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Solving two-dimensional ϕ^4 field (complex scalar) theory by discretized light-front quantization

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The discretized light-front quantization method is applied to ϕ^4 field (complex scalar) theory in $1+1$ dimensions. The interaction Hamiltonian is constructed in its normal-ordered form, and calculations are performed with and without finite-mass renormalization in the charge-0 sector of the field. It is found that, like real scalar theory, finite-mass renormalization prevents the phase transition by restricting the theory to the weak-coupling region. A comparison of the results with and without mass renormalization demonstrates the same estimate of the critical coupling for which the mass gap vanishes. The invariant mass of various states is calculated as a function of bare coupling. In the weak-coupling region where extrapolation to the continuum limit is easily found, there is evidence for scattering, but there is no two-particle bound state in agreement with the well-known result established for constructive quantum field theory. Also, no multiparticle bound states are found. The essential outcome is that the results valid for real-scalar theories are found to be valid for complex scalar theory also.

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I. INTRODUCTION

Discretized light-front quantization (DLFQ) was proposed recently [1,2] as a method to solve relativistic field-theory problems nonperturbatively. This method has been applied to solving some relativistic field-theory problems [1,2], which have been studied previously by other means. Among the popular field-theory models, two-dimensional ϕ^4 theory has received much attention [3] since many of its properties have been established rigorously from the viewpoint of constructive quantum field theory [4,5]. Real-scalar two-dimensional ϕ^4 $[(\phi^4)_2]$ theory has been solved with the DLFQ scheme in the literature [6]. An investigation with the DLFQ scheme of the $(\phi^4)_2$ theory in the complex scalar region of the field has not yet been attempted. So a straightforward application of the DLFQ method to solving relativistic complex-scalar $(\phi^4)_2$ theory is investigated in this paper.

This paper is organized as follows. In Sec. II the interaction Lagrangian density is suitably chosen to work in the complex scalar region of the field. The light-front quantization of self-coupled scalar-field models for complex scalar theory is reviewed. Discretization is introduced and the momentum and Hamiltonian operator are constructed. In Sec. III the points to be investigated with the DLFQ method are discussed somewhat and the relevant issues, such as choosing the Fock-shape basis in the charge-0 sector of the field, the implication of the mass renormalization to keep the finite mass-gap constraint, etc., are also discussed, along with the underlying theoretical significance. In Sec. IV numerical results are established, and the essential outcome arising out of the solution of the problem is pointed out. A comparison of the results with those valid for the real-scalar theory obtained by the DLFQ method, and also by other means, is made. The summary of the main results and the conclusions are presented in Sec. V.

II. REVIEW OF LIGHT-FRONT QUANTIZATION

Light-front quantization was proposed originally by Dirac [7] from his work on the forms of relativistic dynamics, and rediscovered by Weinberg [8] in the context of the covariant formulation of time-ordered perturbation theory. The formal foundation of the light-front quantization approach to quantum field theories was laid by Yan and collaborators [9–12]. Certain results from Refs. [9,10] are reviewed for the case of the complex-scalar $(\phi^4)_2$ theory.

The description of real-scalar $(\phi^4)_2$ theory as given in Ref. [6] does not allow a distinction between particles and antiparticles. Particles and antiparticles have to carry some opposite-charge quantum number, whatever the nature of this charge may be. Classically, the minimal coupling prescription requires at least a complex field. In the quantum case let us therefore introduce a doublet of Hermitian fields ϕ_1 and ϕ_2 represented by the complex quantity

$$\phi=(\phi_1+i\phi_2)/\sqrt{2}$$

and its Hermitian conjugate.

I start with the Lagrangian density

$$\mathcal{L}=(\partial_\mu\phi)^\dagger(\partial^\mu\phi)-m^2\phi^\dagger\phi-\frac{\lambda}{4!}\frac{1}{2}(\phi^{\dagger 4}+6\phi^\dagger\phi^\dagger\phi\phi+\phi^4)\ . \tag{2.1}$$

λ is chosen greater than zero so that the Hamiltonian is bounded. The mass parameter m^2 is chosen positive so that the vacuum state is the normal vacuum at least for small coupling.

It is well known [9] that the number of independent variables describing a dynamical system in the light-front formulation is only one-half of that given in the conventional equal-time reformulation. The equations of motion and the commutation relations between true dynamical variables are derived from Schwinger's action principle [13].

In 1+1 dimensions, the equations of motion are

$$4\partial_+\partial_-\phi+m^2\phi+\frac{\lambda}{4!}\frac{1}{2}(4\phi^{\dagger 3}+12\phi^\dagger\phi\phi)=0 \tag{2.2}$$

and

$$4\partial_+\partial_-\phi^\dagger+m^2\phi^\dagger+\frac{\lambda}{4!}\frac{1}{2}(4\phi^3+12\phi^\dagger\phi\phi^\dagger)=0 . \tag{2.3}$$

Here

$$\partial_+=\frac{\partial}{\partial x^+}$$

and

$$\partial_-=\frac{\partial}{\partial x^-} ,$$

where

$$x^+=x^0+x^1 ,$$

and

$$x^-=x^0-x^1 .$$

The metric tensor $g^{\mu\nu}$ is given by

$$g^{++}=g^{--}=0,\quad g^{+-}=g^{-+}=2 .$$

Suppose one has arbitrarily fixed both ϕ and $\phi^\dagger$ at some particular light-cone time $x^+=x_0^+$ on the interval $x^-\in(-L,L)$. The equations of motion (2.2) and (2.3) can be integrated as

$$\partial_+\phi(x^-,x^+)=-\frac{1}{4}\int_{-L}^{+L}dy^-\epsilon(y^--x^-)\left\{m^2\phi(y^-,x^+)+\frac{\lambda}{4!}\frac{1}{2}[4\phi^{\dagger 3}(y^-,x^+)+12\phi^\dagger(y^-,x^+)\phi(y^-,x^+)\phi(y^-,x^+)]\right\} \tag{2.4}$$

and

$$\partial_+\phi^\dagger(x^-,x^+)=-\frac{1}{4}\int_{-L}^{+L}dy^-\epsilon(y^--x^-)\left\{m^2\phi^\dagger(y^-,x^+)+\frac{\lambda}{4!}\frac{1}{2}[4\phi^3(y^-,x^+)+12\phi^\dagger(y^-,x^+)\phi(y^-,x^+)\phi^\dagger(y^-,x^+)]\right\} \tag{2.5}$$

where ϵ is the antisymmetric step function:

$$\epsilon'(x)=-2\delta(x) .$$

Thus, the equal light-cone time canonical commutation relations are given by

$$[\phi(x^-,x^+),\phi(y^-,x^+)]|_{x^+}=-i\tfrac{1}{4}\epsilon(x^--y^-) , \tag{2.6}$$

$$[\phi^\dagger(x^-,x^+),\phi^\dagger(y^-,x^+)]|_{x^+}=-i\tfrac{1}{4}\epsilon(x^--y^-) , \tag{2.7}$$

and

$$[\phi(x^-,x^+),\phi^\dagger(y^-,x^+)]|_{x^+}=-i\tfrac{1}{2}\epsilon(x^--y^-) . \tag{2.8}$$

The stress tensor $T^{\mu\nu}$ is constructed from the Lagrangian density $\mathcal{L}$ by

$$T^{\mu\nu}=\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\partial^\nu\phi-g^{\mu\nu}\mathcal{L} . \tag{2.9}$$

Thus

$$T^{++}=(\partial^+\phi)^\dagger(\partial^+\phi) , \tag{2.10}$$

where

$$\partial^+=2\partial_-=2\frac{\partial}{\partial x^-} ,$$

$$\partial^-=2\partial_+=2\frac{\partial}{\partial x^+} ,$$

and

$$T^{+-}=2m^2\phi^\dagger\phi+2\frac{\lambda}{4!}\frac{1}{2}(\phi^{\dagger 4}+6\phi^\dagger\phi^\dagger\phi\phi+\phi^4) . \tag{2.11}$$

It is noted that both T^{++} and T^{+-} are positive definite. From the stress tensor $T^{\mu\nu}$ we construct the energy-momentum operator P^μ:

$$P^\mu=\tfrac{1}{2}\int dx^-T^{+\mu} . \tag{2.12}$$

The conservation of $T^{\mu\nu}(\partial_\mu T^{\mu\nu}=0)$ implies that $T^{+\mu}$ is conserved, which, in turn, implies that $T^{+\mu}$ and hence P^+ and P^- are independent of the light-cone time x^+. Thus, both P^+ and P^- are positive definite, conserved, and independent of the light-cone time.

The free-field solutions $\phi_0(x^+,x^-)$ and $\phi_0^\dagger(x^+,x^-)$, in terms of the free-field annihilation operator a (defined by $a|\mathrm{vac}\rangle=0$) and the creation operator $a^\dagger$, can be written as

$$\phi_0(x^+,x^-)=\frac{1}{2\pi}\int\frac{dk^+}{2k^+}[a(k)e^{-ik\cdot x}+b^\dagger(k)e^{ik\cdot x}], \tag{2.13}$$

$$\phi_0^\dagger(x^+,x^-)=\frac{1}{2\pi}\int\frac{dk^\dagger}{2k^+}[b(k)e^{-ik\cdot x}+a^\dagger(k)e^{ik\cdot x}] . \tag{2.14}$$

The commutation relations between the annihilation and creation operators are

$$[a(k^+),a^\dagger(k'^+)]=[b(k^+),b^\dagger(k'^+)]$$
$$=2\pi 2k^+\delta(k^+-k'^+) , \tag{2.15}$$

with all other commutators vanishing. Now, following the conventions of Pauli and Brodsky [1], I construct the light-front momentum and energy operator in the discretized version. Discretization is introduced by the replacement

$$k^+ \to k_n^+ = \frac{2\pi}{L} n, \quad n = 1,2,3,\ldots,\Lambda \; .$$

Λ determines the highest possible value of k^+ for each fixed L. Since $k^+ = k^0 + k^1$, k^+ can be zero for a massive particle only when $k^1 \to -\infty$.

An important point is that the above construction omits the zero-momentum states ($k^+=0$) and neglects what is referred to as the zero-mode problem. The inclusion of this $k^+=0$ state could be a significant issue and would require an extension to the method.

In the discretized version, the free-field solutions are given by

$$\phi_0(x^+,x^-) = \frac{1}{\sqrt{4\pi}} \sum_{n=1}^{\Lambda} \frac{1}{\sqrt{n}} (a_n e^{-ik_\mu^{(n)}x^\mu} + b_n^\dagger e^{ik_\mu^{(n)}x^\mu}) \tag{2.16}$$

and

$$\phi_0^\dagger(x^+,x^-) = \frac{1}{\sqrt{4\pi}} \sum_{n=1}^{\Lambda} \frac{1}{\sqrt{n}} (b_n e^{-ik_\mu^{(n)}x^\mu} + a_n^\dagger e^{ik_\mu^{(n)}x^\mu}) \; . \tag{2.17}$$

The factors $1/\sqrt{4\pi}$ and $1/\sqrt{n}$ are introduced to maintain the following commutation relations:

$$[a_n, a_m^\dagger] = \delta_{n,m} \; , \tag{2.18}$$

$$[b_n, b_m^\dagger] = \delta_{n,m} \; . \tag{2.19}$$

At $x^+ = x_0^+ = 0$, the interacting field ϕ is chosen to coincide with ϕ_0:

$$\phi(x^-,0) = \phi_0(x^-,0) \; .$$

It is convenient to introduce the dimensionless variable

$$\xi = \frac{\pi x^-}{L} \; . \tag{2.20}$$

Then

$$\tfrac{1}{2} k^+ x^- = n\xi \; .$$

The operators K and H are introduced such that

$$P^+ = \frac{2\pi}{L} K \tag{2.21}$$

and

$$P^- = \frac{L}{2\pi} H \; . \tag{2.22}$$

K is the dimensionless momentum operator and H is the Hamiltonian operator with dimensions of mass squared. The invariant-mass operator $M^2 = P^+P^- = KH$ is independent of L. In the discretized version, the momentum K and the Hamiltonian H are given by

$$K = \tfrac{1}{2} \sum_{n=1}^{\Lambda} (n a_n^\dagger a_n + n b_n^\dagger b_n) \tag{2.23}$$

and

$$H = H_0 + H_1 + H_2 + H_3 + H_4 \; , \tag{2.24}$$

where

$$H_0 = \sum_n \frac{1}{n} a_n^\dagger a_n \left[m^2 + \frac{\lambda}{4\pi} \frac{1}{2} \sum_k \frac{1}{k} \right] + \sum_n \frac{1}{n} b_n^\dagger b_n \left[m^2 + \frac{\lambda}{4\pi} \frac{1}{2} \sum_k \frac{1}{k} \right] , \tag{2.25a}$$

$$H_1 = \frac{1}{8} \frac{\lambda}{4\pi} \sum_{klmn} \frac{a_k^\dagger a_l^\dagger a_m a_n + b_n^\dagger b_m^\dagger b_l b_k}{\sqrt{klmn}} \delta_{m+n,k+l} + \frac{1}{8} \frac{\lambda}{4\pi} \sum_{klmn} \frac{a_k^\dagger a_l^\dagger b_m b_n + b_k^\dagger b_l^\dagger a_m a_n}{\sqrt{klmn}} \delta_{m+n,k+l} \; , \tag{2.25b}$$

$$H_2 = \frac{1}{2} \frac{\lambda}{4\pi} \sum_{klmn} \frac{b_k^\dagger a_l^\dagger b_m a_n}{\sqrt{klmn}} \delta_{k+l,m+n} \; , \tag{2.25c}$$

$$H_3 = \frac{1}{12} \frac{\lambda}{4\pi} \sum_{klmn} \frac{a_k^\dagger b_l b_m b_n + a_n^\dagger a_m^\dagger a_l^\dagger b_k}{\sqrt{klmn}} \delta_{k,l+m+n} + \frac{1}{12} \frac{\lambda}{4\pi} \sum_{klmn} \frac{b_k^\dagger a_l a_m a_n + b_n^\dagger b_m^\dagger b_l^\dagger a_k}{\sqrt{klmn}} \delta_{k,l+m+n} \; , \tag{2.25d}$$

$$H_4 = \frac{1}{4} \frac{\lambda}{4\pi} \sum_{klmn} \frac{b_k^\dagger b_l b_m a_n + b_n^\dagger a_m^\dagger a_l^\dagger a_k}{\sqrt{klmn}} \delta_{k,l+m+n} + \frac{1}{4} \frac{\lambda}{4\pi} \sum_{klmn} \frac{a_k^\dagger a_l a_m b_n + a_n^\dagger b_m^\dagger b_l^\dagger b_k}{\sqrt{klmn}} \delta_{k,l+m+n} \; . \tag{2.25e}$$

The results obtained as solutions lead to divergences as per expectation. As in real-scalar theory [6], in complex-scalar $(\phi^4)_2$ theory the only divergent graph is the "tadpole" (one-loop self-energy), which is logarithmically divergent [14]. The logarithmically divergent additive term to m^2 in H_0 is the DLFQ manifestation of the tadpole contribution. This divergence can readily be removed by considering the normal-ordered Hamiltonian [15], which is adopted for numerical work.

III. RELEVANT DISCUSSIONS

For a given finite K, I get a finite number of basis states (provided I neglect the zero-mode problem). The Fock-space basis is chosen with the normal (perturbative) vacuum as the lowest-energy state in the spectrum. The exact spectra is only obtained as $K \to \infty$, the continuum limit. As expressed in Table I, the dimensionality of the Hamiltonian matrix increases rapidly with increasing K. So because of major computational difficulties in obtaining numerical solutions due to the large dimensionality of the Hamiltonian matrix, I restrict the discussions to large but finite values of K. Of course, I will show how my results with the DLFQ method approach the continuum limit as a function of the coupling constant λ.

TABLE I. The dimensionality N of the Hamiltonian matrix as a function of K.

K	N
2	1
4	4
6	11
8	27
10	63
12	141
16	614

I have neglected the $k^+=0$ states which should, in principle, be included even at finite values of K. This might open the possibility of a zero-momentum condensate and would require an extension to the method.

I concentrate my numerical discussions on the charge-0 sector of the field. The corresponding Fock-space basis states are constructed by putting equal numbers of particles and antiparticles with their respective momenta.

In the remainder of this paper I will show how complex scalar theory yields results consistent with real-scalar theory, and also somewhat consistent with what is known from quantization in ordinary space-time. I will show evidence for a lack of multiparticle bound states. In addition, I demonstrate that the method is currently limited to the weak-coupling region.

I treat the mass parameter in the Lagrangian as the adjustable bare mass, and the lowest eigenvalue of the invariant-mass matrix as the fixed physical mass. K is a bounded operator which commutes with the Hamiltonian H. I construct the invariant mass-squared operator $M^2=KH$, which is independent on the box length L.

In order to avoid divergences I need only the renormalization of the mass. This can be done by normal ordering with respect to the mass parameter appearing in the Lagrangian. There are various methods incorporating the processes of mass renormalization. In Ref. [1] Pauli and Brodsky followed the mass-renormalization scheme introduced by Brooks and Frautschi [16] for that same model in ordinary space-time. I followed the mass-renormalization scheme implied by Harindranath and Vary [6]. For given values of λ and m^2_{phys} one diagonalizes the Hamiltonian matrix for an initial guess for the bare mass m^2 and obtains the lowest eigenvalue e_1. Then iteration is performed to solve the nonlinear equation

$$e_1[m^2,\lambda]-m^2_{\text{phys}}=0$$

until convergence is achieved to within a required accuracy (m^2_{phys} was chosen to be 1.0). By definition, this mass-renormalization scheme preserves the mass gap, and in my theory I now show that it restricts the solution to the weak-coupling region.

IV. NUMERICAL ANALYSIS AND RESULTS

A general state in the Fock-space basis in the charge-0 sector of the field is denoted as

$$|n_1^{m_1},n_2^{m_2},\ldots,n_N^{m_N};\bar{n}_1^{\bar{m}_1},\bar{n}_2^{\bar{m}_2},\ldots,\bar{n}_N^{\bar{m}_N}\rangle \ .$$

This represents a state with m_1 quanta (particles) with n_1 units of momentum and $\bar{m}_1$ quanta (antiparticles) with $\bar{n}_1$ units of momentum such that the number of particles $(m_1+m_2+\cdots+m_N)$ is equal to the number of antiparticles $(\bar{m}_1+\bar{m}_2+\cdots+\bar{m}_N)$ for any state. Also, for a given K, the relation

$$K=n_1m_1+n_2m_2+\cdots+n_Nm_N$$
$$+\bar{n}_1\bar{m}_1+\bar{n}_2\bar{m}_2+\cdots+\bar{n}_N\bar{m}_N$$

is satisfied for each state.

The square of the physical mass of each quanta of the field is denoted as m^2_{phys}. The finite-mass renormalization is implemented by insisting that for each value of K the lowest excitation (in other words, the mass gap with respect to the perturbative vacuum) has the invariant mass m_{phys}. The Fock-space dimension is not only finite, it can be as small as 1. Since a 2×2 matrix can be trivially diagonalized, a number of cases can be treated analytically. In the following, I shall increase the resolution stepwise in order to see how the invariant-mass spectrum gains complexity as a function of K.

(a) $K=0$. Since zero modes ($k^+=0$ states) are neglected, the only basis state is the vacuum state $|\text{vac}\rangle$:

$$K|\text{vac}\rangle=0|\text{vac}\rangle \ .$$

Hence $M^2|\text{vac}\rangle=0|\text{vac}\rangle$. Thus $|\text{vac}\rangle$ is the only state with $M^2=0$. This Fock-space vacuum is identical with the physical vacuum.

(b) $K=1$. This case is not an issue in the charge-0 sector of the field since I cannot place both particles and antiparticles with an integral unit of momentum in any state.

(c) $K=2$. I have a single state

$$|1\rangle=|1^1;\bar{1}^1\rangle \ ,$$

with

$$M_1^2=\langle 1|M^2|1\rangle=4m^2+\frac{\lambda}{4\pi}=4m^2_{\text{phys}}+\frac{\lambda}{4\pi} \ .$$

For $K=2$, finite-mass renormalization is not an issue. $|1\rangle$ is the state containing one particle and one antiparticle which are at rest with respect to each other. Since λ is greater than zero M_1 is greater than $2m_{\text{phys}}$, thus proving that complex scalar $(\phi^4)_2$ theory has no two-particle bound states. This is in agreement with the well-known result for the real scalar $(\phi^4)_2$ theory [5].

(d) $K\geq4$. Exact spectra are obtained only in the continuum limit. The limit is attained when I take $K\to\infty$, since then the fractional momenta ($x_i=k_i^+/K$) carried by the constituent particles vary continuously from 0 to 1 in each state. But, practically, the dimensionality of the Hamiltonian matrix grows rapidly as K increases, as illustrated in Table I.

Because of the large computational difficulties arising from the large dimension of the Hamiltonian matrix for increasingly large values of K, I concentrate my discussion on large but finite values of K. I adopt the notation that states are identified by their $\lambda=0$ structure. The ratio of the mass (M) of the lowest four-particle state

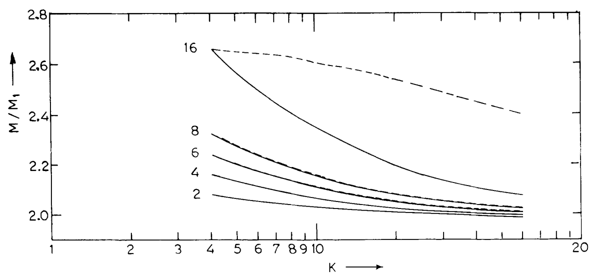

FIG. 1. Ratio of the mass (M) of the lowest four-particle state $|(K/4)^4\rangle$ to the mass (M_1) of the lowest two-particle state $|(K/2)^2\rangle$, as a function of K for different values of λ. Solid lines: with mass renormalization; dashed lines: without mass renormalization. Smooth lines are drawn through results obtained at values of K which are multiples of 4.

$|(K/4)^4\rangle$ (two particles and two antiparticles) to the mass ($\sqrt{e_1}=M_1$) of the lowest two-particle state $|(K/2)^2\rangle$ (one particle and one antiparticle) is plotted in Fig. 1 as a function of K for different values of λ. I choose $m=1.0$ for the calculations without mass renormalization. The results from Fig. 1 indicate that the invariant mass of the state (M/M_1) approaches $2m_{\text{phys}}$ for large K in the weak-coupling region ($\lambda \leq 8$). Convergence becomes much slower as the coupling becomes stronger. The results begin to depend significantly on whether mass renormalization is adopted when λ exceeds about 8. In the theory under discussion the only dimensionless parameter is λ/m^2. In Fig. 1, for $\lambda=16$ the dashed curve represents $\lambda/m^2=16$ for all K, while the solid curve represents $\lambda/m^2=12$ at $K=8$. Its value decreases with increasing K.

I now study the Fock-space decomposition of the state $|(K/4)^4\rangle$ for $\lambda=2$ as a function of K. Let me denote the square of the coefficient of the state $|(K/4)^4\rangle$ by C_0, and the sum of the squares of the coefficients of all two-particle state components of this state by C_1. In Table II I present C_0 and C_1 as functions of K. The fact that C_0 differs from unity with increasing K indicates the presence of scattering in the continuum limit. C_1 remains close to unity, indicating that the dominant mixing of the four-particle state $|(K/4)^4\rangle$ is with other four-particle states (all of which have higher invariant masses at $\lambda=0$). For real-scalar ϕ^4 theory the renormalized coupling is nonvanishing in 1+1 dimensions [17]. My results for complex scalar theory are also consistent with that conclusion.

The matrix diagonalization gives the invariant mass

TABLE II. C_0, the square of the coefficient of the state $|(K/4)^4\rangle$, and C_1, the sum of the squares of the coefficients of all four-particle state components of this state, as a function of K at $\lambda=2.0$.

K	C_0	C_1
4	0.9990	0.9990
8	0.9915	0.9981
12	0.9865	0.9979
16	0.9798	0.9979

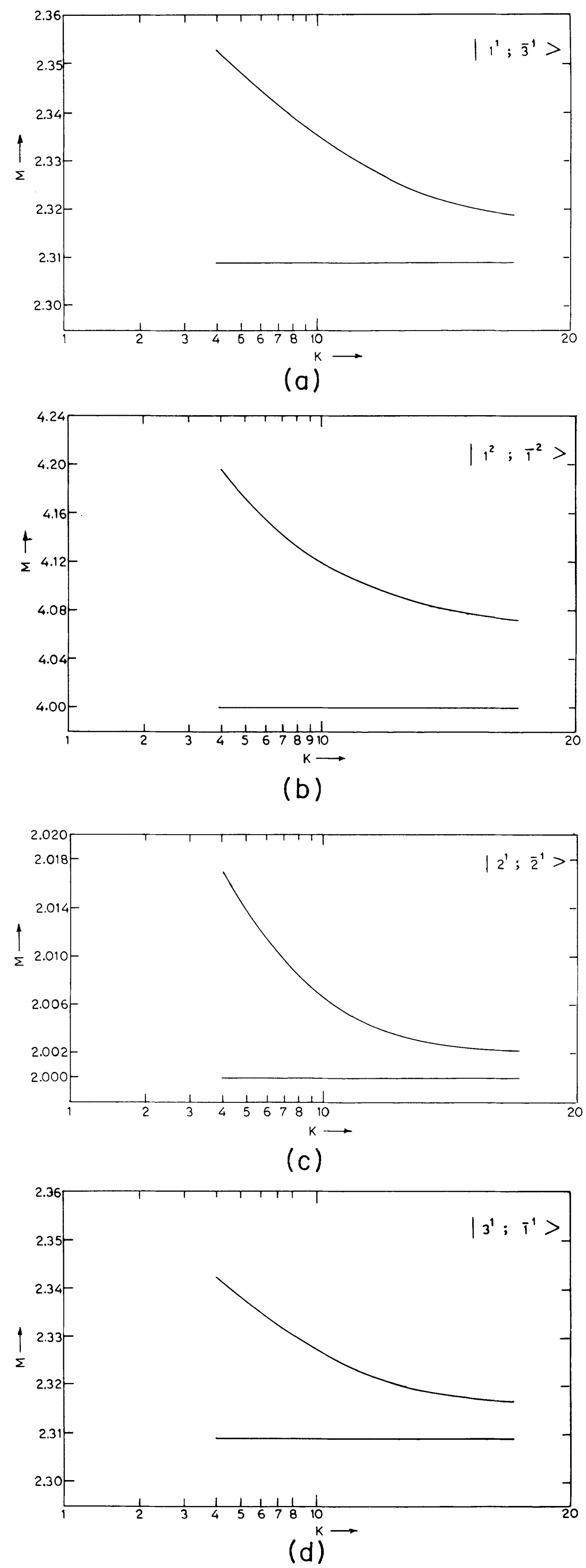

FIG. 2. Invariant mass of all the states at $K=4$ (single-particle $m_{\text{phys}}^2=1.0$) as a function of k for $\lambda=2.0$. The horizontal reference line shows the mass at $\lambda=0$. Smooth lines are drawn through results obtained at values of K which are multiples of 4.

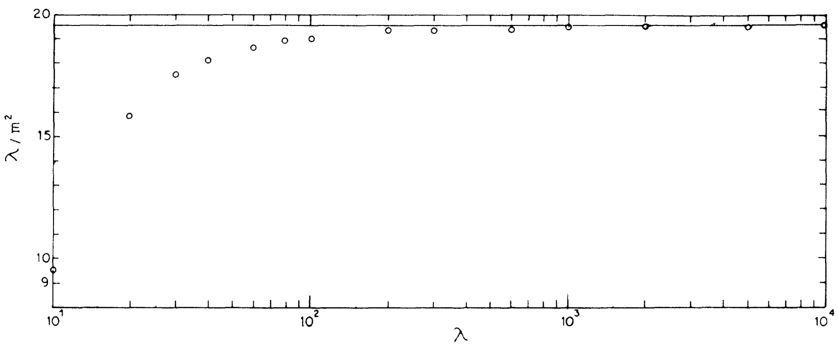

FIG. 3. The intrinsic dimensionless coupling λ/m^2 as a function of λ in the scheme with $m^2_{\text{phys}}=1.0$. The bare coupling at which the mass gap vanishes in the calculation (the critical coupling λ_c) with fixed mass parameters in the Lagrangian ($m=1.0$) is shown as a horizontal line. Both results correspond to $K=8$.

and Fock-space composition of many multiparticle states. The lowest excitation is the state with one particle and one antiparticle with the mass of either fixed at $m_{\text{phys}}=m=1.0$. At $K=4$, there are 3 two-particle (one particle and one antiparticle) states and 1 four-particle state (two particles and two antiparticles). All these states reappear at K values which are integer multiples of 4. I select the case with mass renormalization for the moment. In Fig. 2 the invariant masses of these four states are plotted as functions of K for $\lambda=2.0$. As mentioned earlier, states are identified by their Fock-space structure at $\lambda=0$, and the mass at $\lambda=0$ is shown as a horizontal reference line. The results for the four-particle state indicate the lack of bound states in the continuum limit, since the convergence is similar to that of the two-particle state.

I now examine the behavior of the theory with increased coupling, both with and without the finite-mass renormalization. Without mass renormalization, the DLFQ results yield an invariant-mass squared e_1 of the lowest excitation state (one particle and one antiparticle in the present case), which decreases with increasing λ and eventually becomes negative for all values of $K\geq 4$. The calculations which incorporate the finite-mass renormalization define the value of the mass gap, and thus avoid this vanishing mass gap. I consider the results at $K=8$ as a typical example. Here the mass gap is found to vanish at the critical coupling $\lambda_C=19.57$. This is shown as a horizontal reference line in Fig. 3. The values of the intrinsic dimensionless coupling λ/m^2 with mass renormalization are plotted as a function of λ in Fig. 3 and are clearly seen to approach λ_C as $\lambda\rightarrow\infty$. Thus, it is impossible to go to a strong-coupling (to exceed λ_C) region by adopting mass renormalization. Thus finite-mass renormalization prevents the phase transition by restricting the theory to the weak-coupling region.

The actual value of λ_c changes with K. In the treatment of the real-scalar theory in Ref. [6], they obtained the results for $K=16$ with the corresponding value of $\lambda_C=43.9$. This should be compared with Chang's Hartree result [14] of 54.3 when expressed in conventions of Ref. [6] for the coupling constant. For comparison, another numerical method [18] gives $22.8\leq\lambda_C\leq 51.6$. The significance of my result for λ_c is not so much of a concern since the discussions are restricted to a single phase of the theory.

V. CONCLUSIONS

DLFQ, a recently proposed method to solve relativistic field-theory problems nonperturbatively, is applied to $(\phi^4)_2$ theory in the complex scalar region of the field. The results are in agreement with those of real scalar theory with properties established for constructive quantum field theory. The exact physical spectrum emerges only after I take the limit $K\rightarrow\infty$. In this effort I have obtained results with modest values of K, which are sufficient to sense the continuum limit for weak coupling. Finite-mass renormalization restricts the discussion to a single phase of the theory.

The value of dimensionless coupling λ/m^2 approaches λ_C (the critical coupling at which the mass gap vanishes) but remains below it.

With a fixed mass parameter in the Lagrangian, as I increase the value of the coupling strength, the mass gap vanishes. This indicates the nontrivial vacuum structure of the theory.

Lastly, I emphasize the fact that I have neglected the zero-mode problem ($k^+=0$ states). This might lead to the presence of zero-momentum condensates, which is essential for a study of the vacuum structure, and thus to an extension of the method.

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