

Quantum black holes in two dimensions

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We show that a whole class of quantum actions for dilaton gravity, which reduce to the Callan-Giddings-Harvey-Strominger theory in the classical limit, can be written as a Liouville-like theory. In a subclass of this, the field space singularity observed by several authors is absent, regardless of the number of matter fields, and in addition it is such that the dilaton-gravity functional integration range (the real line) transforms into itself for the Liouville theory fields. We also discuss some problems associated with the usual calculation of Hawking radiation, which stem from the neglect of back reaction. We give an alternative argument incorporating back reaction but find that the rate is still asymptotically constant. The latter is due to the fact that the quantum theory has no lower bound in energy and Hawking radiation takes positive Bondi (or Arnowitt-Deser-Misner) mass solutions to an arbitrarily large negative mass.

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I. INTRODUCTION

The theory of dilation gravity coupled to scalar fields proposed by Callan, Giddings, Harvey, and Strominger [1] (CGHS) has generated a flurry of activity on black-hole physics. What one has is a simple toy model, within which the puzzling questions associated with Hawking radiation [2] can be addressed in a systematic way. In the original work of CGHS, as well as in several subsequent papers, it was assumed that quantum effects to leading order could be included by just adding a piece to the action which reproduced the conformal anomaly. However, it was later realized that the consistent quantization of the theory in the conformal gauge required that the cosmological constant term and/or the kinetic terms should get renormalized in a dilaton-dependent manner so that the theory becomes a conformal field theory (CFT) [3,4]. This requirement that the theory be an exact CFT (though not necessarily a soluble one) is not a matter of choice. It is a necessary consequence of general covariance. In other words, dilaton gravity coupled to matter fields must be a CFT in exactly the same way that string theory [i.e., ordinary two-dimensional (2D) gravity coupled to matter fields] is a CFT.

In this paper we will first review this argument and then consider the generalization of previous solutions to the conformal invariance conditions. We show that there is a subset of models which are free of the quantum black-hole singularity pointed out in [5,6] and which are such that the original range of integration for the conformal factor and the dilation is transformed into itself for the Liouville theory fields. We will also argue that the calculations of Hawking radiation that have been given in the literature are inconsistent with the constraints and equations of motion of the theory in that they neglect

back reaction. There is no sensible approximation scheme in which the latter can be ignored. We then show that when the exact solution of the system of equations coming from a quantum-corrected action is considered, the results differ from previous calculations. However, it turns out that one cannot see the radiation turning off in this theory; the (Bondi) mass of the solutions of the theory can be arbitrarily negative, and the Hawking process causes a positive-mass solution to decay indefinitely to an infinitely negative mass. Although Liouville theory has a positive-definite spectrum, the same is not true of the Liouville-like theory that is obtained from the CGHS theory. It is possible that the origin of the problem lies in the CGHS theory itself, but a more rigorous quantum treatment of the Bondi mass may resolve this question.

In the next section we review the quantization of the CGHS theory. In the third section we discuss a class of solutions to the integrability conditions for the constraints and present arguments for taking the resulting exact conformal field theory as a quantum theory of dilaton gravity. In the fourth section we demonstrate explicitly how the classical singularities are tamed by quantum effects. In the fifth section we review the CGHS calculation of Hawking radiation in this model. In the sixth section we give an alternative calculation which is consistent with the constraints (this is basically a detailed version of a calculation contained in the second paper [3]), and in the final section we make some concluding remarks.

II. QUANTIZATION

The CGHS theory is defined by the classical action

$$S = \frac{1}{4\pi} \int d^2\sigma \sqrt{-g} \left[e^{-2\phi} [R + 4(\nabla\phi)^2 + 4\lambda^2] - \frac{1}{2} \sum_{i=1}^N (\nabla f^i)^2 \right]. \quad (2.1)$$

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In the above G is the 2D metric, R is its curvature scalar, ϕ is the dilaton, and the f^i are N scalar matter fields. This action may be obtained as a low-energy effective action from string theory, in which case the f fields will arise from the Ramond-Ramond sector. Note that the (zero-mass) tachyon of 2D string theory is excluded from this action. If this field had been coupled then the theory would not be solvable even at the classical level.¹

The quantum field theory of this classical action may be defined as

$$Z = \int \frac{[dg]_{\hat{g}} [d\phi]_{\hat{g}} [df]_{\hat{g}}}{[dV]} e^{iS[g,\phi,f]}, \quad (2.2)$$

where dV is the volume element of the diffeomorphism group.

The metrics which define these measures are usually given by

$$\begin{aligned} \|\delta g\|_{\hat{g}}^2 &= \int d^2\sigma \sqrt{-g} g^{\alpha\gamma} g^{\beta\delta} (\delta g_{\alpha\beta} \delta g_{\gamma\delta} + \delta g_{\alpha\gamma} \delta g_{\beta\delta}), \\ \|\delta\phi\|_{\hat{g}}^2 &= \int d^2\sigma \sqrt{-g} \delta\phi^2, \\ \|\delta f\|_{\hat{g}}^2 &= \int d^2\sigma \sqrt{-g} \delta_{ij} \delta f^i \delta f^j. \end{aligned} \quad (2.3)$$

However, we can be more general in these definitions as long as 2D diffeomorphism invariance is preserved. Now let us gauge fix to the conformal gauge $g = e^{2\rho} \hat{g}$ and rewrite the measures with respect to the fiducial metric \hat{g} . Following the work of David and of Distler and Kawai [8], we may expect the action to get renormalized, except that unlike in their case the renormalization will be dilaton dependent (since the coupling is $e^{2\phi}$). Thus, in general we may expect the gauge-fixed path integral to be written as [9,7,3]²

$$\begin{aligned} Z &= \int [dX^\mu]_{\hat{g}} [df]_{\hat{g}} [(db)[dc]]_{\hat{g}} \\ &\quad \times \exp[iI(X,\hat{g}) + iS(f,\hat{g}) + iS(b,c,\hat{g})], \end{aligned} \quad (2.4)$$

where

$$\begin{aligned} I[X,\hat{g}] &= -\frac{1}{4\pi} \int \sqrt{-\hat{g}} [\frac{1}{2} \hat{g}^{ab} G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \\ &\quad + \hat{R} \Phi(X) + T(X)]. \end{aligned} \quad (2.5)$$

$S(b,c,\hat{g})$ is the Faddeev-Popov ghost action, and we have written $(\phi,\rho) = X^\mu$. Note that all the measures in (2.4) are defined with respect to the 2D metric \hat{g} and that in particular the measure $[dX^\mu]$ is derived from the natural metric on the space $\|\delta X_\mu\|^2 = \int d^2\sigma \sqrt{-\hat{g}} G_{\mu\nu} \delta X^\mu \delta X^\nu$. In the limit of weak coupling ($e^{2\phi} \ll 1$) we have

$$\begin{aligned} I \rightarrow \frac{1}{4\pi} \int d^2\sigma \sqrt{-\hat{g}} \{ e^{-2\phi} [4(\hat{\nabla}\phi)^2 - 4\hat{\nabla}\phi \cdot \hat{\nabla}\rho] - \kappa \hat{\nabla}\rho \cdot \hat{\nabla}\rho \\ + \hat{R} (e^{-2\phi} - \kappa\rho) - 4\lambda^2 e^{2(\rho-\phi)} \}. \end{aligned} \quad (2.6)$$

¹For a discussion of how in this case 2D black-hole solutions are affected far away from the black hole by the presence of the tachyon see [7].

²For alternative approaches to the quantization see [10].

This is obtained from (2.1) by putting $g = e^{2\rho} \hat{g}$ and including a very specific high-order term, namely, the usual conformal anomaly term. κ in the above is equal to $[26 - (N+2)]/6 = (24-N)/6$ if one includes the contribution of the transformation of the measure for ϕ and ρ .³

I is a generalized σ -model action, and we have kept only renormalizable terms. The σ -model action introduces three (dilaton-dependent) coupling functions G, Φ , and T , respectively, the field-space metric, dilaton, and tachyon. The only *a priori* restriction arises from the fact that the functional integral for Z in (2.4) must be independent of the fiducial metric \hat{g} , as is obvious from the expression (2.2) for it. This implies that the following constraints should be satisfied:

$$\langle T_{\pm\pm} + t_{\pm\pm} \rangle = 0 \quad (2.7)$$

and

$$\langle T_{+-} + t_{+-} \rangle = 0, \quad (2.8)$$

where $T_{\mu\nu}$ is the stress tensor for the dilaton-gravity and matter sectors, and $t_{\mu\nu}$ is the stress tensor for the ghost sector. [Equation (2.8) is equivalent to the equation of motion for ρ and so is not an additional constraint.] Furthermore, one has to satisfy the integrability conditions for these constraints, namely, that they generate a Virasoro algebra with zero central charge.⁴ As is well known (see for instance [11] and references therein), this is equivalent to the requirement that the β functions [12] corresponding to the coupling functions G, Φ , and, T vanish:

$$\begin{aligned} \beta_{\mu\nu} &= \mathcal{R}_{\mu\nu} + 2\nabla_\mu^G \partial_\nu \Phi - \partial_\mu T \partial_\nu T + \dots, \\ \beta_\Phi &= -\mathcal{R} + 4G^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - 4\nabla_G^2 \Phi + \frac{(N+2)-26}{3} \\ &\quad + G^{\mu\nu} \partial_\mu T \partial_\nu T - 2T^2 + \dots, \\ \beta_T &= -2\nabla_G^2 T + 4G^{\mu\nu} \partial_\mu \Phi \partial_\nu T - 4T + \dots, \end{aligned} \quad (2.9)$$

where \mathcal{R} is the curvature of the metric G . These equations have to be solved under the boundary conditions that in the weak-coupling limit ($e^{2\phi} \ll 1$) we get, comparing (2.5) with (2.6),

$$G_{\phi\phi} = -8e^{-2\phi}, \quad G_{\phi\rho} = 4e^{-2\phi}, \quad G_{\rho\rho} = 2\kappa, \quad (2.10)$$

$$\Phi = -e^{-2\phi} + \kappa\rho, \quad T = -4\lambda^2 e^{2(\rho-\phi)}. \quad (2.11)$$

III. FROM CGHS THEORY TO LIOUVILLE THEORY

Let us first discuss the renormalization of the field-space metric and dilaton (G and Φ) and postpone the discussion of the tachyon T . The (renormalized) field space

³We will justify this in more detail later on.

⁴In effect this means that the field space must be exactly like the target space of string theory, though here we do not give this space a space-time interpretation. The only space-time in the theory is the original one parametrized by the coordinated σ .

metric may be parametrized as

$$ds^2 = -8e^{-2\phi}[1+h(\phi)]d\phi^2 + 8e^{-2\phi}[1+\bar{h}(\phi)]d\rho d\phi + 2\kappa(1+\bar{h})d\rho^2, \quad (3.1)$$

where h, \bar{h} , and \bar{h} are $O(e^{2\phi})$. If we are going to consider only $O(e^{2\phi})$ effects, then we should certainly set \bar{h} to zero. But even if we consider the renormalization functions h and \bar{h} to all orders, it is consistent to limit ourselves to the class of quantum versions of the CGHS theory which have $\bar{h}=0$, provided that we satisfy the β -function equations. This corresponds to confining ourselves to theories in which the field-space curvature $\mathcal{R}=0$. In this case we can transform this metric to Minkowski form. First put⁵

$$y = \rho - \kappa^{-1}e^{-2\phi} + \frac{2}{\kappa} \int d\phi e^{-2\phi} \bar{h}(\phi). \quad (3.2)$$

Then the metric becomes

$$ds^2 = -\frac{8}{\kappa} P^2(\phi) d\phi^2 + 2\kappa dy^2,$$

where

$$P(\phi) = e^{-2\phi} [(1+\bar{h})^2 + \kappa e^{2\phi}(1+h)]^{1/2}. \quad (3.3)$$

Putting

$$x = \int d\phi P(\phi), \quad (3.4)$$

we have

$$ds^2 = -\frac{8}{\kappa} dx^2 + 2\kappa dy^2. \quad (3.5)$$

With this form of the metric, ignoring $O(T^2)$ terms, we find from the first (graviton) β -function equation in (2.9) that $\partial_\mu \partial_\nu \Phi = 0$. In the other words, Φ is linear in x, y . Demanding that we recover the CGHS Φ given in (2.11) in the weak-coupling limit, we find the unique solution

$$\Phi = \kappa y. \quad (3.6)$$

Substituting in the second (dilaton) equation in (2.9), we then get

$$\kappa = \frac{24-N}{6}. \quad (3.7)$$

To determine T we consider the third equation of (2.9), to linear order, and get

$$\frac{\kappa}{4} \partial_x^2 T - \frac{1}{\kappa} \partial_y^2 T + 2\partial_y T - 4T = 0. \quad (3.8)$$

This has solutions of the form $T = e^{\beta x + \alpha y}$, where $\frac{1}{4}\kappa\beta^2 - (1/\kappa)\alpha^2 + 2\alpha - 4 = 0$. Now we need to impose the boundary condition that we recover the CGHS tachyon given in (2.11) in the weak-coupling limit. To do so we

expand the expression for x [(3.4), (3.3)] to get

$$x \simeq -\frac{1}{2}e^{-2\phi} + \int d\phi e^{-2\phi} \bar{h} + \frac{\kappa}{2}\phi + O(e^{2\phi}). \quad (3.9)$$

Then we find that $-(4/\kappa)x + 2y = 2\rho - 2\phi + O(e^{2\phi})$, so that the unique solution (confining ourselves to multiplicative renormalizations) obeying the required boundary condition is

$$T = -4\lambda^2 \exp\left[-\frac{4}{\kappa}x + 2y\right]. \quad (3.10)$$

For $\kappa > 0$ there is another (additive) term⁶ satisfying the boundary condition, namely,

$$T_{np} = \mu e^{(4/\sqrt{\kappa})x} \simeq \mu \exp\left[-\frac{2}{\sqrt{\kappa}}e^{-2\phi}\right].$$

This is in fact a nonperturbative ambiguity. We will set $\mu=0$ in the rest of the paper. In any case, it is absent for $\kappa < 0$, since in that case we will have an oscillatory solution which will not vanish in the classical limit.

It is convenient now to introduce rescaled fields,

$$X = 2 \left[\frac{2}{|\kappa|} \right]^{1/2} x, \quad Y = \sqrt{2|\kappa|} y, \quad (3.11)$$

in terms of which the metric and the tachyon become

$$ds^2 = \mp dX^2 \pm dY^2,$$

$$T = -4\lambda^2 \exp[\mp \sqrt{2/|\kappa|}(X \mp Y)].$$

In the above and in the equations below, upper/lower signs correspond to having $\kappa > 0/\kappa < 0$, respectively. In terms of the new field variables the functional integral becomes

$$Z = \int [dX][dY][df][db][dc] \exp(iS[X, Y, f] + iS_{\text{ghost}}), \quad (3.12)$$

where

$$S = \frac{1}{4\pi} \int d^2\sigma \left\{ \mp \partial_+ X \partial_- X \pm \partial_+ Y \partial_- Y + \sum_i \partial_+ f^i \partial_- f^i + 2\lambda^2 \exp\left[\mp \left[\frac{2}{|\kappa|} \right]^{1/2} (X \mp Y)\right] \right\}. \quad (3.13)$$

Several comments need to be made about this functional integral. First and most obviously, there is the question of the range of the integration. As we see from (3.4) and (3.3), in general the range of integration in X will not extend over the whole real line. What we then have is an approximate solution to the β -function equations (2.9) valid only to leading order in the σ -model (α') expansion and to leading order in the weak field expansion in T . On

⁵We will only consider theories with $\kappa \neq 0$, i.e., $N \neq 24$.

⁶I wish to thank Andy Strominger for pointing this out to me [13].

the other hand, if we *define* the quantum theory by (3.12) with the range of integration for X being the whole real line, we have⁷ a solution to the exact β -function equations. Thus, this definition of quantum dilaton-gravity theory, even if somewhat unorthodox, is a very compelling one. It is on the same footing as, for instance, the definition of (2+1)-dimensional quantum gravity given by Witten [14] in which the functional integral is taken over all values of the vielbein field. Also let us point out that if we restrict the range of integration to be consistent with the original definition of the quantum theory, then since we only have a solution to the leading-order β -function equations, it seems as if we will need an infinite number of terms to satisfy the exact conformal invariance conditions. It is plausible to suppose that this theory is equivalent to the one above with the unrestricted range of integration. This argument is also reinforced by the fact that, as in the usual Liouville theory, the integration range is effectively cut off (albeit softly) by the Liouville potential term. Finally (and perhaps this is the most compelling reason for the quantum Liouville-like conformal field theory), there exist choices of h and \bar{h} for which, when the integration ranges for ϕ and ρ are as usual taken over the whole real line, the same is true for the ranges for X and Y [see case (d) at the end of this section].

The second comment is with regard to the approximation in which the dilaton and graviton loops can be ignored. By rescaling and translating the fields X, Y it is easily seen that $\hbar = \kappa$, so that the semiclassical approximation is valid only for large κ . Thus, one might be inclined to believe that any (even qualitative) conclusions derived for the $N < 24$ theory [3] are drastically effected by dilaton-graviton loop corrections. On the other hand, for $N = 1$ we get $\kappa = 3.8$, which is of the same order as the relevant parameter in QCD where the approximation works quite well.

Finally we comment on the different possibilities for the functions h and \bar{h} . Three special cases have so far been discussed in the literature.

(a) $h = \bar{h} = 0$. That is, the field-space metric of the classical CGHS Lagrangian is not renormalized. However, in this case the cosmological constant term T is renormalized.

(b) $h = -e^{2\phi}, \bar{h} = -2e^{2\phi}$. This is the case proposed by Strominger [16]. In this case both the metric G and the tachyon T are renormalized.

(c) $h = 0, \bar{h} = -(\kappa/4)e^{4\phi}$. This is the case considered in [17] where P^2 is a perfect square [see (3.3) from which we find $P = e^{-2\phi}(1 + \frac{1}{4}\kappa e^{2\phi})$]. In this case the metric is (obviously) renormalized, but the tachyon is not [as is easily seen from (3.10) and the expressions for x and y with the above value of P].

(d) In all of the above cases the transformation (3.4) has a singularity when $\kappa < 0$. For instance, in case (a) it is at $e^{2\phi} = -\kappa^{-1}$. It is, however, quite easy to find a class of models which have no such singularity. Put $\bar{h} = ae^{2\phi}$ and

$h = be^{2\phi}$. Then, putting $e^{2\phi} = z$, the condition for the absence of a singularity is that the quadratic equation $z^2 P^2 = (a^2 + \kappa b)z^2 + (2a + \kappa)z + 1 = 0$ has no real roots. I.e., we must choose $\kappa^2 + 4(a - b)\kappa < 0$. Obviously there are many solutions to these conditions, but this class is of particular importance since members of it naturally allow the range of integration in the X, Y variables to go over the whole real line. The simplest member of this class has $h = 0$ and $\bar{h} = -\frac{1}{2}\kappa e^{2\phi}$. In this case we have, from (3.3), (3.4), and (3.2),

$$x = \int d\phi e^{-2\phi} \left[1 + \frac{\kappa^2}{4} e^{4\phi} \right]^{1/2} \\ = -\frac{1}{2} \left[\frac{\kappa^2}{4} + e^{-4\phi} \right]^{1/2} + \frac{|\kappa|}{4} \operatorname{arcsinh} \left[\frac{|\kappa|}{2} e^{2\phi} \right], \quad (3.14)$$

and

$$y = \rho - \kappa^{-1} e^{-2\phi} - \phi. \quad (3.15)$$

Clearly as ϕ, ρ range from $-\infty$ to $+\infty$ so do x and y .

IV. EXACT SOLUTIONS

The equations of motion coming from (3.13) are⁸

$$\partial_+ \partial_- f = 0, \quad (4.1)$$

$$\partial_+ \partial_- X = \lambda^2 \left[\frac{2}{|\kappa|} \right]^{1/2} \exp \left[\left[\frac{2}{|\kappa|} \right]^{1/2} (X + Y) \right], \quad (4.2)$$

$$\partial_+ \partial_- Y = -\lambda^2 \left[\frac{2}{|\kappa|} \right]^{1/2} \exp \left[\left[\frac{2}{|\kappa|} \right]^{1/2} (X + Y) \right].$$

We have taken the case with the lower signs in (3.13) so that the discussion is for $N > 24$. There is no qualitative difference in the other case, so it is unnecessary to write it out explicitly.⁹ These equations are easily solved. From (4.2) we have $\partial_+ \partial_- (X + Y) = 0$, so that $X + Y = \sqrt{|\kappa|}/2 [g_+(\sigma^+) + g_-(\sigma^-)]$ where g_{\pm} are arbitrary chiral functions. Substituting into the X equation of motion and integrating we have

$$X = - \left[\frac{2}{|\kappa|} \right]^{1/2} [u_+(\sigma^+) + u_-(\sigma^-)] \\ + \lambda^2 \left[\frac{2}{|\kappa|} \right]^{1/2} \int^{\sigma^+} d\sigma^+ e^{g_+(\sigma^+)} \int^{\sigma^-} d\sigma^- e^{g_-(\sigma^-)} \\ = -Y + \left[\frac{|\kappa|}{2} \right]^{1/2} (g_+ + g_-), \quad (4.3)$$

where u_{\pm} are arbitrary chiral functions to be determined by the boundary conditions.

⁸In this section and in Sec. VI, wherever it is appropriate, all equations are to be understood as being valid inside the functional integral, i.e., as expectation values of quantum operators. Since, following the arguments of Ref. [15], the theory can be mapped into a free theory, it is plausible that the only quantum effects come from normal ordering.

⁹It is also contained in [4] and the second paper of [3].

⁷The theory is very much like Liouville theory, which is an exact CFT [15]. In fact, it is less singular than Liouville, so one expects it to be an exact CFT as well.

By a coordinate choice we can set $g_{\pm}=0$. In these coordinates (the analogue of Kruskal-Szekeres coordinates for the black hole) we get

$$X = -Y = - \left[\frac{2}{|\kappa|} \right]^{1/2} (u - \lambda^2 \sigma^+ \sigma^-), \quad (4.4)$$

where $u = u_+ + u_-$.

These solutions are, of course, the same as those of CGHS, except that they are for X and Y and all effects of the quantum anomalies are now incorporated in the expressions for them in terms of ρ and ϕ . To be explicit, consider the case (d) discussed at the end of the last section ($h=0$, $\bar{h} = -\frac{1}{2}\kappa e^{2\phi}$):

$$\begin{aligned} X &= 2 \left[\frac{2}{|\kappa|} \right]^{1/2} \int d\phi e^{-2\phi} \left[1 + \frac{\kappa^2}{4} e^{4\phi} \right]^{1/2} \\ &= \sqrt{2|\kappa|} \int d\phi \left[1 + \frac{4}{\kappa^2} e^{-4\phi} \right]^{1/2} \end{aligned}$$

and

$$Y = \sqrt{2|\kappa|} \rho + \left[\frac{2}{|\kappa|} \right]^{1/2} e^{-2\phi} - \sqrt{2|\kappa|} \phi.$$

In the weak-coupling limit ($e^{2\phi} \ll 1$) we have from (4.4) the classical solution

$$e^{-2\phi} = e^{-2\rho} = u - \lambda^2 \sigma^+ \sigma^-, \quad (4.5)$$

which exhibits the classical (black-hole type) singularity on the curve where the right-hand side vanishes. But the singularity is in the strong-coupling region where we have to use the strong-coupling expansion (from the second line of the above equation for X)

$$X \simeq \sqrt{2|\kappa|} \left[\phi - \frac{e^{-4\phi}}{\kappa^2} + \dots \right].$$

Then we have, from (4.4),

$$\phi \simeq \kappa^{-1} (u - \lambda^2 \sigma^+ \sigma^-)$$

and

$$\rho \simeq \frac{1}{\kappa} \exp[-2\kappa^{-1} (u - \lambda^2 \sigma^+ \sigma^-)].$$

The metric ($e^{2\rho}$) is clearly nonsingular at the classical singularity. Differentiating the solution for X with respect to σ_{\pm} we get

$$2e^{-2\phi} \partial_{\pm} \phi = - \frac{\partial_{\pm} u_{\pm} - \lambda^2 \sigma^{\mp}}{\bar{P}(\phi)}, \quad (4.6)$$

where $\bar{P} = e^{2\phi} P$, P being defined by (3.3). This equation gives the trajectory of the apparent horizon ($\partial_+ \phi = 0$) introduced in [6] (once the unknown function u is determined) as

$$\sigma^- = \frac{1}{\lambda^2} \partial_+ u_+(\sigma^+). \quad (4.7)$$

By differentiating the solution for Y and using (4.6) and

the expression for Y in terms of ρ , ϕ , we have

$$\begin{aligned} \kappa \partial_- \partial_+ \rho &= (1 + \bar{h}) \partial_- \bar{h} (\partial_+ u_+ - \lambda^2 \sigma^-) \bar{P}^{-1} \\ &\quad - (1 + \bar{h}) (\partial_+ u_+ - \lambda^2 \sigma^-) \partial_- \bar{P} \bar{P}^{-2} \\ &\quad + \lambda^2 \left[1 - \frac{1 + \bar{h}}{\bar{P}} \right]. \end{aligned} \quad (4.8)$$

From this expression the curvature $R = 8e^{-2\rho} \partial_+ \partial_+ \rho$ is easily seen to be nonsingular at the classical singularity in all the cases (a)–(d) discussed at the end of Sec. III (as is obvious from the fact that the metric is nonsingular there), and furthermore, in case (d) it is seen that there are no curvature singularities anywhere for either sign of κ .

V. PROBLEMS IN CALCULATING HAWKING RADIATION

Before we calculate Hawking radiation we would like to comment on previous calculations of this phenomenon in 2D dilaton gravity. These comments may have a bearing on the original calculation [2] in 4D as well.

In the CGHS calculation the stress-tensor anomaly is added to the classical stress-tensor trace to give

$$\begin{aligned} T_{-+} &= e^{-2\phi} (2\partial_+ \partial_- \phi - 4\partial_+ \phi \partial_- \phi) \\ &\quad - \lambda^2 e^{2\rho - 2\phi} - \frac{1}{6} N \partial_+ \partial_- \rho. \end{aligned}$$

From the conservation equation for the stress tensor the remaining components of the stress tensor are then determined to be

$$T_{\pm\pm} = e^{-2\phi} (4\partial_{\pm} \rho \partial_{\pm} \phi - 2\partial_{\pm}^2 \phi) + T_{\pm\pm}^f,$$

with the quantum (one-loop) part of the stress tensor being given by

$$T_{\pm\pm}^f = -\frac{1}{6} N [\partial_{\pm} \rho \partial_{\pm} \rho - \partial_{\pm}^2 \rho + t_{\pm}(\sigma^{\pm})],$$

where t_{\pm} are arbitrary chiral functions to be determined by the boundary conditions. Of course, in a consistent quantization ghosts have to be included and $N \rightarrow N - 24$ [16,3,4] and t_{\pm} must be related to the ghost stress tensor [3,4], but we will ignore this for the moment. The usual argument then goes as follows. To leading order, Hawking radiation may be computed by substituting the classical solution (corresponding to the formation of a black hole due to an incoming matter shock wave along $\sigma^+ = \sigma_0^+$) into the quantum piece of the stress tensor T^f , and then imposing boundary conditions. In terms of the asymptotically Minkowski coordinates $\bar{\sigma}^+ = (1/\lambda) \ln(\lambda \sigma^+)$, $\bar{\sigma}^- = -(1/\lambda) \ln(-\lambda \sigma^- - a/\lambda)$, the classical solution is

$$2\rho = -\ln \left[1 + \frac{a}{\lambda} \exp(\lambda \sigma^-) \right], \quad \sigma^+ < \sigma_0^+,$$

$$2\rho = -\ln \left[1 + \frac{a}{\lambda} \exp[\lambda(\sigma^- - \sigma^+ + \sigma_0^+)] \right], \quad \sigma^+ > \sigma_0^+.$$

Substituting this in T_{-+}^f and demanding that the latter vanishes for $\sigma^+ < \sigma_0^+$, one determines $t_-(\sigma^-)$

$= -\frac{1}{4}\lambda^2\{1-1/[1+(a/\lambda)e^{\lambda\sigma^-}]^2\}$. Then observing that $\partial\rho, \partial^2\rho \rightarrow 0$ when $\sigma^+ \rightarrow \infty$ (\mathcal{J}_+^R in the Penrose diagram), we have

$$T_{--}^f \rightarrow \frac{N}{24}\lambda^2 \left[1 - \frac{1}{[1+(a/\lambda)e^{\lambda\sigma^-}]^2} \right].$$

This determines the Hawking radiation rate at timelike future infinity to be $N/24\lambda^2$, in agreement with earlier calculations (see for instance [18]).

This calculation, however, neglects back reaction. This is, of course, true for all previous calculations of Hawking radiation. In the original calculations [2,18] one quantized in a fixed-background metric, which means that back reaction is ignored. But there is no sensible approximation in which back reaction can be ignored. Back reaction is of the same order as the radiation. Within the context of this toy model and our explicit solution of it, this problem can be resolved. But before we do it let us elaborate on this question further.

This point is that the one-loop- (matter-) corrected theory has an action [Eq. (23) of [1]] and associated equations of motion and constraints. Aside from the dilaton equation, these correspond to (2.8) and (2.7) and read, in this notation,

$$\begin{aligned} T_{-+} &= T_{-+}^{\text{cl}} + T_{-+}^f = 0, \\ T_{\mp\mp} &= T_{\mp\mp}^{\text{cl}} + T_{\mp\mp}^f = 0. \end{aligned} \quad (5.1)$$

One has to now find a consistent solution to this set of equations (and the ϕ equation of motion). Such a solution will have a classical piece plus a one-loop quantum correction. Now, in calculating T^f to order \hbar it is sufficient to substitute the classical part of the solution into it. But to the same order one should keep the result of substituting the $O(\hbar)$ correction to the classical solution into T^{cl} . One should not just keep the former as Hawking radiation and ignore the latter. In fact, the classical solution by definition satisfies the classical equations $T_{-+}^{\text{cl}} = 0, T_{\pm\pm}^{\text{cl}} = 0$, so that in order to satisfy (5.1), the leading quantum correction to the classical solution when substituted into T^{cl} must give a value which exactly cancels the value obtained by substituting the classical solution into T^f . The CGHS calculation, of course, agrees with the calculations involving quantization in a fixed background since keeping the background fixed is tantamount to ignoring the quantum correction to the classical solution and is of course inconsistent with the quantum-corrected equations of motion and constraint.

A related point is that the energy-momentum-conservation equation and the equation of motion for ρ make $T_{\pm\pm}$ chiral fields as in conformal field theory.¹⁰ This is because in any conformal gauge the stress-tensor-conservation law (which is a consequence of general covariance and the matter-dilaton equations of motion) takes the form

$$\partial_{\pm} T_{\mp\mp} + \partial_{\mp} T_{\pm\pm} - 2\partial_{\mp}\rho T_{\pm\pm} = 0,$$

and the equation of motion for ρ is equivalent to the first equation of (5.1), so that $\partial_{\pm} T_{\mp\mp} = 0$. Since this is automatically true for t it is also true separately for the nonghost part of the stress tensor. Now how can we identify the ‘‘radiation’’ part of the stress tensor? As we argued earlier it does not make sense to just subtract off the ‘‘classical’’ part of the stress tensor. One can subtract the classical value of the classical stress tensor (i.e., the value when the classical solution is substituted into it). But by definition this is zero, so we are left with the whole stress tensor. Also, as we have seen, T_{--} is independent of σ^+ and hence cannot be zero in the region $\sigma^+ < \sigma_0^+$ and nonzero for $\sigma^+ > \sigma_0^+$. Indeed, since T in this section is defined to include the ghost contribution (the translation is $-\frac{1}{6}Nt_{\pm} \rightarrow t_{\pm\pm}$) it is zero everywhere, for that is the equation of constraint [second equation of (5.1)].

VI. A PROPOSAL FOR CALCULATING HAWKING RADIATION

How then can we identify Hawking radiation? In general relativity there is a definition of the energy left in a system which is asymptotically flat, after radiation has been emitted for a certain time. This is the so-called Bondi mass. This is defined relative to some reference static solution and must be given in asymptotically Minkowski coordinates. So if $\delta T_{\mu\nu}$ is the first variation of the stress tensor around the reference solution, then for a solution (static or nonstatic) which asymptotically approaches the static solution at future null infinity, the Bondi mass is given as (σ^{\pm} are the asymptotically Minkowski coordinates)

$$M(\bar{\sigma}^-) = \int^{\mathcal{J}_R^+} d\bar{\sigma} + \delta T_+^0 = - \int^{\mathcal{J}_R^+} d\bar{\sigma}^+ (\delta T_{++} + \delta T_{+-}). \quad (6.1)$$

In the above the integral is to be evaluated at the future null infinity line \mathcal{J}_R^+ , i.e., at $\bar{\sigma}^+ \rightarrow \infty$. Now the linearized stress tensor satisfies the linearized conservation equation

$$\partial_{\mp} \delta T_{\pm\pm} + \partial_{\pm} \delta T_{\mp\mp} = 0. \quad (6.2)$$

Using this we find, from (6.1),

$$\begin{aligned} \partial_- M(\bar{\sigma}^-) &= - \int^{\mathcal{J}_R^+} d\bar{\sigma}^+ (\partial_- \delta T_{++} + \partial_- \delta T_{+-}) \\ &= + \int^{\mathcal{J}_R^+} d\bar{\sigma}^+ (\partial_+ \delta T_{+-} + \partial_+ \delta T_{--}) \\ &= (\delta T_{+-} + \delta T_{--})_{\mathcal{J}_R^+}. \end{aligned} \quad (6.3)$$

This equation gives the rate of decay of the Bondi mass. We may therefore identify the negative of the right-hand side as the radiation flowing out to future null infinity.

To proceed we need the exact solutions of our quantum-correlated equations of motion (4.3) or (4.4). Once a coordinate system is chosen, these solutions are given in terms of two unknown chiral functions $u_{\pm}(\sigma^{\pm})$ which need to be determined from the constraint equa-

¹⁰Indeed the theory, is, as we argued earlier, a conformal field theory.

tions and the boundary conditions. As we argued in the last section, the boundary conditions that are usually used do not make sense because of the chirality of the stress tensor, so we have to proceed in an alternative manner. Let us first impose the constraint equations (2.7).

The stress tensor calculated from (3.13) is

$$\begin{aligned} T_{\pm\pm} &= \frac{1}{2}(\partial_{\pm}X \partial_{\pm}X - \partial_{\pm}Y \partial_{\pm}Y) \\ &+ \left(\frac{|\kappa|}{2}\right)^{1/2} \partial_{\pm}^2 Y + \frac{1}{2} \sum_i \partial_{\pm} f^i \partial_{\pm} f^i \\ &= e^{-2\phi} [4\partial_{\pm}\phi \partial_{\pm}\rho - 2\partial_{\pm}^2\phi + O(\kappa e^{2\phi})] \\ &+ \frac{1}{2} \sum_i \partial_{\pm} f^i \partial_{\pm} f^i + \kappa(\partial_{\pm}\rho \partial_{\pm}\rho - \partial_{\pm}^2\rho), \end{aligned} \quad (6.4)$$

and

$$T_{+-} = -\sqrt{\kappa/2} \partial_+ \partial_- Y - \lambda^2 \exp[\sqrt{2/|\kappa}|(X+Y)]. \quad (6.5)$$

In the coordinate system in which g_{\pm} are zero we have from (4.3)

$$\begin{aligned} T_{\pm\pm} &= \frac{1}{2} \sum_i \partial_{\pm} f^i \partial_{\pm} f^i + \sqrt{\kappa/2} \partial_{\pm}^2 Y \\ &= \frac{1}{2} \sum_i \partial_{\pm} f^i \partial_{\pm} f^i + \partial_{\pm}^2 u_{\pm}. \end{aligned}$$

Hence, the constraint equations (2.7) become

$$\partial_{\pm}^2 u_{\pm} + \frac{1}{2} \sum_i \partial_{\pm} f^i \partial_{\pm} f^i + t_{\pm\pm} = 0. \quad (6.6)$$

Now we have the problem of determining the ghost stress tensor t . This, as well as the nonghost stress tensors $T^{X,Y}$, T^f transform like connections under coordinate transformation because of the conformal anomaly. It is only the sum which transforms as a tensor (since the conformal anomalies cancel between the two). Thus, under a conformal coordinate transformation $\sigma^{\pm} \rightarrow \sigma'^{\pm} = f^{\pm}(\sigma^{\pm})$,

$$\begin{aligned} T'^f_{\pm\pm}(\sigma') &= \left(\frac{\partial f^{\pm}}{\partial \sigma^{\pm}}\right)^{-2} \left[T^f_{\pm\pm}(\sigma) + \frac{N}{12} Df^{\pm} \right], \\ T'^{X,Y}_{\pm\pm} &= \left(\frac{\partial f^{\pm}}{\partial \sigma^{\pm}}\right)^{-2} \left[T^{X,Y}_{\pm\pm}(\sigma) + \frac{26-N}{12} Df^{\pm} \right], \\ t'_{\pm\pm}(\sigma') &= \left(\frac{\partial f^{\pm}}{\partial \sigma^{\pm}}\right)^{-2} \left[t_{\pm\pm}(\sigma) + \frac{-26}{12} Df^{\pm} \right], \end{aligned} \quad (6.7)$$

where Df is the Schwartz derivative defined by

$$Df = \frac{f'''}{f'} - \frac{3}{2} \left(\frac{f''}{f'} \right)^2.$$

At this point we do not know how to proceed without making an assumption about the boundary conditions. We assume that in a preferred coordinate system, namely, one which covers the whole space (i.e., including the region behind the classical horizon) and is asymptotically Minkowski, the expectation value of the matter stress tensor T^f_{--} vanishes. These coordinates are related to the Kruskal-Szekeres coordinates by

$$\hat{\sigma}^+ = \frac{1}{\lambda} \ln(\lambda\sigma^+), \quad \hat{\sigma}^- = -\frac{1}{\lambda} \ln(-\lambda\sigma^-). \quad (6.8)$$

This condition seems to correspond to Hawking's boundary condition, and the reasoning is that there should be no f -particle energy coming in from \mathcal{J}^- . Now from the point of view of the exact theory the total (including ghosts) stress tensor is zero, so that it is difficult to see what objective meaning this condition has. Nevertheless, in order to be as close as possible to the original calculation, let us impose

$$\hat{T}^f_{--} = 0, \quad \hat{T}^{X,Y}_{--} + \hat{t}_{--} = 0.$$

The latter follows from the first equation and the constraint. However, it still leaves us the freedom of choosing the separate values of the ghost and X, Y stress tensors. Let us put $\hat{t}_{--} = -\alpha\lambda^2/24 = -\hat{T}^{X,Y}_{--}$; i.e., we have in the $\hat{\sigma}$ frame an arbitrary constant influx of ghost stress energy balanced by a constant outflow of X, Y stress energy. On \mathcal{J}^- there is incoming f stress energy, which following CGHS [1] we take to be $\hat{T}^f_{++} = a\lambda\sigma_0^+ \delta(\hat{\sigma} - \hat{\sigma}_0)$. Then we may take $\hat{t}_{++} = -\alpha\lambda^2/24$ and $\hat{T}^{X,Y}_{++} = -a\lambda\sigma_0^+ \delta(\hat{\sigma} - \hat{\sigma}_0) + \alpha\lambda^2/24$ to be consistent with the constraints.

Then by putting $\sigma' = \hat{\sigma}$ in (6.7), we get in the σ frame

$$\begin{aligned} t_{\pm\pm} &= \frac{26-\alpha}{24} \frac{1}{\sigma^{\pm 2}}, \quad T^f_{--} = -\frac{N}{24} \frac{1}{\sigma^{-2}}, \\ T^f_{++} &= a\delta(\sigma^+ - \sigma_0^+) - \frac{N}{24}. \end{aligned}$$

Using these values in (6.6) we find

$$\begin{aligned} u_+ &= a_+ + b_+ \sigma^+ - a(\sigma^+ - \sigma_0^+) \theta(\sigma^+ - \sigma_0^+) - \frac{\bar{N}}{24} \ln|\sigma^+|, \\ u_- &= a_- + b_- \sigma^- - \frac{\bar{N}}{24} \ln|\sigma^-|, \end{aligned} \quad (6.9)$$

where

$$\bar{N} = N + \alpha - 26. \quad (6.10)$$

We now need a reference static solution. This is obtained from (4.4) and (6.10) by putting $a = a_{\pm} = b_{\pm} = 0$ in the latter:

$$X_0 = -Y_0 = \sqrt{2/|\kappa|} \left[\lambda^2 \sigma^+ \sigma^- + \frac{\bar{N}}{24} \ln(-\sigma^+ \sigma^-) \right], \quad f = 0.$$

This solution is in Kruskal-Szekeres coordinates, and we need to transform this into the asymptotically Minkowski coordinates¹¹ $\bar{\sigma}^{\pm}$ defined by $\sigma^+ = (1/\lambda)e^{\lambda\bar{\sigma}^+}$, $\sigma^- = -(1/\lambda)e^{-\lambda\bar{\sigma}^-}$. Under a coordinate transformation, X transforms as a scalar and [since $\rho(\sigma) \rightarrow \rho(\bar{\sigma}) + \frac{1}{2}\lambda(\bar{\sigma}^+ - \bar{\sigma}^-)$] Y transforms as $Y(\sigma) \rightarrow Y(\bar{\sigma}) + \sqrt{|\kappa|/2}\lambda(\bar{\sigma}^+ - \bar{\sigma}^-)$. Hence we have, in the new coordinate system,

¹¹For the static solution these are the same as $\hat{\sigma}$ defined

$$\begin{aligned}
X_0 &= -\sqrt{2/|\kappa|} \left[e^{\lambda(\bar{\sigma}^+ - \bar{\sigma}^-)} - \frac{\bar{N}}{24} \lambda(\bar{\sigma}^+ - \bar{\sigma}^-) + \frac{\bar{N}}{24} \ln \lambda^2 \right] \\
&= -Y_0 + \sqrt{|\kappa|/2} \lambda(\bar{\sigma}^+ - \bar{\sigma}^-). \tag{6.11}
\end{aligned}$$

This solution corresponds to the linear dilaton solution of the classical equations. To obtain the Bondi mass of a general solution which asymptotically tends to the above static solution we need to linearize the stress tensors around the latter. From (6.4) and (6.5) we have, using (6.11),

$$\begin{aligned}
\delta T_{++} + \delta T_{--} &= -\sqrt{2/|\kappa|} \partial_+ [\lambda e^{\lambda(\bar{\sigma}^+ - \bar{\sigma}^-)} (\delta X + \delta Y)] \\
&\quad + \sqrt{2/|\kappa|} \frac{\bar{N}}{24} \lambda \partial_+ (\delta X + \delta Y) \\
&\quad - \sqrt{|\kappa|/2} \lambda \partial_+ \delta Y \\
&\quad + \sqrt{|\kappa|/2} \lambda (\partial_+ \delta Y - \partial_- \delta Y).
\end{aligned}$$

Substituting into (6.1) we get

$$\begin{aligned}
M(\bar{\sigma}^-) &= -\int d\bar{\sigma}^+ (\delta T_{++} + \delta T_{--}) \\
&= [\sqrt{2/|\kappa|} \lambda e^{\bar{\sigma}^+ - \bar{\sigma}^-} (\delta X + \delta Y) \\
&\quad - \sqrt{2/|\kappa|} \frac{\bar{N}}{24} \lambda (\delta X + \delta Y) \\
&\quad + \sqrt{|\kappa|/2} \lambda \delta Y + \sqrt{|\kappa|/2} (\partial_+ \delta Y - \partial_- \delta Y)]_{\bar{g}^+}. \tag{6.12}
\end{aligned}$$

Using (3.4) and (3.2) we find that when $e^{2\phi} \ll 1$ this expression tends (not surprisingly) to the expression given by CGHS [Eq. (26) of [1]] except for the ghost terms.

Static solutions corresponding to black holes (in the classical limit) are obtained by putting $a = b_{\pm} = 0$ and $a_{\pm} \neq 0$. Then for $\bar{\sigma}^+ \gg 1$, $-\delta Y = \sqrt{2/|\kappa|} (a_+ + a_-) = \delta X$, and we have, from (6.12), a constant Bondi [Arnowitt-Deser-Misner (ADM)] mass

$$M(\bar{\sigma}^-) = \lambda(a_+ + a_-).$$

The parameters a_{\pm} can be of either sign, and hence we may have negative-mass solutions of the theory. This is not surprising since the Hamiltonian is not positive definite. Of course, the classical theory has such solutions too, but there these correspond to naked singularities, whereas here these are nonsingular solutions (as we argued in Sec. IV).¹² However, one might ask whether it is the case that we cannot generate these unphysical solutions dynamically by starting with positive-mass solutions, in which case we might choose to ignore them. Unfortunately, this is not the case. To see this let us compute the Bondi mass of the analogue of the dynamic CGHS solution corresponding to the formation of a black

hole by an incoming matter shock wave and its decay by Hawking radiation. This solution is obtained in the σ frame by putting $a_{\pm} = b_{\pm} = 0$, $a \neq 0$, in (6.6) and substituting in (4.4). Then in the region outside the classical horizon we transform to the asymptotically Minkowski coordinates σ , defined by

$$\sigma^+ = \frac{1}{\lambda} e^{\lambda \bar{\sigma}^+}, \quad \sigma^- = -\frac{1}{\lambda} e^{-\lambda \bar{\sigma}^-} - \frac{a}{\lambda^2},$$

to get

$$\begin{aligned}
X &= -\left[\frac{2}{|\kappa|} \right]^{1/2} \left\{ \frac{M_0}{\lambda} \theta(\bar{\sigma}^+ - \bar{\sigma}_0^+) + e^{\lambda(\bar{\sigma}^+ - \bar{\sigma}^-)} \right. \\
&\quad \left. - \frac{\bar{N}}{24} \ln \left[\frac{e^{-\lambda \bar{\sigma}^+}}{\lambda} \left(\frac{e^{-\lambda \bar{\sigma}^-}}{\lambda} + \frac{a}{\lambda^2} \right) \right] \right\} \\
&= -Y + \sqrt{|\kappa|/2} \lambda(\bar{\sigma}^+ - \bar{\sigma}^-),
\end{aligned}$$

where we have put $M_0 = \lambda a \sigma_0^+$, the mass of the classical black hole. Comparing with the static solution we find

$$\begin{aligned}
\delta X = \delta Y &= -\left[\frac{2}{|\kappa|} \right]^{1/2} \left\{ \frac{M_0}{\lambda} \theta(\bar{\sigma}^+ - \bar{\sigma}_0^+) \right. \\
&\quad \left. - \frac{\bar{N}}{24} \ln \left[1 + \frac{a}{\lambda} e^{+\lambda \bar{\sigma}^-} \right] \right\}.
\end{aligned}$$

Substituting into (6.12) we get

$$\begin{aligned}
M(\bar{\sigma}^-) &= M_0 - \frac{\bar{N}}{24} \lambda \ln \left[1 + \frac{a}{\lambda} e^{\lambda \bar{\sigma}^-} \right] \\
&\quad - \frac{\bar{N}}{24} \frac{\lambda}{1 + (\lambda/a) e^{-\lambda \bar{\sigma}^-}}.
\end{aligned}$$

In the infinite (light-cone time) past $\bar{\sigma}^- \rightarrow -\infty$ the Bondi mass tends to the classical black-hole mass M_0 , but at future infinity $\bar{\sigma}^- \rightarrow +\infty$ one gets an infinitely negative value.

This unphysical conclusion is equivalent to the statement that the Hawking radiation rate does not go to zero asymptotically. This rate may be calculated either from the left-hand side, or as the negative of the right-hand side, of (6.3),

$$-\frac{dM(\bar{\sigma}^-)}{d\bar{\sigma}^-} = \frac{\bar{N}}{24} \frac{\lambda^2}{[1 + (\lambda/a) e^{-\lambda \bar{\sigma}^-}]^2} \rightarrow \frac{\bar{N}}{24} \lambda^2. \tag{6.13}$$

Now so far α has been kept arbitrary, but perhaps the most natural choice is $\alpha = 26$, so that [see (6.10)] $\bar{N} = N$. This choice corresponds to the decoupling of the ghosts from the Hawking radiation which is positive regardless of the number of matter fields. This agrees with the two-dimensional analogue of the original Hawking result [2], as well as that of [1] asymptotically, but back reaction still modifies the σ^- dependence. Unfortunately, although the formalism allows this value of \bar{N} it is certainly not the only possibility. Perhaps this choice has to be made on physical grounds. It is also possible that our analysis of the Bondi mass is not the complete quantum

¹²This point has been emphasized by Giddings and Strominger [19].

mechanical story, and a proper treatment would resolve both this issue as well as the question of positivity.

VII. CONCLUSIONS

What progress have we made in understanding quantum black holes, and in particular the phenomenon of Hawking radiation, from this work?

First, let us stress that even if we leave aside our argument for regarding the Liouville-like theory as the complete quantum theory, it still gives us the only consistent treatment of the semiclassical theory (i.e., to first order in $\kappa e^{2\phi}$). As we pointed out in Sec. III, if we just include the leading-order corrections then the field-space curvature is zero, and one immediately has a soluble semiclassical theory. All of the above calculations are then still valid, except that we cannot draw some of the conclusions that we have drawn from them. Thus, we can no longer explicitly demonstrate the taming of the classical singularity, and of course there is no need to conclude that Hawking radiation does not stop, and that positive-mass black holes radiate into negative-mass solutions. Nevertheless, one has a consistent semiclassical picture of black-hole radiation and back reaction. In particular, it should be emphasized again that our remarks about the inconsistencies associated with the usual calculation of Hawking radiation, which ignores back reaction, are valid already at the semiclassical level. To belabor the point, the calculations with the Liouville-like theory when interpreted in terms of the ρ, ϕ variables, and when considered as being valid to $\mathcal{O}(\kappa e^{2\phi})$, are the correct semiclassical results coming from the classical CGHS theory. In particular, they show that the semiclassical physics is obtained whatever options are chosen for the functions h, \bar{h} (as discussed in Sec. III) simply because \bar{h} is zero at the semiclassical level. In other words, we may use the exactly soluble conformal field theory to make the calculations, provided we interpret the result as being valid only at the semiclassical level.

Second, we have shown that there is a class of quantum dilaton-gravity theories, namely, those for which the field-space curvature is zero to all orders in $e^{2\phi}$ [$\bar{h}=0$, see discussion after Eq. (3.1)], whose exact quantum treatment is possible since they can be transformed into a Liouville-like theory.¹³ These theories allow for the first time a complete quantum-mechanical treatment (including the effects of dilaton-graviton loops) of a theory of gravity with classical black-hole solutions. Unfortunately, as we have shown, these theories may not be physical. It is an open question whether it is possible to find a soluble theory with $\bar{h}\neq 0$ which does not have this problem, but we believe that this is unlikely. After all, as we have explicitly demonstrated, quantum mechanics does what one expects it to do; namely, it tames the classical singularities, including the naked ones. Unfortunately it thereby eliminates the usual argument (in the classical theory) for eliminating negative-mass solutions on the grounds that such spaces are not globally hyperbolic. It is likely that the problem is not so much with the soluble class of models that we have treated as with the original classical dilaton-gravity theory itself, which does not have a positive-definite field-space metric. On the other hand it is also possible that the fault lies with our rather heuristic treatment of the Bondi mass in the quantum theory, and that a rigorous quantum treatment may resolve this issue.

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¹³The objection raised by some authors on the range of integration has been answered in Sec. III. In particular, for subclass (d) there can be no objection on these grounds.

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