Vacuum density fluctuations in extended chaotic inflation

Nathalie Deruelle
Département d'Astrophysique Relativiste et de Cosmologie, Centre National de la Recherche Scientifique, Observatoire de Paris, 92195 Meudon, France
and Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, England

Carsten Gundlach
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, England

David Langlois
Département d'Astrophysique Relativiste et de Cosmologie, Centre National de la Recherche Scientifique, Observatoire de Paris, 92195 Meudon, France
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An inflaton (scalar field) with the potential $c\sigma^{2n}$ is coupled to gravity within the Jordan-Brans-Dicke theory. The corresponding inflationary model (that is, a flat Friedmann-Robertson-Walker solution with a slowly varying inflaton) is constructed for all values of the coupling β of the inflaton to the dilaton (Brans-Dicke scalar field). The linearized perturbations of the metric, the dilaton, *and* the inflaton are then quantized within a gauge-invariant formalism. The power spectrum of the vacuum density fluctuations is calculated as a function of c,n, and β. It is the juxtaposition of two powers of the wave number corresponding, respectively, to the contribution of the inflaton and the dilaton. We find the value of β for which the dilaton contribution dominates on observable cosmological scales.

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I. INTRODUCTION

In this paper we consider a cosmological model of extended inflation where a scalar field (the "inflaton") is coupled to gravity in the framework of the Jordan-Brans-Dicke (JBD) theory [1–3]. The action is

$$S=\int d^4x\sqrt{-\bar{g}}\left[\frac{\psi}{2}\bar{R}-\frac{\omega}{2}\psi^{-1}\bar{g}^{\alpha\beta}\partial_\alpha\psi\,\partial_\beta\psi -\tfrac{1}{2}\bar{g}^{\alpha\beta}\partial_\alpha\sigma\,\partial_\beta\sigma-V_I(\sigma)\right]. \quad (1)$$

Here $\bar{R}$ is the curvature scalar and $\bar{g}$ the determinant of the metric tensor $\bar{g}_{\mu\nu}$; σ is the inflaton and $V_I(\sigma)$ its potential; ω is the JBD coupling constant and ψ the Brans-Dicke scalar field. ($1/8\pi\psi\equiv G_{\text{eff}}$ can be viewed as an effective gravitational "constant.") The action (1) has been expressed in terms of the metric tensor $\bar{g}_{\mu\nu}$, called the "Jordan frame" [4,5], to which the matter fields, here σ, are minimally coupled. The dynamics are more simply described in the "Einstein frame" [3–5], obtained by a conformal transformation $g_{\mu\nu}\equiv\kappa^2\psi\bar{g}_{\mu\nu}$, with $\kappa^2\equiv 8\pi G$ (G being Newton's constant), which brings the action into the Einstein-Hilbert form

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R-\frac{1}{2}g^{\alpha\beta}\partial_\alpha\varphi\,\partial_\beta\varphi -\frac{1}{2}e^{-\beta\kappa\varphi}g^{\alpha\beta}\partial_\alpha\sigma\,\partial_\beta\sigma-e^{-2\beta\kappa\varphi}V_I(\sigma)\right], \quad (2)$$

where $\beta\equiv(\omega+3/2)^{-1/2}$ and where the "dilaton" φ is defined by $\kappa^2\psi\equiv\exp(\beta\kappa\varphi)$. (We have chosen here a definition of β which corresponds to 2β in Ref. [6], to $2/\kappa f$ in [5], and to $1/\kappa\varphi_0$ used by us in [7].)

General relativity is the limiting case $\beta=0$, and solar system experiments impose that $\omega>500$ ($\Longleftrightarrow\beta<0.045$) [8]. We shall, however, consider a wider range for β, as the solar system constraint is not as stringent as it appears. One can, for example, postulate that the dilaton couples more strongly to the inflaton than to baryonic matter. (This violates the weak equivalence principle but does not conflict with experiment [6,4].) One can also circumvent the constraint by invoking one-loop quantum effects that give rise to a dilaton-inflaton interaction potential [5], which plays a role after inflation in such a way that this modified JBD theory coincides with general relativity today; in that case (1) and (2) hold approximately for all β during the inflationary stage. Finally, one can also replace, as in hyperextended inflation [9], the coupling $e^{\beta\kappa\varphi}g_{\mu\nu}$ by $f(\varphi)g_{\mu\nu}$. Assuming then that $f(\varphi)$ tends to a constant at large φ, or in other words that the theory coincides with general relativity at late times, is yet another way of relaxing the constraint. The action (1) and (2) does not include that case, for a study of which we refer to [10]. [A classification of scalar-tensor theories, extensions of (1) and (2), is given in [11].]

The first cosmological model of the type described by (1) and (2) that was shown to be viable [12,13] was extended old inflation [14], that is, the case where σ in (1) and (2) is first trapped in a false vacuum $V_I(\sigma)=V_f$ and then decays to a true vacuum state $V_I(\sigma)=V_t$ via quantum tunneling. Now, if the theory of gravitation in the primordial Universe is the JBD theory, then, as emphasized in Ref. [15], one should also consider more general potentials for the inflaton field. In this paper we shall therefore consider, as a generic example, the model of extended chaotic inflation first proposed in [15], where $V_I(\sigma)=c\sigma^{2n}$. In these models there is no quantum tunneling, but we have to deal with two coupled scalar fields which both evolve in time.

In the study of density perturbations in models of extended inflation, one must take into account two "matter" perturbations, that of the inflaton and the dilaton, together with the "scalar" [16] perturbations of the metric. In a previous paper [7] we generalized Mukhanov's [17] treatment of single-field inflation (see also [18]), and expressed the Lagrangian to quadratic order in the perturbations as a function of the two true physical degrees of freedom v and w, gauge-invariant quantities associated with the dilaton and inflaton perturbations, respectively. We also expressed Φ, the gauge-invariant version of the Newtonian potential, and a related gauge-invariant density perturbation δ, in terms of v and w. (Φ and δ are Bardeen's [16] Φ_H and ϵ_m.) Finally we showed that, in the case of extended old inflation (when the inflaton is trapped), the inflaton perturbation w does not contribute to δ, as had been assumed to hold approximatively in previous analyses [19,20].

In this paper we investigate the case where both matter perturbations contribute. We quantize the two coupled degrees of freedom v and w and obtain the power spectrum of the vacuum density fluctuations δ.

We use a slow-rolling approximation to describe the evolution of the background, but in contrast with [15] we do not assume that β is small. From the quadratic Lagrangian we then deduce the classical equations of motion for v and w. In the canonical quantization scheme, v and w become coupled quantum operators and can be written as linear combinations of creation and annihilation operators. The coefficient, or mode functions, are complex classical solutions with initial conditions determined by the canonical commutation relations and the choice of a Fock vacuum state. It is possible in principle to compute the mode functions numerically and thus derive the power spectrum of the density fluctuations δ. Analytic results can be given, however, by expanding them in a small parameter that characterizes the slow-rolling approximation. To leading order, the equations of motion for v and w decouple and the power spectrum of the vacuum density fluctuations δ is easy to obtain.

The plan of the paper is the following. In Sec. II we construct the inflationary background solution of the model (2). In Sec. III we derive the equations of motion for the perturbations v and w during inflation. In Sec. IV we quantize these perturbations in the Heisenberg picture. In Sec. V we discuss the evolution of the density perturbation after inflation. In Sec. VI we comment on the resulting power spectrum of vacuum density fluctuations at late times. Section VII contains our conclusions.

II. THE BACKGROUND SOLUTION

We assume a flat Friedmann-Robertson-Walker (FRW) universe with homogeneous fields σ and φ. The evolution of the system is described by three independent equations of motion, e.g., the (00) component of the Einstein equations and the two Klein-Gordon equations for the scalar fields. They read, in the Einstein frame,

$$H^2=\frac{\kappa^2}{3}\left(\tfrac{1}{2}\dot{\varphi}^2+\tfrac{1}{2}e^{-\beta\kappa\varphi}\dot{\sigma}^2+e^{-2\beta\kappa\varphi}c\sigma^{2n}\right) , \quad (3)$$

$$\ddot{\varphi}+3H\dot{\varphi}+\tfrac{1}{2}\beta\kappa e^{-\beta\kappa\varphi}\dot{\sigma}^2-2\beta\kappa e^{-2\beta\kappa\varphi}c\sigma^{2n}=0 , \quad (4)$$

$$\ddot{\sigma}+(3H-\beta\kappa\dot{\varphi})\dot{\sigma}+e^{-\beta\kappa\varphi}2nc\sigma^{2n-1}=0 , \quad (5)$$

where a dot means a derivative with respect to the cosmic time t and $H\equiv\dot{a}/a$ is the Hubble "constant," a being the scale factor.

Standard chaotic inflation [21] is characterized by a "slow-rolling" regime when the scale factor changes much more rapidly than the inflaton σ and, hence, the Hubble constant. Here we shall consider the regime when only σ is slowly varying compared to the scale factor, while the dilaton φ and the Hubble constant may be rapidly varying. In other words, we shall neglect $\dot{\sigma}^2$ and $\ddot{\sigma}$ in Eqs. (3)–(5), but not $\ddot{\varphi}$ and $\dot{\varphi}^2$. Then the equations of motion simplify to the following set, where in (4) $\ddot{\varphi}$ is consistently approximated by $-\beta\kappa\dot{\varphi}^2$:

$$H=\kappa^2We^{-\beta\kappa\varphi}\sigma^n=W\frac{\sigma^n}{\psi} , \quad (6)$$

$$\dot{\varphi}=2\beta\kappa We^{-\beta\kappa\varphi}\sigma^n \Longleftrightarrow \dot{\psi}=2\beta^2W\sigma^n , \quad (7)$$

$$\dot{\sigma}=-2nW\sigma^{n-1} , \quad (8)$$

with

$$W\equiv\left[\frac{c}{(3-2\beta^2)\kappa^2}\right]^{1/2} . \quad (9)$$

[In [15], $\dot{\varphi}^2$ and $\ddot{\varphi}$ were neglected, which yields (6)–(9) with $\beta=0$ in (9). Therefore our results agree with those of Ref. [15] in the most interesting case of small β.] The condition for inflation is $\ddot{a}>0$, which requires that $\beta^2<\frac{1}{2}$. We shall therefore allow β^2 to range from 0 to $\frac{1}{2}$. The approximations (6)–(9) are valid for $f^2\ll 1$, where

$$f^2\equiv e^{\beta\kappa\varphi}\frac{1}{\kappa^2\sigma^2}=\frac{\psi}{\sigma^2} . \quad (10)$$

(This criterion for slow rolling, rewritten as $8\pi G_{\text{eff}}\sigma^2\gg 1$, is the same as in standard chaotic inflation.) The approximation improves as we approach early times. Indeed, (10) and (6)–(8) give

$$d(\ln f)=(\beta^2+2nf^2)d(\ln a) , \quad (11)$$

and hence $f\to 0$ as $a\to 0$, logarithmically in a for $\beta=0$, and as a power of a for $\beta\neq 0$.

A pertinent question is whether the solutions of (6)–(9) are stable against small perturbations, i.e., whether they are attractors at least locally. We can rewrite (4) and (5) using (3) as a dynamical system:

$$\dot{\varphi}=2\beta\kappa We^{-\beta\kappa\varphi}\sigma^n(1+x) , \quad (12)$$

$$\dot{\sigma}=-2nW\sigma^{n-1}(1+y) , \quad (13)$$

$$\dot{x}=\kappa^2 W e^{-\beta\kappa\varphi}\sigma^n[(-3+2\beta^2+n^2f^2)x-2n^2f^2y+O(x^2,y^2,xy)+O(f^4)] , \quad (14)$$

$$\dot{y}=\kappa^2 W e^{-\beta\kappa\varphi}\sigma^n\{(n^2-2n)f^2+[-3+2\beta^2+(n^2-4n)f^2]y+O(x^2,y^2,xy)+O(f^4)\} . \quad (15)$$

The first two equations define phase-space coordinates x and y, which measure the deviation from the slow-rolling approximation. The inhomogeneous terms in $\dot{x}$ and $\dot{y}$ are at least of order f^2, while the coefficients of the terms linear in x and y are negative. Therefore the solution of (6)–(9), although not a solution of the exact equations of motion (3)–(5), remains close to a true solution which is locally an attractor.

Now we solve the approximate equations of motion. Equation (8) is easily integrated to give $\sigma(t)$. For $n\neq 0$, σ can be used as a time variable and Eq. (7) can be integrated as

$$\psi(\sigma)=\psi_e-\frac{\beta^2\sigma^2}{2n} . \quad (16)$$

Finally Eqs. (6) and (7) give

$$a(\sigma)=a_e\left[\frac{\psi}{\psi_e}\right]^{1/2\beta^2}=a_e\left[1-\frac{\beta^2\sigma^2}{2n\psi_e}\right]^{1/2\beta^2} . \quad (17)$$

The two integration constants ψ_e and a_e are fixed, in a complete cosmological model, by tracing the evolution of a and ψ back from the present epoch, where $a=a_0=1$ and $\psi=\psi_0=\kappa^{-2}$.

We now check that in the limit $\beta\rightarrow 0$ we recover the slow-rolling approximation in standard chaotic inflation [21]. The limit of (16) is in fact $\psi=\psi_e=\psi_0=\kappa^{-2}$. The limit of (17) is

$$a=a_e\exp\left[-\frac{\kappa^2\sigma^2}{4n}\right] \quad \text{as } \beta\rightarrow 0 . \quad (18)$$

The case of extended old inflation corresponds to $n=0$. Equations (6) and (7) then hold exactly and we recover the results of [14]: $\psi(t)=\psi_e+2\beta^2W(t-t_e)$, and $a(t)$ given by (17).

Although the background equations of motion (in the slow-rolling approximation) are now completely solved, we need some further work on them because in the quantization of perturbations the appropriate time parameter will be the conformal time η. We therefore have to express the background parameters a, H, and f as functions of η, again in the slow-rolling approximation $f^2\ll 1$. We find from the slow-rolling equations that

$$\eta\equiv\int a^{-1}dt=\int -\frac{1}{1-2\beta^2}\left[1-\frac{2n^2f^2}{1-2\beta^2}\right]^{-1}\frac{d}{dt}(h^{-1})dt , \quad (19)$$

where $h\equiv aH$. If $n=0$ (extended old inflation), or if we neglect the f^2 term, we can integrate and obtain

$$h=-\frac{1}{(1-2\beta^2)\eta} , \quad (20)$$

where the origin of η has been fixed so that h becomes infinite as $\eta\rightarrow 0$ from below. Integration of (20) gives

$$a(\eta)=a_e\left[\frac{\eta}{\eta_\sigma}\right]^{1/(1-2\beta^2)} , \quad (21)$$

$$\psi(\eta)=\psi_e\left[\frac{\eta}{\eta_\sigma}\right]^{-2\beta^2(1-2\beta^2)} , \quad (22)$$

where η_σ is *a priori* a constant. The dependence of h, a, and ψ on η is then the same as for extended old inflation, where σ is exactly constant. Now, as σ is not constant for $n\neq 0$, these expressions are not consistent with (6)–(8). The reason for that discrepancy is that in obtaining (20) we have neglected not f^2, but an integral of f^2 over time. The error we make in this way may become large over a long period of time. To compensate for it we can readjust the constant η_σ by hand by allowing it to be slowly varying as a function of σ (adiabatic approximation). The explicit dependence is obtained by forcing (20)–(22) to coincide with our former solutions in cosmic time t. Substituting (6) and (20)–(22) in the identity $h=aH$ and solving for η_σ yields

$$\eta_\sigma=-\frac{\psi_e}{(1-2\beta^2)a_eW\sigma^n} . \quad (23)$$

Using this σ-dependent value ensures the consistency of (20)–(22) with the slow-rolling approximation. Equations (6) and (22), for example, give

$$H=-\frac{1}{(1-2\beta^2)\eta_\sigma a_e}\left[\frac{\eta}{\eta_\sigma}\right]^{2\beta^2/(1-2\beta^2)} , \quad (24)$$

which agrees up to an f^2 term with the expression derived directly from (21). The same applies for $\dot{\varphi}$. Finally, substituting (22) into (10), we obtain

$$f^2=\frac{\psi_e}{\sigma^2}\left[\frac{\eta}{\eta_\sigma}\right]^{-2\beta^2/(1-2\beta^2)} . \quad (25)$$

To summarize, Eqs. (20)–(25) are strictly equivalent to (10), (16), and (17) in the case of old extended inflation, where $n=0$ and $\sigma=$const, and adiabatically approximate them in the general case.

III. THE EQUATIONS OF MOTION FOR THE PERTURBATIONS

In [7] we generalized Mukhanov's [17] treatment of single-field inflation and expressed the Lagrangian for the perturbations of the metric and the two scalar fields of extended inflation as a function of the two physical degrees of freedom v and w, which are, respectively, the gauge-invariant quantities associated with the dilaton and inflaton perturbations. This Lagrangian is of the form

$$L=\tfrac{1}{2}v'^2+\tfrac{1}{2}w'^2-\tfrac{1}{2}\partial_i v\,\partial^i v-\tfrac{1}{2}\partial_i w\,\partial^i w-Dvw'-\tfrac{1}{2}Av^2-Bvw-\tfrac{1}{2}Cw^2 , \quad (26)$$

where the prime denotes the derivative with respect to conformal time η and $i=1,2,3$. The explicit expressions for the coefficients as functions of the background solutions are given in [7]. The gauge-invariant density perturbation δ, which is the relative density perturbation on comoving hypersurfaces (through which the energy flux vanishes), is given by a Poisson equation

$$\delta=\frac{2}{3}\frac{\Delta\Phi}{(aH)^2}\ , \tag{27}$$

where Δ denotes the Laplacian with respect to the comoving spatial coordinates and where the gauge-invariant quantity Φ can be interpreted as the gravitational potential on conformally Newtonian hypersurfaces (see [16], where $\Phi\equiv\Phi_H$ and $\delta\equiv\epsilon_m$). The explicit expression for $\Delta\Phi$ as a function of v and w is given in [7]. The equations of motion for v and w derived from the Lagrangian (26) are

$$v''+Dw'+Av+Bw-\Delta v=0\ , \tag{28}$$

$$w''-Dv'-D'v+Bv+Cw-\Delta w=0\ . \tag{29}$$

For $V_I(\sigma)=c\sigma^{2n}$, and in the slow-rolling approximation [(8), (16), and (17)] to the background solution, the coefficients of the Lagrangian reduce to

$$A=-2h^2(1-\beta^2)\ , \tag{30}$$

$$C=-2h^2(1-\beta^2)(1-\tfrac{3}{2}\beta^2)\ , \tag{31}$$

$$B=2n\beta(1-\beta^2)fh^2\ , \tag{32}$$

$$D=-2n\beta fh\ , \tag{33}$$

$$D'=-2n\beta(1-\beta^2)fh^2=-B\ . \tag{34}$$

As in the background equations of motion, terms of order f^2 have been neglected compared to terms of order 1. Specializing the expression for $\Delta\Phi$ in [7] to extended chaotic inflation in the slow-rolling approximation, we find

$$\Delta\Phi=\frac{\kappa h}{a}\{\beta(v'-hv)-nf[w'-(1-\beta^2)hw]\}\ . \tag{35}$$

IV. VACUUM FLUCTUATIONS

We now construct the quantum theory of the perturbations in the Heisenberg picture. The classical quantities v and w become quantum operators that we decompose on a basis of annihilation and creation operators with time-dependent c-number coefficients, the "mode functions" (for further details, see [18]):

$$\begin{bmatrix} v(x,\eta) \\ w(x,\eta)\end{bmatrix}\equiv\int\frac{d^3k}{(2\pi)^{3/2}}\left[e^{-ikx}Q(\eta)\begin{bmatrix} a_{1k} \\ a_{2k}\end{bmatrix}+e^{ikx}Q^*(\eta)\begin{bmatrix} a^\dagger_{1k} \\ a^\dagger_{2k}\end{bmatrix}\right]\ , \tag{36}$$

where a dagger denotes the Hermitian conjugate, and where the matrix Q of mode functions is

$$Q(\eta)=\begin{bmatrix} v_{1k} & v_{2k} \\ w_{1k} & w_{2k}\end{bmatrix}\ , \tag{37}$$

the columns being two linearly independent complex solutions of the equations of motion (28)–(34) in Fourier space.

We define the canonical momenta from the Lagrangian (26) as

$$\pi_v\equiv\frac{\partial L}{\partial v'}=v',\quad \pi_w\equiv\frac{\partial L}{\partial w'}=w'-Dv\ . \tag{38}$$

The quantization is performed by imposing the canonical commutation relations

$$\begin{aligned}[v(x,\eta),\pi_v(x',\eta)]&=i\delta^{(3)}(x-x')\ ,\\ [w(x,\eta),\pi_w(x',\eta)]&=i\delta^{(3)}(x-x')\ ,\end{aligned} \tag{39}$$

while all other pairs commute, if the matrices of mode functions obey

$$[a_{ik},a^\dagger_{jk'}]=\delta_{ij}\delta^3(k-k'),\quad i,j=1,2\ , \tag{40}$$

while all other pairs commute if the matrices of mode functions obey

$$PP^+-P^*P^t=0,\quad QQ^+-Q^*Q^t=0,\quad QP^+-Q^*P^t=i\ , \tag{41}$$

where t denotes transposition of a matrix, an asterisk complex conjugation, a plus their composition, and where P is the matrix of momenta associated to the matrix Q by (38).

To build a Hilbert space of quantum states, we first define the Fock vacuum $|0_Q\rangle$ by

$$a_{ki}|0_Q\rangle=0\quad\text{for all } k,i\ . \tag{42}$$

The vacuum depends on the choice of mode functions. We choose it to be the ground state in the asymptotic past. Now as $\eta\to-\infty$ the coefficients $A,B,C,$ and D vanish with f and h, and the equations of motion (28) and (29) decouple into two harmonic-oscillator equations. Therefore the asymptotic ground state, which we denote by $|0\rangle$, corresponds to the mode functions whose behavior is

$$Q\to\begin{bmatrix} e^{-ik\eta}/\sqrt{2k} & 0 \\ 0 & e^{-ik\eta}/\sqrt{2k}\end{bmatrix}\quad\text{as }\eta\to-\infty\ . \tag{43}$$

Finally we define the quantum operators Φ and δ from (27) and (35) where v and w are replaced by their quantum equivalent (36). This completes the quantization scheme.

It remains to compute the mode functions, that is, to solve the classical equations of motion (28)–(34). It is possible in principle to determine them numerically. However, one can view the slow-rolling approximation as keeping the two first terms of an expansion in f. The corresponding expansion of the mode functions in powers of f is analytically tractable. In solving the perturbation equations of motion it is inconsistent, however, to go further than $O(f)$, as the error in the coefficients (30)–(34) themselves is of $O(f^2)$. To lowest order, $O(f^0)$, the problem simplifies to two decoupled equations, which read, in the adiabatic approximation where h is given by (20),

$$D_v v^{(0)}_{ik}=0,\quad D_w w^{(0)}_{ik}=0\ , \tag{44}$$

with

$$D \equiv \frac{d^2}{d\eta^2} + \left[k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2} \right] ,$$
$$\nu_v \equiv \frac{3}{2} + 2\gamma, \quad \nu_w \equiv \frac{3}{2} + \gamma, \quad \gamma \equiv \frac{\beta^2}{1-2\beta^2} . \tag{45}$$

The initial conditions are given by (43). The solution is

$$v_{1k}^{(0)} = N_v \sqrt{-k\eta} H_{\nu_v}^{(1)}(-k\eta) ,$$
$$w_{2k}^{(0)} = N_w \sqrt{-k\eta} H_{\nu_w}^{(1)}(-k\eta) , \tag{46}$$

where $H_\nu^{(1)}$ is the Hankel function of the first kind [22] and where

$$N = \left[\frac{\pi}{4k} \right]^{1/2} \exp \left[i \left[\frac{\pi}{2}\nu + \frac{\pi}{4} \right] \right] ,$$

while $v_{2k}^{(0)} = w_{1k}^{(0)} = 0$.

We now have all the ingredients to compute, in the adiabatic approximation, the power spectrum $\langle \Phi^2 \rangle_k$ of the vacuum expectation value of Φ^2. We find, from (35)

$$\langle \Phi^2 \rangle_k \equiv \int d^3x \, e^{ikx} \langle 0 | \Phi(0,\eta) \Phi(x,\eta) | 0 \rangle$$
$$= k^{-4} \left[\frac{\kappa h}{a} \right]^2 (\beta^2 |\tilde{v}_{1k}^{(0)}|^2 + n^2 f^2 |\tilde{w}_{2k}^{(0)}|^2) , \tag{47}$$

where

$$\tilde{v}_{1k}^{(0)} \equiv v_1^{(0)\prime} - h v_{1k}^{(0)} = -k N_v \sqrt{-k\eta} H_{\nu_v - 1}^{(1)}(-k\eta) ,$$
$$\tilde{w}_{2k}^{(0)} \equiv w_{2k}^{(0)\prime} - (1-\beta^2) h w_{2k}^{(0)}$$
$$= -k N_w \sqrt{-k\eta} H_{\nu_w - 1}^{(1)}(-k\eta) . \tag{48}$$

$\tilde{v}$ and $\tilde{w}$ arise by use of the identity $(d/dz)H_\nu(z) + (\nu/z)H_\nu(z) = H_{\nu-1}(z)$, valid for any Bessel function $H(z)$.

In (47) we have written down only the contributions from $v^{(0)}$ and $w^{(0)}$. This means that we have included a term of order f^2, but not others which are also of order f^2, for example, $2\beta n f \, \mathrm{Re} \tilde{v}_{1k}^{(0)} w_{1k}^{(1)} + 2\beta n f \, \mathrm{Re} \tilde{w}_{2k}^{(0)} v_{2k}^{(1)}$. (In an appendix we show how $v^{(1)}$ and $w^{(1)}$ can be calculated and that they are of order $\beta n f$.) Still, these terms are really the leading contributions to $\langle \Phi^2 \rangle_k$. This is due to the presence of two potentially small parameters. Evaluated at $-k\eta \simeq 1$, nf takes a definite value, depending on the value of β (and weakly on k), which may be smaller or larger than β.

For $\beta^2 \gg n^2 f^2$ at $-k\eta \simeq 1$, the first term in (47) is the dominant one. For $\beta^2 \ll n^2 f^2$, the second term in (47) is the dominant one and the first one is a small correction, suppressed by β^2. For $\beta^2 \simeq n^2 f^2$ both terms in (47) will be equally dominant. Finally, in the limiting cases of extended old inflation ($n=0$) and of standard chaotic inflation ($\beta=0$) only the first or the second term, respectively, do not vanish altogether. In summary, the leading term in $\langle \Phi^2 \rangle_k$ will be either the first or second term in (47), or both in a special case, while terms in $v^{(1)}$ and $w^{(1)}$ are always smaller. Taking them into account would be inconsistent with the $O(f^2)$ error in the background solution. [This becomes obvious when one considers the contribution $\beta^2 v_{1k}^{(0)} v_{1k}^{(2)}$, which would also have to be included to make the $O(f^2)$ in (47) complete, and which is already beyond the adiabatic approximation.]

We are interested in the asymptotic behavior of the fluctuations after they have "crossed the Hubble radius," that is, when $k^2 \ll h^2$ or equivalently when $|k\eta| \ll 1$ [see (20)]. We then use the asymptotic power-law expression for the Hankel functions, $H_\nu^{(1)}(z) \simeq -i\pi^{-1} 2^\nu \Gamma(\nu) z^{-\nu}$ for $|z| \ll 1$, and obtain

$$\langle \Phi^2 \rangle_k \to \frac{k^{-3}}{2} \kappa^2 H^2 [\bar{N}_v \beta^2 (-k\eta)^{-4\beta^2/(1-2\beta^2)} + \bar{N}_w n^2 f^2 (-k\eta)^{-2\beta^2/(1-2\beta^2)}] , \tag{49}$$

with

$$\bar{N} \equiv \frac{1}{2\pi} |2^{\nu-1} \Gamma(\nu-1)|^2 = 1 + O(\beta^2) ,$$

for $f^2 \ll 1$ and $-k\eta \ll 1$. Finally, if we substitute the explicit expressions for H and f, (24) and (25), Eq. (49) reads

$$\langle \Phi^2 \rangle_k \to \frac{k^{-3}}{2} \kappa^2 [(1-2\beta^2)\eta_\sigma a_e]^{-2} \times \left[\bar{N}_v \beta^2 (-k\eta_\sigma)^{-4\beta^2/(1-2\beta^2)} + \bar{N}_w \frac{n^2 \psi_e}{\sigma^2} (-k\eta_\sigma)^{-2\beta^2/(1-2\beta^2)} \right] . \tag{50}$$

We see that all the powers in η have canceled and that $\langle \Phi^2 \rangle_k$ is a function of σ only. It is therefore strictly constant when $n=0$ (extended old inflation) or nearly frozen, in the sense of the adiabatic approximation, in the general case. The spectrum (50), together with the spectrum of the vacuum density fluctuations, $\langle \delta^2 \rangle_k = \frac{4}{9}(k/aH)^4 \langle \Phi^2 \rangle_k$, is our main result. We shall devote the rest of the paper to tracing its evolution after inflation.

V. THE EVOLUTION OF THE PERTURBATIONS AFTER HUBBLE RADIUS CROSSING

All perturbations of cosmological interest, with wavelengths between 1 and 3000 Mpc today, cross the Hubble radius long before the end of inflation (see, e.g., [23]). For these perturbations, (49) and (50) apply from the first Hubble radius crossing until the end of slow rolling. But we want to know $\langle \Phi^2 \rangle_k$ when the perturbation reenters the Hubble radius. By then the Universe has gone through the end of inflation, through a period of reheating, of radiation, and for small k, of dust domination.

While a perturbation component of given k is "outside the Hubble radius," i.e., while $k^2 \ll h^2$, one can ignore spatial gradient terms in its equation of motion, and it will evolve as the $k=0$ component. The period after the last perturbation of interest has "left the Hubble radius" and before the first "reenters the Hubble radius" will therefore affect only the overall amplitude of the perturbation spectrum, not its shape.

Beyond that knowledge, which follows only from assuming the validity of linear perturbation theory, we can also estimate the overall amplification factor. From the Einstein equations and Bianchi identities alone one can deduce an equation of motion for the Fourier components of the gauge-invariant density perturbation δ [Eq. (4.9) of [16]]:

$$\delta_k'' + \left[1+3\frac{p'}{\rho'}-6\frac{p}{\rho}\right]h\delta_k' + \left[9\frac{p'}{\rho'}-12\frac{p}{\rho}+\frac{9}{2}\frac{p^2}{\rho^2}-\frac{3}{2}\right]h^2\delta_k$$
$$=k^2\left[-\frac{p'}{\rho'}\delta_k-\frac{p}{\rho}\eta_k+\frac{2}{3}\frac{p}{\rho}\pi_k\right]+2h^2\left[3\frac{p^2}{\rho^2}+3\frac{p'}{\rho'}-2\frac{p}{\rho}\right]\pi_k-2h\frac{p}{\rho}\pi_k' , \quad (51)$$

where p and ρ are the pressure and density of the background effective perfect fluid. The quantity $\eta\equiv\delta p/p-(p'/\rho')(\delta\rho/\rho)$ describes the anadiabatic part of the pressure perturbation, and π the anisotropic stress perturbation. (These are η and $\pi_T^{(0)}$ of [16].) Both, like δ, are gauge invariant. In general η and π are not determined by the Einstein and Bianchi equations but by the remaining equations of motion for the matter. If the matter is a single scalar field, then the Bianchi identities give its equation of motion (the Klein-Gordon equation). One can therefore determine η and π as functions of δ and obtains, in fact, $\eta=[1-(p'/\rho')(\rho/p)]\delta$ and $\pi=0$ [24]. When the matter is two scalar fields as in (2) or as in double inflation [25], we have shown in [7] that $\pi=0$. [This follows from Eq. (6a) in [7] with Eq. (4.4) in [16].] As for the η term in (51), it can be neglected for components outside the Hubble radius unless it is very large compared to δ. In double inflation one expects that η becomes large at the transition between the first and second period of inflation because the background equation of state changes abruptly and the perturbations may not be able to follow adiabatically. By contrast, the background evolves smoothly in extended chaotic inflation until reheating, so that η should remain small. Finally, during reheating η is probably large, but by then the factor k^2/h^2 is also much smaller.

During reheating π may also be nonvanishing. There is some reason to believe, though, that its effect on δ is unimportant. Bardeen [16] has solved the homogeneous part of (51) analytically for $p/\rho=p'/\rho'=$const, and has solved the inhomogeneous equation with the help of a retarded Green's function. He finds that for $k^2\ll h^2$ the source term π excites only the decaying mode of δ. One must take into account, however, that $p/\rho=p'/\rho'=$const is not a good approximation during reheating.

We therefore neglect the right-hand side of (51) for $k^2\ll h^2$. The equation is then an equation in δ alone, a first integral of which was found in [24] (see also [26]). Using relation (27), this first integral ζ can be written in terms of Φ and Φ′ as

$$\zeta=\frac{2}{3}\frac{\rho}{p+\rho}(\Phi+h^{-1}\Phi')+\Phi . \quad (52)$$

The constancy of ζ outside the horizon has been claimed [23,27] without all the qualifications we have stressed here. It was checked numerically in a variety of examples with one or more scalar fields in [28]. The only case in that study where ζ was not constant to great precision was that of double inflation. In all cases it remained constant during reheating, which was simulated by a heuristic damping term coupling σ to a radiation fluid. Assuming therefore that in our model ζ is constant while the component is outside the Hubble radius, we can compute the spectrum of density perturbations after inflation, at the time when the components reenter the Hubble radius.

VI. THE POWER SPECTRUM OF THE DENSITY FLUCTUATIONS

After quantization the quantity ζ (52) becomes an operator. Since we assumed its constancy while the wavelength is outside the Hubble radius, its power spectrum at the time of second Hubble radius crossing is

$$\langle\zeta^2\rangle_k|_{2\mathrm{HC}}=\left[\frac{2}{3}\frac{\rho}{p+\rho}+1\right]^2\langle\Phi^2\rangle_k , \quad (53)$$

where the right-hand side is evaluated during the slow-rolling phase after the first Hubble radius crossing, when Φ is adiabatically constant. We have

$$\frac{2}{3}\frac{\rho}{p+\rho}=-\frac{H^2}{\dot H}=(2\beta^2+2n^2f^2)^{-1} , \quad (54)$$

where the first equality holds in any flat FRW universe and the second in the slow-rolling approximation. Substituting (54) into (53) together with the expressions (49) and (50) for $\langle\Phi^2\rangle_k$, we obtain

$$\langle\zeta^2\rangle_k|_{2\mathrm{HC}}=\frac{k^{-3}}{2}\left[\frac{1}{2\beta^2+2n^2f^2}+1\right]^2\kappa^2H^2[\bar N_v\beta^2(-k\eta)^{-4\beta^2/(1-2\beta^2)}+\bar N_w n^2f^2(-k\eta)^{-2\beta^2/(1-2\beta^2)}]$$
$$=\frac{k^{-3}}{2}\left[\frac{1}{2\beta^2+2n^2f^2}+1\right]^2\frac{\kappa^2}{[(1-2\beta^2)\eta_\sigma a_e]^2}\left[\bar N_v\beta^2(-k\eta_\sigma)^{-4\beta^2/(1-2\beta^2)}+\bar N_w\frac{n^2\psi_e}{\sigma^2}(-k\eta_\sigma)^{-2\beta^2/(1-2\beta^2)}\right]. \quad (55)$$

In the case of extended old inflation ($n=0, \sigma=\text{const}$) the right-hand side of (55) is strictly constant, as it should be during the whole slow-rolling phase after the first Hubble radius crossing. In the general case it varies slowly with $\sigma(t)$, due to the fact that we describe the evolution of the background in an adiabatic approximation. Since the adiabatic approximation improves with early times [see (10)] the error made is minimal if we evaluate the right-hand side of (55) as early as possible, that is, at the time of the first Hubble radius crossing.

From (52) and (55) the power spectrum for the density perturbations is easily found. For long wavelengths, for example, which reenter the Hubble radius in a dust-dominated universe when the gravitational potential Φ is nearly constant (see, e.g., [29]), we have $\zeta \simeq \frac{5}{3}\Phi$, and (27) yields

$$\langle \delta^2 \rangle_k |_{2HC} = \frac{4}{25} \left[\frac{k}{aH} \right]^4 \langle \zeta^2 \rangle_k \ . \tag{56}$$

Let us first check that Eqs. (55) and (56) agree with known results in chaotic and extended old inflation. For $\beta=0$ (standard chaotic inflation) (55) reduces to (since $f^2 \ll 1$)

$$\langle \zeta^2 \rangle_k |_{2HC} = k^{-3} \frac{\kappa^2}{8n^2} \frac{H^2}{f^2} \bigg|_{1HC} \quad (\beta=0) \ , \tag{57}$$

which can be rewritten, using (6), (8), and (10), in the familiar form

$$\langle \zeta^2 \rangle_k |_{2HC} = \frac{1}{2} k^{-3} \left[\frac{H^2}{\dot{\sigma}} \right]^2 \bigg|_{1HC} \quad (\beta=0) \ , \tag{58}$$

which is the standard result (see, e.g., [23] and references therein). The other limit, $n=0$, corresponds to extended old inflation. Equation (55) then becomes, using (23),

$$\langle \zeta^2 \rangle_k |_{2HC} = \frac{(2\beta^2+1)^2}{8\beta^2} \kappa^2 \left[\frac{W}{\psi_e} \right]^2 \times \left[\frac{(1-\beta^2) a_e W}{\psi_e} \right]^{4\beta^2/(1-2\beta^2)} \times \bar{N}_v k^{(3-2\beta^2)/(1-2\beta^2)} \quad (n=0) \ , \tag{59}$$

where $\kappa W \equiv \sqrt{c/(3-2\beta^2)}$. To relate (59) to previous results [19,26], we approximate the end of inflation by $\psi=\psi_e, a=a_e$. We can then set $\psi_e=\kappa^{-2}$, assuming as an approximation that the dilaton will no longer evolve after inflation, and $a_e = T_{\text{reh}}/T_0$, using the fact that the radiation present in the Universe is such that $aT=T_0=2.75$ K and the hypothesis that the Universe reheated to a temperature T_{reh} at the end of inflation. A short calculation then gives for $\delta_\lambda|_{2HC}$, the average value of the density fluctuations on a scale $\lambda=1/k$ at second horizon crossing:

$$\delta_\lambda|_{2HC} \equiv \sqrt{k^3 \langle \delta^2 \rangle_k |_{2HC}} = \tfrac{2}{5} k^{3/2} \sqrt{\langle \zeta^2 \rangle_k |_{2HC}} = \frac{|\Gamma(\mu)|}{10\sqrt{2\pi}} \frac{2\beta^2+1}{\beta^2} \left[\frac{3-2\beta^2}{4V_I \beta^2} \right]^{-\beta^2/2(1-2\beta^2)} \times \left[\frac{1-2\beta^2}{\beta} \frac{T_0}{T_{\text{reh}}} \lambda \right]^{2\beta^2/(1-2\beta^2)} \ , \tag{60}$$

which is identical with Eq. (4.16) in [20], with $c=V_I$ and $\bar{N}_v \equiv (1/2\pi)|2^\mu \Gamma(\mu)|^2$ with $\mu \equiv (1+2\beta^2)/(2-4\beta^2)$.

Let us now consider the general case when both terms in (55) contribute, that is, when β is of the same order as f. We have now contributions from both v and w, and both differ from a Harrison-Zeldovich [30] spectrum: where the dilaton contribution dominates the spectrum slowly rises with wavelength $\lambda \equiv 1/k$ as $\lambda^{4\beta^2}$, and as $\lambda^{2\beta^2}$ where the inflaton contribution dominates. We shall calculate here for what β the switchover occurs at a λ in the cosmological range, say from 1 to 3000 Mpc. This will require a fine-tuning of β. More importantly, it indicates up to what upper limit on β the dilaton contribution to δ can be neglected.

To optimize the adiabatic approximation, the right-hand side of (55) has to be evaluated at first Hubble radius crossing, that is, when $k=aH$, which, using (17), reads

$$k = a_e \left[1 - \frac{\beta^2 \sigma^2_{1HC}}{2n\psi_e} \right]^{1/2\beta^2} H_{1HC} \ , \tag{61}$$

where H_{1HC} can be given as a function of σ_{1HC} by (6) and (16). For small β, Hubble radius crossing corresponds to $-k\eta=1$. The switchover in (55) can therefore be defined by $\beta^2=n^2 f^2$. Using (10) and (16) this reads

$$\sigma^2|_{1HC} = \frac{2n^2}{2+n} \frac{\psi_e}{\beta^2} \ . \tag{62}$$

Substituting (62) into (61), we obtain

$$\beta_c^2 = \frac{\ln[(n+2)/2]}{2N_\lambda} \ , \tag{63}$$

where $e^{N_\lambda} \equiv a_e \lambda H_{1HC} = a_e/a_{1HC}$. To estimate N_λ we characterize, in a rough approximation, the end of inflation by $a=a_e$, and set $a_e=T_{\text{reh}}/T_0$ with $T_0=2.75$ K, assuming that the Universe reheated instantaneously to a radiation temperature T_{reh} at the end of inflation. Furthermore, we can make the rough approximation $H_{1HC} \simeq H_e$ in (63) as H_{1HC} depends only logarithmically on λ. Now H_e and T_{reh} are related by the Einstein equation $3H_e^2 = \kappa^2 \rho = \kappa^2 b T^4_{\text{reh}}$, where b is the radiation density (Stefan) constant:

$$N_\lambda \simeq \ln \left[\lambda \left[\frac{\kappa^2 b}{3} \right]^{1/2} T_{\text{reh}} T_0 \right] \simeq 40 + \ln \left[\frac{\lambda}{\text{Mpc}} \right] + \ln \left[\frac{T_{\text{reh}}}{10^{10}\ \text{GeV}} \right] \ .$$

(A more elaborate model, distinguishing $H_{1\mathrm{HC}}$ from H_e and allowing for delayed reheating, and therefore for $\psi_e \neq \psi_0$, would change N_λ by a few units. In our rough approximation the $\ln\lambda$ and $\ln T_{\mathrm{reh}}$ corrections are therefore not significant.) We see that the switchover from a density spectrum dominated by the dilaton to the inflaton occurs at observable cosmological scales if

$$\beta^2 \simeq \beta_c^2 \simeq 0.009 \quad (n=2) .$$

Finally, it is interesting to see how far our results support the approximate expression $\langle \zeta^2 \rangle \simeq H^2/\dot{\phi}$, where ϕ is any scalar field and where the expression is evaluated in the conformal frame in which ϕ is minimally coupled, e.g., φ in the Einstein frame (2) and σ in the Jordan frame (1), and at the moment of Hubble radius crossing in that frame.

If $0<\beta \ll \beta_c$, we obtain approximately $(k^{-3}/2)(H^2/\dot{\sigma})^3$, evaluated in the Jordan frame. Other terms must be considered artifacts of the adiabatic approximation. (This is because in the adiabatic approximation we neglect not only f^2 compared to 1, but even the time dependence of a term like η^{f^2}, and because in the case we are considering here $\beta^2 < n^2 f^2$.)

If $\beta \gg \beta_c$, we obtain

$$\langle 0|\zeta^2|0\rangle_k = k^{-3}\frac{1}{2}N_v \left[\frac{H^2}{\dot{\varphi}}\right]^2 \Bigg|_{\text{Einstein frame}} \times(-k\eta)^{-4\beta^2/(1-2\beta^2)} \tag{64}$$

for $f^2 \ll 1$ and $-k\eta \ll 1$. The explicit power of η renders (64) constant in the adiabatic approximation because it compensates the power of η arising from $H^2/\dot{\varphi}$, which is rapidly varying on its own. We recover the usual expression by evaluating (64) at $-k\eta=1$.

To summarize, we recover the usual expression $H^2/\dot{\phi}$ only when perturbations from one of the scalar fields are dominant, and by evaluating our expression for ζ at $-k\eta=1$.

VII. CONCLUSIONS

We have calculated the inflationary solution of a scalar field σ with the potential $c\sigma^{2n}$ coupled to the Brans-Dicke theory, and the vacuum density fluctuations on that background. Our quantization prescription, more precise than in previous work, is a generalization of [17] (see also [18]), which has been applied previously to extended old inflation [22].

For the background we have used the slow-rolling approximation, in which the change of σ with time is used only adiabatically, but our approximation is better for large JBD coupling β than that of [15]. In order to treat the perturbation equations of motion analytically, we had to use conformal time η as the independent variable. The transition from cosmic time t (or equivalently σ), which is the natural independent variable for the background evolution, required an additional approximation we have called the adiabatic approximation. Although we must expect the perturbation variable ζ to be precisely constant for perturbation modes which are "outside the Hubble radius," our results are not manifestly constant. This problem does not arise from the presence of the Jordan-Brans-Dicke theory, but is already present in standard new or chaotic inflation. (In standard chaotic inflation, for example, ζ is proportional to σ^{n+1}.) The remedy used implicitly has always been to evaluate the nonconstant expression for ζ as soon as ζ is expected to be constant, that is, at Hubble distance crossing. In this paper we have not improved upon the adiabatic approximation, but generalized it to the presence of the JBD theory.

The power spectrum of density fluctuations rises with the wavelength as $\lambda^{2\beta^2/(1-2\beta^2)}$ for small wavelengths and as $\lambda^{4\beta^2/(1-2\beta^2)}$ for large wavelengths. It is continuous at the transition. In the two segments, the dilaton and inflaton fluctuations, respectively, can be neglected. If the change of power law is to occur between 1 and 3000 Mpc or, in other words, if both dilaton and inflaton perturbations are to contribute equally to the density fluctuations on a cosmological scale, β must have the fine-tuned value $\beta_c \simeq 0.1$. In itself this is not likely, but a value of β both greater or less than this value is not in conflict with observation, as reviewed in the Introduction.

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APPENDIX: THE MODE FUNCTIONS TO $O(f)$

Treating $v_{1k}^{(1)}$, etc. and B and D as $O(f)$, the next-to-leading order in the expansion of the mode functions obeys

$$D_v v_{1k}^{(1)}=0 , \tag{A1}$$

$$D_w w_{2k}^{(1)}=0 , \tag{A2}$$

$$D_v v_{2k}^{(1)}=-Dw_{2k}^{(0)\prime}-Bw_{2k}^{(0)}=2n\beta f h \tilde{w}_{2k}^{(0)} , \tag{A3}$$

$$\begin{aligned} D_w w_{1k}^{(1)} &= Dv_{1k}^{(0)\prime}+(D'-B)v_{1k}^{(0)} \\ &= -2n\beta f h[\tilde{v}_{1k}^{(0)}+3(1-\beta^2)hv_{1k}^{(0)}] . \end{aligned} \tag{A4}$$

Putting boundary conditions on the two homogeneous equations is straightforward. We know that all solutions oscillate at a constant amplitude as $\eta \to -\infty$. In the same limit, $f\to 0$, so that the solutions of the homogeneous equations cannot be nontrivally of $O(f)$. Therefore we must choose $v_{1k}^{(1)}=w_{2k}^{(1)}=0$. By the same argument, the solutions of the two inhomogeneous equations must vanish as $\eta\to-\infty$ in order to be of $O(f)$. Again that fixes the boundary conditions uniquely. Explicit solutions of the inhomogeneous equation $Dv=s$ can be given as $v(\eta)=\int G(\eta,\tau)s(\tau)d\tau$. The correct boundary conditions are obeyed by the retarded Green's function for D, which is

$$G(\eta,\tau)=\begin{cases}0 \text{ if } \eta<\tau,\\ -\dfrac{\pi\sqrt{-k\eta}\sqrt{-k\tau}}{2k\sin\nu\pi}[J_\nu(-k\eta)J_{-\nu}(-k\tau)-J_{-\nu}(-k\eta)J_\nu(-k\tau)] \text{ otherwise .}\end{cases} \tag{A5}$$

The explicit solution in this form is clearly of order $\beta n f$. As $G(\eta,\eta)=0$, it is easy to see that $\tilde{v}(\eta)=\int \tilde{G}(\eta,\tau)s(\tau)d\tau$, where the differential operator we have denoted by a tilde [see Eq. (48)] acts on $G(\eta,\tau)$ with respect to its argument η in the same way as on any solution of D, that is, by changing the order by 1. In order to make progress analytically, we need to approximate f in (25) as a power of η, that is, we must treat η_σ as a constant. Even so, the necessary indefinite integrals over a product of two Bessel functions of different order and a power are not known in closed form. At best we can obtain an approximation for $-k\eta \ll 1$ in closed form.

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