Primordial magnetic fields from pseudo Goldstone bosons

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The existence of large-scale magnetic fields in galaxies is well established, but there is no accepted mechanism for generating a primordial field which could grow into what is observed today. We discuss a model which attempts to account for the necessary primordial field by invoking a pseudo-Goldstone boson coupled to electromagnetism. The evolution of this boson during inflation generates a magnetic field; however, it seems difficult on rather general grounds to obtain fields of sufficient strength on astrophysically interesting scales.

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I. INTRODUCTION

The existence of a magnetic field of $\sim 10^{-6}$ G in our Galaxy and other galaxies is well established [1]. The explanation of how such a magnetic field arose is, however, far from certain. While the creation and evolution of stellar magnetic fields is fairly well understood, the extension of these theories to galaxies suffers from problems relating to both scale length and time scales.

Zeldovich, Ruzmaikin, and Sokoloff [1] and Parker [2] discuss the origin and effects of magnetic fields in the Universe and draw the conclusion that the galactic field arises from a dynamo mechanism. The dynamo model requires a seed field at the epoch of galaxy formation which is coherent over a scale of ~ 1 Mpc. We can parametrize the strength of a primordial field by $r = \rho_B/\rho_\gamma$, the ratio of the energy density $\rho_B = B^2/8\pi$ in the magnetic field relative to that of the background radiation ρ_γ. (This ratio is constant while the Universe is a good conductor, which is almost always [3].) Then the field required to seed a galactic dynamo satisfies $r \gtrsim 10^{-31}$, corresponding to an intergalactic field at the epoch of galaxy formation ($z \sim$ 3–5) of $\gtrsim 10^{-20}$ G. The implications of this requirement in terms of the origin of such fields and their possible effect on star formation (or the early history of the Galaxy in general) are discussed by Rees [4].

Kulsrud [5] has argued that the galactic dynamo explanation is fundamentally flawed in that the mean-square deviations of the magnetic field will grow much faster than the mean field itself, resulting in a disordered field with a much smaller mean field strength than would naively be expected. Kulsrud then discusses the possibility that the magnetic field originated in the early Universe and was embedded in the medium out of which the galaxies ultimately formed. In this case, Kulsrud argues that the intergalactic field would need to be $\gtrsim 10^{-12}$ G at the epoch of galaxy formation (giving $r \gtrsim 10^{-15}$) in order to account for the observed interstellar field.

Any dynamo theory requires a mechanism for generating the required seed field; however, compelling mechanisms have been elusive (see, for example, [6].) The hot plasma in the early Universe is highly conducting and thus should strongly inhibit the growth of a magnetic field, even to $r \sim 10^{-31}$. Furthermore, any hypothetical process at work in the very early Universe must be able to produce fields with characteristic length scales much larger then the horizon at that time, in order to correspond to galactic scales today.

The advent of inflation [7] has opened the door to new possibilities for generating a primordial magnetic field. There are two key features of an inflationary universe that make the possibility of creating a magnetic field during this time particularly attractive. First, if there were an inflationary epoch at very early times in the Universe, the exponential expansion would have reduced the conductivity to a negligible value by reducing the charged particle density, thus allowing the creation of a substantial magnetic field. If this field were then frozen into the plasma created during the subsequent reheating of the Universe, it would be supported by the effectively infinite conductivity of the plasma so that its strength would decay only as the inverse square of the scale factor.

Second, if inflation did occur, then the entire observable Universe today was, at some point in the early Universe, contained entirely within the particle horizon. It would then be possible to use physical mechanisms operating on a scale smaller than the horizon to generate magnetic fields that are coherent over macroscopic scales today, an opportunity which is not available in models of the early Universe without inflation.

Turner and Widrow [3] (TW) investigated the possibility that quantum fluctuations during an inflationary epoch might have generated a magnetic field that could be sustained after the wavelength of interest crossed beyond the horizon and thus give the observed field today. TW considered coupling the electromagnetic field to the curvature tensor so as to amplify the fluctuation-induced field, but found satisfactory results could be obtained only at the expense of breaking gauge invariance.

Ratra [8] has argued that it is possible to generate a magnetic field with a present field strength of $\gtrsim 10^{-10}$

G on a scale of 1/1000 the present Hubble radius by coupling a scalar field Φ to the electromagnetic potential A_μ through a term of the form $e^\Phi F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength. This would be sufficient to explain the galactic magnetic field, but it remains to be seen whether or not this coupling could arise naturally in realistic particle physics models.

TW also suggested that the magnetic field could be sustained by coupling the EM field strength to a pseudoscalar axion field ϕ via an interaction term in the Lagrange density of the form

$$\mathcal{L} = \frac{g_{\phi\gamma\gamma}}{4}\phi F_{\mu\nu}\widetilde{F}^{\mu\nu} , \tag{1.1}$$

where $\widetilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is the dual of $F_{\mu\nu}$, and $g_{\phi\gamma\gamma} = (\alpha/2\pi)/f$, where f is a coupling constant with units of mass and α is the fine structure constant. However, they did not complete the necessary analysis to show whether or not this might indeed be the case.

An interaction of this form has been studied by Carroll and Field [9] (see also [10]), who found that the evolution of a Fourier mode of the magnetic field with wave number k is governed by

$$\frac{d^2F_\pm}{d\eta^2} + \left(k^2 \pm g_{\phi\gamma\gamma}k\frac{d\phi}{d\eta}\right)F_\pm = 0 , \tag{1.2}$$

where $F_\pm = a^2(B_y \pm iB_z)$ (the $\pm$ refers to different circular polarization modes of the magnetic field), $d\eta = dt/a$, and a is the scale factor of the Universe (normalized so that $a_0 = 1$ where a_0 is the value of a today). One (or both) of the polarization modes will be unstable for $k < g_{\phi\gamma\gamma}|d\phi/d\eta|$, where both polarization modes can be unstable to exponential growth if ϕ is oscillatory. Thus, if such a scalar field exists during inflation (perhaps the inflaton itself [11]) with the above coupling, this might provide a mechanism for generating a substantial magnetic field.

Here we will consider a generalization of the possibility suggested by TW [3], coupling the photon to an arbitrary pseudo Goldstone boson (PGB) rather than the axion of QCD. The PGB is characterized by a spontaneous symmetry-breaking scale f (as above) and a soft explicit symmetry-breaking scale Λ (see Sec. II). We find that significant growth occurs only at a temperature near Λ, and that the magnetic field strength thus generated cannot give an astrophysically interesting field at the end of inflation.

We should point out that, while our notation throughout this paper suggests that we are working with the photon of the standard $U(1)_{\rm em}$ symmetry, the photon as a separate U(1) gauge boson will not exist at high temperature, since the SU(2) × U(1) gauge symmetry will not have been broken. Nevertheless, our results should be correct up to factors of order unity simply because the U(1) hypercharge symmetry projects onto the photon with a multiplicative factor of $\cos\theta_W \approx 0.88$ at the electroweak phase transition.

We will use units in which $\hbar = c = k_B = 1$, such that $G = m_{\rm pl}^{-2}$, where $m_{\rm pl} \approx 1.22 \times 10^{19}$ GeV is the Planck mass.

II. THE SETUP

In this section we briefly review the essentials of inflation, as well as the physics of pseudo Goldstone bosons and their couplings.

Throughout this paper, we will assume that the Universe is in a spatially flat Friedmann-Robertson-Walker (FRW) cosmology in which the metric is given by

$$ds^2 = a^2(\eta)(-d\eta^2 + d\mathbf{x}^2) , \tag{2.1}$$

where $\mathbf{x}$ represents the standard Cartesian three-space (comoving) coordinates. In addition, we will assume that the Universe is a perfect fluid with the equation of state $p = \gamma\rho$, where p is the pressure, ρ is the total energy density, and γ is a constant. Using this equation and energy-momentum conservation, it is straightforward to show that $\rho \propto a^{-3(1+\gamma)}$ which, from Einstein's equation, gives $a \propto t^{2/3(1+\gamma)}$. In order to explain the horizon problem, we require that, at some time in the past, the scale factor was growing faster than the horizon (H^{-1}, where $H = \dot{a}/a$ is the Hubble parameter and an overdot denotes differentiation with respect to physical time t). Thus, we require $-1 \leq \gamma < -\frac{1}{3}$. For simplicity in the following discussion we will restrict ourselves to inflation in which $\gamma = -1$. This value for γ gives the best possible conditions for generating a magnetic field, since the amount of inflation from the time that a given comoving wavelength crosses outside the horizon is minimal in this case, so this does not limit the validity of our results.

At the end of inflation, the Universe enters a reheating phase, in which the energy density is matter dominated. As a simplifying assumption, we take the process of reheating to be instantaneous, such that the Universe goes directly from inflation to radiation domination. Once again this is a best-possible assumption, since the magnetic field will decay more rapidly (relative to the total energy density) during a matter-dominated phase, in which $\rho \propto a^{-3}$.

Standard inflation is characterized by two parameters: the mass scale for the total energy density $M = \rho^{1/4}$ (note that this is a constant since ρ is constant during inflation with $\gamma = -1$); and the temperature $T_{\rm RH}$ to which the Universe reheats at the end of inflation. H is then given by

$$H^2 = \frac{4\pi}{3m_{\rm pl}^2}M^4 . \tag{2.2}$$

If we assume that the Universe expands adiabatically after inflation so that the entropy per comoving volume element remains constant, it can be shown (see, e.g., [12]) that the total expansion from the time a given comoving wavelength λ (which is equal to the physical wavelength today due to the normalization of a) crosses outside the horizon until the end of inflation is given by

$$\frac{a_{\rm inf}}{a_\lambda} \simeq 10^{26}\frac{\lambda}{\rm Mpc}\left(\frac{M^2T_{\rm RH}}{m_{\rm pl}^3}\right)^{\frac{1}{3}} , \tag{2.3}$$

where $a_{\rm inf}$ is the value of a at the end of inflation (but before reheating), and a_λ is the value of a at the time

λ crosses outside the horizon. Expressing this in terms of the number of e-foldings N_λ ($a_{\rm inf}/a_\lambda = e^{N_\lambda}$) in the expansion, we have

$$N_\lambda \simeq 48 + \ln\left(\frac{\lambda}{\rm Mpc}\right) + \frac{2}{3}\ln\left(\frac{M}{10^{14}\ {\rm GeV}}\right) + \frac{1}{3}\ln\left(\frac{T_{\rm RH}}{10^{14}\ {\rm GeV}}\right) . \qquad (2.4)$$

Our assumption that reheating lasts for a negligible time amounts to setting $M = T_{\rm RH}$.

In this paper we are concerned with the pseudo Goldstone boson ϕ of a spontaneously broken symmetry. PGB's are characterized by two mass scales: a large mass f at which the global symmetry from which the PGB's arise is spontaneously broken, and a smaller scale Λ at which the symmetry is explicitly broken. For concreteness, we imagine the breakdown of a global U(1) symmetry, resulting in the familiar Mexican hat — the radial degree of freedom gets a vacuum expectation value of order f, and the angular degree of freedom becomes a massless boson ϕ. The hat is tilted by a small term of order Λ; the formerly massless scalar ϕ becomes a PGB with a mass of order

$$m \approx \Lambda^2/f . \qquad (2.5)$$

Cosmological constraints on the parameters f and Λ have been studied in [13].

In many models, PGB's interact with fermions by coupling to the axial-vector current (for a review of PGB's and their couplings, see [14]):

$$\mathcal{L}_{\rm int} = \frac{1}{4f}\bar\psi\gamma^\mu\gamma_5\psi\partial_\mu\phi = \frac{1}{4f}J_5^\mu\partial_\mu\phi ; \qquad (2.6)$$

well-known examples include pions and axions. Since the symmetry associated with the axial-vector current (that is, chiral symmetry) is anomalously broken, the current itself is not conserved:

$$\partial_\mu J_5^\mu = \frac{\alpha}{2\pi}F_{\mu\nu}\widetilde{F}^{\mu\nu} , \qquad (2.7)$$

where α is the fine structure constant. Integrating by parts, we find that the anomaly (2.7) induces a coupling between ϕ and the electromagnetic field of the form (1.1).

A similar situation occurs in the low-energy limit of string theory, which involves an antisymmetric two-index tensor field $B_{\mu\nu}$ [14,15]. The Lagrangian for $B_{\mu\nu}$ includes a kinetic term $H_{\mu\nu\rho}H^{\mu\nu\rho}$, where $H_{\mu\nu\rho}$ is an antisymmetric field strength tensor. The demand that the theory be free of anomalies requires that the definition of $H_{\mu\nu\rho}$ include a term involving gauge bosons (which we take to be Abelian for simplicity):

$$H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]} - A_{[\mu}F_{\nu\rho]} , \qquad (2.8)$$

where square brackets denote antisymmetrization. In four dimensions the equations of motion for $B_{\mu\nu}$ allow us to recast its dynamics (at least semiclassically) in terms of a pseudoscalar ϕ by the identification

$$\partial_\mu\phi = \frac{1}{f}\epsilon_{\mu\alpha\beta\gamma}H^{\alpha\beta\gamma} . \qquad (2.9)$$

The interaction between this pseudoscalar and electromagnetism, as implied by (2.8) and (2.9), can be described by an effective Lagrangian of the form (1.1). Thus, string theory offers the possibility of PGB's of the type we discuss.

III. BASIC REQUIREMENTS

We envision a scenario in which the desired magnetic field is entirely created during inflation and then frozen into the post-reheat plasma, so that the field strength decays as the inverse square of the scale factor after reheating. Thus, under the assumption that the dominant component of background photons is created at reheating, both ρ_B and ρ_γ will decay as a^{-4}, such that $r = \rho_B/\rho_\gamma$ is essentially constant after inflation. As mentioned in the Introduction, we seek $r = r_0 \gtrsim 10^{-31}$ (to seed a galactic dynamo) or $\gtrsim 10^{-15}$ (to directly account for the observed magnetic field) at scales of $\sim$ 1 Mpc today.

For convenience we define r during inflation as the ratio of ρ_B to the total (vacuum-dominated) energy density. From above, we can readily calculate the required value of r at the time that the wavelength of interest (comoving scale 1 Mpc) crosses outside the horizon in order to generate astrophysically interesting fields today. This is given by

$$r_{\rm Mpc} = \left(\frac{a_{\rm inf}}{a_{\rm Mpc}}\right)^4 r_0 , \qquad (3.1)$$

or, using (2.3),

$$r_{\rm Mpc} = 10^{104}\left(\frac{M}{m_{\rm pl}}\right)^4 r_0 . \qquad (3.2)$$

Since $r < 1$, we can use this to calculate the maximum allowed value for M such that there would be any hope for meeting the above condition. Using the values given above for r_0, this gives $M \lesssim 10$ GeV for a galactic dynamo, or $\lesssim 1$ MeV to directly account for the field. It is important to realize that these are extreme upper bounds that can only be obtained under ideal situations (i.e., the magnetic field energy being comparable to that in the vacuum at the time the wavelength of interest crosses the horizon, and with negligible reheating). Even so, the upper bound for seeding the galactic dynamo is, at best, marginal (that is, while current constraints on the time at which inflation might occur *could* allow it to occur as late as a temperature of $\sim$ 1 GeV since we merely require the Universe to be radiation dominated by the epoch of primordial nucleosynthesis [3], constraints arising from baryogenesis will probably require $M \gtrsim 200$ GeV, at least). Thus, even if the magnetic field has an energy density comparable to the background energy density when 1 Mpc crosses the horizon during inflation, it will be too weak to explain the observed magnetic field if its energy decays according to the normal a^{-4}.

In other words, in order to generate a significant magnetic field during inflation, we require "superadiabatic growth" — that is, a mechanism that will continue to increase the energy density (or, at least, decrease the rate of decay) of the magnetic field at a wavelength of

1 Mpc *after* 1 Mpc becomes superhorizon sized. At first sight, this may seem impossible due to the fact that such a mechanism must apparently act in a noncausal way. However, it is possible that inflation may create a field that is coherent over scales much larger than the horizon, and that this field can subsequently generate a magnetic field that is also coherent at superhorizon scales simply by classical field interactions.

IV. EQUATIONS OF MOTION

The Lagrange density for the photon and scalar field is

$$\mathcal{L} = -\sqrt{g}\left(\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + V(\phi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g_{\phi\gamma\gamma}}{4}\phi F_{\mu\nu}\widetilde{F}^{\mu\nu}\right) , \tag{4.1}$$

where $g = -\det(g_{\mu\nu})$ and ∇_μ denotes the covariant derivative. As mentioned previously, we are considering only the U(1) fields and ignoring any possible effects from the non-Abelian gauge fields.

The equations of motion for ϕ are

$$-\nabla^\mu\nabla_\mu\phi + \frac{dV(\phi)}{d\phi} = -\frac{g_{\phi\gamma\gamma}}{4}F_{\mu\nu}\widetilde{F}^{\mu\nu} , \tag{4.2}$$

and the equations of motion for $F_{\mu\nu}$ are

$$\nabla_\mu F^{\mu\nu} = -g_{\phi\gamma\gamma}(\nabla_\mu\phi)\widetilde{F}^{\mu\nu} , \tag{4.3}$$

along with the Bianchi identity

$$\nabla_\mu \widetilde{F}^{\mu\nu} = 0 . \tag{4.4}$$

The equations of motion are more transparent if we define the **E** and **B** fields by

$$F^{\mu\nu} = a^{-2}\begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix} . \tag{4.5}$$

Then (4.2) becomes, after expanding the covariant derivatives,

$$\frac{\partial^2\phi}{\partial\eta^2} + 2aH\frac{\partial\phi}{\partial\eta} - \nabla^2\phi + a^2\frac{dV(\phi)}{d\phi} = g_{\phi\gamma\gamma}a^2\mathbf{E}\cdot\mathbf{B} , \tag{4.6}$$

where ∇ represents the usual three-space gradient (for comoving coordinates). Similarly, (4.3) becomes

$$\frac{\partial}{\partial\eta}(a^2\mathbf{E}) - \nabla\times(a^2\mathbf{B}) = -g_{\phi\gamma\gamma}\frac{\partial\phi}{\partial\eta}a^2\mathbf{B} - g_{\phi\gamma\gamma}(\nabla\phi)\times a^2\mathbf{E} , \tag{4.7}$$

with

$$\nabla\cdot\mathbf{E} = -g_{\phi\gamma\gamma}(\nabla\phi)\cdot\mathbf{B} , \tag{4.8}$$

while the Bianchi identity becomes

$$\frac{\partial}{\partial\eta}(a^2\mathbf{B}) + \nabla\times(a^2\mathbf{E}) = 0 , \tag{4.9}$$

with

$$\nabla\cdot\mathbf{B} = 0 . \tag{4.10}$$

Since we are interested in the specific case where the background space-time is inflating, we make the assumption that the spatial derivatives of ϕ are negligible compared to the other terms (if this is not the case at the beginning of inflation, any spatial inhomogeneities will quickly be inflated away and this assumption will quickly become very accurate). Then, eliminating **E** in the above equations, we have

$$\left(\frac{\partial^2}{\partial\eta^2} - \nabla^2 - g_{\phi\gamma\gamma}\frac{d\phi}{d\eta}\nabla\times\right)(a^2\mathbf{B}) = 0 . \tag{4.11}$$

Taking the spatial Fourier transform of this equation so that

$$\mathbf{B}(\eta,\mathbf{k}) = \frac{1}{2\pi}\int e^{i\mathbf{k}\cdot\mathbf{x}}\mathbf{B}(\eta,\mathbf{x})d^3\mathbf{x} , \tag{4.12}$$

and writing $\mathbf{F} = a^2\mathbf{B}$, we then have

$$\frac{\partial^2\mathbf{F}}{\partial\eta^2} + k^2\mathbf{F} - g_{\phi\gamma\gamma}\frac{d\phi}{d\eta}i\mathbf{k}\times\mathbf{F} = 0 . \tag{4.13}$$

Finally, if we take **k** to point along the x axis and define $F_\pm = F_y \pm iF_z$, this becomes

$$\frac{\partial^2 F_\pm}{\partial\eta^2} + \left(k^2 \pm g_{\phi\gamma\gamma}\frac{d\phi}{d\eta}k\right)F_\pm = 0 . \tag{4.14}$$

We can similarly manipulate the equations in an attempt to produce an expression for the evolution of **E**. It turns out, however, that we cannot uncouple the **E** field from the **B** field. We have, in short,

$$\left(\frac{\partial^2}{\partial\eta^2} - \nabla^2 - g_{\phi\gamma\gamma}\frac{d\phi}{d\eta}\nabla\times\right)(a^2\mathbf{E}) = -g_{\phi\gamma\gamma}\frac{d^2\phi}{d\eta^2}a^2\mathbf{B} , \tag{4.15}$$

which, after taking the space Fourier transform and defining $\mathbf{G} = a^2\mathbf{E}$ and $G_\pm = G_y \pm iG_z$, becomes

$$\frac{\partial^2 G_\pm}{\partial\eta^2} + \left(k^2 \pm g_{\phi\gamma\gamma}\frac{d\phi}{d\eta}k\right)G_\pm = -g_{\phi\gamma\gamma}\frac{d^2\phi}{d\eta^2}F_\pm . \tag{4.16}$$

In order to determine the evolution of **E** and **B**, we need to know how ϕ evolves. We look first at the case where **E** and **B** make a negligible contribution to the equation of motion for ϕ. Furthermore, we will consider the evolution after the time when the explicit symmetry breaking for the PGB becomes important (at a temperature scale Λ). The potential for the angular degree of freedom in a tilted Mexican hat is

$$V(\phi) = \Lambda^4[1 - \cos(\phi/f)] . \tag{4.17}$$

Since the details of the potential do not affect our results, we will expand to lowest order: $V(\phi) \sim (\Lambda^4/2f^2)\phi^2$. We again ignore spatial derivatives in ϕ, as well as any back reaction from the electromagnetic fields, to give

$$\frac{d^2\phi}{dt^2} + 3H\frac{d\phi}{dt} + \frac{\Lambda^4}{f^2}\phi = 0 , \tag{4.18}$$

where we have used $d\eta = dt/a$ to write this in terms of physical time rather than conformal time. The general solution to (4.18) will be approximately

$$\phi(t) \approx f \exp\left[-\frac{1}{2}\left(3H \pm \sqrt{9H^2 - 4\frac{\Lambda^4}{f^2}}\right)(t - t_0)\right] \propto \begin{cases} a^{-3/2}\sin\left(\frac{\Lambda^2}{f}t\right) & \text{for } \frac{\Lambda^2}{f} \gg H, \\ \exp\left(-\frac{\Lambda^4}{3Hf^2}t\right) & \text{for } \frac{\Lambda^2}{f} \ll H, \end{cases} \tag{4.19}$$

where we have used the fact that $a \propto \exp Ht$.

Looking again at (4.14) we see that we will only have a growing mode for the magnetic field if

$$\frac{d\phi}{d\eta} = a\frac{d\phi}{dt} > \frac{k}{g_{\phi\gamma\gamma}} . \tag{4.20}$$

Furthermore, since we want this growth to be as large as possible, we will choose k such that $k \sim g_{\phi\gamma\gamma} a\dot{\phi}_{\max}$ at the time of interest. For our initial analysis, we will assume $\Lambda^2/f \gg H$ so that ϕ is oscillating rapidly compared to changes in a. Also, this implies that a is essentially constant over several oscillations in ϕ, and thus we can write $\Delta\eta \approx \Delta t/a$ where a is constant for time intervals $\Delta t \sim f/\Lambda^2$.

In order to estimate the total growth in $F_\pm$, we note that, for a fraction ϵ (where ϵ is not necessarily small, although numerical integration of these equations for some cases indicates that $\epsilon \sim 0.1$) of each period, we can write $F_\pm \propto e^{\alpha\Delta\eta}$ where $\alpha = \sqrt{g_{\phi\gamma\gamma} k\, d\phi/d\eta} \sim a g_{\phi\gamma\gamma}\Lambda^2$ and $\Delta\eta \approx \Delta t/a \sim \epsilon f/a\Lambda^2$. Furthermore, this will continue for a time $\sim H^{-1}$ (since this is the time scale on which a and the amplitude of ϕ are changing), or for a total of $n \sim H^{-1}\Lambda^2/f$ oscillations, from which we can estimate the total growth in $F_\pm$ as

$$\frac{F_{\pm,f}}{F_{\pm,i}} \sim \exp\left(\frac{\epsilon g_{\phi\gamma\gamma}\Lambda^2}{H}\right) . \tag{4.21}$$

Of course, the exponent may contain other factors of order unity, but this estimate allows us to understand the dependence of the growth in the magnetic field on the parameters in the problem. Note that the analysis is similar if $\Lambda^2/f \sim H$, but then the growth only occurs for $n \sim \Lambda^2/Hf \sim 1$ oscillations. Since the amount of growth per oscillation remains roughly the same, this case can only result in less growth than the rapidly oscillating case. Similarly, in the overdamped case ($\Lambda^2/f \ll H$), ϕ does not change rapidly enough to generate any appreciable growth.

Since we are interested specifically in long-wavelength magnetic fields, we would also like to know the wavelength at which we get the most growth by this mechanism. This follows from our assumption that the maximum growth occurs at $k \approx a g_{\phi\gamma\gamma}\dot{\phi}_{\max} \approx a g_{\phi\gamma\gamma}\Lambda^2$, which gives

$$\lambda \approx \frac{2\pi}{a g_{\phi\gamma\gamma}\Lambda^2} , \tag{4.22}$$

where λ is the wavelength today. More importantly, at the time the growth occurs the ratio of the wavelength to the horizon length is given by

$$\frac{a\lambda}{H^{-1}} = \frac{2\pi a H}{k} \approx \frac{2\pi H}{g_{\phi\gamma\gamma}\Lambda^2} . \tag{4.23}$$

But, looking back at (4.21), we see that this ratio is, essentially, just the inverse of the factor in the exponent, i.e., the larger the growth in the magnetic field, the smaller the wavelength at which it occurs. Further, since an increase in the wavelength by a factor β will result in a decrease in the exponent for the growth by a factor $\beta^{-1/2}$, it is apparent that significant growth in the magnetic field will only occur for wavelengths that are subhorizon sized.

We have shown that we cannot have growth in the magnetic field for superhorizon-sized wavelengths when the back effects of the **E** and **B** fields on ϕ are small, but there is still the possibility that the interaction between the fields and ϕ could allow the magnetic field to be sustained such that it does not decrease as a^{-2}. However, from (4.7) and (4.8), we see that, if ϕ decays as $a^{-3/2}$ (that is, ϕ behaves as a nonrelativistic fluid), then the right-hand sides (RHS's) of these equations will rapidly become negligible compared to the individual terms on the LHS's. We then recover the source-free Maxwell equations, from which **B** still decreases as a^{-2}. (The situation is only exacerbated if ϕ behaves as a relativistic fluid.) In short, there seems to be no way of sustaining the magnetic field at superhorizon-sized wavelengths.

V. SUMMARY

In this paper we have considered a mechanism for creating a large-scale magnetic field during inflation, proposed by TW, in which the magnetic field is coupled to a pseudo Goldstone boson. We showed first that the scale required of the magnetic field in order to explain the galactic magnetic field (~ 1 Mpc) implied that the growth had to occur at superhorizon-sized wavelengths since the uncoupled equations of motion for a U(1) gauge field imply that the energy density in the field would simply decay too quickly to be significant at the end of inflation if there was no enhancement in the field for wavelengths larger than the horizon.

We then considered the classical evolution of a U(1) gauge field coupled to a PGB when the back effects of the field on the PGB were negligible and showed that such a coupling can, in fact, produce growth in the field. However, such growth can occur only at subhorizon wavelengths, and thus does not provide a solution to the above problem. In the more general case when we allow the back effects of the U(1) field to be significant, we still have the problem that any natural decay of the ϕ field ultimately allows the U(1) field to uncouple from the PGB leaving us once again with a free U(1) field. Hence, it seems to be impossible to create a significant magnetic field by simply coupling the magnetic field to a PGB during inflation.

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