Spin-$\frac{3}{2}$ perturbations of the Kerr-Newman solution

G. F. Torres del Castillo
Departamento de Física Matemática, Instituto de Ciencias de la Universidad Autónoma de Puebla, 72000 Puebla, México

G. Silva-Ortigoza
Departamento de Física, Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, Apartado Postal 14-740, 07000 México, Distrito Federal, México

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The equations for the spin-$\frac{3}{2}$ perturbations of a rotating charged black hole given by the linearized O(2) extended supergravity model are solved, expressing their complete solution in terms of two Debye potentials that obey the same differential equation. The equation governing the radial parts of the Debye potentials is transformed into a Schrödinger-type equation with a short-range real potential. It is also shown that there exist spin-$\frac{3}{2}$ perturbations corresponding only to ingoing or only to outgoing waves.

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I. INTRODUCTION

The O(2) extended supergravity field equations [1] give a consistent coupling of an O(2) doublet of spin-$\frac{3}{2}$ fields, electromagnetism, and gravity, and they reduce to the usual Einstein-Maxwell equations when the spin-$\frac{3}{2}$ fields vanish. By linearizing the O(2) extended supergravity field equations with respect to the spin-$\frac{3}{2}$ fields about a solution with vanishing spin-$\frac{3}{2}$ fields, one obtains a consistent set of equations for an O(2) doublet of spin-$\frac{3}{2}$ fields on a background solution of the Einstein-Maxwell equations [2]. In this approximation, the torsion vanishes, the supersymmetry transformations affect only the spin-$\frac{3}{2}$ fields, and the anticommutative character of the spin-$\frac{3}{2}$ fields does not show up. This framework has been employed by Aichelburg and Güven [2] to examine the spin-$\frac{3}{2}$ perturbations of the Kerr-Newman solution; by analyzing two gauge-invariant scalars made out of the spin-$\frac{3}{2}$ fields, they showed that there are no nontrivial, stationary, regular spin-$\frac{3}{2}$ perturbations of the charged black holes, except in the case of the extreme Reissner-Nordström solution [3].

In this paper we give the expression for the complete spin-$\frac{3}{2}$ perturbations of the Kerr-Newman solution using the fact that the solution of the equations for the spin-$\frac{3}{2}$ fields can be written in terms of two Deybe potentials that satisfy identical differential equations, provided that one principal null direction of the background electromagnetic field is geodetic and shear free [4]. In Sec. II the basic equations are given and the equation for the Debye potentials is solved by separation of variables. In Sec. III the equation governing the radial parts of the Debye potentials is transformed into a wave equation from which the reflection and transmission coefficients for incident waves can be obtained. In Sec. IV it is shown that, for certain complex frequencies, the charged black holes admit spin-$\frac{3}{2}$ perturbations that are purely ingoing or purely outgoing. In order to facilitate the comparison with previous works, we follow closely Chandrasekhar's [5] notation.

II. EQUATIONS FOR THE SPIN-$\frac{3}{2}$ PERTURBATIONS OF THE KERR-NEWMAN BLACK HOLES

To first order in the spin-$\frac{3}{2}$ fields, the O(2) extended supergravity field equations are [2,6]

$$\Phi_{ABC'D'}=2\varphi_{AB}\varphi^{*}_{C'D'} , \tag{1a}$$

$$\Lambda=0 , \tag{1b}$$

$$\nabla^{A}{}_{C'}\varphi_{AB}=0 , \tag{1c}$$

$$\nabla_{CD'}\psi^{jA}{}_{AB'}=\nabla_{AB'}\psi^{jA}{}_{CD'}+i\sqrt{2}\varepsilon^{jk}\varphi^{A}{}_{C}\psi^{*k}{}_{D'B'A} \tag{1d}$$

($j,k=1,2$), where $\Phi_{ABC'D'}$ corresponds to the trace-free part of the Ricci tensor, $\Lambda=R/24$, φ_{AB} is the electromagnetic spinor, ε^{jk} is the Levi-Civita symbol, and $\psi^{*j}{}_{A'B'C}=(\psi^{j}{}_{ABC'})^{*}$. Equations (1a)–(1c) are just the Einstein-Maxwell equations without a cosmological constant. Equation (1d) can also be written in the form [6]

$$H^{j}{}_{ABC}=H^{j}{}_{(ABC)}, \quad H^{j}{}_{AB'C'}=0 , \tag{2}$$

where

$$\begin{aligned} H^{jA}{}_{BC} &\equiv \nabla_{(B|S'|}\psi^{jA}{}_{C)}{}^{S'}-i\sqrt{2}\varepsilon^{jk}\varphi^{A}{}_{(B}\psi^{*k}{}_{|S'|}{}^{S'}{}_{C)} , \\ H^{jA}{}_{B'C'} &\equiv \nabla_{R(B'}\psi^{jAR}{}_{C')}-i\sqrt{2}\varepsilon^{jk}\varphi^{A}{}_{R}\psi^{*k}{}_{(B'C')}{}^{R} . \end{aligned} \tag{3}$$

(The parentheses denote symmetrization on the indices enclosed and the indices between bars are excluded from the symmetrization.)

In the linearized version of the $N=2$ supergravity field equations given by Eqs. (1), the supersymmetry transformations act only on the spin-$\frac{3}{2}$ fields. These transformations are given by

$$\psi^{j}{}_{ABC'}\rightarrow\psi^{j}{}_{ABC'}+\nabla_{BC'}\alpha^{j}{}_{A}-i\sqrt{2}\varepsilon^{jk}\varphi_{AB}\alpha^{*k}{}_{C'} , \tag{4}$$

where $\alpha^{j}{}_{A}$ is a pair of arbitrary spinor fields and $\alpha^{*k}{}_{C'}=(\alpha^{k}{}_{C})^{*}$. Then, from Eqs. (3) it follows that, modulo Eqs. (1a)–(1c), under the transformations (4),

$$H^j{}_{ABC} \to H^j{}_{ABC} - \Psi_{ABC}{}^D \alpha^j{}_D + i\sqrt{2}\varepsilon^{jk}\alpha^{*k}{}_{S'}\nabla_A{}^{S'}\varphi_{BC} \,, \qquad H^j{}_{AB'C'} \to H^j{}_{AB'C'} \,, \tag{5}$$

where Ψ_{ABCD} is the Weyl spinor. Equations (1) and (3) lead to the equations

$$\nabla^{AR'} H^j{}_{ABC} = \Psi_{ABCD}\psi^{jADR'} + i\sqrt{2}\varepsilon^{jk}\psi^{*k}{}_{S'}{}^{R'A}\nabla_B{}^{S'}\varphi_{AC} \,, \tag{6}$$

whose integrability conditions are satisfied identically.

If the principal null directions of the electromagnetic field are geodetic, shear free, and nonproportional (as in the case of the Kerr-Newman solution), by choosing a spin frame o^A, ι^A such that o^A and ι^A are principal spinors of φ_{AB}, then the only nonvanishing components of φ_{AB} and Ψ_{ABCD} are φ_1 and Ψ_2, respectively, and Eqs. (5) imply that $H^j{}_0 \equiv H^j{}_{ABC} o^A o^B o^C$ and $H^j{}_3 \equiv H^j{}_{ABC}\iota^A\iota^B\iota^C$ are invariant under the transformations (4). From Eq. (6) it follows that these gauge-invariant components satisfy the decoupled equations

$$\begin{aligned} &[(D - 2\varepsilon + \varepsilon^* - 3\tilde{\rho} - \tilde{\rho}^*)(\underset{\sim}{\Delta} - 3\gamma + \mu) \\ &\quad - (\delta - 2\beta - \alpha^* - 3\tau + \pi^*)(\delta^* - 3\alpha + \pi) - \Psi_2] H^j{}_0 = 0 \,, \\ &[(\underset{\sim}{\Delta} + 2\gamma - \gamma^* + 3\mu + \mu^*)(D + 3\varepsilon - \tilde{\rho}) \\ &\quad - (\delta^* + 2\alpha + \beta^* + 3\pi - \tau^*)(\delta + 3\beta - \tau) - \Psi_2] H^j{}_3 = 0 \,, \end{aligned} \tag{7}$$

which can be solved by separation of variables [6].

In the specific case of the Kerr-Newman solution, using the Kinnersley tetrad and the Boyer-Lindquist coordinates as in Ref. [5], the separable solutions of Eqs. (7) are given by

$$\begin{aligned} H^j{}_0 &= R^j{}_{+3/2}(r) S_{+3/2}(\theta) e^{i(\sigma t + m\phi)} \,, \\ H^j{}_3 &= -\frac{1}{2\sqrt{2}(\tilde{\rho}^*)^3} R^j{}_{-3/2}(r) S_{-3/2}(\theta) e^{i(\sigma t + m\phi)} \,, \end{aligned} \tag{8}$$

where σ is the frequency of the waves, m is a half-integer, $\tilde{\rho} \equiv r + ia\cos\theta$, a is the specific angular momentum of the black hole, and the one-variable functions $R^j{}_{\pm 3/2}, S_{\pm 3/2}$ satisfy the ordinary differential equations [2,6]

$$\begin{aligned} &(\Delta \mathcal{D}_{-1/2}\mathcal{D}_0^\dagger - 4i\sigma r)\Delta^{3/2} R^j{}_{+3/2} = \lambda \Delta^{3/2} R^j{}_{+3/2} \,, \\ &(\Delta \mathcal{D}^\dagger_{-1/2}\mathcal{D}_0 + 4i\sigma r) R^j{}_{-3/2} = \lambda R^j{}_{-3/2} \,, \end{aligned} \tag{9a}$$

$$\begin{aligned} &(\mathcal{L}^\dagger_{-1/2}\mathcal{L}_{3/2} + 4a\sigma\cos\theta) S_{+3/2} = -\lambda S_{+3/2} \,, \\ &(\mathcal{L}_{-1/2}\mathcal{L}^\dagger_{3/2} - 4a\sigma\cos\theta) S_{-3/2} = -\lambda S_{-3/2} \,, \end{aligned} \tag{9b}$$

where λ is a separation constant [5],

$$\begin{aligned} &\mathcal{D}_n \equiv \partial_r + \frac{iK}{\Delta} + 2n\frac{r-M}{\Delta}, \quad \mathcal{D}_n^\dagger \equiv \partial_r - \frac{iK}{\Delta} + 2n\frac{r-M}{\Delta} \,, \\ &\mathcal{L}_n \equiv \partial_\theta + Q + n\cot\theta, \quad \mathcal{L}_n^\dagger \equiv \partial_\theta - Q + n\cot\theta \,, \end{aligned} \tag{10}$$

and

$$\begin{aligned} K &\equiv (r^2 + a^2)\sigma + am \,, \\ Q &\equiv a\sigma\sin\theta + m\csc\theta \,, \\ \Delta &\equiv r^2 - 2Mr + a^2 + e^2 \,. \end{aligned} \tag{11}$$

Here M and e denote the mass and charge of the black hole, respectively.

From Eqs. (9a) it follows that the functions $R^j{}_{\pm 3/2}$ can be chosen in such a way that [6]

$$\begin{aligned} \Delta^{3/2}\mathcal{D}_0^\dagger\mathcal{D}_0^\dagger\mathcal{D}_0^\dagger\Delta^{3/2} R^j{}_{+3/2} &= C_1 R^j{}_{-3/2} \,, \\ \Delta^{3/2}\mathcal{D}_0\mathcal{D}_0\mathcal{D}_0 R^j{}_{-3/2} &= C_2 \Delta^{3/2} R^j{}_{+3/2} \,, \end{aligned} \tag{12}$$

where

$$C_1 C_2 = \lambda^3 + \lambda^2 + 16\sigma^2(a^2 + e^2 - \lambda\alpha^2) \,, \tag{13}$$

and $\alpha^2 \equiv a^2 + (am/\sigma)$. For real σ, $\Delta^{3/2}R^j{}_{+3/2}$ and $R^j{}_{-3/2}$ satisfy complex-conjugate equations and, therefore, they can be normalized in such a way that (for real σ) $C_2 = C_1^*$. Similarly, the functions $S_{\pm 3/2}$ obey the relations

$$\begin{aligned} \mathcal{L}_{-1/2}\mathcal{L}_{1/2}\mathcal{L}_{3/2} S_{+3/2} &= -B S_{-3/2} \,, \\ \mathcal{L}^\dagger_{-1/2}\mathcal{L}^\dagger_{1/2}\mathcal{L}^\dagger_{3/2} S_{-3/2} &= B S_{+3/2} \,, \end{aligned} \tag{14}$$

where

$$B^2 = \lambda^3 + \lambda^2 + 16\sigma^2(a^2 - \lambda\alpha^2) \,. \tag{15}$$

When $a\sigma$ is equal to zero, the angular functions $S_{\pm 3/2}(\theta)e^{im\phi}$ are spin-weighted spherical harmonics and λ takes the values $(j - \frac{1}{2})(j + \frac{3}{2})$ where j is a half-integer greater than, or equal to, $\frac{3}{2}$.

As shown in Ref. [4], if one principal null direction of the electromagnetic field is geodetic and shear free then the complete solution of Eq. (1d) can be written in terms of two gauge-invariant scalar potentials ψ^j. The potentials ψ^j associated with the geodetic and shear free principal null direction of φ_{AB} defined by o^A are governed by

$$\begin{aligned} &[(\underset{\sim}{\Delta} + 2\gamma - \gamma^* + \mu^*)(D + 3\varepsilon + 2\tilde{\rho}) \\ &\quad - (\delta^* + 2\alpha + \beta^* - \tau^*)(\delta + 3\beta + 2\tau) - \Psi_2]\psi^j = 0 \,. \end{aligned} \tag{16}$$

The most general solution of Eq. (1d) is given locally by

$$\begin{aligned} \psi^{*j}{}_{1'1'1} &= (\delta + 2\beta + \alpha^* + \tau)(\delta + 3\beta)\psi^j - \lambda^*(D + 3\varepsilon)\psi^j \\ &\quad - i\sqrt{2}\varepsilon^{jk}\varphi_1^*(\delta^* + 3\beta^*)\psi^{k*} \,, \\ \psi^{*j}{}_{1'0'1} &= (D + 2\varepsilon + \varepsilon^* + \tilde{\rho})(\delta + 3\beta)\psi^j - \pi^*(D + 3\varepsilon)\psi^j \,, \\ \psi^{*j}{}_{0'1'1} &= (\delta + 2\beta - \alpha^* + \tau)(D + 3\varepsilon)\psi^j + \tilde{\rho}^*(\delta + 3\beta)\psi^j \,, \\ \psi^{*j}{}_{0'0'1} &= (D + 2\varepsilon - \varepsilon^* + \tilde{\rho})(D + 3\varepsilon)\psi^j \,, \\ \psi^{*j}{}_{1'1'0} &= -i\sqrt{2}\varepsilon^{jk}\varphi_1^*(D + 3\varepsilon^*)\psi^{k*} \,, \\ \psi^{*j}{}_{1'0'0} &= \psi^{*j}{}_{0'1'0} = \psi^{*j}{}_{0'0'0} = 0 \,, \end{aligned} \tag{17}$$

up to the gauge transformations (4).

Equation (16) admits separable solutions of the form

$$\psi^j = R^j{}_{-3/2}(r) S_{-3/2}(\theta) e^{i(\sigma t + m\phi)} \,, \tag{18}$$

where $R^j{}_{-3/2}$ and $S_{-3/2}$ obey Eqs. (9). Substituting Eq. (18) into Eqs. (17) one obtains all the components of the spin-$\frac{3}{2}$ fields with the correct relative normalization. In particular, using Eqs. (3), (9), (12), (14), (17), and (18) we find that the gauge-invariant components $H^j{}_0$ and $H^j{}_3$

generated by (18) are

$$(H^j{}_0)^* = C_2 R^j{}_{+3/2}(r) S_{-3/2}(\theta) e^{i(\sigma t + m\phi)} ,$$
$$(H^j{}_3)^* = \frac{1}{2\sqrt{2}(\bar{\rho})^3} \{ B R^j{}_{-3/2}(r) S_{+3/2}(\theta) e^{i(\sigma t + m\phi)} + 4e\sigma^* \varepsilon^{jk} [R^k{}_{-3/2}(r) S_{-3/2}(\theta) \times e^{i(\sigma t + m\phi)}]^* \} , \tag{19}$$

(cf. Ref. [7]).

III. THE POTENTIAL BARRIER FOR THE SPIN-$\frac{3}{2}$ FIELDS

It is a remarkable fact that, despite the coupling of the O(2) doublet $\psi^j{}_{ABC'}$ with itself and with the electromagnetic field given by the last term in Eq. (1d), the equations governing the spin-$\frac{3}{2}$ perturbations of the Kerr-Newman solution differ from those corresponding to the Kerr metric [8] only by the replacement of a^2 by a^2+e^2 in some few places [see Eqs. (11) and (13)]. Since Eqs. (9a) are almost identical to the equations for the spin-$\frac{3}{2}$ perturbations of the Kerr metric considered in Ref. [8], we can readily reduce Eqs. (9a) to Schrödinger-type equations by making some slight changes in the results of Ref. [8].

Following Chandrasekhar [5,9], we make use of the independent variable r_* defined by $dr_*/dr = \rho^2/\Delta$, where

$$\rho^2 \equiv r^2 + \alpha^2, \quad \alpha^2 \equiv a^2 + (am/\sigma) , \tag{20}$$

and of the variables

$$Y^j = \rho^{-2} R^j{}_{-3/2} . \tag{21}$$

From Eqs. (9a) and (10) one finds that the functions Y^j satisfy

$$\Lambda^2 Y^j + P\Lambda_+ Y^j - QY^j = 0 , \tag{22}$$

where

$$\Lambda_\pm \equiv \frac{d}{dr_*} \pm i\sigma ,$$
$$\Lambda^2 \equiv \Lambda_+\Lambda_- = \Lambda_-\Lambda_+ = \frac{d^2}{dr_*^2} + \sigma^2 , \tag{23}$$

and [10]

$$P = \frac{3}{\rho^4}[2r\Delta - \rho^2(r-M)] ,$$
$$Q = \frac{\Delta}{\rho^6}(\lambda\rho^2 + 2Mr - 2a^2 - 2e^2) . \tag{24}$$

[Since the complete solution to Eq. (1d) is generated by (18) it suffices to consider the equation for $R^j{}_{-3/2}$ only; an entirely similar reduction can be made with the equation for $R^j{}_{+3/2}$.] Equation (22) can be transformed into the Schrödinger-type equation

$$\Lambda^2 Z^j = VZ^j \tag{25}$$

by means of the substitutions

$$Y^j = \left[R - \frac{dT_1}{dr_*} \right] Z^j + (T_1 - 2i\sigma)\Lambda_- Z^j ,$$
$$\frac{\Delta^{3/2}}{\rho^6} K Z^j = RY^j - (T_1 - 2i\sigma)\Lambda_+ Y^j , \tag{26}$$

where [10]

$$R = Q + \frac{\Delta^{3/2}}{\rho^6}\beta_2 = \frac{\Delta^{3/2}}{\rho^6}(F + \beta_2), \quad F = \frac{\rho^6}{\Delta^{3/2}} Q ,$$
$$T_1 = \frac{1}{F - \beta_2}\left[\frac{dF}{dr_*} - \kappa_2 \right], \quad \beta_2^2 = 4(a^2 + e^2 - \lambda\alpha^2) , \tag{27}$$
$$\kappa_2^2 = \lambda^3 + \lambda^2, \quad K = -4\sigma^2\beta_2 - 2i\sigma\kappa_2 ,$$

and [11]

$$V = Q - \frac{dT_1}{dr_*} . \tag{28}$$

As in the case of the spin-$\frac{3}{2}$ perturbations of the Kerr metric, the potential V is real (for real σ) and of short range; therefore the reflection and transmission coefficients can be defined as in Ref. [8]. Furthermore, from Eqs. (13) and (27) one finds that the constants $K^{(+\sigma)}$ and $K^{(-\sigma)}$, corresponding to the frequencies σ and $-\sigma$, satisfy the relation

$$K^{(+\sigma)} K^{(-\sigma)} = 4\sigma^2 C_1 C_2 , \tag{29}$$

which is analogous to those obtained for the perturbations of the Kerr metric by massless fields of spin $\frac{1}{2}$, 1, $\frac{3}{2}$, and 2 [5,9,8].

IV. INGOING OR OUTGOING WAVES

The presence of a nonvanishing background electromagnetic field allows the existence of solutions to Eq. (1d) that correspond only to ingoing waves $(H^j{}_0 \neq 0, H^j{}_3 = 0)$ or only to outgoing waves $(H^j{}_3 \neq 0, H^j{}_0 = 0)$. From Eqs. (12) it follows that in order to have $R^j{}_{-3/2} = 0$ with $R^j{}_{+3/2}$ different from zero, it is necessary that C_2 be equal to zero. Then, Eq. (12) gives

$$\mathcal{D}_0^\dagger \mathcal{D}_0^\dagger \mathcal{D}_0^\dagger \Delta^{3/2} R^j{}_{+3/2} = 0 . \tag{30}$$

Therefore, taking into account the fact that

$$\mathcal{D}_0^\dagger = e^{i\sigma r_*} \partial_r e^{-i\sigma r_*} , \tag{31}$$

the general solution of Eq. (30) has the form

$$\Delta^{3/2} R^j{}_{+3/2}(r) = (a^j r^2 + b^j r + c^j) e^{i\sigma r_*} , \tag{32}$$

where a^j, b^j, and c^j are constants of integration. By substituting Eq. (32) into Eq. (9a) one finds that

$$\Delta^{3/2} R^j{}_{+3/2}(r) = A^j \left[2i\sigma r^2 - \lambda r - \frac{i}{4\sigma}(\lambda^2 + \lambda - 8\sigma^2\alpha^2 - 4i\sigma M) \right] \times e^{i\sigma r_*} , \tag{33}$$

where A^j are arbitrary constants and σ is such that

$C_2=0$. Similarly, assuming that $R^j{}_{+3/2}=0$, from Eqs. (12) and (9a) we obtain

$$R^j{}_{-3/2}(r) = B^j\left[-2i\sigma r^2-\lambda r+\frac{i}{4\sigma}(\lambda^2+\lambda-8\sigma^2\alpha^2+4i\sigma M)\right] \times e^{-i\sigma r_*}, \quad (34)$$

where B^j are arbitrary constants and σ is such that $C_1=0$.

When $a=0$ (in which case the Kerr-Newman background solution reduces to the Reissner-Nordström solution), from Eq. (13) one finds that the vanishing of C_1 or C_2 implies that

$$\sigma=\pm\frac{i}{4e}(j-\tfrac{1}{2})(j+\tfrac{1}{2})(j+\tfrac{3}{2}), \quad (35)$$

where j is a half-integer greater than, or equal to, $\frac{3}{2}$.

The foregoing relations are analogous to those obtained in Ref. [12] for the case of the algebraically special (gravitational) perturbations of the Kerr metric. In fact, as in Ref. [12], the same solutions can also be derived by means of the transformation theory given in the preceding section [13].

V. CONCLUDING REMARKS

The results presented here, as well as those of Ref. [8], show that the equations given by the linearized supergravity lead to spin-$\frac{3}{2}$ perturbations of the black hole solutions that possess many remarkable properties analogous to those previously found in the analysis of the perturbations of the black holes [5,9,12].

Equations (19) show that, when $e\sigma\neq0$, a separable expression for the gauge-invariant components $H^j{}_0$ is accompanied by expressions for $H^j{}_3$ that cannot be written as the product of one-variable functions. A similar behavior is found for Ψ_0 and Ψ_4 in the case of the gravitational perturbations of the Schwarzschild metric [14] and of the Reissner-Nordström solution [7].

As pointed out in Ref. [7], there exist singular Debye potentials that generate well-behaved perturbations; such singular potentials are required to generate solutions of Eq. (1d) with vanishing $H^j{}_0$ and $H^j{}_3$.

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