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Nonequilibrium effect of the neutrino distribution on primordial helium synthesis

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We study the effect of the deviation of the electron-neutrino spectrum from the thermal equilibirum distribution, originating from residual interactions between neutrinos and electrons which have different temperatures after decoupling. It is found that this effect causes an appreciable spectral distortion of the order of 1% or more in the higher-energy side of the distribution, when the temperature drops below 1 MeV. The resulting modification in the helium abundance, however, is small, and only of the order of $\Delta Y \approx 1.3\times 10^{-4}$.

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I. INTRODUCTION

The correct prediction of the cosmic abundances of light elements has been regarded as a great success of the standard hot universe model [1–4]. The only free parameter, the baryon-to-photon ratio N_B/N_γ, deduced from a comparison between the prediction and the observation of the primordial abundances of d, ^{3}He, ^{4}He, and ^{7}Li, now converges to quite a narrow range. It is interesting to ask, however, to what accuracy such an agreement holds when more precise estimates become available for the primordial elemental abundances. For the primordial helium, for instance, the latest value from H II galaxies is $Y_{\rm obs}=0.229\pm0.004$ [5] with a relative error at a 2% level. Taking a small error seriously, this is marginally consistent with the standard calculation with three neutrino species and with N_B/N_γ determined from $^3\text{He}+d$ and ^{7}Li: $Y=0.236-0.243$ [3]. In view of the availability of precision values for the primordial helium abundance today and more to come in the near future, we feel it worthwhile to reexamine the calculation of primordial nucleosynthesis in more detail.

After the freezing of the neutron-to-proton ratio, the calculation is quite accurately carried out with the standard code, and there seems also little uncertainty in the nuclear reaction rates used. In the calculation of the n/p ratio, however, all authors have assumed the equilibrium Fermi distribution for the electron-neutrino spectrum. We consider that this assumption is worth examining: neutrinos decouple from the primeval plasma at a temperature $T\sim3$ MeV for ν_e and 5 MeV for ν_μ and ν_τ. Around this epoch there is no doubt that the neutrino spectrum is described well by the Fermi distribution. After this epoch, however, the temperatures of neutrinos and of the $e^\pm$ and γ plasma become different because of the annihilation of e^+e^- pairs that heats up the electromagnetic component of the plasma. The relative temperature difference is about 0.9×10^{-3} at 3 MeV, about 1.6×10^{-2} at $T=0.7$ MeV, and reaches eventually the well-known value of 29% [4]. Although equilibrium ceases at a few MeV, some thermal contact between electrons and neutrinos remains, especially for a high-energy tail of the neutrino spectrum due to stronger interactions between them at a higher energy. This would distort the equilibrium Fermi distribution. In fact, we find that this distortion amounts to as large as 1% or more for the higher-energy side of the spectrum. This motivates us to examine the change of the n/p ratio caused by this distortion.

Actually there have been a few authors who noticed the effect driven by the temperature difference between the photon and the neutrino components [6–8]. These authors, however, considered only average heating of the neutrino gas owing to residual interactions between electrons and neutrinos, and assumed that the effect is renormalized into the change of the effective neutrino temperature. What we really need to see is, however, the effect of the distorted spectrum, which cannot simply be absorbed into the temperature. In this paper we study the nonequilibrium effect on the n/p ratio by directly solving kinetic equations. A brief communication of our central result is published in Ref. [9].

II. KINETIC EQUATIONS FOR NEUTRINO SPECTRUM

The kinetic equation that governs the ν_e phase-space distribution in the expanding Universe has the form

$$\left[\frac{\partial}{\partial t}-Hp\frac{\partial}{\partial p}\right]n_\nu(t,p)=S\ , \tag{1}$$

where $H=1/2t$ is the expansion parameter $\dot a/a$, and $p=E$ is the neutrino momentum with the mass of neutrinos assumed to be negligible. The collision term S is given, for example, for $\nu\bar\nu\leftrightarrow e^+e^-$ by the integral

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$$S=\frac{(2\pi)^4}{2p}\int d\tau(e^-)d\tau(e^+)d\tau(\bar{\nu})\delta^4(p_++p_--p-\bar{p})|A(\nu\bar{\nu}\leftrightarrow e^+e^-)|^2$$
$$\times[n_{e^+}n_{e^-}(1-n_\nu)(1-n_{\bar{\nu}})-n_\nu n_{\bar{\nu}}(1-n_{e^+})(1-n_{e^-})]\ , \quad (2)$$

where p_+, p_-, p, and $\bar{p}$ are the momenta of e^+, e^-, ν, and $\bar{\nu}$, and $d\tau(e^-)=d^3p_-/(2\pi)^3 2E_-$, etc., is the phase space volume element for the respective particles. The amplitude in the integrand is written

$$|A(\nu\bar{\nu}\leftrightarrow e^+e^-)|^2=128G_F^2[g_L^2(pp_+)^2+g_R^2(pp_-)^2+g_Lg_Rm^2(p\bar{p})]\ , \quad (3)$$

with $G_F=1.03\times10^{-5}/m_N^2$ the Fermi coupling constant, $g_L=\frac{1}{2}+\sin^2\theta_W$, $g_R=\sin^2\theta_W$, ($\sin^2\theta_W=0.23$), and m the electron mass. There are also contributions to S from elastic scattering $\nu e^\pm\leftrightarrow\nu e^\pm$, etc., which will be taken into account afterwards.

For the energy region that concerns us the number densities of neutrinos and electrons are small enough so that we can approximate the Fermi distribution by the Boltzmann distribution, especially when we are interested in small correction terms. For electrons and positrons the Coulomb and Thomson scattering processes are fast enough, and their distribution is given by the equilibrium form

$$n_e=\exp(-E_e/T_\gamma)\simeq\exp(-E_e/T)\left[1+\frac{E_e}{T}\frac{\Delta T}{T}\right]\ . \quad (4)$$

Here the temperature of the $e\gamma$ plasma, T_γ, differs from the neutrino temperature T by $\Delta T=T_\gamma-T$. We write the neutrino distribution in the form

$$n_\nu=\exp(-E_\nu/T)[1+\delta(p,t)]\ , \quad (5)$$

with $\delta(p,t)$ the spectral distortion due to neutrino heating by electrons and positrons.

Since $n_\nu n_{\bar{\nu}}$ and $n_{e^+}n_{e^-}$ depend only on the total energy $E_{e^+}+E_{e^-}$, we can carry out the integration over dp_+dp_-, which yields

$$(2\pi)^4\int|A|^2d\tau(e^+)d\tau(e^-)\delta^4(p_++p_--p-\bar{p})$$
$$=\frac{16}{\pi}G_F^2(p\bar{p})\left[\frac{g_L^2+g_R^2}{3}(p\bar{p}-m^2/2)+m^2g_Lg_R\right]\times\left[1-\frac{2m^2}{p\bar{p}}\right]^{1/2}\ . \quad (6)$$

By inspecting the integral, we find that the effect of the electron mass is negligible. This simplifies the kinetic equation for δ, which now takes the form

$$\left[\frac{\partial}{\partial t}-HE\frac{\partial}{\partial E}\right]\delta(t,E)=\frac{16G_F^2}{3\pi^3}(g_L^2+g_R^2)\left[\frac{\Delta T}{T}T^3E(E+4T)-ET^4\delta(E,t)-\frac{1}{6}E\int dE'\delta(E',t)(E')^3\exp(-E'/T)\right]\ . \quad (7)$$

This equation is not easy to handle, but we shall see in what follows that δ is a small quantity in the temperature range responsible for determining the neutron to proton ratio and the δ term which appears in the second term in the right-hand side can be ignored as a first approximation. This means that the system is far from equilibrium for the temperature that concerns us; $e^+e^-\rightarrow\nu\bar{\nu}$ contributes largely, while the inverse process that would restore equilibrium is much smaller. An account of the inverse process would only slightly diminish our final result.

We have to take into account also heating of neutrinos by elastic νe^- and νe^+ scattering. These processes conserve the neutrino numbers, but modify the spectrum. The kinetic equation is then

$$\left[\frac{\partial}{\partial t}-HE\frac{\partial}{\partial E}\right]\delta(E,t)\approx\frac{16G_F^2(g_L^2+g_R^2)}{3\pi^3}\frac{\Delta T}{T}T^3E[E+4T+\tfrac{7}{4}(E-4T)]\ , \quad (8)$$

where the terms proportional to δ are ignored in the right-hand side. By noting that $\dot{T}=-HT$ we can easily integrate Eq. (8) and obtain

$$\delta(E/T,t)=\frac{16G_F^2(g_L^2+g_R^2)}{3\pi^3}\frac{E}{T}\left[\frac{E}{T}+4+\frac{7}{4}\left[\frac{E}{T}-4\right]\right]\times\int_{t_i}^{t}dt\ T^5\frac{\Delta T}{T}\ . \quad (9)$$

We write this to be

$$\delta(E/T,T)\approx0.031\frac{E}{T}\left[\frac{11}{4}\frac{E}{T}-3\right]\int_\eta^{\eta_i}d\eta\,\eta^2\frac{\Delta\eta}{\eta}\ , \quad (10)$$

with the use of

$$t=(90/32\pi^3g)^{1/2}m_{\rm Pl}T^{-2}$$

(g is the number of relativistic degrees of freedom and $m_{\rm Pl}$ is the Planck mass). Here η is the temperature in units of MeV and η_i is its initial value corresponding to decoupling of ν_e from the plasma. $\Delta\eta/\eta\equiv\Delta T/T$ is given by [4]

$$\Delta T/T=\left[\frac{11}{4}\right]^{1/3}\left[1+\frac{45}{2\pi^4}\int_0^\infty dx\frac{x^2[(x^2+y^2)^{1/2}+x^2(x^2+y^2)^{-1/2}/3]}{e^{(x^2+y^2)^{1/2}}+1}\right]^{-1/3}-1\ . \tag{11}$$

We note that

$$(\Delta T/T)T^2\simeq 0.60\times 10^{-2}(\mathrm{MeV})^2$$

for wide range of T from 3 MeV down to 0.5 MeV (within 3%; the error is only 10% even at 0.3 MeV). With $T_i\approx 3-4$ MeV (see below), this yields approximately

$$\delta\approx 6\times 10^{-4}(E/T)(11E/4T-3) \tag{12}$$

at $T\approx 0.6$ MeV.

Precisely speaking the decoupling temperature η_i is determined from internal consistency. If we would retain the terms proportional to δ in the right-hand side of the kinetic equation, the initial temperature is automatically set by solving the equation. The full kinetic equation takes the form

$$\left[\frac{\partial}{\partial t}-HE\frac{\partial}{\partial E}\right]\delta(E,t)=\frac{16G_F^2(g_L^2+g_R^2)}{\pi^3}\left[A\frac{\Delta T}{T}-B\delta(E)+\int dE'C\delta(E')\right]. \tag{13}$$

The expression for the coefficient A is already given in Eq. (8), and the contribution from $e^+e^-\leftrightarrow\nu_e\bar{\nu}_e$ to B and C is given in Eq. (7). Let us note that its contribution to C is negative. We must also take into account the contribution from neutrino elastic scattering on other neutrinos and $\nu_e\bar{\nu}_e$ annihilation into other neutrino species. This also gives a positive contribution to B and a negative contribution to C, which practically doubles the contribution from $\nu_e\bar{\nu}_e\leftrightarrow e^+e^-$ alone.

On the other hand, νe elastic scattering gives the contribution

$$B_{\nu e}=ET^4\ , \tag{14}$$

and

$$\left[\int dE'C\delta(E')\right]_{\nu e}=K(T/E)T^4\int_0^E dE'\delta(E')[1-e^{-E'/T}K(T/E')/2]+[1-e^{E/T}K(T/E)/2]\int_E^\infty dE'\delta(E')e^{-E'/T}K(T/E')\ , \tag{15}$$

where $K(z)=1+2z+2z^2$. We see that these terms give positive contributions both to B and C on the same order [if an integration is made with a factor $\exp(-E/T)$, the two contributions are equal]. So these terms almost compensate each other, and the net contribution is small.

It is not a simple task to solve Eq. (13), but our purpose here is to find the effective decoupling temperature η_i. To this end it is sufficient to retain only the annihilation process keeping in mind that the integral over E' effectively doubles B. With this simplification we obtain

$$\left[\frac{\partial}{\partial t}-HE\frac{\partial}{\partial E}\right]\delta(E,t)=\frac{16G_F^2(g_L^2+g_R^2)}{3\pi^3}ET^4\times\left[\frac{\Delta T}{T}\left[\frac{11}{4}\frac{E}{T}-3\right]-2\delta(E,t)\right]. \tag{16}$$

This equation can easily be solved to give

$$\delta(E,t)=0.031\frac{E}{T}\left[\frac{11}{4}\frac{E}{T}-3\right]e^{0.018\eta^3(E/T)}\times\int_\eta^\infty d\eta'\eta'^2\frac{\Delta T}{T}e^{-0.018(E/T)\eta^3}\ . \tag{17}$$

The integral is effectively cut off at

$$\eta_i\simeq 3.8\ \mathrm{MeV}(T/E)^{1/3} \tag{18}$$

and we recover essentially the result given in Eq. (12).

In summary we find that the deviation of the ν_e spectrum from the Fermi distribution increases as E^2 with the neutrino energy and is of the order of 1% at the thermally averaged neutrino energy $E\approx 3T$. One may then expect that the neutron-to-proton ratio is sensitive to the high energy tail of the spectrum where the distortion is even larger.

III. THE NEUTRON-TO-PROTON RATIO

The helium abundance is basically determined by the neutron to proton ratio n/p, which is fixed by the competition of $n+\nu\leftrightarrow p+e^-$ and $n+e^+\leftrightarrow p+\bar{\nu}$ with the expansion rate of the Universe. The kinetic equation that governs the evolution of neutron number density is given by

$$\frac{dr_n}{dt}=-\frac{(1+3g_A^2)G_F^2}{\pi^3}r_n\left[\int_0^\infty dE\,E^2(E+\Delta M)[(E+\Delta M)^2-m^2]^{1/2}[n_\nu(E)+n_e(E+\Delta M)]\right.$$
$$\left.+\int_m^\infty dE\,E(E^2-m^2)^{1/2}(E+\Delta M)^2[n_e(E)+n_\nu(E+\Delta M)]\right]$$
$$+\frac{(1+3g_A)G_F^2}{\pi^3}\left[\int_0^\infty dE\,E^2(E+\Delta M)[(E+\Delta M)^2-m^2]^{1/2}n_e(E+\Delta M)\right.$$
$$\left.+\int_m^\infty dE\,E(E^2-m^2)^{1/2}(E+\Delta M)^2 n_\nu(E+\Delta M)\right], \tag{19}$$

where $r_n=r_n(t)$ is the fraction of the neutron number against the total number of baryons (so that $r_n+r_p=1$), $\Delta M=1.29$ MeV is the neutron-proton mass difference, and $g_A=1.26$ is the axial-vector coupling constant. Here we ignore the term representing the decay of neutrons, which is not essential for our argument.

In the standard calculation it is assumed that n_ν and n_e are given by the equilibrium Fermi distributions with temperatures T and T_γ. Here we will take account of the deviation of the neutrino spectrum from equilibrium as calculated in the previous section [see Eqs. (12) and (17)]. Putting Eq. (5) into (19) we obtain

$$\frac{dr_n}{dT}=-0.05T^2\int_0^\infty dx\,x^2\left[x+\frac{\Delta M}{T}\right]^2 e^{-x}\left\{e^{-\Delta M/T}\left[1+\frac{1}{2}\delta(E+\Delta M)\right]\right.$$
$$\left.-r_n\left[1+\frac{\delta(E)}{2}+e^{-\Delta M/T}\left[1+\frac{\delta(E+\Delta M)}{2}\right]\right]\right\}, \tag{20}$$

where $x=E/T$ and $\delta(x,T)$ is given by Eq. (10) of Sec. II. When $(T^2\Delta T/T)$ stays constant, Eq. (10) can be explicitly integrated, and Eq. (20) reads

$$\frac{dr_n}{dT}=-0.05T^2\{J_1[e^{-\Delta M/T}-r_n(1+e^{-\Delta M/T})]+\epsilon e^{-\Delta M/T}J_3(1-r_n)-\epsilon J_2 r_n\}\ . \tag{21}$$

Here

$$J_1=24+12\beta+2\beta^2\ ,$$
$$J_2=\int_T^\infty d\eta[1980z^{-7}+660(\beta-\tfrac{6}{11})z^{-6}+66(\beta-\tfrac{21}{11})z^{-5}-18\beta^2z^{-4}]\ ,$$
$$J_3=\int_T^\infty d\eta\exp[-0.018\beta(\eta^3-T^3)][1980z^{-7}+1320(\beta-\tfrac{3}{11})z^{-6}+396\beta(\beta-\tfrac{18}{33})z^{-5}$$
$$+66\beta^2(\beta-9/11)z^{-4}+(\tfrac{11}{2})\beta^3(\beta-\tfrac{12}{11})z^{-3}]\ , \tag{21$'$}$$
$$z=1+0.018(\eta^3-T^3)\ ,$$
$$\beta=\Delta M/T\ ,$$

and $\epsilon=0.91\times10^{-4}$. We integrate Eq. (21) numerically with the fourth-order Runge-Kutta algorithm. Below 0.3 MeV the approximation $(\Delta T/T)T^2$ = const is not valid, but the effect is small and it can be easily taken into account in the final answer. In this way, we find the deviation of r_n from the standard value $r_n(\epsilon=0)$ to be 0.9×10^{-4} at low enough temperatures. This is indeed a very small number, compared with what we naively expect from the deviation of the neutrino spectrum from the Fermi distribution.

This small value may be understood in the following way. Equation (20) has the form

$$\frac{dr_n}{dT}=P-\Gamma r_n\ . \tag{22}$$

Both terms P and Γ vary by about 1% due to distortion of the neutrino spectrum. Correspondingly, if one calculates the change of the n/p-freezing temperature induced by the variation of Γ, it would produce the variation of r_n equal to 0.5%. However, the variation of P gives an effect of the opposite sign. At larger temperatures the P term dominates and r_n becomes larger than the value in the standard model, while at low temperatures the contribution from Γ dominates, which makes r_n smaller than the standard value. As our numerical calculation shows, these two contributions cancel each other at $T\approx0.52$ MeV. Below this temperature the variation of r_n with time is tiny. Accordingly, the influence of the nonequilibrium distribution of neutrinos on r_n is very small.

IV. DISCUSSION

We have shown that the neutrino spectrum appreciably deviates from the Fermi distribution by about 1% at

$\langle E/T \rangle \approx 3$ and the deviation increases as E^2 towards the higher-energy tail of the neutrino spectrum, due to residual interactions between neutrinos and electrons (positrons), which have slightly different temperatures after decoupling of neutrinos from the plasma. Its contribution to the neutron to proton ratio is very small, however, and it changes the helium abundance only by the amount of $\Delta Y = -1.3 \times 10^{-4}$. This value may nominally be compared with those obtained by Dicus *et al.* [6], $+3\times10^{-4}$, and by Herrera and Hacyan [7], -2×10^{-4} (note that the signs do not agree with each other), and also by Rana and Mitra [8], -3×10^{-3}, which is certainly too large. All authors estimated the effect as a shift of the effective neutrino temperature and hence of the freezing temperature of β equilibrium. Our emphasis here is on the point that one cannot absorb the effect into the shift of the freezing temperature. This may be demonstrated by the fact that the correction to the n/p ratio is temperature dependent. For instance, according to our calculations, ΔY would be $+1.1\times10^{-4}$ if we adopt r_n near the freezing temperature usually accepted for beta equilibrium, $T \approx 0.7$ MeV.

Anyway the effect seems too small ($\Delta Y/Y = -0.05\%$) to be observationally relevant. The standard calculation assuming the equilibrium distribution is sufficiently accurate as a matter of fact for the purpose of estimating the helium abundance. If the discrepancy between the prediction and the observation were actually present for the primordial helium abundance, we must seek for the reason somewhere else.

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