Planck-scale corrections to axion models

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It has been argued that quantum gravitational effects will violate all nonlocal symmetries. Peccei-Quinn symmetries must therefore be an "accidental" or automatic consequence of local gauge symmetry. Moreover, higher-dimensional operators suppressed by powers of $M_{\rm Pl}$ are expected to explicitly violate the Peccei-Quinn symmetry. Unless these operators are of dimension $d \geq 10$, axion models do not solve the strong *CP* problem in a natural fashion. A small gravitationally induced contribution to the axion mass has little if any effect on the density of relic axions. If $d = 10$, 11, or 12 these operators can solve the axion domain-wall problem, and we describe a simple class of Kim-Shifman-Vainshtein-Zakharov axion models where this occurs. We also study the astrophysics and cosmology of "heavy axions" in models where $5 \leq d \leq 10$.

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I. INTRODUCTION

It has been argued [1] that quantum gravitational effects should lead to explicit violation of all global symmetries in the effective theory describing physics below the Planck scale. If this is correct, two conclusions follow. First, the renormalizable terms of that effective theory can be invariant under such a global symmetry only as an "accidental" consequence of gauge invariance. Second, such a global symmetry must be explicitly broken by nonrenormalizable (i.e., $d > 4$) terms in that effective theory, which will be suppressed by powers of the Planck mass. These conclusions have powerful implications for theories which involve Goldstone or pseudo Goldstone particles. The implications for "texture" models [2] have been very recently explored in papers [3] which have inspired this work [4]. Here we study the implications for axion theories [5–7].

The main conclusions of this paper are as follows. (1) The Peccei-Quinn symmetry [8] must be an "automatic" consequence of gauge invariance [9]. Gravitationally induced operators that explicitly violate the Peccei-Quinn symmetry must be of dimension 10 or higher if axions are to provide a natural solution to the strong *CP* problem. Therefore gauge invariance must forbid operators of this type with $d < 10$. (3) Although a small gravitationally induced mass might conceivably reduce the relic density of axions for $d \geq 10$, we find that the effects are not likely to relax the cosmological constraints on the axion decay constant [10,11] by more than a factor of 2. (4) If the new operators have dimension 10, 11, or 12, they can play an essential role in avoiding an "axion domain-wall problem" [12] by explicitly breaking the degeneracy between the distinct QCD vacua. (5) It is possible to construct models which illustrate these points in a simple and straightforward way. (6) For $5 \leq d \leq 9$, although the axion is not a natural solution to the strong *CP* problem, axions may nevertheless exist. If so, they would be heavier than in a standard axion model and this leads to a revised discussion of their astrophysical and cosmological effects.

The rest of this paper is organized as follows. In Sec. II we discuss the role that higher-dimensional operators play in destroying the Peccei-Quinn mechanism. In Sec. III we discuss the cosmological implications of these operators as they pertain to the relic density of axions and the stability of networks of axionic strings and domain walls. In Sec. IV we describe a simple class of models that illustrate how the Peccei-Quinn symmetry may arise automatically from gauge symmetry. These models lend themselves to a transparent analysis of the axion domain-wall problem. The astrophysical and cosmological constraints on heavy axions are presented in Sec. V. A brief summary follows in Sec. VI.

It should be emphasized that many of the points made here have been made previously, although not in exactly the same context or detail. The issue of Planck-mass-suppressed higher-dimensional operators as a problem for axion models has been mentioned before [13]. The use of explicit symmetry breaking to solve the domain-wall problem has been studied, usually using soft ($d \leq 4$) symmetry breaking, in the context of grand unified theories [14]. The simple model presented in Sec. IV has not, to our knowledge, been discussed before. The subsequent analysis of the axion domain-wall problem is similar to that found in Refs. [15] and [16] with, however, some novel features leading to new possibilities for solving the domain-wall problem. The literature on the astrophysics and cosmology of axions is quite extensive [17,18]. Frieman and Jaffe [19] extended these arguments to more general pseudo Nambu-Goldstone bosons. In this paper we examine the consequences of including $d > 4$ gravitationally induced operators, albeit solely in the context of axion models.

II. EFFECTS OF GRAVITATIONALLY INDUCED OPERATORS ON AXION MODELS

We begin with the zero-temperature effective Lagrangian for a simple model that contains the essential axion physics we wish to study:

$$L=|\partial\phi|^2-\lambda(|\phi|^2-f_a^2)^2+\Lambda^4\cos(Na/f_a)+\frac{|\phi|^{n+3}(ge^{-i\delta}\phi+ge^{i\delta}\phi^*)}{2M_{\rm Pl}^n}\ . \qquad (1)$$

Here the axion field a is defined by $\phi=(f_a+\tilde{\phi})e^{ia/f_a}$, where $\langle\tilde{\phi}\rangle=0$. Λ is a scale of order 100 MeV, and the cosine term in which it appears represents the effects of QCD instantons. The last term is a dimension $n+4$ operator that is supposed to represent the effects of quantum gravity. In the absence of that term, the minimum would lie at $a=0$, which we take to be a point at which $\bar{\theta}\lesssim 10^{-9}$, as required by experiment. δ is a phase, defined relative to the phase in the quark mass matrix, which has no reason to be small. For small a one can expand the potential $V(a)$ as

$$V(a)\cong\frac{1}{2}\Lambda^4\left[\frac{Na}{f_a}\right]^2+\frac{1}{2}g\frac{f_a^{n+4}}{M_{\rm Pl}^n}\left[\left[\frac{a}{f_a}\right]^2\cos\delta-\frac{2a}{f_a}\sin\delta\right]\ . \qquad (2)$$

It is convenient to define $m_a=N\Lambda^2/f_a$ to be the QCD contribution to the axion mass and $m_*=[g\cos\delta f_a^{n+2}/M_{\rm Pl}^n]^{1/2}$ to be the gravitational contribution. Then $V(a)$ can be rewritten as

$$V\cong\tfrac{1}{2}m_{\rm phys}^2a^2-m_{\rm phys}^2\bar{\theta}f_a a\ , \qquad (3)$$

where

$$m_{\rm phys}^2=m_a^2+m_*^2\ ,\qquad \bar{\theta}=\left\langle\frac{a}{f_a}\right\rangle=\frac{m_*^2\tan\delta}{m_a^2+m_*^2}\ . \qquad (4)$$

We have assumed that a and, implicitly, m_*/m_a are small. The experimental limit on $\bar{\theta}$ implies that

$$m_*^2/m_a^2\le 10^{-9} \qquad (5)$$

or

$$g\frac{f_a^{n+4}}{M_{\rm Pl}^n\Lambda^4}\lesssim 10^{-9}\ . \qquad (6)$$

This is conveniently written as

$$\log_{10}f_a\lesssim\frac{19n-13-\log_{10}g}{n+4}\ , \qquad (7)$$

where we have adopted the convention $\log_{10}f_a\equiv\log_{10}(f_a/1\ \text{GeV})$, used $M_{\rm Pl}\approx 10^{19}$ GeV, $\Lambda\approx 0.1$ GeV, and assumed that δ is of order 1.

Using the constraint from supernova 1987A [21] that $f_a\gtrsim 10^{10}$ GeV, one finds that, for $g\sim 1$,

$$n\ge 6\ . \qquad (8)$$

We suggest that this is a nontrivial constraint on axion models.

It is possible to accommodate smaller values of n, if one is willing to assume that either g or δ is small. Without a specific model, this seems rather contrived. For example, to allow $n=5$ would require $|g|\lesssim 10^{-8}$, and $n=1$ requires $|g|\lesssim 10^{-43}$. Depending on the details of the model, the gravitational operator may be naturally suppressed by a symmetry factor that may be as small as $1/(n+4)!=1/d!$, but this is not small enough to affect our conclusions [20]. If δ happens to be less than 10^{-9}, then the strong CP problem is solved by fiat, for any n. In such a case, the mass of axion would be dominated by m_* for $n\le 5$. This leads to some interesting observations about constraints on axion models that we will explore in Sec. V.

III. COSMOLOGICAL CONSIDERATIONS

There are two cosmological issues that arise in ordinary axion models. First, one requires the relic density of axions to not overclose the Universe. Second, if the QCD instantons leave several degenerate minima, then a system of walls and strings results. If some method for eliminating this network is not present in the model, then the energy density in the network will overclose the Universe. The explicit breaking of the Peccei-Quinn symmetry by the gravitational operators mildly alters the first calculation and may provide a new method for eliminating the string-wall network.

A. Relic density of axions

There are two scenarios in which one may estimate the relic density of axions depending upon the possible occurrence and timing of a period of inflation. If the Peccei-Quinn transition occurs before or during an inflationary epoch, then the Kibble mechanism is inoperative. There will be no axionic strings, and domain walls are unlikely [22]. In this case the original estimates of the relic density of axions [10] from coherent oscillations of the axion field apply, although the amplitude of those oscillations is presumably a free parameter depending on the initial alignment angle θ_i of the axion field. We will assume that $\theta_i\sim 1$.

In the usual discussion of the misalignment case, the relic axion density is determined when the axion mass "turns on" at a temperature $T=T_1$, determined by the condition $m_a(T_1)\sim 3H(T_1)$. Numerically, this yields $T_1\sim 10\Lambda$. The factor of 10, taken from more detailed calculations of the temperature-dependent axion mass, has a weak dependence on f_a, which we shall ignore [23]. The number density of nonrelativistic axions at that time is $n_a\sim f_a^2H\sim f_a^2T_1^2/M_{\rm Pl}$, which may be compared to the photon number density T_1^3. The ratio $n_a/n_\gamma\sim f_a^2H(T_1)/T_1^3$ eventually determines the density in axions today, which is given approximately by $\Omega_a\sim f_a/10^{12}$ GeV.

Let us consider how a nonzero m_* affects this discussion. If $m_*\lesssim m_a(T_1)$, then m_* is never dynamically important, but if $m_*\gtrsim m_a(T_1)$, then the axion field will start to oscillate sooner, which, as we will see, reduces the relic density of axions. The constraint from strong CP violation, $m_*^2\lesssim 10^{-9}m_a^2$, leads to a relation between m_* and $H(T_1)$:

$$m_* \lesssim 10^{-4.5}\frac{\Lambda^2}{f_a} = 10^{-8}\frac{M_{\rm Pl}}{f_a}[3H(10\Lambda)] . \tag{9}$$

The numerical factor uses $H^2(T)=(8\pi/3)(\pi^2 g_*/30)T^4/M_{\rm Pl}^2$, where $g_* \sim 60$ is the number of relativistic degrees of freedom at T_1. For f_a in the range $10^{11} \lesssim f_a \lesssim 10^{12}$ GeV, one may have axions consistent with astrophysical and laboratory constraints, and their relic abundance is not modified by the presence of m_*. It is, however, possible to have $m_* > m_a(T_1)$ for the small range $10^{10} \lesssim f_a \lesssim 10^{11}$ GeV, and we look at the consequences.

When $m_* > m_a(T_1)$ the gravitationally generated axion mass causes oscillations to commence earlier than in the usual scenario. At that time the temperature is given by $T_* \approx \frac{1}{6}(M_{\rm Pl} m_*)^{1/2}$. The ratio of n_a to n_γ is now $f_a^2 H(T_*)/T_*^3$. Keeping in mind that $H \sim T^2$ and assuming that $g_*(T_*) \approx g_*(T_1)$, the effect of m_* is simply to multiply the comoving axion density of the usual scenario by a factor T_1/T_*, which is less than 1 for the case being considered. The axion contribution toward the closure density is then

$$\Omega_0 \sim \frac{10\Lambda}{T_*}\frac{f_a}{10^{12}\ \text{GeV}} . \tag{10}$$

We conclude that the relic axion abundance may be reduced in the presence of m_*. It does not, however, seem possible to relax the cosmological constraint on f_a since $10^{12} > 10^{11}$. The effects of m_* should not affect plans to build a detector of cosmological axions [24].

If no inflationary epoch is relevant, then the relic axion density is due to the evaporation of a network of axionic strings and domain walls [12]. We summarize the cosmological evolution that leads to these features.

When $T \sim f_a$, axion strings form by the Kibble mechanism. These have mass per length of order f_a^2. When QCD instanton effects become important at $T \sim \Lambda$, domain walls form that are bounded by these strings. A wall will have thickness of order $m_a^{-1} \sim f_a/\Lambda^2$ and mass per area of order $\mu \sim f_a \Lambda^2$. The number of walls connected to each string is given by N, the number of distinct degenerate minima in the instanton-generated potential for the axion. N is an integer that depends on such details of the underlying theory as the fermion content and Peccei-Quinn charge assignments. As one circumnavigates an axion string, the phase a/f_a changes by a multiple of 2π. For the minimal string with $\Delta(a/f_a)=2\pi$, a path around the string will visit successively the N distinct minima $(a/f_a)=2\pi/N$. Thus each such minimal string will have N axion domain walls emerging from it. If $N=1$, it has been shown by Vilenkin and Everett [25] that the network of strings and walls will be able to evaporate as soon as the axion mass is dynamically active, but if $N>1$, the network will persist and eventually overclose the Universe.

Let us deal with the $N=1$ case first. Davis [11] has argued that, in the absence of inflation, the density of axions from a decaying $N=1$ string-wall network will exceed that estimated for the coherent oscillations by a factor of ~ 100, pushing the cosmological constraint on f_a down to $\sim 10^{10}$ GeV. This is marginally in conflict with the supernova constraint and has been interpreted as evidence either for inflation or against axions [26].

In the present case a gravitationally induced mass may allow the network to evaporate earlier. As argued above, for a given f_a, this reduces the density of axions. Or keeping the density of axions fixed, one may reduce the constraint on f_a. Using the maximum-allowed m_*, $T_* = \frac{1}{6}(m_* M_{\rm Pl})^{1/2}$, and Eq. (10), we find that $f_a \lesssim 10^{10.3}$ GeV from the requirement that $\Omega_a < 1$. We conclude that if decaying axion strings are the dominant source of axions today, then including the potential effects of a gravitationally induced contribution to the axion mass relaxes the cosmological constraint on f_a by at most a factor of ~ 2. This makes any attempt to "close the axion window" by overlapping the astrophysical and cosmological constraints more difficult, but not by a large amount.

The analyses in this section depend only on the magnitude of m_*, but not on the source. As such, we conclude that the relic density of axions is not likely to be seriously altered in any model where explicit breaking in the Peccei-Quinn mechanism is weak enough to allow the strong CP problem to be solved in a natural way. Finally, we note that if the explicit CP-violating parameter δ is small, the constraint from Eqs. (4) and (5) is weakened, and the effects discussed here may be more interesting.

B. Networks of axionic strings and walls

We turn now to more complex string-wall networks for $N>1$, which may persist. Various solutions to this "axion domain-wall problem" have been proposed in the literature [27] including the idea that explicit violation of the Peccei-Quinn symmetry could lift the discrete degeneracy of the axion vacuum. Let us look at this idea in the present context.

The gravitationally induced terms in the Lagrangian typically split the energy densities for the minima by

$$\Delta\rho \sim \frac{2\pi}{N} g \frac{f_a^{n+4}}{M_{\rm Pl}^n} . \tag{11}$$

This implies a pressure difference between the different domains, which can act to push the walls around. The acceleration of the walls is given by

$$a_{\rm wall} \sim \Delta\rho/\mu \sim \frac{2\pi}{N}\frac{g f_a^{n+4}}{M_{\rm Pl}^n}/(f_a\Lambda^2) . \tag{12}$$

The time for a wall to become relativistic is

$$\tau \sim a_{\rm wall}^{-1} \sim \frac{N}{2\pi g}\frac{M_{\rm Pl}^n \Lambda^2}{f_a^{n+3}} . \tag{13}$$

The regions sitting in the $N-1$ minima that are not the true vacuum should shrink away in a time of order τ. At that point the network of strings and domain walls has effectively become an $N=1$ network; i.e., the N walls connected to a minimal axion string fuse to become a single wall. The temperature of the Universe at this point is

$$T_w \sim \sqrt{M_{\rm Pl}/\tau} \sim \left[\frac{f_a^{n+3}}{M_{\rm Pl}^{n-1}\Lambda^2} \frac{2\pi g}{N} \right]^{1/2} \tag{14}$$

or, comparing T_w to Λ,

$$\log_{10}\frac{T_w}{\Lambda} \sim \frac{1}{2}[(n+3)\log_{10}f_a - 19(n-1)+4] \ . \tag{15}$$

From the point of view of minimally disturbing the standard cosmology, it is desirable to have the network disappear as quickly as possible. We therefore want to increase f_a in Eq. (15) as much as possible consistent with the constraints both from consideration of the relic density of axions ($f_a < 10^{12}$ GeV) and from consideration of the strong *CP* problem [Eq. (7)].

As a fairly conservative requirement, we require $T_w/\Lambda \gtrsim 10^{-2}$, so that the network will disappear before nucleosynthesis. Plugging in the limiting value from Eq. (7) leads to a weak constraint on n, $69-5n>0$. A stronger constraint comes from $f_a < 10^{12}$ GeV, which gives $n \le 9$, with the $n=9$ case being marginal. We conclude that it is possible to solve the axion domain-wall problem by invoking gravitationally induced explicit symmetry breaking with $d=n+4=10$, 11, or 12.

There is an assumption in this argument that the energy density stored in the axionic domain walls is released into the Universe in a benign fashion. Before the network dissipates, its energy density increases relative to the energy density in nonrelativistic particles. Assuming a minimum of one string-wall per horizon, the energy density in walls is

$$\rho_{\rm wall} \sim f_a \Lambda^2 H \sim f_a \Lambda^2 T^2/M_{\rm Pl} \ . \tag{16}$$

By comparison, the energy density in nonrelativistic matter scales as T^3, and so the importance of the network increases as $1/T$. Vilenkin and Everett assumed that after the network was cut apart it would dissipate by graviton radiation, which would present no problems for cosmology. This seems plausible, but we know of no proof. It seems likely that axion radiation will be suppressed since by this time the curvature of the walls is much smaller than the axion Compton wavelength. However, if the energy density does get turned into nonrelativistic axions, then the relic axion density may exceed that estimated in the misalignment or $N=1$ string scenarios, which could make for another axion energy problem.

We note that this discussion has large uncertainties in both directions. If the network of strings and walls is not minimal, then the energy density will be greater. On the other hand, if axions are radiated, their spectrum may be relativistic. As pointed out by Harari and Sikivie [28] for $N=1$ strings, the number density of axions is decreased in this case and they eventually provide a smaller contribution to Ω_a.

In addition to providing a driving force for the walls after $T \sim \Lambda$, if $m_* > m_a(T_1)$, then the walls are present and felt before the QCD instanton potential turns on. In this case a string-wall network forms earlier, with an N' appropriate for the gravitationally induced explicit breaking. When QCD effects turn on, the network will be transmuted into the network described above. The danger of such a scenario is that the early set of walls may act to stabilize the string network, increasing the number density of strings and, potentially, the final number density of axions. The condition to avoid this problem is $T_* \lesssim T_1$, which is the converse to the condition considered in Sec. III A to relax the constraints on f_a. According to Eq. (9), this condition is almost always met. Even if it is not, without a detailed knowledge of how an $N>1$ string-wall network evolves it is not possible to comment on the severity of the problem.

IV. ILLUSTRATIVE FAMILY OF MODELS

Consider a model with the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)'$. Add to the standard model fields [which we take to be neutral under $U(1)'$] the following left-handed quarks and antiquarks:

$$(p+q)\times Q_0 + q\times \overline{Q}_p + p\times \overline{Q}_{-q} \ .$$

The subscripts refer to $U(1)'$ charge, and the integer in front of the Q's refers to the number of copies. These fields are assumed to be singlets under $SU(2)_L \times U(1)_Y$, and $Q(\overline{Q})$ are $3(\overline{3})$ under $SU(3)_c$. The $SU(3)_c^3$, $SU(3)_c^2 \times U(1)'$, and $SU(3)_c \times U(1)'^2$ anomalies trivially cancel. To cancel the $U(1)'^3$ anomaly, it is only necessary to add fermions, which are neutral under everything except $U(1)'$. Let there be scalar fields S_p and S_q that are standard-model singlets and have $U(1)'$ charges p and q, respectively, and let them acquire vacuum expectation values (VEV's) V_p and V_q. All of the extra quarks acquire masses from the terms $Q_0\overline{Q}_p\langle S_p^* \rangle$ and $Q_0\overline{Q}_{-q}\langle S_q \rangle$. Let p and q be relatively prime. Then the lowest-dimensional term that is allowed by $U(1)'$, which "knows" about the relative phase of S_p and S_q, is

$$O_{p+q} \equiv (S_p^*)^q (S_q)^p \ , \tag{17}$$

which has dimension $d=q+p\equiv n+4$. If $q+p>4$, the effective renormalizable Lagrangian has an accidental global U(1) symmetry. That is, neglecting the term O_{p+q}, there are two U(1) symmetries corresponding to rephasing S_p and S_q. One combination of these is local and anomaly-free, namely, $U(1)'$, while the other is global and anomalous. This latter is the Peccei-Quinn symmetry. f_a is given approximately by the smaller of V_p and V_q, which we can take to be V_q without loss of generality. If $10^{10} < V_q < 10^{12}$ GeV, the model is an invisible axion theory of the Kim-Shifman-Vainshtein-Zakharov (KSVZ) type [6]. If we choose p and q relatively prime and $p+q \ge 10$, then $\overline{\theta}$ comes out to be sufficiently small, in spite of gravitationally induced terms.

The cosmology of these models is instructive. If we take p and q to be relatively prime, it is easy to see that QCD instantons break $U(1)_{\rm PQ}$ to a unique vacuum (i.e., $N=1$). Since p and q are relatively prime, there exist integers a and b such that $bp+aq=1$. The rotation $S_p \to S_p\exp\{i2\pi a\}$, $S_q \to S_q \exp\{-i2\pi b\}$ changes $\overline{\theta}$ by $\Delta\overline{\theta} = 2\pi(bp+aq)$ (recall that S_q gives mass to p flavors of quarks and S_p^* gives mass to q flavors of quarks). So this rotation, which takes the fields back to themselves, changes $\Delta\bar{\theta}$ by exactly 2π. Thus $N=1$.

This argument was given first in Ref. [15], but as shown in Ref. [16], the uniqueness of the vacuum does not guarantee that there will be no domain-wall problem. Although the argument shows that one can mathematically construct a string for which $\Delta\theta_p=2\pi a$ and $\Delta\theta_q=-2\pi b$, which has one domain wall coming out of it (θ_p and θ_q are henceforth the phases of S_p and S_q, respectively), these are not necessarily the strings that form in the early Universe. What happens is the following. When $T\sim V_p$, S_p develops a VEV that breaks U(1)'. *Local* strings form that we will call S_p strings. The lowest-energy S_p strings have $\Delta\theta_p=2\pi$. Later, when $T\sim V_q$. S_q develops a VEV and Goldstone bosons appear. Where there is already a *local* S_p string, the phase θ_q will try to adjust itself to minimize its kinetic energy. One can easily show, if $\Delta\theta_p=2\pi n$ around the local S_p string and $\Delta\theta_q\simeq 2\pi m$, that the energy due to winding is proportional to $(nq-mp)^2$. Moreover, $\Delta\bar{\theta}=2\pi(nq-mp)$. Therefore, for the minimal ($n=1$) S_p strings, which we assume to be the most abundant, m adjusts itself to be that value which minimizes $(q-mp)^2$. The number of walls that ultimately will be bounded by such a string is

$$N_{\rm wall}(S_p \text{ string})=|q-mp|_{\min} \ . \tag{18}$$

where the minimum is with respect to varying the integer m. The strings we have discussed are not the only strings that will form. Even where no local S_p string exists, when $\langle S_q\rangle\neq 0$ develops, S_q strings will form by the Kibble mechanism. These have $n=0$ and, if they are minimal, $m=\pm 1$. For these, $\Delta\bar{\theta}=\pm 2\pi p$. Therefore

$$N_{\rm wall}(S_q \text{ string})=|p| \ . \tag{19}$$

What is required [25] to chop up the network of domain walls that form later at $T\sim\Lambda$ is a preexisting distribution of strings with $N=1$. Thus a domain-wall problem does not arise if one of the following conditions is satisfied:

$$|p|=1 \ , \tag{20a}$$

$$|q|=1 \ , \tag{20b}$$

$$\exists m \ni |q-mp|=1 \ . \tag{20c}$$

The foregoing analysis differs from that in Ref. [16] in that here one of the two U(1) symmetries is local. If both are global, then S_p strings will have $\Delta\theta_p=2\pi$, $\Delta\theta_q=0$, and the S_q strings will have $\Delta\theta_p=0$, $\Delta\theta_q=2\pi$. As a result, the domain-wall problem is avoided only if condition Eq. (20a) or (20b) is satisfied, but not for Eq. (20c).

Condition Eq. (20c) means that there is no domain-wall problem if $p=1$, 2, 3, 4, or 6 and p and q are relatively prime. This comes close to the kind of automatic solution to the domain-wall problem envisioned in Ref. [15].

We should point out that in the case we have been considering instanton effects give a potential proportional to $(S_p^*)^q(S_q)^p+\text{H.c.}$, which is precisely the same as the form of the lowest-dimensional gravitationally induced operator. Thus these gravitationally induced operators cannot in this case lift the N-fold degeneracy of the axion vacuum discussed in the previous section.

Let us now turn to the case where p and q have largest common divisor k. Let $p=\tilde{p}k$ and $q=\tilde{q}k$. This model is then just equivalent to a model with k families of the form $(\tilde{p}+\tilde{q})\times Q_0+\tilde{q}\times\bar{Q}_{\tilde{p}}+\tilde{p}\times\bar{Q}_{-\tilde{q}}$, where $\tilde{p}$ and $\tilde{q}$ are relatively prime. The above analysis changes in two crucial ways. Now the vacuum is not unique, but has a k-fold degeneracy. Also, $N_{\rm wall}(S_{\tilde{p}} \text{ string})=k|\tilde{q}-m\tilde{p}|_{\min}$ and $N_{\rm wall}(S_p \text{ string})=k|\tilde{p}|$. It would seem that a domain-wall problem is unavoidable. However, whereas QCD instantons lead to a potential proportional to $[(S_{\tilde{p}}^*)^{\tilde{q}}(S_{\tilde{q}})^{\tilde{p}}]^k$, gravitational effects can induce an operator $(S_{\tilde{p}}^*)^{\tilde{q}}(S_{\tilde{q}})^{\tilde{p}}$. This operator lifts the k-fold degeneracy of the axion vacuum. Thus, when walls form, if $n=\tilde{p}+\tilde{q}-4=6$, 7, or 8 and f_a satisfies the constraint outlined following Eq. (15), then the $k-1$ false vacua will be squeezed out, and k walls will fuse to form one wall. The conditions for solving the domain-wall problem then reduce to those given in Eqs. (20a), except p and q are there to be replaced by $\tilde{p}=p/k$ and $\tilde{q}=q/k$. Let us give some concrete examples.

(1) $p=3$, $q=7$, $V_q=10^{10}$ GeV. Here $d=n+4=3+7=10$, and so $\bar{\theta}$ is sufficiently small, and for $m=2$, $|7-3m|=1$; so no domain-wall problem exists.

(2) $p=5$, $q=7$, $V_q=10^{10}$ GeV. Again, $d=5+7\geq 10$, and so $\bar{\theta}$ is small enough, but no m exists such that $|7-5m|=1$; so that domain walls kill this model.

(3) $p=3$, $q=9$. Here $k=3$, $\tilde{p}=1$, $\tilde{q}=3$, and so $d=1+3<10$, and $\bar{\theta}$ is too big.

(4) $p=15$, $q=18$. Here $k=3$, $\tilde{p}=5$, $\tilde{q}=6$, $d=5+6>10$, and $\bar{\theta}$ is small enough. For $m=1$, $|6-5m|=1$, and domain walls are not a problem provided the gravitational pressure can act to squeeze out the $k-1$ false vacua as discussed in Sec. III B.

V. ALIGNMENT AND "HEAVY" NAMBU-GOLDSTONE BOSONS

It is possible, either by accident or design, that $|\delta|<10^{-9}$. To be honest, we do not know of any theoretical construct where this occurs as an automatic consequence of the model, but if one wishes to consider axion models with large m_*, then this is required by the experimental upper limits on the neutron electric dipole moment. There are then no laboratory constraints on m_* from considerations of θ, and so we turn to cosmology and astrophysics. First, we note that the usual relation between the axion mass and its coupling to matter is altered. The coupling to matter g_a is still proportional to $1/f_a\sim m_a/\Lambda^2$, whereas the physical axion mass is now given by m_*. As a result, the usual constraints, e.g., from stellar evolution or the relic density of axions, must be reworked. The main result is that by making m_* large one can avoid the usual constraints from cosmology.

Much of the content of this section is not specific to axions. We are working in the regime where the QCD instanton contribution to the axion mass in dynamically unimportant. Thus, apart from the relation to the strong CP problem, our remarks apply to any theory where a global symmetry breaks spontaneously with a large VEV and is explicitly broken by gravitational effects. The interactions with matter are somewhat model dependent, but if the scalar field which acquires a VEV is weakly mixed with the Higgs field of the standard model, matter couplings will be similar in strength to those in the axion model. Interactions with the gauge bosons depend on the presence of anomalies. Frieman and Jaffe [19] have considered axionlike pseudo Nambu-Goldstone bosons that may arise in, e.g., technicolor theories, but they did not include the effects of higher-dimensional operators. Some of the results given here may be readily applied as extensions to their work.

For most of this section, we take the dimension of the gravitational terms to be $n \leq 6$. For $n<6$, alignment is required. For $n \geq 6$, one may use all the known astrophysical constraints for $f_a \lesssim 10^{12}$ and no alignment is necessary. For $n \leq 6$, we are dealing almost exclusively with "heavy" axions. The axion mass is given by $m^2_{\rm phys} = m_*^2 + m_a^2$. The condition for $m_* \gtrsim m_a$ is $f_a \gtrsim \Lambda^{(4/n+4)} M_{\rm Pl}^{(n/n+4)} g^{-(1/n+4)}$ or

$$\log_{10} f_a \gtrsim \frac{19n - 4 - \log_{10} g}{n+4} \, . \tag{21}$$

The constraint on f_a from Eq. (21) is shown in Fig. 1 by bold marks on the line for each value of n. The style of the marks is indicated in the legend by $m_* = m_a$. In the region to the left of these marks, $m_a > m_*$, and any of the usual constraints on axion models apply. The strong *CP* problem would be solved for f_a to the left of the light marks labeled $\bar{\theta}$. As already explained, these values are excluded for $n<6$. Figure 1 also contains a summary of the results from the rest of this section.

In locating the marks in Fig. 1, we have taken $g=1$. Other values of g will result in different constraints, but unless g is much different from 1, the effect will not alter any of our conclusions. For example, the symmetry factor [20] naturally gives

$$\ln g \lesssim -(n+4)[\ln(n+4) - 1] \, ,$$

which is small compared to the factor of 19 which results from $\log_{10} M_{\rm Pl}/{\rm GeV}$.

A. Astrophysical constraints

The strongest astrophysical constraint on axion models comes from considering the cooling of the neutron-star cores of supernovae [21]. Axions produced in the core may escape freely, robbing the supernova of energy to power the late (10 sec) neutrino emission. Since this emission was in fact observed, one may place a bound on the axion-nucleon coupling, which is generally believed to correspond to $f_a \lesssim 10^{10}$ GeV. The exact value of this bound is somewhat uncertain, depending upon uncertain knowledge of the details of supernovae and the emission rates for axions from quasidegenerate nuclear material [29].

In the present context it is also possible to avoid axion emission by making the axion heavy [19], thus providing an $\exp(-m_*/T_c)$ suppression to the emission. For core temperatures $T_c \sim 50$ MeV, we estimate that it should be sufficient to have $m_* \gtrsim 0.3$ GeV. Given $m_* = f_a g^{1/2} (f_a/M_{\rm Pl})^{n/2}$, we find that

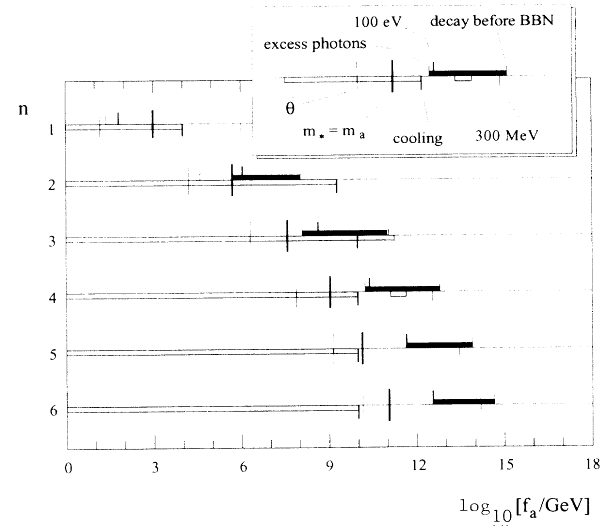

FIG. 1. Summary of the astrophysical and cosmological constraints on "heavy" axions for different n, the power of $M_{\rm Pl}$ suppressing the explicit breaking of the Peccei-Quinn symmetry. Values of f_a relevant for astrophysical considerations are shown below the line for each n, while those of relevance for cosmology are above the line. The range of validity of the analyses presented is $m_* = m_a$, denoted by the bold mark which extends above and below each line. The value of f_a necessary to solve the strong *CP* problem is denoted similarly by the light mark labeled $\bar{\theta}$ in the legend. All the constraints in this figure are drawn taking the dimensionless strength of the gravitationally induced symmetry breaking to be $g=1$. The gray bars show constraints arising from astrophysical considerations. For $f_a < 10^{10}$ GeV the constraints arise from the cooling of supernovae, except for $n=1$, where it is derived from considering helium ignition in red giants. The lower limit to f_a from the cooling constraint is shown by a strong mark descending from the central line, labeled "cooling." For $f_a > 10^{10}$ GeV the constraints arise from considering a γ-ray flux from the decay of $\sim$100-MeV axions emitted from supernova cores. Any supernova constraint will cutoff when the axion mass exceeds $\sim$300 MeV, denoted by the light descending mark labeled "300 MeV." We extend the astrophysical constraints into the region where $m_* < m_a$ based on our knowledge of these constraints in the usual axion scenario. The black bars show regions constrained by cosmological considerations. The strong ascending mark labeled "100 eV" denotes $m_* = 100$ eV. Values of f_a to the right of this mark are constrained by considerations of the relic density of axions, unless the axions decay. To the right of the light ascending mark labeled "decay before BBN," axions decay before the time of big-bang nucleosynthesis. Axions lighter than 100 eV may decay into an observable flux of photons. The light ascender marked "excess photons" shows the limiting value of f_a necessary to keep the axion stable long enough to avoid this problem. Although we do not extend the cosmological constraint bar below the $m_* = m_a$ mark, we still position ascending marks as if m_* dominated the mass. There is no cosmological constraint for $n=1$ since the axions decouple after they become nonrelativistic in the early Universe, and their number density is negligible.

$$\log_{10} f_a \gtrsim \frac{19n-1-\log_{10} g}{n+2} . \tag{22}$$

This constraint is shown by the light descending marks labeled 300 MeV in Fig. 1. For $n=1,2$ the astrophysical constraint is softened from its usual value of $f_a > 10^{10}$ GeV, but for $n \geq 3$ the cooling constraint is unaffected by the axion mass. Concerning the $n=1$ case, we recall that for $f_a \lesssim 10^7$ GeV axions are coupled sufficiently strongly to matter that they are trapped inside the dense core [30] and may only be emitted from the cooler surface. In this case there is no cooling constraint from supernovae and one would need to consider the next most stringent stellar constraint: helium ignition in red giants. Then the relevant temperatures are more like 10 keV, which would give a correspondingly weaker constraint that $f_a \gtrsim 10^4$ GeV, in order that the axion be heavy ($m_* \gtrsim 200$ keV) in an $n=1$ model. The question of trapping in red giants is discussed by Raffelt [18], who concludes that for axions nominally coupled to electrons ($g_{ae} \approx m_e / f_a$) the axions would be trapped for $f_a \lesssim 10^3$ GeV. There is then an excluded region of $10^3 < f_a < 10^4$ GeV, which we will take as the astrophysical constraint for $n=1$. For $f_a < 10^3$ GeV the axion mass is dominated by m_a and the usual arguments constrain such models, including limits on f_a based upon the absence of a signal in experiments where axions are produced in the laboratory. The astrophysical limits from stellar cooling are shown as the lightly shaded bars in Fig. 1, which lie to the left of $f_a = 10^{10}$ GeV.

It is also possible to obtain astrophysical limits by considering axion decays into photons. In axion models with large f_a, this is not usually interesting because of the axion's small mass and long lifetime. Here the axion mass may be large and hence decay faster, and some new constraints arise from the absence of an excess γ-ray flux associated with supernovae.

The time integral of the γ-ray flux expected from SN 1987A is

$$\Phi \approx \frac{f E_{\text{tot}}}{\langle \epsilon \rangle} \frac{1}{4\pi d^2} P_{\text{dec}} , \tag{23}$$

where $E_{\text{tot}} \approx 10^{53}$ erg is the energy budget available for axion emission, $\langle \epsilon \rangle \approx 0.1$ GeV is the average energy per axion, f is the fraction of the energy available to axions that is actually emitted as axions (see below), $d \approx 50$ kpc is the distance to the supernova, and P_{dec} is the probability for an axion to decay in flight, to photons. Since cooling is saturated for $f_a = 10^{10}$ GeV and is proportional to the production cross section above that, we take

$$f = \text{Min}[1, (10^{10}\ \text{GeV}/f_a)^2 e^{-m_*/T_c}] .$$

The exponential factor accounts for the cutoff to the emission due to the axion mass. We take $T_c = 50$ MeV for this discussion. The decay probability is given by

$$P_{\text{dec}} = \Gamma_{a\gamma\gamma} t / \gamma_{\text{rel}} , \tag{24}$$

where γ_{rel} is the relativistic time dilation factor, $\Gamma_{a\gamma\gamma}$ is the decay rate into photons, and t is the flight time of the axion. The two-photon decay rate is

$$\Gamma_{a\gamma\gamma} \sim \alpha^2 \frac{m_*^3}{f_a^2} . \tag{25}$$

The "flight" time of the photon is the shorter of the distance to the supernova, $t_{\text{SN}} \approx 5 \times 10^{12}$ sec, and the decay

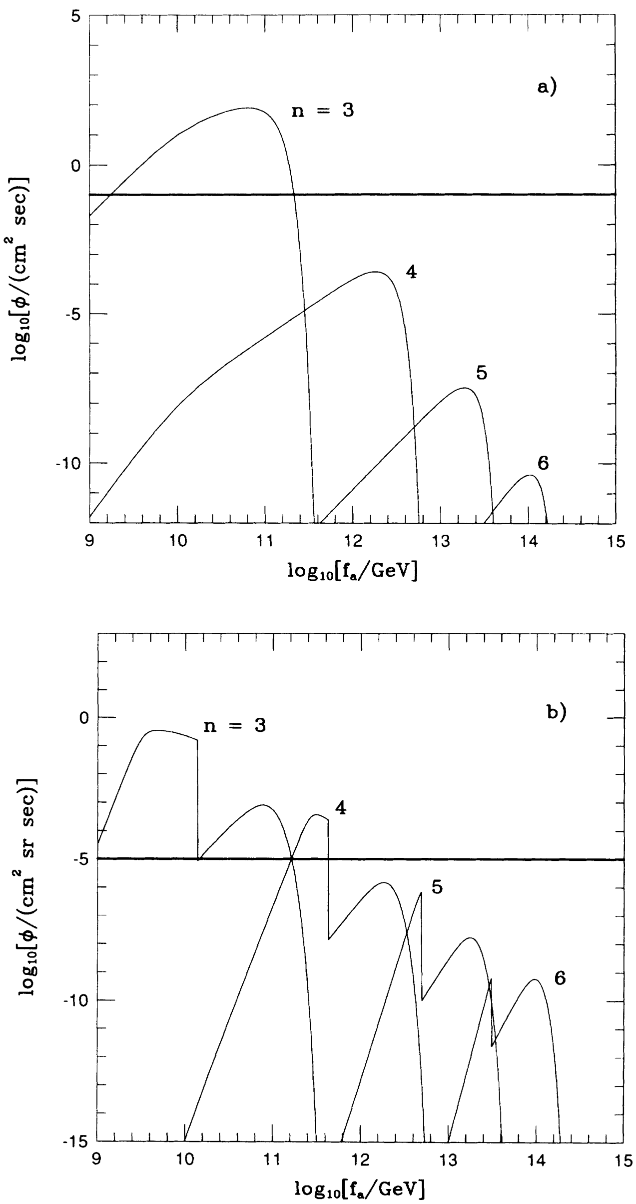

FIG. 2. γ-ray fluxes expected from the decay of heavy axions emitted from supernovae. (a) The flux expected from SN 1987A for $n=3$, 4, 5, or 6 as a function of f_a. The horizontal line shows the background rate from the gamma ray spectrometer [31] for the energy band 25–100 MeV, which we use as a constraint. (b) The γ-ray intensity expected from summing over all supernovae in the history of the Universe. The constraint shown is the high galactic ($b=90$) intensity reported by the SAS-2 group [34]. The structure in each curve is due to competition between the two-photon and e^+e^- decay channels for the axion. This structure is absent in (a) because the decay length is generally greater than the distance to the supernova for $m_* \sim m_e$.

length of the axion, $t_{\rm dec}$; specifically, we use $t=t_{\rm SN}t_{\rm dec}/(t_{\rm SN}+t_{\rm dec})$. The decay rate of the axion must be Lorentz dilated and must include the possibility of decay into other channels, $t_{\rm dec}=\gamma_{\rm rel}/(\sum_i \Gamma_i)$. We include decay into an electron-positron pair with a rate

$$\Gamma_{ae\bar{e}} \sim \frac{m_* m_e^2}{f_a^2} \ . \tag{26}$$

Other fermions do not contribute significantly in the regime where $m_* < 0.3$ GeV, and we will similarly ignore the decay into two gluons.

The γ-ray flux is spread out over a time Δt, which is dominated by the velocity dispersion of the heavy axions, $\Delta t \approx t/(2\gamma_{\rm rel}^2)$. The instantaneous flux of axions is given by $\phi \approx \Phi/\Delta t$, and this may be compared with the background. In Fig. 2(a) we show ϕ for various values of n and f_a.

Results from the gamma ray spectrometer (GRS) at the Solar Maximum Mission place a limit [31] on the γ-ray flux integrated over 10 sec from SN 1987A of $\Phi < \Phi_* \approx 0.6$ cm^{-2} in the energy band $25 < E_\gamma < 100$ MeV. However, with a significant spread in arrival times, it is more appropriate to compare with the background count rate for the GRS, which is roughly 0.1 cm^{-2} sec^{-1} in this energy band. This level is shown as the horizontal line in Fig. 2. Comparison with the $n=3$ curve yields a new constraint on f_a for $n=3$, but not for other n.

It is also possible to consider the cosmological γ-ray background from all past supernovae. The argument is similar. The predicted flux per steradian is

$$\phi \approx \frac{\Gamma_{\rm SN} t_U}{4\pi} \frac{f E_{\rm tot}}{\langle \epsilon \rangle} P_{\rm dec} \ , \tag{27}$$

where $\Gamma_{\rm SN}=\Gamma_{84}10^{-84}$ cm^{-3} sec^{-1} is the rate of supernovae occurrence averaged over the history of the Universe [32] and t_U is the age of the Universe. We use $\Gamma_{84}=1$ and $t_U=10^{10}$ yr (see Woosley, Wilson, and Mayle [33] for a discussion of $\Gamma_{\rm SN}$). The predicted flux is shown in Fig. 2(b). This should be compared with the lowest observed γ-ray background. Near the galactic poles the background integrated from 100 to 200 MeV is approximately [34] $\phi_B \approx 10^{-5}$ cm^{-2} sec^{-1} sr^{-1}. Comparison with Fig. 2(b) again leads to a constraint for $n=3$ and also to a weaker constraint for $n=4$. The dips in these spectra are due to competition from the electron decay channel. Those dips are not present in the SN 1987A fluxes because the decay length is greater than the distance to the supernova.

The constraints derived from supernova-produced axions decaying into photons are shown in Fig. 1 as the $f_a > 10^{10}$ GeV extensions to the shaded bars for $n=3,4$. For $n=4$ this extension is disconnected from the cooling constraint.

B. Cosmological constraints

The cosmological constraint on axion models arises from requiring the relic density of axions to be less than the critical density needed for closure of the Universe. There are two contributions to the relic axion density: (a) a thermal distribution and (b) a coherent field energy. The coherent field energy may show up in two forms, either due to a random misalignment of the phase of the ϕ field relative to δ or in association with topological defects such as strings and walls.

1. Thermal relics

We begin with the thermal density. The energy density from this contribution is not usually significant for ordinary axion models because the axions are so light, although it may play a role in some hadronic axion models where low value of f_a may be allowed [35]. Similarly, there is not ordinarily a constraint on the relic abundance of thermal axions from big-bang nucleosynthesis since they freeze out early and contribute only a fraction of a relativistic species to the density of the Universe. The first statement must be reexamined for heavy axions, although the second still holds.

The first issue is to determine when the axions were last in thermal equilibrium with the rest of the plasma. The condition for thermal equilibrium is that $\Gamma/H \sim TM_{\rm Pl}\sigma \gtrsim 1$, where Γ is a typical interaction rate to emit or absorb an axion from a given momentum state, H is the Hubble parameter, and σ is a typical cross section for axion emission and absorption off the other particles in the plasma. We write the cross sections for emission and absorption of axions by ordinary matter as $\sigma=\epsilon_A/f_a^2$. The value of ϵ_A is order α_s^3 when the processes $q+a \leftrightarrow q+g$ are mediated through the $aG\,{}^*G$ interaction in a Primakoff-type process. Here G is the SU(3) color field strength and ${}^*G$ is its dual. Since color is a non-Abelian gauge group, there are a host of diagrams that may mediate $a+g \leftrightarrow g+g$ interactions, which should also be order α_s^3 in strength. Axion-glue "Compton" scattering from quarks is also possible with a strength $\epsilon_A \sim \alpha_s m_q^2/T^2$. Photons can replace the gluons by adding factors of α/α_s. The glue interactions should be dominant with $\epsilon_A \sim 1$ due to the strong coupling. Then freeze-out for the axions occurs at a temperature $T_F \sim f_a^2/(\epsilon_A M_{\rm Pl})$. This is sufficiently late in the history of the Universe that axions may be assumed to have been in thermal equilibrium. At the same time it is sufficiently early for plausible f_a that big-bang nucleosynthesis is not threatened.

The next issue to settle is whether or not the axions decouple while they are relativistic. Comparing T_F with m_*, axions decouple after becoming nonrelativistic if

$$\left[\frac{f_a}{M_{\rm Pl}}\right]^{(2-n)/2} < \epsilon_A g^{1/2} \ . \tag{28}$$

If $n=1$, axions become heavy before they decouple. As a result, their number density today is negligible, being suppressed by a factor of $\exp(-m_*/T_F)$. On the other hand, for $n=3$, 4, or 5, axions are as abundant as any relativistic species. For $n=2$, we make the assumption that the right-hand side of Eq. (28) is less than unity, and again a relativistic abundance would follow.

For the cases $n=2$, 3, 4, and 5, if the axions were stable, there would be a constraint on their mass $m_* \lesssim 100$ eV arising from the requirement that their density not overclose the Universe. The exact constraint depends upon the number of relativistic degrees of freedom at axion freeze-out, the amount of entropy produced after freeze-out, etc. [36]. The constraint on the mass translates into a constraint on f_a,

$$\log_{10} f_a \lesssim \frac{19n-14-\log_{10} g}{n+2} , \tag{29}$$

which is shown by the ascending strong marks labeled "100 eV" in Fig. 1. For $n=2$ or 3, this constraint is incompatible with the constraint from astrophysics; however, we have not yet allowed for the possibility of axion decay.

We use the same decay rates as in the astrophysical arguments, except now we include the possibility of decay into $\mu\bar{\mu}$, gg, and $q\bar{q}$ pairs. For $m_* \gtrsim 100$ MeV this gives $\Gamma_a \sim m_*^3/f_a^2$. For $m_* \lesssim 100$ MeV we use a smaller value for decay into photons, $\Gamma_a \sim \alpha^2 m_*^3/f_a^2$, enhanced where possible by decay into e^+e^- pairs.

In general, we expect that the decays will occur after the axions become nonrelativistic. If decays occurred while the axions were relativistic, then inverse decays would keep the axion distribution in thermal equilibrium; but we already know, for $n=2$, 3, 4, or 5, that freeze-out occurs while the axions are relativistic. Given that the decays occur after the axions become nonrelativistic, the energy density in axions is $\rho_a \sim T^3 m_* \sim (m_*/T)\rho_{\rm rad}$; i.e., the energy density is dominated by the axions. It follows that entropy is produced in the decays. Since it is difficult to accommodate a significant amount of entropy production after big-bang nucleosynthesis (BBN), we will constrain the axions to decay before nucleosynthesis. This leads to the constraint $\Gamma_a > H(T_{\rm BBN})$, where $T_{\rm BBN} \approx 1$ MeV is the temperature to be associated with nucleosynthesis [37]. Assuming that axion decay is dominated by decay into gluons, it is necessary that

$$\log_{10} f_a \gtrsim \frac{57n-50-3\log_{10} g}{3n+2} . \tag{30}$$

This constraint is shown by the light ascending marks labeled "decay before BBN" in Fig. 1. In the case $n=2$, the associated m_* is less than the pion mass, and so we use the weaker decay rate into photons. However, since the axions are nonrelativistic and the decays occur before $T=1$ MeV, it follows that the electron channels are open. Except for a small mass range just around 100 MeV, the constraint is not substantially altered.

When the axion decays before the onset of nucleosynthesis, the major effects on cosmology are the dilution of the baryon number by entropy creation and the possible creation of baryon number by B and CP violation in the decay process [38]. All other decay products thermalize. It seems likely that models can be constructed wherein the baryon number is created in the axion decay, thus eliminating the need for specific constraints on the entropy production.

Axion decays provide another constraint when $m_* < 100$ eV. If the decay rate is too fast, a flux of observable photons may occur. The exact constraint will depend upon the mass of the axion since that determines the wavelength for the photons. We look at constraints for axion masses in the range $T_0 < m_* < 100$ eV, where T_0 is the present temperature of the microwave background. Constraints for $m_* > 100$ eV are unnecessary since that range has already been addressed, and axions lighter than $\sim 2.5\times10^{-4}$ eV live long enough that they present no serious constraints, even with our modified relation between mass and f_a.

It is a formidable task to treat this whole range of masses carefully—the photon energies span everything from the microwave to the extreme ultraviolet. Rather than attempt this, we will borrow a result from Ressel and Turner [39], who summarized the diffuse electromagnetic spectrum for photon energies $10^{-9} < E_\gamma < 10^{20}$ eV. As an application of their summary, they gave constraints on the lifetime of neutrinos. Although they give many individual results, in the range $10^{-3} < E_\gamma < 10^2$ eV, one may approximate the lifetime limits by

$$\tau_\nu \sim 10^{16.5} B_r \left[\frac{m_\nu}{10^{-3.25}\ \text{eV}} \right]^{1.5} \text{sec} , \tag{31}$$

where B_r is the radiative branching ratio and m_ν is the neutrino mass. This form overstates their constraints for $E_\gamma \sim 0.1$ eV by two orders of magnitude, but otherwise is within a factor of 3 of their results over the whole range. Their constraint assumes that the neutrinos were in thermal equilibrium until the Universe cooled to $T \sim 1$ MeV as in the standard cosmology. We adapt this result to our axion case by taking $B_r = 0.1$ to allow for the fact that the axions will freeze out earlier and thus be diluted. Using m_* instead of m_ν and the decay rate to axions, we find a constraint on f_a:

$$\log_{10} f_a < \frac{19n-24-\log_{10} g}{n+\frac{10}{9}} . \tag{32}$$

This constraint is shown as a light ascending mark labeled "excess photons" in Fig. 1. For $n=2$, 3, or 4, this mark lies to the left of the strong mark, indicating a mass of 100 eV. For $n=1$ there is no constraint since there are no relic axions to speak of. If plotted, the constraint would lie off the graph. For $n=2$ the constraint lies to the left of the $m_* = m_a$ line, and so the use of m_* in Eq. (32) is inconsistent; however, the associated value of m_* is still greater than T_0, and so we show the mark anyway as it may be useful for a nonaxionic model involving anomalous coupling to photons. For $n=5$ or 6, Eq. (32) implies a mass greater than 100 eV and is therefore not consistent with our use of Ressel and Turner; so we do not show it.

For each n we show the cosmological constraints from relic axions as the dark bar in Fig. 1. The closure argument applies to the right of the $m_* = 100$ eV mark, while the excess photon constraint is to the left.

2. *Coherent field energy*

The next case we consider is that of coherent field energy from random alignment. The treatment is simpler than in the usual axion scenario because the axion mass is not a function of temperature. Just after the axion mass turns on at $T_* \sim (m_* M_{\rm Pl})^{1/2}$, i.e., when $H \sim m_*$, the energy density in axions is $\rho_a(T_*) \approx f_a^2 m_*^2 \approx f_a^2 H^2$. This energy density is equivalent to a number density $n_a \sim f_a^2 m_*$ of axions with mass m_*, which may be compared with the number density in thermal axions. The ratio of the two quantities is

$$R \equiv \frac{n_{\rm coh}}{n_{\rm th}} \sim \left[\frac{f_a}{M_{\rm Pl}}\right]^{(6-n)/4} g^{-1/4} . \tag{33}$$

If $n < 6$, the coherent state axions are not as important as the thermal axions. Even for $n = 6$ this is likely to be so as far as Eq. (33) is concerned; however, in the $n \geq 6$ cases, smaller values of f_a allow the strong *CP* problem to be solved with an axion mass less than 100 eV.

In passing, we note that $T_F < T_*$ for $n \leq 6$, suggesting that the coherent axion state may dissipate. In fact, the proper condition for the coherent state to dissipate is that $\Gamma / H > n_{\rm occup}$, where $n_{\rm occup}$ is the typical occupancy number of the low-momentum states. It turns out that this condition is not satisfied for $n > 1$.

3. *Network of walls and strings*

Next, we consider the topological defects. Let the PQ charge of the lowest-dimensional gravitationally induced operator to break explicitly the symmetry be N'. Consider the network of strings that forms when the Peccei-Quinn symmetry breaks spontaneously at $T \sim f_a$. Each string will eventually be connected to N' walls, but not until the temperature falls to T_* and the mass of the Goldstone bosons is dynamically active. The string tension is $\sigma \sim f_a^2$, while the surface tension for the walls is $\mu \sim f_a^2 m_*$. When $N' = 1$ such a network can dissipate as soon as the walls can accelerate the strings to relativistic velocities. This occurs after a time $\tau \sim \sigma / \mu \sim 1/m_*$, which implies that the string network dissipates as soon as the walls form. Since this is the same time that the coherent field energy due to random misalignment becomes active, these two cases are similar. The energy density from the strings may exceed that from the random alignment by the logarithmic factor $\ln f_a / H$ associated with global strings [11], but that will not alter the conclusion that thermal relics dominate over coherent relics for the $N' = 1$ case.

If $N' > 1$ the string-wall network is stable as long as the different vacua are degenerate. If the degeneracy is lifted, pressure forces can cause the system to collapse, as discussed in Sec. III. The degeneracy may be lifted either by the QCD anomaly or by the next highest term in the gravitational expansion of the Lagrangian $L_m \sim f_a^4 g (f_a / M_{\rm Pl})^n g' (f_a / M_{\rm Pl})^m$. In simple cases the condition for either of these to completely lift the degeneracy is that N and N' are relatively prime, where N is the Peccei-Quinn charge of the term that breaks the degeneracy. If QCD lifts the degeneracy, then the pressure on the walls will be $\sim \Lambda^4$, while the surface tension is $\mu \sim f_a^2 m_*$. The network can dissipate immediately after the turn on of the QCD instantons if

$$\log_{10} f_a < \frac{19n + 34 - \log_{10} g}{6 + n} . \tag{34}$$

If higher-dimensional gravitational terms break the discrete N-fold symmetry, then the pressure is given by L_m and the condition that must be satisfied is

$$\log_{10} f_a > \frac{19(n + 2m) - 42 - \log_{10} g - \log_{10} g'}{n + 2m + 2} . \tag{35}$$

If either Eq. (34) or (35) is satisfied then the $N' > 1$ network can dissipate by the time $T = \Lambda$. Once the network dissipates, it is possible for the axions to decay, and the constraint for this to occur by big-bang nucleosynthesis is already shown in Fig. 1.

On the other hand, if the axions do not decay, the constraint on the axion mass may be more serious than in the $N = 1$ or the random alignment cases. For reasons discussed in Sec. III B, as the Universe evolves after the network forms at T_*, the energy density in walls increases relative to that in nonrelativistic matter. Thus there is no guarantee that thermal axions will dominate the relic axion density and the mass constraint may be strengthened. This argument is subject to all the uncertainties outlined in Sec. III.

Finally, we recall that if the constraints for string-wall systems are not satisfied, then one may always turn to inflation as a mechanism to align the phase of the scalar field over seemingly acausal distances, thus avoiding the presence of the network.

VI. SUMMARY

If we accept the argument that Planck-scale physics will not respect global symmetries, then it follows that global symmetries can only arise as an accidental consequence of an underlying local gauge symmetry. Further, it is expected that any such global symmetry will be explicitly broken by higher-dimensional ($d > 4$) terms in the effective Lagrangian, which will be suppressed by appropriate powers of $M_{\rm Pl}$.

We have examined the consequences of these terms for axion models. The major result is that the dimensionality of these terms must be at least $d = 10$ for axions to provide a natural solution to the strong *CP* problem. The explicit breaking of the Peccei-Quinn symmetry may be used to soften the cosmological constraints from consideration of the density of relic axions and axionic walls and strings. If axions from the decay of strings dominate the density, the constraint on f_a may be relaxed by a factor of about 2. In the cases where $d = 10$, 11, or 12, the problem of axionic domain walls may be solved. It is not difficult to construct explicit models of the KSVZ type that illustrate our arguments.

Section V is given over to a discussion of the effects that a gravitationally induced mass will have on the phenomenology of Goldstone bosons in cosmology and astrophysics. The results of this discussion are summarized in Fig. 1. The major result is that for large f_a the large gravitational mass allows the axion to decay before nucleosynthesis, and so there is not a serious cosmological constraint in this case. Although our arguments are couched in terms of axion models, many of the constraints may be extended to other specific cases. The axion models discussed in this section are not natural solutions to the strong *CP* problem, in that a small parameter (δ) is needed, but the smallness of this parameter is not a factor in the constraints presented.

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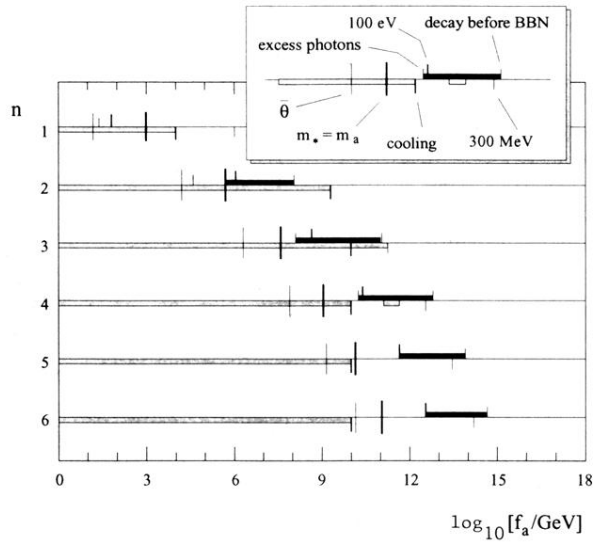

FIG. 1. Summary of the astrophysical and cosmological constraints on "heavy" axions for different n, the power of M_{Pl} suppressing the explicit breaking of the Peccei-Quinn symmetry. Values of f_a relevant for astrophysical considerations are shown below the line for each n, while those of relevance for cosmology are above the line. The range of validity of the analyses presented is $m_* = m_a$, denoted by the bold mark which extends above and below each line. The value of f_a necessary to solve the strong *CP* problem is denoted similarly by the light mark labeled $\bar\theta$ in the legend. All the constraints in this figure are drawn taking the dimensionless strength of the gravitationally induced symmetry breaking to be $g = 1$. The gray bars show constraints arising from astrophysical considerations. For $f_a < 10^{10}$ GeV the constraints arise from the cooling of supernovae, except for $n = 1$, where it is derived from considering helium ignition in red giants. The lower limit to f_a from the cooling constraint is shown by a strong mark descending from the central line, labeled "cooling." For $f_a > 10^{10}$ GeV the constraints arise from considering a γ-ray flux from the decay of $\sim$100-MeV axions emitted from supernova cores. Any supernova constraint will cutoff when the axion mass exceeds $\sim$300 MeV, denoted by the light descending mark labeled "300 MeV." We extend the astrophysical constraints into the region where $m_* < m_a$ based on our knowledge of these constraints in the usual axion scenario. The black bars show regions constrained by cosmological considerations. The strong ascending mark labeled "100 eV" denotes $m_* = 100$ eV. Values of f_a to the right of this mark are constrained by considerations of the relic density of axions, unless the axions decay. To the right of the light ascending mark labeled "decay before BBN," axions decay before the time of big-bang nucleosynthesis. Axions lighter than 100 eV may decay into an observable flux of photons. The light ascender marked "excess photons" shows the limiting value of f_a necessary to keep the axion stable long enough to avoid this problem. Although we do not extend the cosmological constraint bar below the $m_* = m_a$ mark, we still position ascending marks as if m_* dominated the mass. There is no cosmological constraint for $n = 1$ since the axions decouple after they become nonrelativistic in the early Universe, and their number density is negligible.