On background-independent open-string field theory

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A framework for background-independent open-string field theory is proposed. The approach involves using the Batalin-Vilkovisky formalism, in a way suggested by recent developments in closed-string field theory, to implicitly define a gauge-invariant Lagrangian in a hypothetical "space of all open-string world-sheet theories." It is built into the formalism that classical solutions of the string field theory are Becchi-Rouet-Stora-Tyutin- (BRST-) invariant open-string world-sheet theories and that, when expanding around a classical solution, the infinitesimal gauge transformations are generated by the world-sheet BRST operator.

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I. INTRODUCTION

Though gauge-invariant open- and closed-string field theories are now known, the problem of the background dependence of string field theory has not been successfully addressed. This problem is fundamental because it is here that one really has to address the question of what kind of geometrical object the string represents. The world-sheet or σ-model formulation of string theory is the one known formulation in which anything can be done in a manifestly background-independent way. It has therefore been widely suspected that somehow one should do string field theory in the "space of all two-dimensional field theories," by finding an appropriate gauge-invariant Lagrangian on that space. The tangent space to the "space of all two-dimensional field theories" should be the space of all local operators, including operators of very high dimension, time-dependent operators of negative dimension, and operators containing ghost fields. This approach, which has been pursued in [1–7], has two glaring difficulties: (1) Because of the ultraviolet difficulties of quantum field theory, it is hard to define a "space of all two-dimensional field theories" with the desired tangent space (this is why the σ-model approach to string theory is limited in practice to a long-wavelength expansion); (2) one has not known what properties such a space should have to enable the definition of a gauge-invariant Lagrangian.

In the present paper, I will propose a solution to the second problem for the case of open (bosonic) strings, leaving the first problem to the future. Considering open strings means that we consider world-sheet actions of the form $I=I_0+I'$, where I_0 is a fixed bulk action (corresponding to a choice of closed-string background) and I' is a boundary term representing the open strings. For instance, the standard closed-strong background is

$$I_0=\int_\Sigma d^2x\sqrt{h}\left[\frac{1}{8\pi}h^{ij}\partial_i X^\mu\partial_j X_\mu+b^{ij}D_i c_j\right] . \quad (1.1)$$

Here Σ is the world sheet with metric h with coordinates x^k, and c_i and b_{jk} are the usual ghost and antighost fields. This theory has the usual conserved Becchi-Rouet-Stora-Tyutin (BRST) current J^i. The corresponding BRST charge $Q=\oint d\sigma\, J^0$ (σ is an angular parameter on a closed strong and 0 is the normal direction) obeys the usual relations

$$Q^2=0 \quad\text{and}\quad T_{ij}=\{Q,b_{ij}\} , \quad (1.2)$$

with T_{ij} being here the stress tensor. We then take I' to be an arbitrary boundary interaction:

$$I'=\int_{\partial\Sigma}d\sigma\,\mathcal{V} , \quad (1.3)$$

where $\mathcal{V}$ is an arbitrary local operator constructed from X,b,c; in this paper, we consider two $\mathcal{V}$'s equivalent if they differ by a total derivative. A two-dimensional theory with action $I=I_0+I'$, with I_0 defined as above and I' allowed to vary, will be called an open-string world-sheet field theory. Our goal will be to define a gauge-invariant Lagrangian on the space of all such open-string world-sheet theories (or, actually, a space introduced later with some additional degrees of freedom).

This will be easier than it may sound because the Batalin-Vilkovisky formalism [8–12] will do much of the work for us. The use of this formalism was suggested by its role in constructing and understanding classical and quantum closed-string field theory [14], its elegant use in quantizing open-string field theory [15,16] and its role in string-theory Ward identities [17,18]. In particular, while the Batalin-Vilkovisky (BV) formalism was first invented for quantizing gauge-invariant classical field theories that are already known, it was used in closed-string field theory [14] as an aid in finding the unknown theory; that is how we will use it here. The BV formalism also has an interesting analogy with the renormalization group [6].

Here is a brief sketch of the relevant aspects of the BV formalism. (For more information see [10].) One starts with a supermanifold $\mathcal{M}$ with a U(1) symmetry that we will call ghost number, generated by a vector field U. The essential structure on $\mathcal{M}$ is a nondegenerate fermionic two-form ω of $U=-1$, which is closed, $d\omega=0$. One can think of ω as a fermionic symplectic form. As in the usual bosonic case, such an ω has no local invariants; ω can locally be put in the standard form $\omega=\sum_a d\theta_a dq^a$, with q^a and θ_a bosonic and fermionic, respectively.

Just as in the usual case, one can define Poisson brackets

$$\{A,B\}=\frac{\partial_r A}{\partial u^K}\omega^{KL}\frac{\partial_l B}{\partial u^L}\ , \tag{1.4}$$

with ω^{KL} the inverse matrix to ω_{KL} and u^I local coordinates on $\mathcal{M}$. (The subscripts r and l refer to derivatives from the right to left.) These Poisson brackets, which are the BV antibrackets, obey a graded Jacobi identity. (At the cost of some imprecision, I will sometimes refer to ω rather than the Poisson brackets derived from it as the antibracket.) The BV master equation is

$$\{S,S\}=0 \tag{1.5}$$

(which would be vacuous if ω were bosonic). An action function S obeying the master equation is automatically gauge invariant, with the gauge-transformation law

$$\delta u^I=\left[\omega^{IJ}\frac{\partial^2 S}{\partial u^I\partial u^K}+\frac{1}{2}\frac{\partial\omega^{IJ}}{\partial u^K}\frac{\partial S}{\partial u^J}\right]\epsilon^K\ , \tag{1.6}$$

with arbitrary infinitesimal parameters ϵ^K. It is straightforward to see that $\delta S=\epsilon^K\partial_K\{S,S\}/2=0$. [The gauge transformations (1.6) will only close, and are only well defined, independent of the choice of coordinates u^I, modulus "trivial" gauge transformations that vanish on shell. These are of the form $\delta u^I=\lambda^{IJ}\partial S/\partial u^J$, with $\lambda^{IJ}=-\lambda^{JI}$.]

Let $\mathcal{N}$ be the subspace of $\mathcal{M}$ on which $U=0$. We define the "classical action" S_0 to be the restriction of S to $\mathcal{N}$. The classical action has a gauge invariance given, again, by (1.6), with the ϵ^K restricted to have $U=-1$. In usual applications of the BV formalism to gauge fixing, $\mathcal{N}$ and S_0 are given, and the first step is the construction of $\mathcal{M}$ and S (the latter is required to obey a certain cohomological condition as well as the master equation). A general theorem shows that suitable $\mathcal{M}$ and S exist, but their actual construction is usually rather painful. The insight of Thorn [15] and Boccicchio [16] (extending earlier ideas, beginning with Siegel [13], on the role of the ghosts in string theory) was that, in string theory, $\mathcal{M}$ and S are related to $\mathcal{N}$ and S_0 just by relaxing the condition on the ghost number of the fields. Anticipating this structure was a help in developing closed-string field theory, as explained in [14], and will be essential here.

If S is any function, not necessarily obeying the master equation, one can define a vector field V by

$$V^K\omega_{KL}=\frac{\partial_l S}{\partial u^L}\ . \tag{1.7}$$

If S has $U=0$, then V has $U=1$. If we take S as an action functional, then the Euler-Lagrange equations $0=d\omega$ are equivalent to $V^I=0$. As we will see later, the master equation implies that $V^2=0$ or, in components,

$$V^K\frac{\partial}{\partial u^K}V^I=0\ . \tag{1.8}$$

If we let i_V be the operation of contraction with V, then the definition (1.7) of V can be written as

$$i_V\omega=dS\ . \tag{1.9}$$

Under an infinitesimal diffeomorphism $u^I\to u^I+\epsilon V^I$ of $\mathcal{M}$, a two-form ω transforms as $\omega\to\omega+\epsilon(i_Vd+di_V)\omega$. V therefore generates a symmetry of ω precisely if

$$(di_V+i_Vd)\omega=0\ . \tag{1.10}$$

As $d\omega=0$, this reduces to

$$d(i_V\omega)=0\ , \tag{1.11}$$

and so is a consequence of (1.9). Therefore any vector field derived as in (1.7) from a function S generates a symmetry of ω. Conversely, if V is any symmetry of ω, that is, any vector field obeying (1.11), then a function S obeying (1.7) always exists at least locally (and is unique up to an overall additive constant). The possible failure of the global existence of S would be analogous to the multivaluedness of the Wess-Zumino and Chern-Simons functionals in field theory. Since topological questions analogous to this multivaluedness would be out of reach at present in string theory, we will in this paper content ourselves with local construction of S.

Suppose that one is given a vector field V that generates a symmetry of ω and also obeys $V^2=0$. One might wonder if it then follows that the associated function S obeys the master equation. This is not quite true, but almost. The actual situation is that, because of the Jacobi identity of the antibrackets, the map (1.9) from functions to vector fields is a homomorphism of Lie algebras; consequently, V^2 is the vector field derived from the function $\{S,S\}/2$ and vanishes precisely if $\{S,S\}$ is constant.

To verify this, one can begin by writing the equation $V^2=0$ in the form

$$[di_V+i_Vd,i_V]=0\ . \tag{1.12}$$

Equation (1.10) then implies that

$$(di_V+i_Vd)i_V\omega=0\ . \tag{1.13}$$

Using (1.11), we get

$$d(i_Vi_V\omega)=0\ . \tag{1.14}$$

This is equivalent to

$$d\{S,S\}=0\ , \tag{1.15}$$

so that $\{S,S\}$ is a constant, perhaps not zero. Since this argument can also be read backwards, we have verified that $V^2=0$ if and only if $\{S,S\}$ is constant.

Looking back at the proof of gauge invariance, we see that the master equation is stronger than necessary. A function S obeying (1.15) is automatically gauge invariant, with gauge invariance (1.6). The generalization of permitting $\{S,S\}$ to be a nonzero constant is not very interesting in practice for the following reason. If we take S to be an action, then the corresponding Euler-Lagrange equations are $V=0$. If these equations have at least one solution, then by evaluating the constant $\{S,S\}$ at the zero of V, one finds that, in fact, $\{S,S\}=0$. Therefore $\{S,S\}$ can be a nonzero constant only if the classical equations of motion are inconsistent.

I can now explain the strategy for constructing a gauge-invariant open-string Lagrangian. There are two steps. (1) On the space of all open-string world-sheet theories, we will find a fermionic vector field V, of ghost number 1, obeying $V^2=0$. (2) Then we will find, on the same space, V-invariant antibrackets, that is, a V-invariant fermionic symplectic form ω of ghost number -1. The Lagrangian S is then determined (up to an additive constant) from $dS=i_V\omega$; it is gauge invariant for reasons explained above.[1]

Of these two steps, the definition of V is straightforward, as we will see. The definition of ω is less straightforward, and a proper understanding would depend on really understanding what is "the space of all open-string world-sheet theories." I will give only a preliminary, formal definition of ω. At least the discussion should serve to make clear what structures one should want "the space of all two-dimensional field theories" to have.

II. DEFINITION OF V

In this paper our open-string quantum field theories will be formulated on a disk Σ. As one might expect, this is the relevant case in describing the classical Lagrangian. The open-string quantum field theories will be required to be invariant under rigid rotations of the disk, but are not required to have any other symmetries such as conformal invariance. That being so, Σ must be endowed with a metric (not just a conformal structure). Since rotation invariance will eventually be important, we consider a rotationally invariant metric on Σ, say,

$$ds^2=dr^2+f(r)d\theta^2, \quad 0\le r\le 1, \quad 0\le\theta\le 2\pi . \tag{2.1}$$

The choice of f does not matter; a change in f would just induce a reparametrization of the space of possible boundary interactions. In any event the metric on Σ can be held fixed throughout this paper.

As explained in the Introduction, by an open-string world-sheet field theory we mean a two-dimensional theory with action $I=I_0+I'$, where I_0 is the fixed bulk action (1.1) and I' is a boundary interaction that does not necessarily conserve the ghost number. Our first goal in the present section is to describe an anticommuting vector field, of ghost number 1, on the space of such theories. (Later, in defining ω, we will add new degrees of freedom to the open-string field theories. The construction of V is sufficiently natural that it will automatically carry over to the new case.)

One way to explain the definition of V is as follows. An open-string field theory can be described by giving all possible correlation functions of local operators in the *interior* of the disk. Thus the correlation functions we consider are

$$\left\langle \prod_{i=1}^{n} \mathcal{O}_i(P_i) \right\rangle , \tag{2.2}$$

with arbitrary local operators $\mathcal{O}_i$ and P_i in the interior of Σ. The correlation functions (2.2) obey Ward identities. Since we choose the P_i to be *interior* points, the Ward identities are entirely determined by the bulk action I_0 of Eq. (1.1) and are independent of the choice of boundary contribution in the action. The boundary interactions determine not the structure of the Ward identities, but the choice of a specific solution of them. It is reasonable to expect that the space of all solutions of the Ward identities, for all correlation functions in the interior of Σ, can be identified with the space of possible boundary interactions, since, roughly speaking, the boundary interaction determines how a left-moving wave incident on the boundary is scattered and returns as a right-moving wave. We will use this identification of the space of solutions of the Ward identities with the space of open-string theories to define a vector field on the space of theories. We also will give an alternative definition that does not use this identification.

If one is given one solution of the Ward identities, corresponding to one boundary interaction, then another solution of the Ward identities can be found by conjugating by any symmetry of the interior action I_0. An important symmetry is the one generated by the BRST charge Q. Conjugating by Q is particularly simple since $Q^2=0$. If ϵ is an anticommuting c number, we can form a one-parameter family of solutions of the Ward identities with

$$\left\langle \prod_{i=1}^{n} \mathcal{O}_i(P_i) \right\rangle_\epsilon = \left\langle \prod_{i=1}^{n} [\mathcal{O}_i(P_i) - i\epsilon\{Q,\mathcal{O}(P_i)\}] \right\rangle . \tag{2.3}$$

At the tangent-space level, this group action on the space of theories is generated by a vector field V, which is anticommuting and has ghost number 1, since those are the quantum numbers of Q, and obeys $V^2=0$ (or $\{V,V\}=0$) since $Q^2=0$.

Here is an alternative description of V. Let J^i be the conserved BRST current. Let $j=\epsilon_{ij}J^i dx^j$ be the corresponding closed one-form. Let C_α be a circle that winds once around all of the P_i; for instance, C_α may be a circle a distance α from the boundary of Σ for small α. Since j is closed, the contour integral $\oint_{C_\alpha} j$ is invariant under homotopically trivial displacements of C. The term in (2.3) proportional to ϵ is just

$$\left\langle \oint_{C_\alpha} j \prod_i \mathcal{O}_i(P_i) \right\rangle , \tag{2.4}$$

as one sees upon shrinking the contour C_α to pick up terms of the form $\{Q,\mathcal{O}_i\}$. On the other hand, we can evaluate (2.4) by taking the limit as $\alpha\to 0$, so that C_α approaches the boundary of the disk. In this limit, $\int_{C_\alpha} j$ approaches $\int_{\partial\Sigma}\mathcal{V}$ for some local operator $\mathcal{V}$ defined on the boundary. There is no general formula for $\mathcal{V}$; its determination depends on the behavior of local operators (in this case the BRST current) near the boundary of Σ

[1]On the basis of what happens in field theory, I expect that when space-time is not compact, the formula $dS=i_V\omega$ is valid only for variations of the fields of compact support; otherwise, there are additional surface terms in the variation of S. Of course, a formula for the change of S in variations of compact support suffices, together with locality, to determine S up to an additive constant.

and so on the details of the boundary interaction in the open-string field theory. But, in general, we can interpret $\int_{\partial\Sigma}\mathcal{V}$ as a correction to the boundary interaction of the theory, and as such it defines a tangent vector field to the space of all open-string field theories. This is an alternative description of the vector field $\mathcal{V}$ defined in the previous paragraph.

The correction $\int_{\partial\Sigma}\mathcal{V}$ to the boundary Lagrangian resulted from a BRST transformation of that Lagrangian. Therefore $\mathcal{V}$ vanishes when, and only when, the boundary interactions are BRST invariant. The BRST-invariant world-sheet open-string theories are therefore precisely the zeros of V. In other words, the equations

$$V^I=0 \tag{2.5}$$

are the equations of world-sheet BRST invariance. These equations are certainly background independent in the relevant sense; no *a priori* choice of an open-string background entered in the construction. As explained in the Introduction, a gauge-invariant Lagrangian with $V^I=0$ as the equations of motion can be constructed provided we can find V-invariant antibrackets on the space of open-string field theories.

Before undertaking this task, let us make a few remarks about the relation of the vector field V to BRST invariance. At a point at which a vector field does not vanish, there is no invariant way (lacking an affine connection) to differentiate it. However, at a zero of a vector field, that vector field has a well-defined derivative which is a linear transformation of the tangent space. For instance, if V has a zero at, say, $u^K=0$, then we can expand $V^K=\sum_L q^K{}_L u^L+O(u^2)$, and $q^K{}_L$ is naturally defined as a tensor; in fact, it can be regarded as a matrix acting on tangent vectors. Upon expanding the equation $V^2=0$ in powers of u, one finds that $q^K{}_L q^L{}_M=0$ or, more succinctly,

$$q^2=0 \ . \tag{2.6}$$

In the case of the vector field V on the space of open-string world-sheet theories, the tangent space on which the matrix q acts is the space of local operators that can be added to the boundary interaction; so it is closely related to the space of first-quantized open-string states. Thus, essentially, q is an operator of ghost number 1 and square 0 in the open-string Hilbert space; it is, in fact, simply the usual BRST operator for the world-sheet theory with that particular boundary interaction.

What we have come upon here seems to be the natural off-shell framework for BRST invariance. Off shell, one has a vector field V of $V^2=0$. V vanishes precisely on shell, and then the derivative of V is the usual BRST operator q of $q^2=0$. In fact, this structure can be seen, but is perhaps not usually isolated, in conventional versions of string field theory.

III. DEFINITION OF THE ANTIBRACKETS

We now come to the more difficult part of our problem—defining the antibrackets. What will be said here is in no way definitive.

It might be helpful first to explain how the antibrackets are defined on shell; see also [18,17]. We start with a conformally invariant and BRST-invariant world-sheet theory with action $I=I_0+I'$, where

$$I'=\int_{\partial\Sigma}d\sigma\,\mathcal{V} \ , \tag{3.1}$$

for some $\mathcal{V}$. A tangent vector to the space of classical solutions of open-string theory is represented by a spin-1 primary field $\delta\mathcal{V}$. This perturbation must be BRST invariant in the sense that

$$\{Q,\delta\mathcal{V}\}=d\mathcal{O} \ , \tag{3.2}$$

for some $\mathcal{O}$ of ghost number 1. If we are given two such tangent vectors $\delta_i\mathcal{V}$, $i=1,2$, then $\{Q,\delta_i\mathcal{V}\}=d\mathcal{O}_i$ for two operators $\mathcal{O}_i$. Then we can define the antibrackets:

$$\omega(\delta_1\mathcal{V},\delta_2\mathcal{V})=\langle\mathcal{O}_1\mathcal{O}_2\rangle \ . \tag{3.3}$$

Here $\langle\cdots\rangle$ is the expectation value of a product of operators inserted on the disk, in the world-sheet field theory, and the $\mathcal{O}_i$ are inserted at arbitrary points on the boundary of the disk. Conformal invariance ensures that the positions at which the $\mathcal{O}_i$ are inserted do not matter. With a view, however, to the later off-shell generalization, I prefer to write

$$\omega(\delta_1\mathcal{V},\delta_2\mathcal{V})=\oint d\sigma_1\oint d\sigma_2\langle\mathcal{O}_1(\sigma_1)\mathcal{O}_2(\sigma_2)\rangle \ , \tag{3.4}$$

with the length element $d\sigma$ (determined from the metric on Σ) now normalized so that the circumference is 1.

The correlation function in (3.4) is BRST invariant and vanishes if either of the $\mathcal{O}_i$ is of the form of $\{Q,\ldots\}$, and so ω can be regarded as a two-form on the space of classical solutions. ω has ghost number -1 since the ghost number of the vacuum is -3 on the disk, and the shifts $\delta_i\mathcal{V}\to\mathcal{O}_i$, $i=1,2$, have shifted the ghost number by $+2$. Nondegeneracy of ω follows from its relation to the Zamolodchikov metric $g(\cdot,\cdot)$ on the space of conformal field theories. Indeed, if V and W are two spin-1 primary fields containing no ghost or antighost fields, and $\delta_1\mathcal{V}=V$, $\delta_2\mathcal{V}=\partial c\, W$, then $\omega(\delta_1\mathcal{V},\delta_2\mathcal{V})=g(V,W)$. According to the standard analysis of world-sheet BRST cohomology, every tangent vector to the space open-string solutions can be put in the form of $\delta_1\mathcal{V}$ or $\delta_2\mathcal{V}$. The nondegeneracy of ω thus is a consequence of the nondegeneracy of the Zamolodchikov metric. $\omega(\cdot,\cdot)$ is really the correct analog of $g(\cdot,\cdot)$ when one includes the ghosts.

In many respects, $\mathcal{O}$ is more fundamental than $\delta\mathcal{V}$. In string field theory, for instance, the classical string field is an object of ghost number 1, corresponding to $\mathcal{O}$. At the level of states, the relation between $\delta\mathcal{V}$ and $\mathcal{O}$ can be written

$$b_{-1}|\mathcal{O}\rangle=|\delta\mathcal{V}\rangle \ . \tag{3.5}$$

This equation has the immediate consequence

$$b_{-1}|\delta\mathcal{V}\rangle=0 \ . \tag{3.6}$$

I want to reexpress these formulas in terms of operators inserted on the boundary of the disk (rather than states), so that they can be taken off shell. A useful way to do this is as follows. Let v^i be the Killing vector field that generates a rotation of the disk, and let $\epsilon^j{}_k$ be the complex structure of the disk. Since v is a Killing vector field, the operator-valued one-form $b(v)=v^i b_{ij}\epsilon^j{}_k dx^k$ is closed. Let

$$b_\alpha=\oint_{C_\alpha} b(v) , \tag{3.7}$$

where the contour C_α is a distance α from the boundary of the disk. Since $b(v)$ is closed, the operator b_α, inserted in correlation functions, is independent of α except when the contour C_α crosses operator insertions. The operator b_α acts like b_{-1} on an open-string insertion on the boundary of the disk (it acts as $b_0-\bar{b}_0$ on a closed-string insertion at the center of the disk). A version of (3.6) that involves no assumption of conformal or BRST invariance, and hence makes sense off shell, is the statement

$$\lim_{\alpha\to 0} b_\alpha=0 . \tag{3.8}$$

This captures the idea that the operators on the boundary of the disk, which is at $\alpha=0$, are annihilated by b_{-1}. A similar version of (3.5) that makes sense off shell is

$$\lim_{\alpha\to 0} b_\alpha \mathcal{O}(\sigma)=\delta\mathcal{V}(\sigma) , \tag{3.9}$$

with σ an arbitrary point on the boundary of the disk. We will use the symbol b_{-1} as an abbreviation for $\lim_{\alpha\to 0} b_\alpha$, and so write (3.9) as $b_{-1}\mathcal{O}=\delta\mathcal{V}$.

On shell, when $\delta\mathcal{V}$ is given, $\mathcal{O}$ is uniquely determined, either by (3.2) or by the pair of equations

$$\delta\mathcal{V}=b_{-1}\mathcal{O} \tag{3.10}$$

and

$$0=\{Q,\mathcal{O}\} . \tag{3.11}$$

Off shell, neither (3.2) nor (3.11) makes sense. Equation (3.10) still makes sense, but it does not determine $\mathcal{O}$ uniquely. It determines $\mathcal{O}$ only modulo addition of an operator of the form $b_{-1}(\cdots)$. Actually, since we consider $\delta\mathcal{V}$ to be trivial if it is of the form $d(\cdots)$, $\mathcal{O}$ is also indeterminate up to addition of an operator of the form $d(\cdots)$. The possibility of adding a total derivative to δV or $\mathcal{O}$ causes no problem. The indeterminacy that causes a problem is the possibility of adding $b_{-1}(\cdots)$ to $\mathcal{O}$.

We might want to define the antibrackets off shell by the same formula we used on shell:

$$\omega(\delta_1\mathcal{V},\delta_2\mathcal{V})=\oint d\sigma_1\oint d\sigma_2\langle \mathcal{O}_1(\sigma_1)\mathcal{O}_2(\sigma_2)\rangle .$$

But this formula is ambiguous, since the $\mathcal{O}$'s are not uniquely determined by the $\delta\mathcal{V}$'s. I will make a proposal, though far from definitive, for solving this problem.

A. Enlarged space of theories

By comparison to string field theory, it is easy to see the origin of the problem. In string field theory, the basic field is an object of ghost number 1—an $\mathcal{O}$, in our present terminology—and the antibrackets are defined, accordingly, by a two-point function of $\mathcal{O}$'s. Since the perturbation of the (boundary term in the) Lagrangian of the two-dimensional field theory is defined by $\delta\mathcal{V}=b_{-1}\mathcal{O}$, in passing from $\mathcal{O}$ to $\delta\mathcal{V}$, we are throwing away some of the degrees of freedom, namely, the operators annihilated by b_{-1}. To solve the problem, one must find a role in the formalism for those operators. I will simply include them by hand.

Instead of saying that the basic object is a world-sheet Lagrangian of the form

$$I=I_0+\int_{\partial\Sigma} d\sigma\, \mathcal{V} , \tag{3.12}$$

I will henceforth say that the basic object is such a world-sheet Lagrangian together with a local operator $\mathcal{O}$ such that

$$\mathcal{V}=b_{-1}\mathcal{O} . \tag{3.13}$$

The left-hand side is now $\mathcal{V}$, not $\delta\mathcal{V}$, and so we are changing the meaning of $\mathcal{O}$. Since $\mathcal{V}$ is determined by $\mathcal{O}$, we can consider the basic variable to be $\mathcal{O}$, just as in string field theory. (However, just as in string field theory, one defines the statistics of the field to be the natural statistics of $\mathcal{V}$ and the opposite of the natural statistics of $\mathcal{O}$.) Now we can define the antibrackets:

$$\omega(\delta_1\mathcal{O},\delta_2\mathcal{O})=\oint d\sigma_1\oint d\sigma_2\langle \delta_1\mathcal{O}(\sigma_1)\delta_2\mathcal{O}(\sigma_2)\rangle . \tag{3.14}$$

To formally prove that $d\omega=0$, one proceeds as follows. First of all, if $U_i(\sigma_i)$ are any local operators inserted at points $\sigma_i\in\partial\Sigma$, then

$$0=\langle b_{-1}[U_1(\sigma_1)\cdots U_n(\sigma_n)]\rangle . \tag{3.15}$$

This is a consequence of the fact that (as all the operator insertions are on $\partial\Sigma$), the correlation function $\langle b_\alpha\prod_i U_i(\sigma_i)\rangle$ is independent of α. Taking the limit as the contour C_α shrinks to a point, this correlation function vanishes; taking it to approach $\partial\Sigma$, we get (3.15). This Ward identity can be written out in more detail as

$$\begin{aligned}0=&\langle [b_{-1}U_1(\sigma_1)]U_2(\sigma_2)\cdots U_n(\sigma_n)\rangle-(-1)^{\eta_1}\langle U_1(\sigma_1)[b_{-1}U_2(\sigma_2)]\cdots U_n(\sigma_n)\rangle\\ &+(-1)^{\eta_1+\eta_2}\langle U_1(\sigma_1)U_2(\sigma_2)[b_{-1}U_3(\sigma_3)\cdots]\rangle\pm\cdots=0 ,\end{aligned} \tag{3.16}$$

with η_i such that $(-1)^{\eta_i}$ is ∓ 1 for U_i bosonic or fermionic (and ± 1 for $b_{-1}U_i$ bosonic or fermionic). Now if $\mathcal{O}=\mathcal{O}_0+\sum_i t_i\mathcal{O}_i$, then

$$d\omega(\delta_i\mathcal{O},\delta_j\mathcal{O},\delta_k\mathcal{O})=\frac{\partial}{\partial t_i}\omega(\delta_j\mathcal{O},\delta_k\mathcal{O})\pm\text{cyclic permutations} . \tag{3.17}$$

Also, since $\partial/\partial t_i$ is generated by an insertion of $\delta_i\mathcal{V}=b_{-1}\delta_i\mathcal{O}$, we have

$$\frac{\partial}{\partial t_i}\omega(\delta_j\mathcal{O},\delta_k\mathcal{O})=\oint d\sigma_1 d\sigma_2 d\sigma_3\langle[b_{-1}\delta_i\mathcal{O}(\sigma)]\delta_j\mathcal{O}(\sigma_2)\delta_k\mathcal{O}(\sigma_3)\rangle \ . \tag{3.18}$$

Combining these formulas, we see that $d\omega=0$ is a consequence of (3.16). To establish BRST invariance of ω, one must show that $d(i_V\omega)=0$ or, in other words, that

$$0=\frac{\partial}{\partial t_i}\oint d\sigma_1 d\sigma_2\langle\delta_j\mathcal{O}(\sigma_1)\{Q,\mathcal{O}\}(\sigma_2)\rangle\pm i\leftrightarrow j \ . \tag{3.19}$$

This is proved similarly, using the additional facts that $\{b_{-1},Q\}=v^i\partial_i$ (the operator that generates the rotation of the circle) and $\oint d\sigma\, v^i\partial_i\mathcal{O}=0$.

B. Critique

What is unsatisfactory about all this? To begin with, we have been working formally in a "space of all open-string world-sheet theories," totally ignoring the ultraviolet divergences that arise when one starts adding arbitrary local operators (perhaps of very large positive or negative dimension) to the boundary action. Even worse, in my view, we have tacitly accepted that a theory is canonically determined by its Lagrangian, in this case $I=I_0+\int_{\partial\Sigma}d\sigma\,\mathcal{V}$. That is fine for cutoff theories with a particular cutoff in place, but runs into difficulties when one tries to remove the cutoff. In the limit in which one removes the cutoff the theory really depends on both $\mathcal{V}$ and the cutoff procedure that is used.

In our construction, can we work with a cutoff theory or do we need to remove the cutoff? The ingredients we needed were rotation invariance, invariance under b_{-1}, and Q invariance. There is no problem in picking a cutoff (such as a Pauli-Villars regulator in the interior of the disk) that preserves the first two (with a modified definition of b_{-1}), but there is presumably no cutoff that preserves Q. Therefore we need to take the limit of removing the cutoff. With a cutoff in place, one can use the above procedure to define ω and prove $d\omega=0$, but the cutoff ω will not be BRST invariant; one will have to hope to recover BRST invariance of ω in the limit in which the cutoff is removed.

This may well work, if a "space of all world-sheet theories" (with the desired tangent space) does exist. The main point that arouses skepticism is actually the existence of the wished-for theory space. Even if such a space exists, there is something missing (even at a formal level) in my above definition of ω. Because of the cutoff dependence at intermediate stages, an open-string field theory does not really have a naturally defined local operator $\mathcal{V}$ representing the boundary interaction. Even formally, there is some work to be done to explain what type of objects $\mathcal{V}$ and $\mathcal{O}$ are (independently of the particular cutoff procedure) such that the key equation $\mathcal{V}=b_{-1}\mathcal{O}$ makes sense. If this were accomplished, one could perhaps give a direct formal definition of ω manifestly independent of cutoff procedure.

IV. CONCLUSIONS

I hope that I have at least demonstrated in this paper that, in trying to make sense of the "space of all open-string world-sheet field theories," the important structure that this space should possess is a BRST-invariant antibracket. This will automatically lead to a natural, background-independent open-string field theory in which classical solutions are BRST-invariant world-sheet theories and on-shell gauge transformations are generated by the world-sheet BRST operator. The reasons for hoping that the appropriate antibrackets exist are that they exist on shell, they exist in string field theory, and they would exist (as we saw in the last section) if one could totally ignore ultraviolet questions. Moreover, the antibrackets are the one important structure that always exists in (appropriate) gauge fixing of classical field theory. Other structures, such as metrics in field space, etc., may or may not exist, but have no general significance in off-shell classical field theory.

Perhaps it is worth mentioning that although our considerations may appear abstract, they can be made concrete to the extent that one can make sense of the space of open-string field theories. One does not even need the space of *all* open-string field theories, since the considerations of this paper are local in theory space and never involve sums over unknown degrees of freedom. If one understands any concrete family of two-dimensional field theories, one can determine the function S on the parameter space of this family (up to an additive constant) by integrating the formula $V^I\omega_{IJ}=\partial_J S$; this formula can be made entirely concrete (in terms of correlation functions in the given class of theories). I hope to give some examples of this elsewhere.

It seems reasonable to expect that natural antibrackets also exist on the space of all two-dimensional closed-string field theories. It would be nice to understand at least a formal definition (even at the imprecise level of Sec. III). As for defining an anticommuting vector field on the space of closed-string theories, I hope that this can be done by embedding the two-dimensional world sheet as a nontopological defect in a topological theory of higher dimension and by using the higher-dimensional world much as we used the disk in the present paper. Background-independent closed-string field theory may therefore be closer than it appears.

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