Second-order electron mass dispersion relation at finite temperature. II.

Mahnaz Qader, Samina S. Masood, and K. Ahmed
Physics Department, Quaid-i-Azam University, Islamabad, Pakistan
(Received 10 July 1992)

The calculations of electron self-energy up to two-loop order in the framework of a real-time formalism at finite temperature are carried out in detail. Some aspects associated with the renormalization of the electron mass at finite temperature and the resulting evaluation of its dispersion relation are highlighted. This work generalizes one of the earlier works to all temperatures. A comparison of the results with some of the existing ones is also made.

PACS number(s): 12.20.Ds, 11.10.Gh

I. INTRODUCTION

The renormalization of QED at finite temperature up to first order in α has been studied in detail in the literature [1–6]. Dicus, Down, and Kolb [7] have proposed a perturbative expansion of the electron self-energy at high temperature, i.e., $T \gg m_e$, in natural units $\hbar = c = k = 1$ in the usual notation:

$$\Sigma_\beta(p) \sim T \sum_{n=1} c_n \alpha^n \left[\frac{T}{p} \ln \frac{E+p}{E-p} \right]^n , \tag{1.1}$$

with n standing for the order of perturbation. The electron mass dispersion up to order α^2 in a background heat bath has been explicitly calculated earlier at low temperature, i.e., $T \ll m_e$ [8]. In this limit the probability of finding $e^- e^+$ pairs in the medium is suppressed by a factor $e^{-m/T}$ ($\beta = 1/T$) due to the fermion distribution function. Therefore, as a first approximation, in Ref. [8] we ignored the contribution of the fermions and included hot photons only. However, if $T \geq m_e$ and the background fermions possess relativistic energies ($E > m$), then their distribution functions would contribute significantly. This has been the basis of some earlier work in the real-time formalism at the one-loop level [4] which is now being extended to two loops. In this work the choice of the finite-temperature real-time formalism is made because these methods are covariant and can separate the additive finite-temperature contributions. The Feynman rules in vacuum theory hold at finite temperature [9], and the vacuum propagators are simply replaced by the hot propagators of the heat bath. Thus the fermion propagator is written as [10]

$$S_\beta(p) = \frac{i}{\not p - m + i\epsilon} - 2\pi\delta(p^2 - m^2)(\not p + m) n_F(p) \tag{1.2a}$$

and the boson propagator as [10]

$$D_\beta^{\mu\nu}(k) = -g^{\mu\nu} \left[\frac{i}{k^2 + i\epsilon} + 2\pi\delta(k^2) n_B(k) \right] , \tag{1.2b}$$

where the corresponding distribution functions are given by

$$n_F(p) = \frac{1}{e^{\beta|p\cdot u|} + 1} \tag{1.3a}$$

for the fermions and

$$n_B(k) = \frac{1}{e^{\beta|k\cdot u|} - 1} \tag{1.3b}$$

for the bosons, with $u^\mu = (1,0,0,0)$ as the four-velocity of the heat bath. The fermion distribution function serves as a cutoff to ultraviolet singularities so that no such divergence appears even at high temperature. However, an infrared singularity arises as a result of the photon distribution function, in an enhanced form, i.e., $I_A \sim \int_0^\infty (dk/k) n_B(k)$, which becomes $\int_0^\infty dk/k^2$ in the limit $k \to 0$.

The electron mass was reported to be renormalizable up to first order in α for all temperatures in QED [4]. The electron mass dispersion relation at the one-loop level is given by

$$m_{\text{phys}}^2 \simeq m^2 \left[1 - \frac{6\alpha}{\pi} b(m\beta) \right] + \frac{4\alpha}{\pi} m T a(m\beta) + \tfrac{2}{3}\alpha\pi T^2 \left[1 - \frac{6}{\pi^2} c(m\beta) \right] , \tag{1.4}$$

where m^2 is its mass at zero temperature,

$$a(m\beta) = \ln(1 + e^{-m\beta}) , \tag{1.5a}$$

$$b(m\beta) = \sum_{n=1}^{\infty} (-1)^n \mathrm{Ei}(-nm\beta) , \tag{1.5b}$$

$$c(m\beta) = \sum_{n=1}^{\infty} (-1)^n \frac{e^{-nm\beta}}{n^2} . \tag{1.5c}$$

The corresponding self-mass correction up to first order in α at finite temperature is

$$\frac{\delta m^{(1)}}{m} \simeq \frac{\alpha\pi T^2}{3m^2} \left[1 - \frac{6}{\pi^2} c(m\beta) \right] + \frac{2\alpha}{\pi} \frac{T}{m} a(m\beta) - \frac{3\alpha}{\pi} b(m\beta) . \tag{1.6}$$

The calculation of the electron mass dispersion relation at $T \sim m_e$ is important from the point of view of cosmology. The first-order-in-α electron mass correction in Eq. (1.6) has been shown to affect directly the β-decay rate during primordial nucleosynthesis [11]. This leads to the change in certain parameters in cosmology such as the energy density of the early Universe, ρ_T, the helium abundance parameter Y, etc. It is interesting to note that these parameters become slowly varying functions of temperature [12] during nucleosynthesis; otherwise, they remain constant. Similar types of implication are expected at order α^2 because the thermal corrections to the electron mass are expected to modify the above-mentioned parameters of early cosmology in a similar manner.

The two-loop Feynman graphs given in Fig. 1 are calculated to check the renormalizability of QED in a thermal background at higher orders of perturbation. We present here a closed analytic form for $\delta m/m$ calculated perturbatively up to the $O(\alpha^2)$ so that the second-order contribution to the electron mass can be evaluated from these expressions for required ranges of temperature. In the process of calculating the mass renormalization of the electron for these temperatures, we examine and classify all types of singularities which arise in the two-loop finite-temperature perturbative calculations. We further find that, in addition to the temperature-dependent enhanced singularity of type I_A, there are some overlapping singularities which appear as a result of the interference of the ultraviolet singularities in vacuum with either the I_A-type singularity or with the divergence-free terms at finite temperature. Moreover, there arise, as expected, the coincident momenta $\delta(0)$-type singularities at the two-loop level, which can be removed by using the identity [13]

$$\frac{\delta(k^2)}{k^2+i\epsilon}=-\tfrac{1}{2}\delta'(k^2)-i\pi[\delta(k^2)]^2 . \tag{1.7}$$

In this paper we demonstrate that the finite-temperature perturbative techniques so far available in the literature are sufficient to renormalize the electron self-energy at all temperatures in QED up to two-loop order. On the basis of these results, we find that it is hard

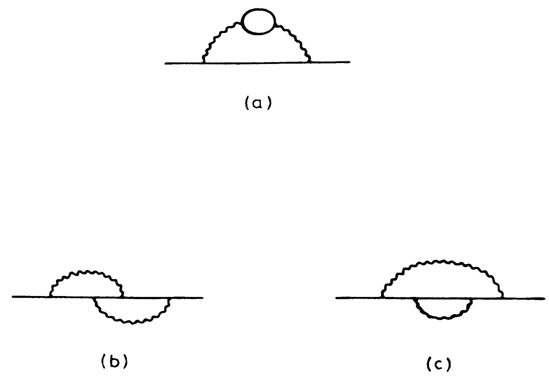

FIG. 1. One-particle-irreducible electron self-energy at second order in α.

to predict a higher-order form of the electron self-energy for the perturbative expansion valid for all orders, in the case of one-particle-irreducible graphs.

After giving the details of the calculations for the two-loop diagrams in Sec. II, we present an explicit analysis for the cancellation of singularities and establish the electron mass renormalization in Sec. III. Section IV comprises a discussion of the results and their interpretation in some interesting limits of temperature.

II. ELECTRON SELF-ENERGY AT $O(\alpha^2)$

As mentioned in Ref. [8], the two-loop Feynman diagrams given in Fig. 1 for the electron self-energy were first calculated by Dicus, Down, and Kolb [7]. They omitted the contribution of the "vacuum polarization insertion" diagram in Fig. 1(a) as it contributes to charge renormalization and not to the electron mass renormalization. Therefore we also give the results of a detailed calculation, taking into account Figs. 1(b) and 1(c) only.

The Feynman graph up to second order in α in Fig. 1(b) with the "overlapping loops" can be written as

$$\Sigma_\beta^{(b)}(p)=e^4\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4l}{(2\pi)^4}\gamma_\mu D_\beta^{\mu\nu}(k)S_\beta(p-k)\gamma_\sigma D_\beta^{\sigma\rho}(l)S_\beta(p-k-l)\gamma_\nu S_\beta(p-l)\gamma_\rho . \tag{2.1}$$

The operator part of the numerator in Eq. (2.1), on using the usual properties of the γ matrices, gives

$$\begin{aligned}N^{(b)}=-4[&2(p^2-p\cdot k-p\cdot l+k\cdot l)(\not p-\not k-\not l)\\&-m(3p^2+k^2+l^2-2p\cdot k-2p\cdot l-2\not p\not k-2\not l\not p+3\not l\not k)-m^2(3\not p-2\not k-2\not l)+m^3] .\end{aligned} \tag{2.2}$$

Since the real part of the electron self-energy contributes to the electron mass, we pick out only the real terms in Eq. (2.1). The finite-temperature part in this equation is, therefore,

$$\Sigma^{(b)}_{T\neq 0}(p)=\frac{e^4}{(2\pi)^7}\int d^4k\int d^4l\,N^{(b)}$$
$$\times\Bigg[\Bigg\{\frac{n_B(k)\delta(k^2)}{l^2\{(p-k)^2-m^2\}\{(p-l)^2-m^2\}\{(p-k-l)^2-m^2\}}$$
$$-\frac{n_F(p-k)\delta\{(p-k)^2-m^2\}}{l^2k^2\{(p-k-l)^2-m^2\}\{(p-l)^2-m^2\}}-\frac{n_F(p-k-l)\delta\{(p-k-l)^2-m^2\}}{2l^2k^2\{(p-k)^2-m^2\}\{(p-l)^2-m^2\}}$$
$$+4\pi^2\Bigg[\frac{n_B(k)\delta(k^2)n_B(l)\delta(l^2)n_F(p-k)\delta\{(p-k)^2-m^2\}}{\{(p-k-l)^2-m^2\}\{(p-l)^2-m^2\}}$$
$$+\frac{n_B(k)\delta(k^2)n_F(p-k)\delta\{(p-k)^2-m^2\}n_F(p-k-l)\delta\{(p-k-l)^2-m^2\}}{l^2\{(p-l)^2-m^2\}}$$
$$-\frac{n_B(l)\delta(l^2)n_B(k)\delta(k^2)n_F(p-k-l)\delta\{(p-k-l)^2-m^2\}}{2\{(p-k)^2-m^2\}\{(p-l)^2-m^2\}}$$
$$+\frac{n_B(l)\delta(l^2)n_F(p-k)\delta\{(p-k)^2-m^2\}n_F(p-k-l)\delta\{(p-k-l)^2-m^2\}}{k^2\{(p-l)^2-m^2\}}$$
$$-\frac{n_B(k)\delta(k^2)n_F(p-k)\delta\{(p-k)^2-m^2\}n_F(p-l)\delta\{(p-l)^2-m^2\}}{l^2\{(p-k-l)^2-m^2\}}$$
$$+\frac{1}{2l^2k^2}n_F(p-k)\delta\{(p-k)^2-m^2\}n_F(p-l)\delta\{(p-l)^2-m^2\}$$
$$\times n_F(p-k-l)\delta\{(p-k-l)^2-m^2\}\Bigg]\Bigg\}+k\leftrightarrow l\Bigg]\,. \tag{2.3}$$

In Eq. (2.3) the terms involving the products of two and four δ functions are neglected because they are imaginary and, therefore, do not contribute to the electron mass dispersion relation. Further, a few terms appearing here with a product of three δ functions vanish since their combined support lies outside the physical region, as noted in Refs. [14,15] also. Similarly, the term with five δ functions also vanishes. The term-by-term integrations of Eq. (2.3) are evaluated in Appendix A. Adding the results in (A1), (A3), (A5), and (A7), the Feynman graph in Fig. 1(b) gives

$$\Sigma^{(b)}_{\beta}(p)=\Sigma^{(b)}_{T=0}(p)+\frac{\alpha^2}{4}\Bigg[\frac{1}{2\pi^3\epsilon}\{3[(2m-\not p)J_A+I+J_B]+(\not p+6m)I_A\}-\frac{3}{\pi^2}(\not p+4m)I_A$$
$$+\frac{1}{8\pi}(\not p+m)I_BI_C+B_1TI_B+(B_2T^2+B_3T+B_4)I_C+B_5T^2+B_6T+B_7\Bigg]\,, \tag{2.4}$$

where

$$\frac{1}{\epsilon}=\frac{1}{\eta}-\gamma-\ln\left[\frac{4\pi\mu^2}{m^2}\right] \tag{2.5}$$

is the usual ultraviolet divergence of the vacuum field theory in the modified minimal subtraction ($\overline{MS}$) scheme of renormalization with the Euler-Mascheroni constant $\gamma=0.5772\ldots$:

$$I_A=8\pi\int_0^\infty\frac{dk}{k}n_B(k)\,, \tag{2.6a}$$

$$I_B=8\pi\sum_{r=1}^{\infty}(-1)^r\int_0^\infty\frac{dk}{k}n_B(k)e^{-r\beta(p-k)}\,, \tag{2.6b}$$

and

$$I_C=8\pi\sum_{r=1}^{\infty}(-1)^r\int_0^\infty\frac{dl}{l}n_B(l)e^{-r\beta l}\,. \tag{2.6c}$$

The functions $B_1,\ldots,B_7$ in Eq. (2.4) are given in (A9), . . . , (A15). B_2, B_3, and B_7 are temperature independent, while B_1, B_4, B_5, and B_6 contain temperature dependence in the form of a series in powers of $e^{-\beta m}$ or $e^{-\beta E}$.

Finally, the Feynman diagram in Fig. 1(c) which contains a "loop within a loop" can be expressed as

$$\Sigma_{\beta}^{(c)}(p)=e^4\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4l}{(2\pi)^4}\gamma_{\mu}D_{\beta}^{\mu\nu}(k)S_{\beta}(p-k)\gamma_{\sigma}D_{\beta}^{\sigma\rho}(l)S_{\beta}(p-k-l)\gamma_{\rho}S_{\beta}(p-k)\gamma_{\nu}\ . \tag{2.7}$$

Simplifying the numerator in this Feynman integral, as before in Eq. (2.1), we get

$$N^{(c)}=4[q^2(\not q+\not l)-2l\cdot q(\not q-2m)-3m^2\not q-m^2\not l+4m^3]\ , \tag{2.8}$$

where we have transformed $p-k\rightarrow q$. The real part of the temperature-dependent contribution to the electron self-energy in Fig. 1(c) is

$$\begin{aligned}\Sigma_{T\neq0}^{(c)}(p)=\frac{e^4}{(2\pi)^7}\int d^4q\int d^4l\,N^{(c)}\Bigg[&\frac{n_F(q)}{2E_q^2l^2(p-q)^2\{(q-l)^2-m^2\}}\left\{\frac{\delta(q_0-E_q)}{q_0+E_q}+\frac{\delta'(q_0-E_q)}{2}\right\}\\&-\frac{n_F(q-l)\delta\{(q-l)^2-m^2\}}{l^2(p-q)^2(q^2-m^2)^2}\\&+\frac{1}{(q^2-m^2)^2\{(q-l)^2-m^2\}}\left\{\frac{n_B(l)\delta(l^2)}{(p-q)^2}+\frac{n_B(p-q)\delta[(p-q)^2]}{l^2}\right\}\\&+4\pi^2\Bigg\{\Bigg[n_F(q-l)\delta\{(q-l)^2-m^2\}\left\{\frac{n_B(l)\delta(l^2)}{(p-q)^2}+\frac{n_B(p-q)\delta[(p-q)^2]}{l^2}\right\}\\&\qquad-n_B(p-q)\delta\{(p-q)^2\}n_B(l)\delta(l^2)\Bigg]\frac{n_F(q)}{2E_q^2}\left[\frac{\delta(q_0-E_q)}{q_0+E_q}+\frac{\delta'(q_0-E_q)}{2}\right]\\&\qquad+\frac{n_B(p-q)\delta[(p-q)^2]}{(q^2-m^2)^2}n_B(l)\delta(l^2)n_F(q-l)\delta[(q-l)^2-m^2]\Bigg\}\Bigg]\ ,\end{aligned} \tag{2.9}$$

where $\delta'(q_0-E_q)$ is the first-order derivative with respect to q_0. The factors such as $\delta(q_0-E_q)$ and $\delta'(q_0-E_q)$ in the numerator and $(q^2-m^2)^2$ in the denominators are obtained on using Eq. (1.7) for the electron propagator with repeated momentum q in Fig. 1(c).

The results of the integrations over the loop momenta l and q in expressions (B3), (B6), (B7), (B10), (B12), and (B15) are added to obtain

$$\begin{aligned}\Sigma_{\beta}^{(c)}(p)=\Sigma_{T=0}^{(c)}(p)&\\+\alpha^2\Bigg[&\frac{1}{\eta}\{C_1I_A+C_2T^2+C_3T+C_4\}\\&+C_5I_A+C_6T^3+C_7T^2+C_8T+C_9\Bigg]\ ,\end{aligned} \tag{2.10}$$

where $C_1,\ldots,C_9$ are given in Eqs. (B16) , . . . , (B24). C_1, C_2, and C_5 do not contain any temperature dependence, while C_3, C_4, $C_6,\ldots,C_9$ depend upon the temperature in the form of a series in powers of $e^{-\beta m}$ or $e^{-\beta E}$.

III. CANCELLATION OF SINGULARITIES

As mentioned in Sec. I, no extra ultraviolet singularities are introduced from the heat bath. We begin with the classification of singularities arising in a two-loop calculation and then give the relevant renormalized results. These singularities are as follows.

(i) The $\delta(0)$-type coincident singularities due to the repeated electron or photon propagators terms as "pathologies" in Ref. [14].

(ii) The temperature-dependent infrared divergences of the type I_A given in Eqs. (2.6a)–(2.6c) since I_B and I_C have the same singular behavior in the infrared region as I_A.

(iii) The overlap of the I_A-type singularities with the usual vacuum ultraviolet ones.

(iv) The interference of the vacuum ultraviolet singularities with the temperature-dependent finite terms.

(v) The high-temperature mass singularities of the form $\ln[(1-v)/(1+v)]$, which persist at the second order in α also.

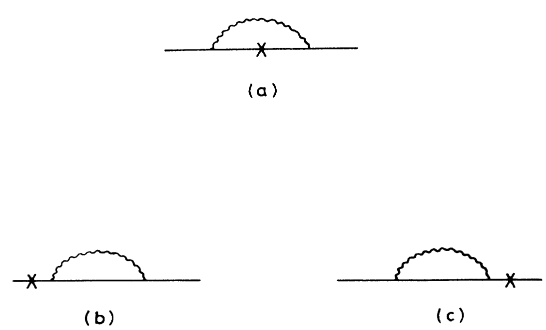

FIG. 2. Mass counterterms at order α^2.

The singularities classified in (i) are removed using the identity in Eq. (1.7). The divergences of form (ii) have been shown in the literature to cancel out in physical processes [16]. We have checked that the divergences of forms (iii) and (iv) are canceled by adding the mass counterterms in Fig. 2. In particular, the divergences appearing in Figs. 1(b) are canceled by adding the mass counterterms from Figs. 2(b) and 2(c), i.e.,

$$\frac{3\alpha^2}{4\pi^3\epsilon}(I+J_B+mJ_A)\ . \tag{3.1}$$

Further, the divergences in Fig. 1(c) are canceled by the mass counterterms in Fig. 2(a) in a similar way. The electron self-energy at order α^2 obtained by adding the mass counterterms to Eqs. (2.4) and (2.10) is now free from any type of infrared or ultraviolet divergence. The self-energy relation, however, contains the terms with mass singularities of type (v) at high temperature.

FIG. 3. One-particle-reducible self-energy of an electron at order α^2.

Following the procedure in Refs. [2,4], the physical mass of the electron, after incorporating the temperature radiative corrections up to the order α^2, becomes

$$\begin{aligned}
m_{\rm phys}^2 = m^2 &+ \alpha\left[\frac{4m}{\pi}Ta(m\beta)-\frac{6}{\pi}m^2b(m\beta)+\tfrac{2}{3}\pi T^2\left\{1-\frac{6}{\pi^2}c(m\beta)\right\}\right]\\
&+2\alpha^2\Bigg[\frac{T^3}{m}\left\{\frac{\mathcal{M}_1}{\pi^2}F_1+\frac{\mathcal{M}_2}{\pi^2}\zeta(3)+\frac{\pi m}{6Ev}F_2\right\}\\
&\quad+T^2\left\{\tfrac{27}{16}v^2F_3+\mathcal{M}_3F_4+\frac{5}{\pi^2}a(m\beta)+\frac{8}{\pi^2}c(m\beta)+\frac{m}{3Ev}F_2+\mathcal{M}_4+\tfrac{3}{2}\mathcal{M}_5(F_5+F_6)+27(\tfrac{1}{4}\mathcal{M}_6F_3+\mathcal{M}_7F_7)\right\}\\
&\quad+mT\Bigg\{8a(m\beta)-\frac{9}{4v^2}F_8-3\mathcal{M}_8F_9+\frac{9E}{4m}F_{10}+\mathcal{M}_9F_{11}+\frac{9E}{2mv^2}F_{12}+2\mathcal{M}_{10}F_{13}-3(\tfrac{3}{4}F_{14}+F_{15})\mathcal{M}_5\\
&\qquad+\frac{5}{\pi^2}a(m\beta)\mathcal{M}_{11}+\mathcal{M}_{12}F_{16}+\frac{\pi}{6}(F_{17}-F_{18})\Bigg\}\\
&\quad-m^2\left\{\mathcal{M}_{13}b(m\beta)-9\mathcal{M}_{14}F_{19}+\frac{5}{2\pi^2}[a(m\beta)-\mathcal{M}_{15}b(m\beta)]-\tfrac{1}{2}F_{20}-\mathcal{M}_{16}F_{21}\right\}\Bigg]\\
&+m^2\alpha^2\left[M_1\left[\frac{T}{m}\right]^4+M_2\left[\frac{T}{m}\right]^3+M_3\left[\frac{T}{m}\right]^2+M_4\left[\frac{T}{m}\right]+M_5\right]\ .
\end{aligned} \tag{3.2}$$

The functions F_i's except F_1 and F_4 again contain the series in $e^{-\beta m}$ and/or $e^{-\beta E}$ such as B_i's and C_i's, whereas the $\mathcal{M}_i$'s are not temperature dependent. The F_i's, $\mathcal{M}_i$'s, and M_i's appearing in Eq. (3.2) are given in Appendix C. The magnitude of the temperature-dependent radiative corrections to the electron mass calculated from Eq. (3.2) is

$$\frac{\delta m}{m}\simeq\frac{1}{m}(\delta m^{(1)}+\delta m^{(2)})+\left[\frac{\delta m^{(1)}}{m}\right]^2+O(\alpha^3)\ , \tag{3.3}$$

where the superscripts (1) and (2) stand for one and two loops, respectively. $\delta m^{(1)}/m$ is already defined in Eq. (1.6), whereas its iterated form $(\delta m^{(1)}/m)^2$ corresponds to the one-particle-reducible Feynman graph in Fig. 3 and is

$$\left[\frac{\delta m^{(1)}}{m}\right]^2\simeq\alpha^2\left[M_1\left[\frac{T}{m}\right]^4+M_2\left[\frac{T}{m}\right]^3+M_3\left[\frac{T}{m}\right]^2+M_4\frac{T}{m}+M_5\right]\ . \tag{3.4}$$

Further,

$$\frac{\delta m^{(2)}}{m} \simeq \alpha^2 \left[\left[\frac{T}{m} \right]^3 \left\{ \frac{\mathcal{M}_1}{\pi^2} F_1 + \frac{\mathcal{M}_2}{\pi^2} \zeta(3) + \frac{\pi m}{6Ev} F_2 \right\} + \left[\frac{T}{m} \right]^2 \left\{ \tfrac{27}{16} v^2 F_3 + \mathcal{M}_3 F_4 + \frac{1}{\pi^2}[5a(m\beta)+8c(m\beta)] + \frac{m}{3Ev} F_2 + \mathcal{M}_4 + \tfrac{3}{2}\mathcal{M}_5(F_5+F_6) + 27(\tfrac{1}{4}\mathcal{M}_6 F_3 + \mathcal{M}_7 F_7) \right\} \right.$$
$$+ \frac{T}{m} \left\{ 8a(m\beta) - \frac{9}{4v^2} F_8 - 3\mathcal{M}_8 F_9 + \frac{9E}{4m} F_{10} + \mathcal{M}_9 F_{11} + \frac{9E}{2mv^2} F_{12} + 2\mathcal{M}_{10} F_{13} - 3(\tfrac{3}{4} F_{14} + F_{15})\mathcal{M}_5 + \frac{5}{\pi^2} a(m\beta)\mathcal{M}_{11} + \mathcal{M}_{12} F_{16} + \frac{\pi}{6}(F_{17} - F_{18}) \right\}$$
$$\left. - m^2 \left\{ \mathcal{M}_{13} b(m\beta) - 9\mathcal{M}_{14} F_{19} + \frac{5}{2\pi^2}[a(m\beta) - \mathcal{M}_{15} b(m\beta)] - \tfrac{1}{2} F_{20} - \mathcal{M}_{16} F_{21} \right\} \right] . \quad (3.5)$$

Equations (3.3)–(3.5) give the electron mass renormalization for all temperatures. The limiting values of the mass renormalization for various temperature ranges $T \ll m_e$, $T \sim m_e$, and $T \gg m_e$ can be evaluated from this result. We carry this out in the next section, wherein we compare our results with those in Ref. [7].

IV. RESULTS AND DISCUSSIONS

It has been discussed in Sec. III that in the hot background all types of singularities appearing up to the $\mathcal{O}(\alpha^2)$ in the electron self-energy are canceled except the high-temperature mass singularities. These mass singularities persist even at the single-loop level at $T \gg m_e$ [2,4,5]. Equation (3.3) gives a general result valid for all temperatures and relativistic electron energies $E_{q,l}$ up to the order $(m/E_{q,l})^3$.

The low-temperature limit is obtained from Eq. (3.3) by neglecting all factors containing the damping exponentials $e^{-\beta m}$ and $e^{-\beta E}$. Therefore, in this limit, the only factors which survive are F_1 and F_4. Since the term containing T^3 is a nonleading term at $T \ll m_e$, it can be neglected, giving

$$m^2_{\text{phys}} \simeq m^2 + \frac{2\alpha\pi T^2}{3} + \frac{2\alpha^2 T^2}{3} \left[\frac{33}{2} + \frac{1}{v} \left[\frac{8m}{E} - 1 \right] + \left[\frac{5}{v} - \frac{1}{2} - \frac{4m}{Ev^2} \right] \ln\frac{1+v}{1-v} + \left\{ 1 + \frac{1}{2} \left[1 + \frac{1}{v} \right] \ln\frac{1+v}{1-v} \right\} F_4 \right] . \quad (4.1)$$

Thus, in Fig. 1, the change in mass due to the temperature relative to that at zero temperature is

$$\frac{\delta m}{m} \simeq \frac{\alpha\pi T^2}{3m^2} + \frac{\alpha^2 T^2}{3m^2} \left[\frac{33}{2} + \frac{1}{v} \left[\frac{8m}{E} - 1 \right] + \left[\frac{5}{v} - \frac{1}{2} - \frac{4m}{Ev^2} \right] \ln\frac{1+v}{1-v} + \left\{ 1 + \frac{1}{2} \left[1 + \frac{1}{v} \right] \ln\frac{1+v}{1-v} \right\} F_4 \right] . \quad (4.2)$$

In Ref. [8] we have given an analogous expression for m^2_{phys}, which was obtained by expanding the loop variables up to $(m/E_{q,l})^2$ only. Therefore relation (4.1) is an improvement over the previous results.

The higher-order perturbative corrections in α become significant at high temperatures since the powers of T increase. The one-particle-irreducible graphs in Fig. 1 give a leading-order contribution to $(\delta m^{(2)}/m) \sim (T/m)^3$. In this range of temperature, as $m\beta \rightarrow 0$, the infinite series in F_i's have to be cut off at the lower end, as in the case of one-loop analysis such that the series remains finite. Therefore

$$\frac{\delta m^{(2)}}{m} \underset{T \gg m_e}{\sim} \alpha^2 \left[\frac{T}{m} \right]^3 \left[\frac{4}{\pi^2} \left\{ \zeta(3) \frac{E}{m} \left[\left\{ 20 \left[1 + \frac{1}{v} \right] + 7v \right\} \times \ln\frac{1+v}{1-v} - \frac{4E}{m} \right] + 2 \left[\frac{10m}{Ev} - \frac{E}{m} + \frac{7E}{mv}(1+v^2) \right] F_1 \right\} + \frac{\pi m}{6Ev} F_2 \right] , \quad (4.3)$$

where

$$F_2(R,\epsilon) \simeq \sum_{r=R}^{\infty} \sum_{n=3}^{r+1} \left\{ 2 - \frac{(-1)^r}{r} \right\} e^{-\beta\epsilon(n + rE/m)} , \quad (4.4)$$

with $\epsilon = m\beta \ll 1$ and R is a sufficiently large number which serves as a cutoff on the lower side of the summation over r. Similarly, the series in F_1 can also be truncated to maintain its finiteness.

On the other hand, the electron self-energy predicted by Dicus, Down, and Kolb [7] gives the leading-order behavior at second order in α for $T \gg m_e$ to be

$$\Sigma_\beta \sim \alpha^2 T \left[\frac{T}{p} \ln \frac{1+v}{1-v} \right]^2 . \tag{4.5}$$

Thus we note that the mass singularity appearing in our calculations of the Feynman graphs in Fig. 1 is less severe as its logarithmic degree of divergence is one less. However, a straightforward calculation of the one-particle-reducible diagram up to order α^2 (Fig. 3) in the high-temperature limit gives

$$\left[\frac{\delta m^{(1)}}{m} \right]^2 \underset{T \gg m_e}{\sim} \left[\frac{\alpha \pi T^2}{2m^2} \right]^2 . \tag{4.6}$$

From relation (4.6) the perturbative expansion in α for the one-particle-reducible diagrams for the electron self-energy can be conjectured to all orders to be

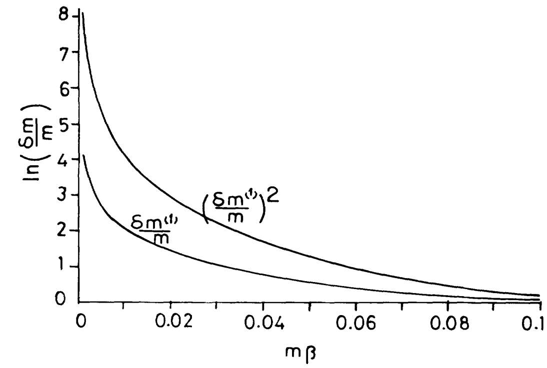

FIG. 4. Plot of $m\beta$ vs $\delta m^{(1)}/m$ and $(\delta m^{(1)}/m)^2$.

$$\left[\frac{\delta m^{(1)}}{m} \right]^n \underset{T \gg m_e}{\sim} \left[\frac{\alpha \pi T^2}{2m^2} \right]^n . \tag{4.7}$$

The values of $(\delta m^{(1)}/m)^2$ and $(\delta m^{(1)}/m)$ are plotted for some particular values of temperature in Fig. 4.

In the intermediate range of temperature $T \sim m_e$, which is particularly interesting from the point of view of cosmology, $m\beta \to 1$. Therefore, substituting the relevant expressions for $\delta m^{(1)}/m$ and $\delta m^{(2)}/m$ in this limit, Eq. (3.3) gives

$$\begin{aligned}
\frac{\delta m}{m} \simeq \alpha & \left[\frac{\pi}{3} \left\{ 1 - \frac{6}{\pi^2} c(1) \right\} + \frac{2}{\pi} a(1) - \frac{3}{\pi} b(1) \right] \\
& + \alpha^2 \left[\frac{3}{\pi^2} \{ 4a(1) + 3b(1) \} b(1) + \frac{\pi^2}{9} \left\{ 1 - \frac{6}{\pi^2} c(1) \right\}^2 + \left\{ 1 - \frac{6}{\pi^2} c(1) \right\} \left\{ \frac{4\pi}{3} a(1) - 2b(1) \right\} - \frac{4}{\pi^2} [a(1)]^2 \right] \\
& + \alpha^2 \Bigg[\left[\frac{T}{m} \right]^3 \left\{ \frac{\mathcal{M}_1}{\pi^2} \mathcal{F}_1 + \frac{\mathcal{M}_2}{\pi^2} \zeta(3) + \frac{\pi m}{6Ev} \mathcal{F}_2 \right\} \\
& \quad + \left[\frac{T}{m} \right]^2 \left\{ \tfrac{27}{16} v^2 \mathcal{F}_3 + \mathcal{M}_3 \mathcal{F}_4 + \frac{1}{\pi^2} [5a(1) + 8c(1)] + \frac{m}{3Ev} \mathcal{F}_2 + \mathcal{M}_4 + \tfrac{3}{2} \mathcal{M}_5 (\mathcal{F}_5 + \mathcal{F}_6) + 27 (\tfrac{1}{4} \mathcal{M}_6 \mathcal{F}_3 + \mathcal{M}_7 \mathcal{F}_7) \right\} \\
& \quad + \frac{T}{m} \Bigg\{ 8a(1) - \frac{9}{4v^2} \mathcal{F}_8 - 3\mathcal{M}_8 \mathcal{F}_9 + \frac{9E}{4m} \mathcal{F}_{10} + \mathcal{M}_9 \mathcal{F}_{11} + \frac{9E}{2mv^2} \mathcal{F}_{12} + 2\mathcal{M}_{10} \mathcal{F}_{13} - 3 (\tfrac{3}{4} \mathcal{F}_{14} + \mathcal{F}_{15}) \mathcal{M}_5 \\
& \qquad + \frac{5}{\pi^2} \mathcal{M}_{11} a(1) + \mathcal{M}_{12} \mathcal{F}_{16} + \frac{\pi}{6} (\mathcal{F}_{17} - \mathcal{F}_{18}) \Bigg\} \\
& \quad - m^2 \left\{ \mathcal{M}_{13} b(1) - 9\mathcal{M}_{14} \mathcal{F}_{19} + \frac{5}{2\pi^2} [a(1) - \mathcal{M}_{15} b(1)] - \tfrac{1}{2} \mathcal{F}_{20} - \mathcal{M}_{16} \mathcal{F}_{21} \right\} \Bigg] ,
\end{aligned} \tag{4.8}$$

where the $\mathcal{F}_i$'s are obtained from the F_i's in the limit $m\beta \to 1$.

ACKNOWLEDGMENTS

Two of the authors (M.Q. and K.A.) would like to acknowledge the National Scientific and Research Development Board, Pakistan for partial financial support during this work. K.A. would also like to thank the Pakistan Science Foundation for provision of funds under a PSF Project.

APPENDIX A

We discuss here the integrations over the loop momenta in Eq. (2.3) term by term. In the first term of Eq. (2.3), the l integration is done by the usual method in quantum field theory in vacuum, whereas the k integration is done by the procedure of Ref. [2] to obtain

$$-\frac{\alpha^2}{4\pi^3}\left[\frac{1}{2\epsilon}\{(\not p+6m)I_A-3\not I\}+\tfrac{3}{2}(\not p+4m)I_A-2\not I\right], \tag{A1}$$

where $1/\epsilon$ is defined in Eq. (2.5) and I_A in Eq. (2.6a). Further,

$$\frac{I^0}{E}=\frac{2\pi^3T^2}{3E^2v}\ln\frac{1+v}{1-v}, \tag{A2a}$$

$$\frac{\mathbf{I}\cdot\mathbf{p}}{\mathbf{p}^2}=\frac{2\pi^3T^2}{3E^2v^3}\left[\ln\frac{1+v}{1-v}-2v\right], \tag{A2b}$$

with

$$v=|\mathbf{p}|/p_0 .$$

The second term in Eq. (2.3) has an l integration similar to that in the first term, but the k integration is done following the method in Ref. [4] to obtain

$$\frac{\alpha^2}{4\pi^3}\left\{\frac{1}{2\epsilon}\{\tfrac{3}{2}(2m-\not p)J_A+3\not J_B\}+\tfrac{1}{2}(12m-5\not p)J_A+2\not J_B\right\}, \tag{A3}$$

where

$$J_A\simeq-8\pi b(m\beta), \tag{A4a}$$

$$\frac{J_B^0}{E}\simeq\pi\left[\frac{4T}{vE^2}\{ma(m\beta)-Tc(m\beta)\}\ln\frac{1+v}{1-v}-12b(m\beta)\right], \tag{A4b}$$

$$\frac{\mathbf{J}_B\cdot\mathbf{p}}{\mathbf{p}^2}\simeq\frac{\pi}{v}\left[\frac{4T}{vE^2}\{ma(m\beta)-Tc(m\beta)\}\left[\ln\frac{1+v}{1-v}-2v\right]+\left[1-\frac{2m^2}{3E^2}\right]b(m\beta)\right]. \tag{A4c}$$

The third term in Eq. (2.3) is integrated by transforming $p-k-l\to\kappa$. Using the standard integral tables [17], we find that after the integration over the azimuthal angle ϕ_l or ϕ_κ the integral vanishes.

The fourth, fifth, and eighth terms in Eq. (2.3) contain the products of δ functions;

$$\delta(k^2)\delta\{(p-k)^2-m^2\},$$

because of which the integration over k vanishes, as also mentioned in Sec. II. In the sixth term in Eq. (2.3), the ϕ_l and l_0 integrations are done using the relevant δ functions and the θ_l integration is carried out using Ref. [17]. Next, the k_0 and θ_k integrations are again similar to their above l-dependent counterparts, whereas the ϕ_k integration simply gives a factor 2π. Then the statistical distribution functions are expanded binomially so that the integrations over the variables $|\mathbf{l}|$ and $|\mathbf{q}|$ give

$$-\frac{\alpha^2}{8}\sum_{n,r,s=1}^{\infty}(-1)^r e^{-r\beta E}\left[\frac{3T^2}{2v^2E^2}f_+(n,r)\{Ef_-(s,r)-mf_+(s,r)\}+\frac{T^2}{v^2}\left\{\frac{1}{E}f_+(s,r)+\frac{1}{m}(1+v^2)f_-(s,r)\right\}+(\not p+m)\frac{I_BI_C}{64\pi^2}+\frac{T}{4\pi}I_B\left\{5f_+(n,r)+\frac{2E}{m}f_-(n,r)\right\}+2T\left\{5f_+(s,r)-\frac{2E}{m}f_-(s,r)\right\}\right], \tag{A5}$$

with

$$f_\pm(n,r)=\frac{1}{n+r}\pm\frac{1}{n-r}, \tag{A6}$$

and $f_\pm(s,r)$ is similar to (A6). I_B and I_C are given in Eqs. (2.6b) and (2.6c).

The seventh term in Eq. (2.3) is integrated over the loop momenta in a similar way as the sixth one in Eq. (2.3) to get

$$-\frac{\alpha^2}{4}\sum_{n,r,s=1}^{\infty}(-1)^{r+s}\left\{\frac{T^2}{E^2v^2}[(\gamma_ip^i-3m)f_+(s,r)-3Ef_-(s,r)]f_+(n,r)+T^2\left[\left\{\frac{1}{m}\left[\gamma_0-\frac{\gamma_ip^i}{Ev^2}\right]-\frac{3m}{E^2v^2}f_+(s,r)\right\}-\frac{3}{m}f_-(s,r)\right]f_-(n,r)-T\left[\left\{\frac{3}{v^2}f_+(n,r)+\frac{3E}{m}f_-(n,r)\right\}g_+(r,s)+\left\{\left[\frac{6E}{m}-17+\frac{3m^2}{E^2v^2}\right]f_+(n,r)-\frac{3E}{m}f_-(n,r)\right\}g_-(s,r)+3\left\{\frac{1}{v^2}f_+(n,r)+\frac{E}{m}(v^2-2)f_-(n,r)\right\}\right.\right.$$

$$\times\left\{\frac{2e^{-rm\beta}}{m}\sinh(sm\beta)+\frac{s}{T}g_-(r,s)+\frac{r}{T}g_+(r,s)\right\}$$

$$-\frac{m^2}{E^2v^2}\{mf_-(n,r)+\tfrac{1}{2}(\gamma_i p^i-3m)f_+(n,r)\}\left\{\frac{2e^{-rm\beta}}{m}\cosh(sm\beta)+\frac{s}{T}g_-(r,s)+\frac{r}{T}g_+(r,s)\right\}\Bigg]\Bigg]\Bigg\}\ , \tag{A7}$$

where

$$g_\pm(r,s)=\mathrm{Ei}\{-(r+s)m\beta\}\pm\mathrm{Ei}\{-(r-s)m\beta\}\ . \tag{A8}$$

The ninth and last terms in Eq. (2.3) are also integrated in a similar way as the sixth and seventh terms. However, on the integration over the three-momenta of the loops, the term vanishes. Adding Eqs. (A1), (A3), (A5), and (A7), we obtain Eq. (2.4), in which

$$B_1=\frac{-3}{8\pi}\sum_{r=1}^{\infty}\sum_{n=1}^{\infty}(-1)^r e^{-r\beta E}f_+(n,r)\theta(n-r)\ , \tag{A9}$$

$$B_2=\frac{1}{2\pi}\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}(-1)^{r+s}\frac{\gamma_0}{m}f_-(s,r)\theta(s-r)\ , \tag{A10}$$

$$B_3=\frac{1}{4\pi}\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}(-1)^{r+s}[2\gamma_0 f_+(s,r)+f_-(s,r)]\theta(s-r)\ , \tag{A11}$$

$$B_4=\frac{1}{4\pi}\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}(-1)^{r+s}[Eg_+(s,r)-m\gamma_0 g_-(s,r)+me^{-rm\beta}\sinh(sm\beta)]\theta(s-r)\ , \tag{A12}$$

$$B_5=\frac{16\pi^3}{3|\mathbf{p}|}+3\sum_{r=1}^{\infty}(-1)^r\sum_{n=1}^{\infty}\sum_{s=1}^{\infty}\left[\frac{e^{-r\beta E}}{2}\left\{\frac{m}{E^2v^2}f_+(n,r)f_+(s,r)-\frac{1}{m}\left[1+\frac{1}{v^2}\right]f_-(n,r)f_-(s,r)\right\}+2(-1)^s\left\{\frac{m}{E^2v^2}f_+(s,r)[f_+(n,r)+f_-(n,r)]-\frac{1}{m}f_-(n,r)f_-(s,r)\right\}\right]\theta(n-r)\theta(s-r)\ , \tag{A13}$$

$$B_6=\sum_{r=1}^{\infty}(-1)^r\sum_{s=1}^{\infty}\left[3e^{-r\beta E}f_+(s,r)-2(-1)^s\sum_{n=1}^{\infty}\left\{\frac{3E}{m}f_+(n,r)g_+(s,r)-\left[17-\frac{3m^2}{E^2v^2}\right]f_+(n,r)g_-(r,s)+\frac{6E^2v^2}{m^2}f_-(n,r)e^{-rm\beta}\sinh(sm\beta)+\frac{m}{E^2v^2}\{(3m-\gamma_i p^i)f_+(n,r)+2mf_-(n,r)\}\times e^{-rm\beta}\cosh(sm\beta)\right\}\right]\theta(n-r)\theta(s-r)\ , \tag{A14}$$

$$B_7=2\sum_{r=1}^{\infty}(-1)^{r+s}\left[\frac{3E^2v^2}{m}f_-(n,r)\{sg_-(r,s)+rg_+(r,s)\}-\frac{m^2}{2E^2v^2}\{(3m-\gamma_i p^i)f_+(n,r)+6mf_-(n,r)\}\{sg_+(r,s)-rg_-(r,s)\}\right]\theta(n-r)\theta(s-r)\ . \tag{A15}$$

APPENDIX B

The integrations in Eq. (2.9) are given here term by term. The first term in Eq. (2.9) is

$$\frac{e^4}{(2\pi)^7}\int\frac{d^4q\,d^4l\,N^{(c)}n_F(E_q)}{2E_q^2 l^2(p-q)^2\{(q-l)^2-m^2\}}\left\{\frac{\delta(q_0-E_q)}{q_0+E_q}+\frac{\delta'(q_0-E_q)}{2}\right\}\ . \tag{B1}$$

The temperature-independent integration in term (B1) is done by dimensional regularization to get

$$\frac{e^4}{64\pi^5}\int \frac{d^4q\, n_F(E_q)}{E_q^2(p-q)^2}\left[\frac{\delta(q_0-E_q)}{q_0+E_q}+\frac{\delta'(q_0-E_q)}{2}\right] \times \left[\frac{1}{2\epsilon}(8m^3-7m^2\not{q}+4mq^2+q^2\not{q})-\left[12m^3-13m^2\not{q}+8mq^2+2q^2\not{q}+\frac{m^4\not{q}}{q^2}\right] + \left[\frac{q^2-m^2}{q^2}\right]\ln\left[\frac{m^2-q^2}{m^2}\right]\left[12m^3-6m^2q-4mq^2+3q^2\not{q}-\frac{m^4\not{q}}{q^2}\right]\right]. \tag{B2}$$

In expression (B2) the integration with $\delta(q_0-E_q)$ is similar to the temperature-dependent one in the second term of Eq. (2.3), giving

$$\frac{m^2e^4}{128\pi^4}\frac{1}{|\mathbf{p}|}\left[\gamma_0\ln\frac{1-v}{1+v}+\frac{\gamma_i p^i}{|\mathbf{p}|}\left[2-\frac{1}{v}\ln\frac{1-v}{1+v}\right]\right]\left[\frac{3}{\epsilon}-10\right]b(m\beta)\ . \tag{B3}$$

The q_0 integration with $\delta'(q_0-E_q)$ in (B2) is done by parts. The integration over the azimuthal angle gives 2π, whereas that over θ_q leads to factors such as

$$\ln\frac{m^2-p_0E_q+|\mathbf{p}||\mathbf{q}|}{m^2-p_0E_q-|\mathbf{p}||\mathbf{q}|} \tag{B4}$$

and

$$\frac{1}{m^2-p_0E_q+|\mathbf{p}||\mathbf{q}|}\pm\frac{1}{m^2-p_0E_q-|\mathbf{p}||\mathbf{q}|}\ . \tag{B5}$$

The factors in (B4) and (B5) are expanded in powers of (m^2/E_q^2) up to second order and integrated over $|\mathbf{q}|$ to obtain

$$\frac{-e^4}{64\pi^4}\left[\frac{1}{\epsilon}\left\{\left[\frac{20m}{|\mathbf{p}|}+3\gamma_0-3\frac{\gamma_i p^i}{|\mathbf{p}|^2}\left[p_0-\frac{m^2}{2|\mathbf{p}|}\ln\frac{1-v}{1+v}\right]\right]Ta(m\beta)\right.\right.$$
$$-\left[40m+\frac{3m^2}{|\mathbf{p}|}\ln\frac{1-v}{1+v}-6p_0\gamma_0-\left\{|\mathbf{p}|+\frac{m^2}{|\mathbf{p}|}\left[p_0\ln\frac{1-v}{1+v}+2\right]\right\}\frac{\gamma_i p^i}{|\mathbf{p}|}\right]b(m\beta)$$
$$\left.+\sum_{n=1}^{\infty}(-1)^n e^{-nm\beta}\left[20p_0-3\gamma_0\left\{m+\frac{p_0^2+|\mathbf{p}|^2}{4|\mathbf{p}|}\right\}+\frac{\gamma_i p^i}{|\mathbf{p}|^2}\left\{\frac{2p_0}{m}(2p_0^2+3|\mathbf{p}|^2)+\frac{m}{|\mathbf{p}|}\left[p_0|\mathbf{p}|-\frac{m^2}{2}\right]\right\}\right]\right\}$$
$$-\frac{5}{|\mathbf{p}|}\left\{2|\mathbf{p}|^2\gamma_0-m^2\frac{\gamma_i p^i}{|\mathbf{p}|}\ln\frac{1-v}{1+v}\right.$$
$$-\left[\gamma_0\ln\frac{1-v}{1+v}+\frac{\gamma_i p^i}{|\mathbf{p}|}\left[2-\frac{1}{v}\ln\frac{1-v}{1+v}\right]\right]\left[ma(m\beta)(m-2T)+2T^2c(m\beta)\right.$$
$$\left.+m^2\sum_{n,r=1}^{\infty}(-1)^{n+r}\frac{e^{-(n+r)m\beta}}{n+r}\right]$$
$$-\left[4m|\mathbf{p}|-2m^2\gamma_0\ln\frac{1-v}{1+v}+\left\{2(m^2-2|\mathbf{p}|^2)-\frac{m^2p_0}{|\mathbf{p}|}\ln\frac{1-v}{1+v}\right\}\frac{\gamma_i p^i}{|\mathbf{p}|}\right]b(m\beta)$$
$$+\left[4mp_0|\mathbf{p}|-\gamma_0|\mathbf{p}|(5p_0^2+|\mathbf{p}|^2)+\frac{\gamma_i p^i}{|\mathbf{p}|}\left\{\left[\frac{m^2}{2}-p_0|\mathbf{p}|\right]|\mathbf{p}|-2p_0(p_0^2+3|\mathbf{p}|^2)\right\}\right]$$
$$\left.\left.\times\sum_{n=1}^{\infty}(-1)^n\frac{e^{-nm\beta}}{m}\right\}\right]\ . \tag{B6}$$

In Eq. (2.9) the second term containing $\delta[(q-l)^2-m^2]$ vanishes in the same way as the third one in Eq. (2.3). The third term in Eq. (2.9) after a simple integration gives

$$\frac{e^4}{(2\pi)^7}\int \frac{d^4q\, d^4l\, N^{(c)} n_B(l)\delta(l^2)}{(p-q)^2(q^2-m^2)^2\{(q-l)^2-m^2\}} = \frac{2\alpha^2 T^2}{3\pi^2 m^2}(\not p - m) \ . \tag{B7}$$

The fourth term in Eq. (2.9) is

$$\frac{e^4}{(2\pi)^7}\int \frac{d^4q\, d^4l\, N^{(c)} n_B(p-q)\delta[(p-q)^2]}{l^2(q^2-m^2)^2\{(q-l)^2-m^2\}} \ , \tag{B8}$$

in which the l integration is done by dimensional regularization and $p-q$ is replaced by q to get

$$\frac{e^4}{(2\pi)^5}\int \frac{d^4q\, n_B(q)\delta(q^2)}{q^2-2p\cdot q}\left[\frac{1}{2\epsilon}\left\{\frac{12m^3-6m^2(\not p-\not q)}{q^2-2p\cdot q}+\not p-\not q+4m\right\}\right.$$
$$+\left\{\frac{20m^3-11m^2(\not p-\not q)}{q^2-2p\cdot q}+2(\not p-\not q)+8m+\frac{m^4(\not p-\not q)}{(q^2-2p\cdot q)(q^2-2p\cdot q+m^2)}\right\}$$
$$+\frac{q^2-2p\cdot q}{q^2-2p\cdot q+m^2}\ln\left[\frac{m^2}{2p\cdot q-q^2}\right]\left\{\frac{8m^3-3m^2(\not p-\not q)}{q^2-2p\cdot q}+3(\not p-\not q)-4m\right.$$
$$\left.\left.-\frac{m^2(\not p-\not q)}{(q^2-2p\cdot q)(q^2-2p\cdot q+m^2)}\right\}\right] \ . \tag{B9}$$

In (B9) the ϕ_q integration gives a factor 2π and the q_0 integration is done using $\delta(q^2)$. After a somewhat lengthy calculation, in which we expand the logarithm in (B9) up to $O(m^3/E_q^3)$, we obtain

$$\frac{\alpha^2}{\pi^2}\left[\frac{1}{4\epsilon}\{\tfrac{1}{8}(28m-5\not p)I_A-I\}+\tfrac{1}{8}(42m-\not p)I_A+\frac{2\pi^2}{3}T^2+\frac{2\pi^2T^2}{3}\left[\frac{\not p}{m^2}+\frac{\gamma_i p^i}{2|\mathbf{p}|}\right]\right.$$
$$+T^2\sum_{n=1}^{\infty}\frac{1}{n^2}\left\{\frac{4(12m-7\not p)}{m^2}+\frac{2\not p}{m^2 v}\ln\frac{1+v}{1-v}+\frac{8}{|\mathbf{p}|}\left[2-\frac{1}{v}\ln\frac{1+v}{1-v}\right]+\frac{\gamma_i p^i}{|\mathbf{p}|^2}\left[4-\frac{2}{v}+\frac{7}{v}\ln\frac{1+v}{1-v}\right]\right.$$
$$\left.+\left[\frac{8\gamma_0}{|\mathbf{p}|}\ln\frac{1-v}{1+v}+\frac{\gamma_i p^i}{\mathbf{p}|^2}\left\{\left[\frac{9}{v}-1\right]\ln\frac{1+v}{1-v}-2\right\}+\frac{4\not p}{m^2}\right][1+\gamma+\ln 2+\ln(nm\beta)]\right\}$$
$$+2T^3\sum_{n=1}^{\infty}\frac{1}{n^3}\left\{\frac{8(12m-7\not p)}{m^4}p_0\ln\frac{1+v}{1-v}+\frac{12\gamma_0}{m^2 v}\ln\frac{1+v}{1-v}-\frac{8p_0^2}{m^4}+\frac{\gamma_i p^i}{m^2|\mathbf{p}|}\left[\frac{2p_0^2+|\mathbf{p}|^2}{|\mathbf{p}|}\right]\ln\frac{1+v}{1-v}\right.$$
$$\left.\left.+\left[\frac{8(12m-7\not p)}{m^2|\mathbf{p}|}(p_0^2+|\mathbf{p}|^2)+\frac{24\gamma_0}{m^2}+\frac{28\gamma_i p^i}{m^3|\mathbf{p}|^2}(p_0^2+|\mathbf{p}|^2-p_0|\mathbf{p}|)\right][\tfrac{3}{2}-\gamma+\ln 2-\ln(nm\beta)]\right\}\right] \ . \tag{B10}$$

The fifth term in Eq. (2.9) is

$$\frac{e^4}{(2\pi)^5}\int \frac{d^4q}{2E_q^2}d^4l\, N^{(c)} n_F(q) n_F(q-l)\delta\{(q-l)^2-m^2\}\frac{n_B(l)\delta(l^2)}{(p-q)^2}\left\{\frac{\delta(q_0-E_q)}{q_0+E_q}+\frac{\delta'(q_0-E_q)}{2}\right\} \ . \tag{B11}$$

In the term (B11), the integration over the azimuthal angle ϕ_q is done using $\delta[(q-l)^2-m^2]$, whereas that over ϕ_l gives 2π. The integrations over θ_l and θ_q are performed using Ref. [17]. The l_0 integration is done with the help of $\delta(l^2)$. The q_0 variable is integrated using $\delta(q_0-E_q)$ in the first part of (B11), whereas, in its second part, the integration is done by parts. Then the statistical distribution functions are expanded in powers of βE and integrated to get

$$\alpha\pi^2 mT\sum_{r,s=1}^{\infty}(-1)^{r+s}\sum_{n=1}^{\infty}\frac{\theta(n-r)}{n-r}\left[-\left\{r\beta\left[\gamma_0-\frac{\gamma_i p^i}{|\mathbf{p}|^2}(p_0-m)\right]+\frac{2}{m}-\frac{\gamma_0 p_0}{m^2}\right\}\mathrm{Ei}\{-(r+s)m\beta\}\right.$$
$$+\left\{\frac{4p_0}{m}-2\gamma_0\frac{|\mathbf{p}|^2}{m^2}-\frac{\gamma_i p^i}{|\mathbf{p}|^2}(p_0-m)+r\beta(2m-p_0\gamma_0-\gamma_i p^i)\right\}$$
$$\left.\times\left\{\frac{e^{-m\beta(r+s)}}{m}+(r+s)\beta\,\mathrm{Ei}[-(r+s)m\beta]\right\}\right] \ . \tag{B12}$$

The sixth term in Eq. (2.9) is given by

$$\frac{e^4}{(2\pi)^5}\int\frac{d^4q}{2E_q^2l^2}d^4l\,N^{(c)}n_F(q-l)\delta\{(q-l)^2-m^2\}n_B(p-q)\delta[(p-q)^2]n_F(q)\left\{\frac{\delta(q_0-E_q)}{q_0+E_q}+\frac{\delta'(q_0-E_q)}{2}\right\}. \tag{B13}$$

The term in (B13) is integrated such that the variable $q-l$ is replaced by l and the ϕ_l integration is done using Ref. [17]. Then the integrations over l_0, ϕ_l, and $|\mathbf{l}|$ are straightforward. The integration over the loop momentum q is done such that the ϕ_q integration gives 2π, and θ_q integration is performed using the δ function in θ_q. The integration over q_0 is done as in (B8) and (B11), whereas that over $|\mathbf{q}|$ is carried out after binomially expanding the distribution functions to get

$$\frac{\alpha^2\pi^3T^2}{3|\mathbf{p}|}\sum_{n,r=1}^{\infty}(-1)^{r+1}e^{-n\beta E}\theta(r-n)\left[\left[\frac{\gamma_ip^i}{|\mathbf{p}|^2}\frac{e^{-m\beta(r-n)}}{\beta(r-n)}+m\left[2-m\frac{\gamma_ip^i}{|\mathbf{p}|^2}\right]\left\{\frac{e^{-m\beta(r-n)}}{m}-(r-n)\beta\,\mathrm{Ei}[-(r-n)m\beta]\right.\right.\right.$$
$$\left.\left.-\beta\left[\sum_{s=1}^{\infty}(-1)^{s+1}\mathrm{Ei}\{-(r-n-s)m\beta\}+\sum_{t=1}^{\infty}\mathrm{Ei}\{-(r-n-t)m\beta\}\right]\right\}\right]$$
$$\left.-\left[\gamma_0-\frac{\gamma_ip^i}{|\mathbf{p}|^2}p_0\right]\left\{\sum_{s=1}^{\infty}(-1)^{s+1}\frac{e^{-m\beta(r-n-s)}}{r-n-s}+\sum_{t=1}^{\infty}\frac{e^{-m\beta(r-n-t)}}{r-n-t}\right\}\right]. \tag{B14}$$

The seventh term in Eq. (2.9), i.e.,

$$\frac{-e^4}{(2\pi)^5}\int\frac{d^4k}{2E_q^2}d^4l\,N^{(c)}n_B(p-q)\delta[(p-q)^2]n_B(l)\delta(l^2)n_F(q)\left\{\frac{\delta(q_0-E_q)}{q_0+E_q}+\frac{\delta'(q_0-E_q)}{2}\right\}, \tag{B15}$$

is also integrated in a similar way as the term (B13). However, it is found that the part containing $\delta(q_0-E_q)$ is exactly canceled by the contribution from the part with $\delta'(q_0-E_q)$ such that (B14) vanishes. The last term in Eq. (2.9) vanishes in a similar way as the last term in Eq. (2.3). Hence adding (B3), (B6), (B7), (B10), (B12), and (B14), we get the self-energy in Eq. (2.10) with

$$C_1=\frac{1}{32\pi^2}(28m-5\not{p}), \tag{B16}$$

$$C_2=-\frac{\pi}{3}\left[\gamma_0\ln\frac{1+v}{1-v}-\frac{\gamma_ip^i}{|\mathbf{p}|}\left[\frac{1}{v}\ln\frac{1+v}{1-v}-2\right]\right], \tag{B17}$$

$$C_3=\frac{-1}{4\pi^2}\left[\frac{20m}{|\mathbf{p}|}+3\gamma_0-3\frac{\gamma_ip^i}{|\mathbf{p}|^2}\left[p_0+\frac{m^2}{2|\mathbf{p}|}\ln\frac{1+v}{1-v}\right]a(m\beta)\right], \tag{B18}$$

$$C_4=\frac{1}{4\pi^2}\left\{\left[40m+\frac{3m^2}{|\mathbf{p}|}\ln\frac{1-v}{1+v}-6p_0\gamma_0-\left\{|\mathbf{p}|+\frac{m^2}{|\mathbf{p}|}\left[\frac{1}{v}\ln\frac{1-v}{1+v}+2\right]\right\}\frac{\gamma_ip^i}{|\mathbf{p}|}\right]b(m\beta)\right.$$
$$-\sum_{n=1}^{\infty}(-1)^ne^{-nm\beta}\left[20p_0-3\gamma_0\left\{m+\frac{(p_0^2+|\mathbf{p}|^2)}{4|\mathbf{p}|}\right\}+\frac{\gamma_ip^i}{|\mathbf{p}|^2}\left\{\frac{2p_0}{m}(2p_0^2+3|\mathbf{p}|^2)+m\left[p_0-\frac{m^2}{2|\mathbf{p}|}\right]\right\}\right]$$
$$\left.+\frac{3m^2}{2|\mathbf{p}|}\left[\gamma_0\ln\frac{1-v}{1+v}-\frac{\gamma_ip^i}{|\mathbf{p}|}\left[2+\frac{1}{v}\ln\frac{1-v}{1+v}\right]\right]b(m\beta)\right\}, \tag{B19}$$

$$C_5=\frac{1}{16\pi^2}(42m-\not{p}), \tag{B20}$$

$$C_6=\frac{4}{\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^3}\left[\frac{4(12m-7\not{p})}{m^4}p_0\ln\frac{1+v}{1-v}-\frac{4p_0^2}{m^4}+\frac{6\gamma_0}{m^2v}\ln\frac{1+v}{1-v}+7\frac{\gamma_ip^i}{|\mathbf{p}|^3}\frac{2p_0^2+|\mathbf{p}|^2}{m^2}\ln\frac{1+v}{1-v}\right.$$
$$\left.+\left\{\frac{12\gamma_0}{m^2}+\frac{4(12m-7\not{p})}{m^4|\mathbf{p}|}(p_0^2+|\mathbf{p}|^2)+\frac{14\gamma_ip^i}{m^2|\mathbf{p}|^3}(p_0^2+|\mathbf{p}|^2-p_0|\mathbf{p}|)\{\tfrac{3}{2}-\gamma+\ln2-\ln(nm\beta)\right\}\right]$$

$$+\frac{\pi}{6|\mathbf{p}|}\sum_{n=1}^{\infty}\sum_{r=1}^{\infty}(-1)^{r+1}e^{-n\beta E}\frac{\gamma_i p^i}{|\mathbf{p}|^2}\frac{e^{-m\beta(r-n)}}{r-n}\theta(r-n)\,, \tag{B21}$$

$$\begin{aligned}C_7=&\frac{2}{3m^2}(\not p-m)+\frac{1}{3}\left[\frac{\not p}{m^2}+\frac{\gamma_i p^i}{2|\mathbf{p}|^2}\right]+\frac{5}{2\pi^2}\frac{c(m\beta)}{|\mathbf{p}|}\left[\gamma_0\ln\frac{1-v}{1+v}+\frac{\gamma_i p^i}{|\mathbf{p}|}\left[2-\frac{1}{v}\ln\frac{1-v}{1+v}\right]\right]\\&+\frac{1}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[\frac{4(12m-7\not p)}{m^2}+\frac{2\not p}{m^2}\left[2+\frac{1}{v}\ln\frac{1+v}{1-v}\right]+\frac{8}{|\mathbf{p}|}\left[2-\frac{1}{v}\ln\frac{1+v}{1-v}\right]\right.\\&\qquad+\frac{\gamma_i p^i}{|\mathbf{p}|^2}\left[2\left[1-\frac{1}{v}\right]+\left[\frac{16}{v}-1\right]\ln\frac{1+v}{1-v}\right]\\&\qquad\left.+\left[\frac{4\not p}{m^2}-\frac{8\gamma_0}{|\mathbf{p}|}\ln\frac{1+v}{1-v}-\frac{\gamma_i p^i}{|\mathbf{p}|^2}\left\{2-\left[1+\frac{9}{v}\right]\ln\frac{1+v}{1-v}\right\}\right][\ln 2-\gamma-\ln(nm\beta)]\right]\\&+\frac{\pi}{6|\mathbf{p}|}\sum_{n=1}^{\infty}\sum_{r=1}^{\infty}(-1)^{r+1}e^{-n\beta E}\left\{\left[2-m\frac{\gamma_i p^i}{|\mathbf{p}|^2}\right]e^{-(r-n)m\beta}\right.\\&\qquad\left.-\left[\gamma_0-p_0\frac{\gamma_i p^i}{|\mathbf{p}|^2}\right]\sum_{s=1}^{\infty}[(-1)^s+1]\frac{e^{-m\beta(r-n-s)}}{r-n-s}\right\}\theta(r-n-s)\,,\end{aligned} \tag{B22}$$

$$\begin{aligned}C_8=&\frac{5m}{2\pi^2|\mathbf{p}|}a(m\beta)\left[\gamma_0\ln\frac{1+v}{1-v}+\frac{\gamma_i p^i}{|\mathbf{p}|}\left[2-\frac{1}{v}\ln\frac{1+v}{1-v}\right]\right]\\&+\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^{r+s}}{n-r}\left[\left[1-\frac{p_0\gamma_0}{2m}\right]\mathrm{Ei}\{-(r+s)m\beta\}+\left\{\frac{2p_0}{m}+\frac{\gamma_0|\mathbf{p}|^2}{m^2}-\frac{\gamma_i p^i}{2|\mathbf{p}|^2}(p_0-m)e^{-m\beta(r+s)}\right\}\right]\\&-\frac{\pi m}{6|\mathbf{p}|}\sum_{n=1}^{\infty}\sum_{r=1}^{\infty}(-1)^{r+1}e^{-n\beta E}\left[2-m\frac{\gamma_i p^i}{|\mathbf{p}|^2}\right]\left[(r-n)\mathrm{Ei}\{-(r-n)m\beta\}-\sum_{s=1}^{\infty}\{(-1)^s+1\}\mathrm{Ei}\{-(r-n-s)m\beta\}\right],\end{aligned} \tag{B23}$$

$$\begin{aligned}C_9=&\frac{5m^2}{4\pi^2|\mathbf{p}|}\left\{\left[\gamma_0\ln\frac{1-v}{1+v}+\frac{\gamma_i p^i}{|\mathbf{p}|}\left[2-\frac{1}{v}\ln\frac{1-v}{1+v}\right]\right]\left[b(m\beta)-a(m\beta)+\sum_{n=1}^{\infty}\sum_{r=1}^{\infty}(-1)^{n+r}\frac{e^{-(n+r)m\beta}}{(n+r)}\right]\right.\\&\qquad\left.+2\gamma_0\frac{|\mathbf{p}|^2}{m^2}+\frac{\gamma_i p^i}{|\mathbf{p}|}\ln\frac{1-v}{1+v}-\left[\frac{4|\mathbf{p}|}{m}+2\gamma_0\ln\frac{1+v}{1-v}+\frac{2\gamma_i p^i}{|\mathbf{p}|}\left[1-\frac{2|\mathbf{p}|^2}{m^2}\right]+\frac{\gamma_i p^i}{|\mathbf{p}|}\frac{1}{v}\ln\frac{1+v}{1-v}\right]b(m\beta)\right\}\\&+\sum_{r=1}^{\infty}\sum_{n=1}^{\infty}\sum_{s=1}^{\infty}\frac{(-1)^{r+s}}{n-r}\left[\frac{mr}{2}\left\{\gamma_0-\frac{\gamma_i p^i}{|\mathbf{p}|^2}(p_0-m)\right\}\mathrm{Ei}\{-(r+s)m\beta\}+\frac{r}{2}(2m-\not p)e^{-m\beta(r+s)}\right.\\&\qquad\left.+(r+s)\left\{2p_0+\frac{2\gamma_0}{m}|\mathbf{p}|^2+\frac{m^2\gamma_i p^i}{2|\mathbf{p}|^2}\left[1-\frac{p_0}{m}\right]\right\}\mathrm{Ei}\{-(r+s)m\beta\}\right].\end{aligned} \tag{B24}$$

APPENDIX C

We list the temperature dependent variables F_i's in Eq. (3.2) as

$$F_1=\sum_{r=1}^{\infty}\frac{1}{r^3}[\tfrac{3}{2}-\gamma+\ln 2-\ln(rm\beta)]\,, \tag{C1}$$

$$F_2=\sum_{r=1}^{\infty}\sum_{n=3}^{r+1}(-1)^{n+r}e^{-\beta(rE+nm)}\,, \tag{C2}$$

$$F_3=\sum_{r=1}^{\infty}(-1)^r e^{-r\beta E}\,, \tag{C3}$$

$$F_4=\ln 2-\gamma-\frac{6}{\pi^2}\sum_{r=1}^{\infty}\frac{\ln(rm\beta)}{r^2}\,, \tag{C4}$$

$$F_5=\sum_{r=1}^{\infty}(-1)^r e^{-r\beta E}\sum_{n=3}^{r+1}\frac{1}{n}\,, \tag{C5}$$

$$F_6=\sum_{r=1}^{\infty}(-1)^r e^{-r\beta E}\sum_{n=3}^{r+1}\frac{1}{n}\sum_{s=3}^{r+1}\frac{1}{s}\,, \tag{C6}$$

$$F_7=\sum_{r=1}^{\infty}(-1)^r e^{-r\beta E}\sum_{s=3}^{r+1}\frac{(-1)^s}{s}\,, \tag{C7}$$

$$F_8=\sum_{r=1}^{\infty}\left[\sum_{s=1}^{r+1}(-1)^s\mathrm{Ei}(-sm\beta)-\sum_{s=3}^{r+1}(-1)^s\mathrm{Ei}(-sm\beta)\right], \tag{C8}$$

$$F_9=\sum_{r=1}^{\infty}\left[\sum_{s=1}^{r+1}(-1)^s\mathrm{Ei}(-sm\beta)-\sum_{s=3}^{r+1}(-1)^s\mathrm{Ei}(-sm\beta)\right]\sum_{n=3}^{r+1}\frac{1}{n}, \tag{C9}$$

$$F_{10}=\sum_{r=1}^{\infty}\left[\sum_{s=1}^{r+1}(-1)^s\mathrm{Ei}(-sm\beta)+\sum_{s=3}^{r+1}(-1)^s\mathrm{Ei}(-sm\beta)\right], \tag{C10}$$

$$F_{11}=\sum_{r=1}^{\infty}\left[\sum_{s=1}^{r+1}(-1)^s\mathrm{Ei}(-sm\beta)+\sum_{s=3}^{r+1}(-1)^s\mathrm{Ei}(-sm\beta)\right]\sum_{n=3}^{r+1}\frac{1}{n}, \tag{C11}$$

$$F_{12}=\sum_{r=1}^{\infty}e^{-rm\beta}\sum_{s=1}^{\infty}\sinh(sm\beta), \tag{C12}$$

$$F_{13}=\sum_{r=1}^{\infty}e^{-rm\beta}\sum_{s=1}^{\infty}\sinh(sm\beta)\sum_{n=3}^{r+1}\frac{1}{n}, \tag{C13}$$

$$F_{14}=\sum_{r=1}^{\infty}e^{-rm\beta}\sum_{s=1}^{\infty}\cosh(sm\beta), \tag{C14}$$

$$F_{15}=\sum_{r=1}^{\infty}e^{-rm\beta}\sum_{s=1}^{\infty}\cosh(sm\beta)\sum_{n=3}^{r+1}\frac{1}{n}, \tag{C15}$$

$$F_{16}=\sum_{r=1}^{\infty}\sum_{n=3}^{r+1}\frac{1}{n}\sum_{s=1}^{r+1}(-1)^s\mathrm{Ei}(-sm\beta), \tag{C16}$$

$$F_{17}=\sum_{r=1}^{\infty}e^{-r\beta E}\sum_{n=3}^{r+1}(-1)^{n+r+1}n\,\mathrm{Ei}(-nm\beta), \tag{C17}$$

$$F_{18}=\sum_{r=1}^{\infty}e^{-r\beta E}\sum_{s=2}^{r+1}[(-1)^r-(-1)^s]\times\sum_{n=1}^{s+1}(-1)^n\mathrm{Ei}(-nm\beta), \tag{C18}$$

$$F_{19}=\sum_{r=1}^{\infty}\sum_{n=3}^{r+1}\frac{1}{n}\left[\sum_{s=3}^{r+1}(-1)^s s\,\mathrm{Ei}(-sm\beta)-\sum_{s=1}^{r+1}(-1)^s s\,\mathrm{Ei}(-sm\beta)\right], \tag{C19}$$

$$F_{20}=\sum_{r=1}^{\infty}\sum_{s=1}^{r+1}(-1)^s[e^{-sm\beta}-\mathrm{Ei}(-sm\beta)]\sum_{n=3}^{r+1}\frac{1}{n}, \tag{C20}$$

$$F_{21}=\sum_{r=1}^{\infty}\sum_{n=3}^{r+1}\frac{1}{n}\sum_{s=1}^{r+1}(-1)^s s\,\mathrm{Ei}(-sm\beta). \tag{C21}$$

The constants $\mathcal{M}_i$'s appearing in Eq. (3.2) are given as

$$\mathcal{M}_1=8\left[\frac{10m}{Ev}-\frac{E}{m}+\frac{7E}{mv}(1+v^2)\right], \tag{C22}$$

$$\mathcal{M}_2=\frac{4E}{m}\left[\left\{20\left[1+\frac{1}{v}\right]+7v\right\}\ln\frac{1+v}{1-v}-\frac{4E}{m}\right], \tag{C23}$$

$$\mathcal{M}_3=\frac{1}{6}\left[2+\left[1+\frac{1}{v}\right]\ln\frac{1+v}{1-v}\right], \tag{C24}$$

$$\mathcal{M}_4=\frac{1}{3}\left[\frac{33}{2}-\frac{1}{v}+\frac{4m}{Ev}\left[2-\frac{1}{v}\ln\frac{1+v}{1-v}\right]+\left[\frac{5}{v}-\frac{1}{2}\right]\ln\frac{1+v}{1-v}\right], \tag{C25}$$

$$\mathcal{M}_5=\frac{1}{v^2}-1, \tag{C26}$$

$$\mathcal{M}_6=4-\frac{3}{v^2}\left[1-\frac{m}{2E}\right], \tag{C27}$$

$$\mathcal{M}_7=1+\frac{3m}{Ev^2}-3\mathcal{M}_5, \tag{C28}$$

$$\mathcal{M}_8=\frac{E}{m}+\frac{1}{v^2}, \tag{C29}$$

$$\mathcal{M}_9=\frac{8E}{m}-17+3\left[\frac{1}{v^2}-1\right], \tag{C30}$$

$$\mathcal{M}_{10}=\frac{E}{m}\left[\frac{3}{v^2}+\frac{3Ev^2}{m}-5\right], \tag{C31}$$

$$\mathcal{M}_{11}=\frac{E}{2m}-1-\frac{m}{4Ev}\ln\frac{1+v}{1-v}, \tag{C32}$$

$$\mathcal{M}_{12}=1-\frac{E^2}{2m^2}, \tag{C33}$$

$$\mathcal{M}_{13}=\frac{16}{3}+11\frac{E^2}{m^2}, \tag{C34}$$

$$\mathcal{M}_{14}=\frac{1}{v^2}\left[\frac{E}{m}+\frac{m^2}{2E^2}\right], \tag{C35}$$

$$\mathcal{M}_{15}=3-\frac{1}{v}-2\frac{E^2v^2}{m^2}-\frac{1}{2v}\ln\frac{1-v}{1+v}, \tag{C36}$$

$$\mathcal{M}_{16}=\frac{2E}{m}-\frac{E^3}{m^3}+\frac{1}{2}. \tag{C37}$$

Moreover, the coefficients of the powers of (T/m) in Eq. (3.4) are

$$M_1=\frac{4}{\pi^2}[c(m\beta)]^2, \tag{C38}$$

$$M_2=-\frac{8}{\pi}c(m\beta)a(m\beta), \tag{C39}$$

$$M_3=\frac{4}{\pi^2}\left\{[a(m\beta)]^2+3b(m\beta)c(m\beta)-\frac{\pi^2}{3}c(m\beta)\right\}, \tag{C40}$$

$$M_4=\frac{12}{\pi^2}a(m\beta)b(m\beta)+\frac{4\pi}{3}a(m\beta), \tag{C41}$$

$$M_5=\frac{9}{\pi^2}[b(m\beta)]^2-2b(m\beta)+\frac{\pi^2}{9}. \tag{C42}$$

[1] H. A. Weldon, Phys. Rev. D **26**, 1394 (1982); **26**, 2789 (1982); G. Peressutti and B. S. Skagerstam, Phys. Lett. **110B**, 406 (1982).

[2] J. F. Donoghue and B. R. Holstein, Phys. Rev. D **28**, 340 (1983); **29**, 3004(E) (1983).

[3] J. F. Donoghue, B. R. Holstein, and R. W. Robinett, Ann. Phys. (N.Y.) **164**, 233 (1985).

[4] K. Ahmed and Samina Saleem (Masood), Phys. Rev. D **35**, 1861 (1987).

[5] W. Keil, Phys. Rev. D **40**, 1176 (1989).

[6] T. Altherr and P. Aurenche, Phys. Rev. D **40**, 4171 (1989).

[7] Duane A. Dicus, David Down, and Edward W. Kolb, Nucl. Phys. **B223**, 525 (1983).

[8] Mahnaz Qader, Samina S. Masood, and K. Ahmed, Phys. Rev. D **44**, 3322 (1991).

[9] C. Bernard, Phys. Rev. D **9**, 3312 (1974); R. Kobes and G. W. Semenoff, Nucl. Phys. **B260**, 714 (1985); **B272**, 319 (1986); P. Landsman and Ch G. Weert, Phys. Rep. **145**, 141 (1987), and references therein.

[10] A. Fetter and J. D. Walecka, *Quantum Theory of Many Particle Systems* (McGraw-Hill, New York, 1971); L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974).

[11] L. Cambier, J. Primack, and M. Sher, Nucl. Phys. **B209**, 273 (1982); D. A. Dicus, E. W. Kolb, A. M. Gleeson, E. C. G. Sudarshan, V. L. Teplitz, and M. S. Turner, Phys. Rev. D **20**, 2694 (1982); R. Baier, E. Pilon, B. Pire, and D. Schiff, Nucl. Phys. **B336**, 157 (1990).

[12] Samina Saleem (Masood), Phys. Rev. D **36**, 2602 (1987).

[13] L. R. Ram Mohan, Phys. Rev. D **14**, 2670 (1976).

[14] A. J. Niemi and G. W. Semenoff, Nucl. Phys. **B230**, 181 (1984).

[15] T. Altherr and T. Becherrawy, Nucl. Phys. **B330**, 174 (1990).

[16] See Refs. [2], [4], and [15]; A. Weldon, Phys. Rev. D **44**, 3955 (1991); T. Grandou, M. LeBellac, and D. Poizat, Phys. Lett. B **249**, 478 (1990); Nucl. Phys. **B358**, 408 (1991); T. Altherr, Phys. Lett. B **262**, 314 (1991); T. Grandou, M. LeBellac, and J. L. Meunier, Z. Phys. C **43**, 575 (1989); J. Cleymans and I. Dadic, *ibid.* **42**, 133 (1989); **43**, 57 (1989).

[17] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, New York, 1972).