\tilde{\psi})^2+m^2\tilde{\psi}^2]\ . \tag{3.4}$$

The mass is $m=b/\sqrt{Q}$ in the five-dimensional case and $m=b/2Q$ in the four-dimensional case. Propagation of excitations below the mass gap is exponentially suppressed. This is in accord with the observation of [4] that there is a barrier surrounding the near-extremal black hole that suppresses emission and absorption.

IV. SEMICLASSICAL EVAPORATION NEAR THE EXTREMAL LIMIT

A dilatonic black hole formed from gravitational collapse will in general emit ordinary Hawking radiation. Holes with arbitrary mass $M>M_{\text{extremal}}$ will thus eventually approach the extremal limit. In this section we will use semiclassical methods to investigate the properties of the radiation near the extremal limit. As will be quantified shortly, these methods break down before the extremal limit is reached due to a large back reaction. However, for large Q, there is nevertheless a range of near-extremal values of M for which the back reaction is negligible. As we will see, in this range the radiation has some rather novel features.

The inverse temperatures

$$\beta_4=8\pi M,\quad \beta_5=2\pi(\sqrt{Q^2+\Delta_5^2}+\Delta_5)^{1/2} \tag{4.1}$$

of the black holes are given by the periodicity of the Euclidean sections. Note that as the extremal limit is approached these temperatures approach finite values

$$T_4=1/4\pi Q,\quad T_5=1/2\pi\sqrt{Q}\ . \tag{4.2}$$

However *at* the extremal limit, no identification is required to make the Euclidean section regular, and the temperature vanishes.

A. Radiation for $Q\gg 1$

Consider the near-extremal range

$$Q\gg\Delta_4\gg 1/Q\quad (D=4)\ ,$$
$$Q\gg\Delta_5\gg 1/\sqrt{Q}\quad (D=5)\ . \tag{4.3}$$

For Δ less than the upper bound the length of the throat exceeds its width. The lower bound is required in order that the back reaction can be neglected. The back reaction is important whenever the energy of an emitted quantum is comparable to the deviation of the mass from extremality since then the geometry changes significantly during the emission of the quantum. Near extremality this deviation is proportional to Δ, so the back reaction becomes important when $\Delta\sim T$; the temperature relations (4.2) then give the lower limits in (4.3). The important case where Δ is less than this bound will be discussed in Sec. V.

In order that the range (4.3) be large we need $Q\gg 1$. This also implies that the temperature is small as compared to the Planck temperature, $T\ll 1$, and that, within the context of string theory, higher mass string modes may be ignored.

As argued in the preceding section, perturbations propagating along the dilation throat into or out of the hole are governed by the action (3.4). The radiation rate for such perturbations can be calculated by the usual Hawking [1] methods. This rate may be estimated for small Δ_4 as follows. (We describe the four-dimensional case, although the five-dimensional case is similar.) The number of particles emitted from the vicinity of the horizon of the hole per time in the energy range $(E,E+dE)$ should be governed by the usual thermal factor

$$dn=\frac{dE}{2\pi}\frac{1}{e^{\beta E}\mp 1} \tag{4.4}$$

(with $\mp$ for bosons or fermions) times the transmission coefficient for a particle of energy E to escape from the vicinity of the horizon to infinity. In the case of an ordinary black hole this latter coefficient is of order unity; however, the effective mass in (3.4) modifies this behavior near extremality. Indeed, propagation of excitations below the mass gap, $E<b/2Q$, is exponentially suppressed with amplitude

$$A(\sigma)\exp(-\sqrt{b^2-4Q^2E^2}\,\sigma)\ . \tag{4.5}$$

Since the throat has an extent $\sim\ln(Q/\Delta)$, this means that the tunneling rate has a suppression factor $\sim\Delta^b$ for energies far below the gap.[7]

This is not, however, sufficient to imply the vanishing of the radiation rate in the extremal limit. The reason is that the temperature in this limit is $T\sim 1/Q$ and therefore there is appreciable probability for producing particles with energies above the mass gap. Since the proper radius of the mouth is also of order Q, the energy needed for a particle of angular momentum l to be transmitted into or out of the throat is given by $E^2\sim b^2/4Q^2+l(l+1)/Q^2$. Therefore the s wave ($l=0$) dominates. A rough estimate of the evaporation rate can then be made by approximating the transmission coefficient by unity for $E^2>[b^2/4+l(l+1)]/Q^2$ and zero otherwise. Then the sum of transmission probabilities over angular momenta is approximately

$$T(b,E)\simeq\sum_l(2l+l)\Theta[\sqrt{4Q^2E^2-b^2}-\sqrt{l(l+1)}]$$
$$\sim 4Q^2E^2-b^2 \tag{4.6}$$

for $E>b/2Q$. This yields an estimate of the rate loss of mass from the hole:

$$\frac{dM}{dt}\sim-\int_{b/2Q}^{\infty}\frac{E\,dE}{e^{4\pi QE}\mp 1}(4Q^2E^2-b^2)\sim-\frac{1}{Q^2}\ . \tag{4.7}$$

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[7]This can equivalently be seen without the field redefinition (3.2) if one takes into account the factor $e^{-2b\phi}$ in the normalization.

Note that the rate approaches a nonzero value in the extremal limit. (This is contrary to [3] but in accord with the independent observations of [15].)

This naive estimate might seem to suggest that evaporation could continue beyond extremality to produce a naked singularity. However, a crucial ingredient has been neglected: the back reaction of the radiation on the geometry of the black hole. As stated above, this will become relevant when $\Delta_4 \sim 1/Q$ ($\Delta_5 \sim 1/\sqrt{Q}$). It is clearly very important, and difficult, to investigate the physics of the black hole in this limit to resolve the issue of what happens in the end stages of the Hawking process. Although we cannot at present give a full treatment of the four- or five-dimensional problem of the back reaction, progress can be made in this direction by exploiting the connection between the higher-dimensional solutions and the two-dimensional black hole. This will be discussed in Sec. V.

B. Radiation for $Q \sim 1$

Now we turn briefly to the question of Hawking radiation for near-extremal five-dimensional black holes with small values of $Q = n$. (Although similar statements could in principle be made for the four-dimensional holes, the exact form of these solutions for small n is not known.) One might expect that quantum effects will allow extremal black holes to bifurcate. For the five-dimensional case supersymmetry suggests that higher Q black holes are neutrally stable, but they might be split up into lower Q black holes by particle scattering. If this is the case, $Q = 1$ black holes would behave the most like fundamental particles and would be the most interesting objects to study. However it is clear that quantum gravity effects are then important. In the context of string theory the low-energy field theory approximation is not valid because the Hawking temperature near $\Delta_5 = 0$,

$$T_5 \simeq \frac{1}{2\pi\sqrt{n\alpha'}} , \tag{4.8}$$

is high enough to excite massive string modes. (Here we have momentarily restored the suppressed factor of α'.) Indeed the Hagedorn temperatures are

$$T_c = \frac{1}{2\pi\sqrt{2\alpha'}} \quad \text{(type II)} ,$$
$$T_c = \frac{1}{(1+\sqrt{2})\pi\sqrt{2\alpha'}} \quad \text{(heterotic)} . \tag{4.9}$$

Thus in the heterotic case the $n = 1,2$ black holes are above the Hagedorn temperature in the heterotic theory, as is the $n = 1$ hole in the type II theory. Furthermore, the $n = 2$ hole is precisely *at* the Hagedorn temperature in the type II theory. This obviously brings some new and fascinating issues, beyond the scope of the present work, into the study of extremal black holes. Recent work [9,13,11] obtaining the five-dimensional extremal black holes as *exact* solutions of string theory may be useful in this regard.

V. THE LOW-ENERGY EFFECTIVE FIELD THEORY

In the preceding section we have argued that very near extremality, $\Delta < T$, the back reaction must be included in examining the subsequent evaporation of the black hole. Though the dynamics in this region are not well understood, it is reasonable to suppose that the energy of an emitted quanta in this region is bounded by $M - M_{\text{extremal}}$, since the emission of a quanta exceeding this bound would result in a naked singularity. For sufficiently small $M - M_{\text{extremal}}$, this implies an exponential suppression for emission of modes which are massive along the throat. The dynamics are then dominated by those modes which are massless along the throat, i.e., have $b = 0$ in (3.1). As we shall see, such massless modes do arise in string theory, despite the generic tendency for dilaton-induced masses.

The regime $\Delta \ll T$ is also the regime of interest for the problem discussed in the introduction of scattering particles off of an extremal black hole. If the energy of the incoming particle is sufficiently low, the extremal black hole can never be excited to a state with $\Delta_4 \sim 1/Q$ ($\Delta_5 \sim 1/\sqrt{Q}$). The hope is that particle-hole scattering can be described by perturbation theory about the extremal black hole.

Since all the relevant dynamics for $\Delta_4 \ll 1/Q$ ($\Delta_5 \ll 1/\sqrt{Q}$) occurs at length scales much greater than Q ($\sqrt{Q}$), it is useful to derive a low-energy effective action to describe these dynamics. The back-reaction problem may then be analyzed in this context. It will turn out that this effective action is partially two dimensional, and leads to a direct and simple relation between the process of absorption and reemission of a particle by a higher-dimensional extremal black hole with the process of formation and reevaporation of a two-dimensional black hole.

A. The throat limit

Let us first consider the throat limits (2.14) [(2.8)] of the four- (five-) dimensional black holes. In that limit the geometry is the product of two-dimensional Minkowski space with a two- (three-)sphere of radius Q ($\sqrt{Q}$). The effective action at scales longer than Q ($\sqrt{Q}$) can be derived by the usual Kaluza-Klein procedure for compactification from four (five) to two dimensions on a two- (three-)sphere. The result is

$$S_2 = \int d^2\sigma \sqrt{-g}\, e^{-2\phi}[R + 4(\nabla\phi)^2 + 4\lambda^2] , \tag{5.1}$$

with $\lambda^2 = 1/4Q^2$ ($1/Q$) for four dimensions (five dimensions). In addition there are two-dimensional gauge fields arising both as relics of higher-dimensional gauge fields and from isometries of the compactification spheres. These gauge fields may lead to dyons and other interesting effects (they do not propagate in two dimensions), but it is consistent to set them to zero as shall be done in the present analysis. Corrections to (5.1) are suppressed by powers of Q.

Equation (5.1) has a two-dimensional black hole solution given by

$$ds^2=-\lambda^{-2}\tanh^2\sigma\, d\tau^2+\lambda^{-2}d\sigma^2\ ,$$
$$e^{-2(\phi-\hat{\phi}_0)}=\lambda^2\cosh^2\sigma\ . \tag{5.2}$$

which is precisely the two-dimensional portion of the higher-dimensional black hole solutions described in the previous sections (as well as, for $\lambda^2=k/2$, the low-energy limit of the exact conformal field theory discussed in [13]). Thus the effective action (5.1) actually describes all the configurations of the horizon limit.

One key to the relation between the two- and higher-dimensional scattering problems alluded to above is now apparent. The masses of the four-dimensional and five-dimensional black holes are related to the value of the dilaton at the horizon by

$$e^{-2\phi_h}=\frac{2e^{-2\phi_0}\Delta_4}{\sqrt{4Q^2+\Delta_4^2}+\Delta_4}\quad\text{and}\quad e^{-2\phi_h}=\frac{2e^{-2\phi_0}\Delta_5}{\sqrt{Q^2+\Delta_5^2}+\Delta_5} \tag{5.3}$$

from (2.2) and (2.10). But recall [13] that the mass of the two-dimensional black hole is given by the value of dilaton at the horizon:

$$M_{2d}\propto e^{-2\phi_h}\propto e^{-2\phi_0}\frac{\Delta_{4,5}}{Q}\ , \tag{5.4}$$

where we work near the extremal limit. Thus the evaporation of or scattering by higher-dimensional black holes near extremality is closely related to analogous processes for two-dimensional black holes near zero mass.[8]

While (5.1) does have interesting black hole solutions, it is a theory with no propagating degrees of freedom; the two constraints and two gauge conditions of two-dimensional gravity absorb the $3+1$ degrees of freedom of the metric plus dilaton. This means that gravitational collapse and Hawking radiation cannot be studied in the theory (5.1). However, within the context of an enlarged theory such as string theory, there are additional massless fields, neglected in (2.9) and (2.1), which do lead to propagating degrees of freedom in (5.1). The precise form of these fields differs between the four- and five-dimensional case, and also depends on the type of string theory under consideration.

For a four-dimensional compactification of heterotic string theory there are for example typically massless scalars M resulting from compactification moduli. These are governed by an action of the form

$$S_M=\int d^4x\sqrt{-g}\,e^{-2\phi}(\nabla M)^2\ , \tag{5.5}$$

[compare (3.1)]. This should be added to (2.9). The factor of $e^{-2\phi}$ appears in front of all terms in the classical action. As in (3.2)–(3.4) it may be eliminated by the field redefinition $\hat{M}=e^{-\phi}M$. Would-be massless contributions to the two-dimensional effective action arise from modes of M which are constant on the compactification two-sphere. However, as in (3.4) it is clear that even this lowest-energy mode does *not* give rise to a massless two-dimensional mode, since the dilaton background produces a mass of order $1/Q$ ($1/\sqrt{Q}$). More generally, massless bosons in four dimensions with the $e^{-2\phi}$ prefactor will not lead to massless two-dimensional fields.

The situation is different in some cases for fermions. The effective action for a four-dimensional heterotic string compactification contains the charged fermion term

$$S_\chi=\int d^4x\sqrt{-g}\,e^{-2\phi}\bar{\chi}\overleftrightarrow{\not{D}}\chi\ , \tag{5.6}$$

where D here is the gauge-covariant derivative. The $e^{-2\phi}$ prefactor may be absorbed by defining $\hat{\chi}=e^{-\phi}\chi$:

$$S_\chi=\int d^4x\sqrt{-g}\,\bar{\hat{\chi}}\overleftrightarrow{\not{D}}\hat{\chi}\ . \tag{5.7}$$

Unlike the case of bosons, this redefinition does not produce an effective mass term.[9] S_χ will lead to a massless field in (5.1) if $\hat{\chi}$ has a zero mode on the two-sphere. An index theorem relates the number of such zero modes to the integral of F over the two-sphere. One finds that (5.1) can be supplemented by a charged Dirac fermions ψ containing both chiralities:

$$S_\psi=\int d^2\sigma\,\bar{\psi}\not{D}\psi\ . \tag{5.8}$$

These can be seen to be the only low-energy modes on the throat for the four-dimensional black holes.

The situation again differs for the five-dimensional black holes. In this case the two-form field strength F is replaced by the three-form H. However since fermions do not couple directly to the corresponding potential B, there are no corresponding zero modes. In fact for heterotic string theory it can be seen that there are no zero modes at all. This is not the case for type II strings. For example, the type II a string contains a Ramond-Ramond four-form field strength $F=dA$ governed by the low-energy action

$$S_F=\int d^{10}x\sqrt{-g}\,F_{MNPQ}F^{MNPQ}\ . \tag{5.9}$$

The usual factor of $e^{-2\phi}$ could be reinstated in (5.9) by redefining A, but this would result in an $A\,d\phi$ term in F and a nonstandard gauge transformation law for A. After reducing to five dimensions, F is equivalent to a single scalar field f defined by

$$F=*\,df\ , \tag{5.10}$$

where $*$ is the Hodge dual. Further reducing to two dimensions on the three-sphere, one obtains the effective action

$$S_f=\int d^2\sigma\sqrt{-g}\,(\nabla f)^2\ . \tag{5.11}$$

Thus f is a massless scalar which moves along the throat.

[8]This is in accord with the suggestion of Witten [13,16] that the two-dimensional vacuum be interpreted as an extremal black hole.

[9]We are grateful to Mark Alford for useful conversations on this issue.

B. Interacting effective field theory

In the previous subsection the effective field theory governing long-wavelength excitations of the throat was derived. The effective field theory governing the AF region outside the mouth is given by (2.1) or (2.9), supplemented by terms describing the other fields. The full effective field theory is obtained by gluing the two field theories together at the black hole mouth, as we now describe.

For notational simplicity we consider only the five-dimensional black hole and only the dynamics of the massless scalar field f, treating the metric and dilaton as fixed backgrounds. Let us separate the scalar field f into f_2 and f_5 which have support on the throat and asymptotic region, respectively. The scalar field on the throat is governed by the action

$$S_2[f_2]=\int d^2\sigma(\nabla f_2)^2 , \tag{5.12}$$

where $-\infty<\tau<\infty$ and $0<\sigma<\infty$. The mouth of the throat is at $\sigma=0$, and boundary conditions there will be discussed shortly. Outside the black hole, the scalar modes f_5 are governed by the five-dimensional effective action

$$S_5[f_5]=\int d^5x(\nabla f_5)^2 . \tag{5.13}$$

Thus the full long-distance action has two pieces: one corresponding to the region where the universe is effectively five dimensional, the other where it is effectively two-dimensional.

The field theories corresponding to the two different regions are matched along the world tube $X^\mu(\tau)+Rn^\mu(\Omega)$ of the black hole mouth. [Here $n^\mu(\Omega)$ is the unit normal to S^3.] The values of the scalar fields f along this world tube must agree as the mouth is approached from inside or outside, and this in general leads to the boundary condition

$$f_2(\tau,\sigma=0,\Omega)=f_5(X^\mu(\tau)+Rn^\mu(\Omega)) . \tag{5.14}$$

Within the functional integral this functional constraint may be enforced by Lagrange multipliers. In the low-energy limit where modes with nonzero angular momentum are neglected, the mouth is replaced by a point and (5.14) reduces to the Ω-independent constraint

$$f_2(0,\tau)=f_5(X^\mu(\tau)) . \tag{5.15}$$

This boundary condition is enforced by introducing a single (time-dependent) Lagrange multiplier, $\beta(\tau)$, into the functional integral:

$$Z=\int \mathcal{D}f_2\mathcal{D}f_5\mathcal{D}\beta\, e^{iS_5+iS_2+iS_\beta} , \tag{5.16}$$

where S_5 and S_2 are given by (5.13) and (5.12), and S_β is given by

$$S_\beta=\int d\tau\,\beta(\tau)f_5(X(\tau))-\int d\tau\,\beta(\tau)f_2(0,\tau) . \tag{5.17}$$

Z involves a weighted integral over all possible Dirichlet-type boundary conditions at the boundary $\sigma=0$ of the two-dimensional field theory. One can then integrate out both the boundary value $f_2(0,\tau)$ and $\beta(\tau)$ to reexpress the functional integral in terms of a two-dimensional field theory with a fixed Dirichlet boundary condition $f_2(0,\tau)=0$ and operator insertions at $\sigma=0$. The result is

$$Z=\int \mathcal{D}_D f_2\mathcal{D}f_5 e^{iS_5+iS_2+iS_I} , \tag{5.18}$$

where

$$S_I=\int d\tau\, f_5(X^\mu(\tau))\partial_\sigma f_2(0,\tau) \tag{5.19}$$

and the subscript D on the integration measure denotes that $f_2(0,\tau)=0$.

More generally one must include effects due to the finite mouth size. The technology for including these effects has been developed in the context of wormhole theory. The most general expression consistent with locality and energy conservation is

$$S_\beta=\int d\tau\, C_{ia}\mathcal{O}_5^i(X(\tau))\mathcal{O}_2^a(0,\tau) , \tag{5.20}$$

where the $\mathcal{O}$'s are complete sets of local operators in the theories and a detailed calculation is required to determine the constants C_{ia}. The coefficients C_{ia} for the angle-dependent modes will be suppressed due to the large effective barriers.

This analysis extends in an obvious way to incorporate the other fields in the theory. S_2 and S_5 and the measure in (5.18) are promoted to the full expressions involving the metric and dilaton. Since (e.g.) the dilaton has no propagating massless excitations along the throat, a low-energy dilaton pulse incident on the black hole cannot enter the throat unless it turns into an f pulse, a process which occurs only at higher orders. The boundary conditions lead to the important constraint that the (fixed) value ϕ_0 of the dilaton zero mode in the asymptotic region match the value of the dilaton at the mouth $\sigma=0$ of the two-dimensional region.

While the four-dimensional case is largely similar, some new features arise due to the well-known peculiarities of the charged Dirac equation in the presence of a magnetic source. This will be the subject of a separate publication [17].

In summary, the low-energy effective action contains an asymptotically flat four- or five-dimensional and a two-dimensional piece joined along the black hole mouth. Interactions between the two pieces are represented by local operators integrated along the mouth world line.

VI. IS THERE A UNITARY *S* MATRIX?

We now return to the issue of how to describe scattering from extremal black holes. In the present context we may wish to consider, for example, scattering of low-energy f particles off of extremal black holes.

There are two distinct possibilities that have previously been discussed in the literature: either black holes exhibit intrinsically nonunitary dynamics or they undergo coherent quantum-mechanical evolution. In the former case it has been argued [2] that black hole dynamics may be described by a probability-conserving but nonunitary $\$$ matrix. Such a matrix linearly maps density matrices to density matrices, but allows an arbitrarily large loss of information or increase in entropy.

Although nonunitary dynamics is a logical possibility that cannot be ruled out at present, there are several reasons to favor the alternative. One of these is the classical thermodynamical result that a fixed entropy should be associated with a black hole of a given mass: this is a result that would be expected if at the fundamental level the black hole were a quantum-mechanical system. Indeed, with the definition $S=-\mathrm{Tr}(\rho\ln\rho)$ of the entropy, an N-state quantum system has a maximum entropy $\ln N$. This latter result is suggestive that not only are black holes quantum systems, but that they have finitely many accessible states at a given mass level. (We will examine a different possibility shortly.)

In the present context this problem naturally divides into two parts. The first is unitarity of the two-dimensional effective field theory of Sec. V A (argued there to describe particle-hole scattering after the f particle enters the throat region). If this two-dimensional theory is not unitary (e.g., due to singularity formation), then it is highly unlikely that low-energy f-particle-hole scattering is unitary. On the other hand, unitarity of the two-dimensional field theory (5.1) does not necessarily imply that there is a unitary S-matrix for f-particle-hole scattering. The reason for this is simple: from the higher-dimensional viewpoint, the state of the two-dimensional field theory inside the black hole mouth is unobservable and should be traced over.[10] More explicitly, one first computes the S matrix for the full S_5+S_2 field theory. This maps the initial density matrix $\rho(0)=\rho_2(0)\otimes\rho_5(0)$ to the final density matrix $S\rho(0)S^\dagger$. One then traces over the unobservable f_2 field theory to obtain the five-dimensional density matrix, $\rho_5=\mathrm{tr}_2[S\rho(0)S^\dagger]$. This will in general correspond to a mixed state.

Some progress on the question of unitarity of the two-dimensional theory was made in [5], in which the process of black hole formation and evaporation in the two-dimensional field theory (5.1) (the two-dimensional analogue of particle-hole scattering) was analyzed. It was suggested that a collapsing f wave (that is an f wave heading toward $\sigma=\infty$) dissipates its energy via Hawking evaporation and that the classical singularity at $\phi=\infty$ (strong coupling) is removed. The absence of any singularities would imply the existence of a unitary S matrix. However more recent work [6,7] has provided strong evidence that, while this strong-coupling singularity may indeed not appear, other types of singularities occur in the quantum theory at a fixed, critical value ϕ_c of ϕ. In the linear dilaton vacuum (which corresponds to the extremal black hole) $\phi=\phi_c$ is a timelike line which separates the vacuum into two regions governed by different dynamics. The singularities found in [6,7] occur when an incoming f particle hits this critical line.[11] The proper interpretation of these singularities is not evident to us at present. However in the present context we note that they might be avoided as follows.[12] The description of f-particle-hole scattering does not require the full two-dimensional field theory, since the latter is glued on to the higher-dimensional theory near the line $\phi=\phi_0$, where ϕ_0 is approximately the higher-dimensional asymptotic value of ϕ. If we choose $\phi_0>\phi_c$, then the singular line of the two-dimensional field theory is avoided entirely. By introducing a large number N of f particles, this may be accomplished within the weak-coupling regime, and the corresponding scattering processes perhaps computed perturbatively. However it is not clear to us at present that the problems do not pop up somewhere else, and this suggestion should be regarded as tentative.

While the resolution of these issues is extremely important, to proceed for the moment we assume that the two-dimensional field theory is unitary, and consider the consequences for higher-dimensional f-particle-hole scattering. In general, whether or not this scattering is effectively unitary depends both on the internal theory of the black hole (as given by the two-dimensional theory of Sec. V A) and on the couplings between this theory and the external world. There are important constraints on these couplings in the present context, as follows. The results of [5] suggest that the interaction time scale for f-particle-hole scattering is of the order of the black hole size; i.e., the interaction Lagrangian of the low-energy field theory is local in both time and space. Therefore the effect of the incident f particle in the low-energy approximation is simply to induce transitions among the extremal black hole ground states. This corresponds to an interaction Lagrangian of the form

$$S_I=\int d\tau\, C_{ia}\mathcal{O}_5^i(X(\tau))T^a\ , \tag{6.1}$$

where the T^a are time-independent operators which act on the black hole ground states.[13] If the number of ground states is N (where N may be infinite) then the T^a can be taken to be the generators of $U(N)$.

If the number of ground states N is finite, this has the important consequence that there can be no real loss of quantum coherence. In that case the final density matrix, though not that of a pure state, is of the form which arises in the example of scattering off of a molecule with N degenerate ground states whose quantum state is initially unknown. This is of course the usual case for real laboratory experiments. Initially, scattering experiments which induce transitions among the molecular ground

[10]This is somewhat different than the Reissner-Nordström case, where the traced-over state of the black hole is inaccessible due to an event horizon, and the consequent loss of unitarity is perhaps physically less disturbing.

[11]These singularities were noted in perturbation theory in [5], but it was not known if they persisted in the nonlinear theory.

[12]The following idea was independently discussed in [7].

[13]Note that a mixed functional integral-operator formalism is being used here. Because the operators T^a do not in general commute with one another and are multiplied by time-dependent functions $\mathcal{O}$, it is necessary to time order the evolution generating exponentials.

states will lead to an increase in the entropy of the experimental apparatus. However, its entropy cannot increase to more than $\ln N$. After many scattering experiments, the quantum state of the molecule is eventually determined, and further experiments do not then lead to a further increase in the entropy of the experimental apparatus. Thus, if the black hole has a finite ground state degeneracy, the incoherence which arises from scattering experiments is no more alarming than that encountered in real laboratory experiments.

While not much is known about the ground states, the possibility of an infinite degeneracy certainly cannot be ruled out. In fact the recent analysis of [5] suggests that the black hole could have an infinite number of ground states labeled by different values of a global conserved charge. While an infinite degeneracy is a necessary condition for effective quantum incoherence (here we mean over and above what ordinarily occurs in the laboratory), it is not a sufficient condition, as can be seen from the following example. Suppose the infinite ground states $|m\rangle$, $m\in Z$ of the black hole are characterized by $q|m\rangle=m|m\rangle$ where q is an operator, and that the only coupling to the observable sector is given by

$$H_i=f^2(X(\tau))q \ , \qquad (6.2)$$

where $X(\tau)$ is the world line of the black hole and f is a quantum field operator which creates f particles. This theory has a superselection rule forbidding transitions among the ground states. Off-diagonal elements of the density matrix cannot be measured, and may be set to zero. The density matrix is then of the form

$$\rho=\sum_m \rho_m |m\rangle\langle m| \ . \qquad (6.3)$$

This means that the black hole is in the state $|m\rangle$ with probability ρ_m. The eigenvalue m of q, which determines the quantum state of the black hole, may then be measured (to arbitrary finite accuracy) in a finite number of f-particle scattering experiments. Once the state is determined, the results of further scattering experiments can be predicted with quantum-mechanical determinacy.

The eigenvalue of q in this example is a new type of quantum number which can label the internal state of an extremal black hole. Classically black holes are characterized by only a few parameters which are generally conserved charges. Quantum mechanically they may carry an additional few varieties of "quantum hair" which can be measured by long-range interference experiments involving strings [18]. We are finding here that there exists the possibility of additional observable parameters characterizing the internal state of the black hole. These are detectable precisely because black holes are not black: the parameters determine how an incoming particle scatters to an outgoing one. The number of such parameters is potentially infinite when one considers all possible scattering experiments. Whether there are sufficiently many of them to completely characterize the internal quantum state of the black hole depends on the details of the scattering process. Like quantum hair, these new parameters exist only at the quantum level. Unlike quantum hair, they can be detected only in a short-range scattering experiment. We therefore refer to them as "quantum whiskers."

We wish to emphasize that it is a logical possibility that the quantum whiskers as specified are determined purely by the hair, either classical or quantum, on the black hole. In that case they would not further distinguish different black holes. But it is also possible that the quantum whiskers are independent quantities that can take on infinitely many values, perhaps providing a memory of the initial collapsing state which formed the black hole. A more detailed dynamical analysis is required to answer this question.

The coupling given in (6.2) is of course very special. The general situation is as follows. The linear combination of operators $q_A\equiv H_{iAa}q^a$ appearing in the coupling H_i of the black hole to the external world form an algebra of observables that generates a subgroup G of the group $U(N)$ of unitary transformations of the N-dimensional (where N may be infinite) space of black hole ground states. G need not be simple or finite dimensional. The space of ground states then decomposes into irreducible representations V_r of dimension d_r of G. The general density matrix describing the black hole assigns a probability ρ_r for the black hole to be in the representation V_r (off-diagonal elements which mix representations can again be set to zero.) If a finite number of representations occur, the irreducible representation can be determined (e.g., by measuring the Casimirs) after a finite number of experiments. The irreducible representation is a conserved quantum whisker labeling the black hole. (The quantum whiskers corresponding to values of the other observables are not necessarily conserved, as they may be changed in scattering experiments.) If the irreducible representation so determined is finite dimensional, the subsequent increase in entropy of the experimental apparatus is bounded as before by $\ln d_r$. The quantum state of the system can eventually be determined, and quantum coherence is not effectively lost. An infinite value of d_r is a necessary condition for an effective loss of quantum coherence. We do not know the sufficient conditions: we hope to return to this question in a future publication.

Finally, there is an important question we have been postponing: why should there be superselection sectors among the black hole ground states? Of course they might arise as a consequence of symmetries, but we have no strong reason to believe that this is the case. Rather we are arguing that it is a logical and interesting possibility. One piece of evidence is the following: according to Hawking the black hole entropy is proportional to the area and in particular is finite. This entropy should be identified as $\ln d_r$, the entropy within a given superselection sector. On the other hand the possibility of arbitrary values of global charges for black holes suggests, as in [5], that the number of ground states is infinite. This is consistent with finite d_r only if there are indeed superselection sectors. Thus apparently something must give: either (a) there are an infinite number of states within each superselection sector, quantum coherence is effectively lost and Hawking's area law for entropy is incorrect, (b) the superselection sectors have a finite number of states, the area law is correct and quantum coherence is not truly lost, or (c) the black hole does not behave like a quantum system.

VII. CONCLUDING COMMENT

In conclusion, we would like to comment on the generality of the previous discussion on quantum whiskers and incoherence. It might appear that our discussion depended heavily on specific properties of dilatonic black holes and did not apply for example to the Reissner-Nordström case. At the beginning of the previous section we assumed, partially motivated by [5], that the black hole is described by a quantum state, and that the only potential source of information loss is tracing over the quantum state. More generally, the whole system might be described by a density matrix together with a $\mathcal{S}$-matrix governing its evolution. However, these matrices are subject to powerful physical constraints such as probability conservation, locality and energy conservation. It is plausible, though we have been unable to prove this, that every system compatible with these constraints can be obtained by tracing over some internal, unmeasurable quantum system modeling the black hole (some results in this direction can be found in [19]). If this is true, then our considerations are quite general and apply to scattering off of any type of extremal black hole. If it is not true, it would be very interesting to characterize counterexamples in which particle-hole scattering can truly lead to information loss without violating energy or probability conservation, or storing the information in the black hole.

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