# Supersymmetry as a cosmic censor

Renata Kallosh,* Andrei Linde,† Tomás Ortín,‡ Amanda Peet,§
and Antoine Van Proeyen**
*Physics Department, Stanford University, Stanford, California 94305*
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In supersymmetric theories the mass of any state is bounded below by the values of some of its charges. The corresponding bounds in the case of Schwarzschild ($M \geq 0$ ) and Reissner-Nordström ($M \geq |q|$) black holes are known to coincide with the requirement that naked singularities be absent. Here we investigate $[U(1)]^2$ charged dilaton black holes in this context. The extreme solutions are shown to saturate the supersymmetry bound of $N = 4$, $d = 4$ supergravity, or dimensionally reduced superstring theory. Specifically, we have shown that extreme dilaton black holes, with electric and magnetic charges, admit supercovariantly constant spinors. The supersymmetric positivity bound for dilaton black holes is given by $M \geq \frac{1}{\sqrt{2}}(|Q|+|P|)$. This condition for dilaton black holes coincides with the cosmic censorship requirement that the singularities be hidden, as was the case for other asymptotically flat static black-hole solutions. We conjecture that the bounds from supersymmetry and cosmic censorship will coincide for more general solutions as well. The temperature, entropy, and singularity of the stringy black hole are discussed in connection with the extreme limit and restoration of supersymmetry. The Euclidean action (entropy) of the extreme black hole is given by $2\pi|PQ|$. We argue that this result is not altered by higher-order corrections in the supersymmetric theory. In the Lorentzian signature, quantum corrections to the effective on-shell action in the extreme black-hole background are also absent. When a black hole reaches its extreme limit, the thermal description breaks down. It cannot continue to evaporate by emitting (uncharged) elementary particles, since this would violate the supersymmetric positivity bound. We speculate on the possibility that an extreme black hole may "evaporate" by emitting smaller extreme black holes.

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## I. INTRODUCTION

Evaporation of black holes is one of the most interesting effects of nonperturbative quantum gravity. Investigation of this process may help us to understand the nature of singularities in gravitational theory, the problem of information loss during the process of black-hole evaporation and the interplay between quantum and thermal descriptions of processes near black holes. All of these problems become especially urgent in the theory of the last stages of black-hole evaporation. For a Schwarzschild black hole, this happens when its mass approaches the Planck value $M_P$. In this case any semiclassical description of the black hole becomes impossible. Therefore, despite many attempts, we still do not have a complete understanding of the last stages of black-hole evaporation.

Recently there have been many new attempts to study quantum effects near black holes. Most of them are related to more complicated black holes, such as magnetically or electrically charged black holes, dilaton black holes, etc. First of all, this provides a more complete picture of black-hole physics in the context of theories of elementary particles and/or superstrings. Moreover, some aspects of the theory of such black holes prove to be simpler than the corresponding aspects of ordinary black holes. For example, evaporation of a charged Reissner-Nordström black hole stops when its mass, measured in units of the Planck mass $M_P$, approaches the absolute value of its charge $|q|$. Thus, for a sufficiently large charge, one may study the last stages of black-hole evaporation when the mass of the black hole is much larger than $M_P$ and quantum fluctuations of the metric are not too strong.[1]

However, strongly charged black holes may discharge by creation of pairs of charged elementary particles. To isolate these effects from the effects of quantum gravity, in which we are mainly interested, one may consider models without charged elementary particles. In such models electric and magnetic fields are not produced by charged particles, but flow from infinity or from the singularity.

*On leave from Lebedev Physical Institute, Moscow. Electronic address: kallosh@physics.stanford.edu
†On leave from Lebedev Physical Institute, Moscow. Electronic address: linde@physics.stanford.edu
‡Electronic address: tomaso@slacvm.bitnet
§Electronic address: peet@slacvm.bitnet
**Permanent address: Instituut voor Theoretische Fysica, Universiteit Leuven, B-3001 Leuven, Belgium. Onderzoeksleider, N.F.W.O. electronic address: fgbda19@blekul11

[1]Throughout this paper we will work in a system of units where $M_P = 1$.

 

At the classical level, there is no difference between a Reissner-Nordström black hole with charged particles in its center and a black hole with a spherically symmetric electric field originating from the singularity. In both cases we will have the same theory of black-hole evaporation, but in the last case we do not need to be concerned with extra complications, such as particle production in strong electric fields, charge quantization, etc.

This idea proves to be especially productive if one can find a way of embedding the original bosonic theory in a supersymmetric theory. In such a case one has the same description of the classical properties of the black hole and of its evaporation, but higher-order quantum corrections are under much better control. For example, an extreme Reissner-Nordström black hole with mass $M = |q|$ may be embedded in $N = 2$ supergravity [1, 2]. All higher-order quantum corrections to the effective action of $N = 2$ supergravity in the field of the extreme black hole could be shown to vanish if the theory had no anomalies [3]. In particular, the effective action would have no imaginary part, which would mean no particle creation in the field of an extreme Reissner-Nordström black hole. These and some other properties of supersymmetric black holes indicate that they may prove to be a unique laboratory for investigation of black-hole physics and quantum gravity in general. Indeed, until we started studying supersymmetric black holes, we had no example of a nontrivial Lorentzian four-dimensional background where all quantum gravity corrections to the effective action vanished.

The theory of $N = 2$ supergravity, however, has one-loop anomalies [4]. Therefore the formal proof of the absence of quantum corrections for the classical extreme Reissner-Nordström black hole is not sufficient. One is led to try to find a supersymmetric embedding of charged dilaton black holes [5–8] in dimensionally reduced string theory or $d = 4, N = 4$ supergravity, where the anomalies can be cancelled.

Surprisingly enough, until now, no such supersymmetric embedding has been found. It was argued in [2] that the lower bound on the mass of such black holes does not follow from supersymmetry, as the supersymmetric theory investigated in [2] was a Kaluza-Klein compactification of five-dimensional supergravity. In [7] it was found that there was again no suitable embedding if one takes the vector field to be in a Yang-Mills multiplet. Moreover, supersymmetry is usually related to zero temperature. However, the thermal properties of dilaton black holes are somewhat unusual (the corresponding literature contains several contradictory statements on this issue, see, e.g., [6–8]), and there was no clear signal for supersymmetry from the temperature, unlike the extreme Reissner-Nordström case where $T = 0$.

Such dilaton black holes, if supersymmetrically embedded in supergravity and/or superstring theory, would be especially interesting since they might lead to new insights into perturbative and nonperturbative quantum effects in these theories.

In this paper we will investigate dilaton black holes in this context. We will consider $U(1)\otimes U(1)$ dilaton black holes, with electric and magnetic charges and without axion. They do not coincide with dual dilaton dyons [9], which are dual rotations of the purely electric or purely magnetic charged dilaton black holes. Under that dual rotation, the second charge arises together with the axion, but the metric, causal structure, and thermodynamic properties of the dual dilaton dyon are the same as in the purely electric or purely magnetic solution without axion. In the solution, the existence of a second charge does change the metric, causal structure, and thermodynamic properties of the solution. The importance of the second charge is related to the existence of two central charges in $N = 4$ supersymmetry. In particular, we will show that *extreme dilaton black holes are supersymmetric in the context of N=4 supergravity.* We will also show that the lower bound on the dilaton black-hole mass imposed by cosmic censorship,

$$M^2 + \Sigma^2 \geq P^2 + Q^2 \ , \tag{1}$$

does coincide with the bound which can be derived from supersymmetry. In Eq. (1) $\Sigma, P, Q$ are the dilaton, magnetic, and electric charges, respectively. Equality in (1) shows that, at the extremal value of the mass, supersymmetry leads to the balance of gravitational, electromagnetic, and dilatonic forces. Supersymmetric dilatonic *multi*-black-hole solutions, satisfying the force-balance condition, will be exhibited.

The positivity bound (1) implies, in particular, that the black-hole singularity remains hidden by the event horizon until the mass of the black hole decreases to its extreme value. If the black hole has both electric and magnetic charge, the singularity always remains hidden by the event horizon. It approaches the horizon only if the extreme dilaton black hole has purely electric or purely magnetic charge. Even in this case, though, any external observer who does not touch the singularity cannot see it. In this sense, supersymmetry plays the role of a cosmic censor. It keeps the singularity away from the eyes of any observer who does not want to fall into the black hole.

The paper is organized as follows. In Sec. II we describe the relation between the supersymmetric positivity bound and cosmic censorship for classical Schwarzschild and Reissner-Nordström black holes. In Sec. III the spherically symmetric electrically and magnetically charged dilaton black hole is presented as a solution of dimensionally reduced superstring theory. This solution includes (for some particular values of electric, magnetic, and dilaton charges) the classical Schwarzschild and Reissner-Nordström black holes and dilaton black holes with either purely electric, purely magnetic, or both charges present in the solution.

In Sec. IV the extreme multi black holes are described, as well as spherically symmetric electrically and magnetically charged extreme dilaton black holes. It is also explained that the purely electric extreme dilaton black holes are special cases of a metric of Bonnor [10] describing charged dust in equilibrium.[2] These

[2]The purely magnetic ones also fit in to this category, as can be seen by performing a duality transformation.

metrics generalize the Papapetrou-Majumdar class of metrics in the presence of specific sources in the Einstein and Maxwell equations called "charged dust." The Papapetrou-Majumdar metrics are supersymmetric extreme multi-black-hole solutions [1] of Einstein-Maxwell theory.

In Sec. V the thermal properties of the dilaton black holes are discussed, in particular, the temperature, entropy, and specific heat. The extreme dilaton black holes with $PQ \neq 0$ are shown to have zero temperature, whereas their entropy is given by $2\pi|PQ|$. It is shown that the thermal description of stringy black holes near extremality breaks down for all possible values of the charges $P$ and $Q$.

Section VI contains an investigation of the supersymmetric properties of dilaton black holes, in the context of $N = 4$ supersymmetry. It is shown that nonextreme dilaton black holes necessarily break all supersymmetries. The unbroken $N = 1$ supersymmetries of electrically and magnetically charged extreme multi black holes are found. In addition, the unbroken $N = 2$ supersymmetries of purely electric and purely magnetic extreme multi black holes are identified.

In Sec. VII the partition function of the dilaton black holes is calculated in the semiclassical approximation. A nonrenormalization theorem for quantum corrections to the extreme dilaton black-hole partition function is outlined.

In Appendix A we introduce our conventions and compare them with those used by other authors. In Appendix B some speculative ideas about splitting of extreme dilaton black holes are presented.

In the figures, we plot different characteristics of the charged dilaton black hole, such as temperature and entropy, by using the program MATHEMATICA.

## II. SUPERSYMMETRIC POSITIVITY BOUND AND COSMIC CENSORSHIP

In order to make our goals and methods more clear, let us remember some basic facts about ordinary Schwarzschild black holes with metric

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\Omega \ . \tag{2}$$

For $M > 0$, this metric has a singularity at $r_g = 2M$. However, this is just a coordinate singularity, which corresponds to the event horizon where the components of the metric $g_{tt} = g_{rr}^{-1} = (1 - \frac{2M}{r})$ change their sign. The true singularity, where the curvature tensor becomes infinite, is at $r = 0$.

The presence of singularities, i.e., of places where the normal laws of physics formulated in terms of classical space-time break down, is one of the main problems of classical general relativity. However, in many cases this problem is somewhat softened. For example, any observers near the Schwarzschild black hole cannot actually *see* any violation of standard laws of physics until they reach the singularity. Indeed, the change of sign of $g_{tt} = g_{rr}^{-1}$ inside the black hole means that the light cone inside it looks inwards ($T$ region) [11]. It is possible to send a signal towards the singularity, but it is impossible to get any signal backwards. Therefore, we will not have information about the singularity until we fall into it, and then we will not care.

The situation with electrically charged Reissner-Nordström black holes with metric

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega \tag{3}$$

is slightly more complicated. When the mass $M$ of the black hole is larger than the absolute value of its charge $|q|$ the singularity at $r = 0$ is hidden from us by two horizons located at

$$r_+ = M + \sqrt{M^2 - q^2}\ , \quad r_- = M - \sqrt{M^2 - q^2}\ . \tag{4}$$

At each of these horizons $g_{tt}$ changes its sign. An observer deciding to fly to the region $r < r_-$ would be able to see the singularity. However, an outside observer staying at $r > r_-$ cannot see the singularity for the same reason as in the Schwarzschild space: The region between $r_-$ and $r_+$ is a $T$ region, where all signals can go only towards smaller $r$.

The extreme case $M = q$ is special. In this case the two horizons $r_+$ and $r_-$ coincide and the $T$ region between them disappears. But even in this case an outside observer staying at any finite distance from the horizon $r_+ = r_- = M$ will not see the singularity. Moreover, just as in the Schwarzschild space, the observer will not see anything which is hidden under the horizon *or coincides with the horizon.* Indeed, the equation describing the radial motion of a wavefront of light is $ds = 0$: i.e.,

$$\left(\frac{dr}{dt}\right)^2 = \frac{g_{tt}}{g_{rr}}\ . \tag{5}$$

The time taken for a signal to go from $r_1$ to $r_2$ is given by

$$t = \int_{r_1}^{r_2} \sqrt{\frac{g_{rr}}{g_{tt}}}\, dr\ . \tag{6}$$

One can easily check that this time diverges if $r_1$ coincides with $r_g = 2M$ for the Schwarzschild black hole or with $r_+$ for the Reissner-Nordström black hole. Thus the horizon at $r_g$ (at $r_+$) is called an *event horizon*: Our part of the Universe cannot be influenced by any event which may happen in a region covered by the event horizon or coinciding with it. In particular, we cannot see a singularity if it is covered by (or coincides with) the event horizon.

Thus, in the Schwarzschild space the singularity cannot be seen from any place with $r > 0$. In the Reissner-Nordström space the singularity can be seen from a place with $r < r_-$, but it cannot be seen from any place with $r \geq r_-$. However, in a Schwarzschild metric with $M < 0$, as well as in the Reissner-Nordström metric with $M < |q|$, the singularity does not coincide with any horizon, is not hidden by any horizon and, therefore, is visible from any place. There exists a cosmic censorship conjecture, which says that a naked singularity of such type cannot be formed. There are several versions of this conjecture, which differ from each other by specific assumptions concerning initial conditions for gravitational collapse and the structure of the energy-momentum tensor (weak cosmic censorship conjecture [12], strong cosmic censorship conjecture [13]; see, e.g., [14]). Even the definition of a naked singularity is author dependent: Is the singularity naked if it can be seen from the horizon? What if the horizon itself is singular? To avoid unnecessary complications, we will say that the singularity is hidden if it is covered by (or coincides with) the event horizon, i.e., if an observer staying at any finite distance from the horizon cannot see the singularity. We will also consider only the weak cosmic censorship conjecture, which refers to asymptotically flat space-time [14].

In our paper we do not address the complicated problem of the dynamical origin of naked singularities. A general proof of the cosmic censorship conjecture is still absent, and several (rather artificial) exceptions are known. It is very interesting, therefore, that for some supersymmetric theories to be discussed below, the mass bound from the cosmic censorship conjecture in the form mentioned above coincides with the supersymmetric positivity bound, which requires that the mass of the asymptotically flat space-time be larger than or equal to the absolute values of all central charges.

In extended global supersymmetry, the mass of any quantum state is bounded below by the moduli of the eigenvalues of the central charges $z_n$ of the supersymmetry algebra with $N$ spinor operators, $n = 1, 2, \ldots, \frac{N}{2}$ [15, 16]. Consider, specifically, the $N = 4$ theory with two central charges $z_1, z_2$. From the supersymmetry algebra in the rest frame it can be derived that

$$\begin{aligned} \{S_{(1)}, S^*_{(1)}\} &= 2|S_{(1)}|^2 = M - |z_1| \geq 0 \ , \\ \{S_{(2)}, S^*_{(2)}\} &= 2|S_{(2)}|^2 = M - |z_2| \geq 0 \ , \\ \{T_{(1)}, T^*_{(1)}\} &= 2|T_{(1)}|^2 = M + |z_1| \geq 0 \ , \\ \{T_{(2)}, T^*_{(2)}\} &= 2|T_{(2)}|^2 = M + |z_2| \geq 0 \ . \end{aligned} \tag{7}$$

The positivity bound for $M - |z_1|$ and $M - |z_2|$ exists because these combinations of the mass and central charges of some state can be expressed through the square of particular supersymmetry generators acting on that state.

The first bound is saturated, i.e., $M = |z_1|$, if and only if the state is invariant under one-quarter of all the supersymmetries, since the state has to be invariant under the action of $S_1, S_1^*$. The saturation of the second bound $M = |z_2|$ means that the state has to be invariant under another quarter of the supersymmetries $S_2, S_2^*$. Thus, if both bounds are saturated, i.e., $M = |z_1| = |z_2|$, the state has to be invariant under one-half of all supersymmetries. For globally supersymmetric Yang-Mills theory, these bounds are known as Bogomolny bounds for magnetic monopoles. To identify the central charges one can quantize the theory, construct the supersymmetry charges in terms of coordinates and canonical momenta, and calculate the commutators of supersymmetry charges, paying attention to boundary terms as was done in [15] for $N = 2$ globally supersymmetric Yang-Mills theory.

The situation with supersymmetric positivity bounds for theories with local supersymmetries including gravity is in general much more complicated. Here, we will consider only configurations which are asymptotically flat where one can identify the mass as the Arnowitt-Deser-Misner (ADM) or Bondi mass. The positivity of energy in Einstein theory was obtained via supergravity theory by Deser, Teitelboim, and Grisaru [17]. Using the supergravity-type formalism, Witten has presented a proof of the positivity bound for the ADM or Bondi mass of an asymptotically flat space under the assumption of the dominant energy condition. This was developed later to the so-called Witten-Nester-Israel construction [18]. It has been shown in [18] that the mass of an asymptotically flat space-time is non-negative and vanishes only when the space-time is flat. In terms of the Schwarzschild black hole we may interpret the $N = 1$ supersymmetry bound $M \geq 0$ as the statement that the $r = 0$ singularity is inside the horizon, until the bound is saturated, i.e., the mass vanishes. However, when the mass vanishes the space-time becomes trivial. The Schwarzschild black hole does not admit any unbroken supersymmetry; but broken supersymmetry does work as a cosmic censor as it requires $M > 0$.

The positivity bound for $N = 2$ theory was derived in [1], by applying the Witten-Nester-Israel construction [18] to the local $N = 2$ supersymmetry transformation rules of the gravitino. For asymptotically flat solutions of $N = 2$ supergravity the corresponding bound is [1]

$$M \geq \sqrt{Q^2 + P^2} = |z_1| \ , \tag{8}$$

where the central charge $z_1$ has been expressed through the electric and magnetic charges.

For global $N = 2$ supersymmetry of asymptotic states there is one central charge $z_1$ in the positivity bound, according to [15, 16]. Its value

$$z_1 = \sqrt{Q^2 + P^2} \tag{9}$$

has also been identified by Gibbons and Hull by solving the equations

$$\begin{aligned} \delta\Psi_{\mu I} &= (\hat{\nabla}_\mu(g, A)\,\epsilon)_I \\ &= \nabla_\mu(g)\epsilon_I - (2\sqrt{2})^{-1}\sigma^{\rho\sigma}F^+_{\rho\sigma}\epsilon_{IJ}\gamma_\mu\epsilon^J, \quad I = 1, 2 \ , \end{aligned} \tag{10}$$

where the supercovariant connection in Eq. (10) depends on the metric and the vector field (our notation is defined in Appendix A). They have found that extreme Reissner-Nordström black holes with $M = \sqrt{Q^2 + P^2}$ admit supercovariantly constant spinors of $N = 2$, $d = 4$ supergravity (10) and saturate the bound (8). The bound is saturated only for extreme Reissner-Nordström black holes, as a consequence of the fact that they admit supercovariantly constant spinors, defined in Eq. (10). These equations, in the case that they have solutions, define the unbroken part ($N = 1$) of the original $N = 2$ supersymmetry of the theory. *This shows that the solution ad-*

*mits some supersymmetries despite being purely bosonic; the generators of unbroken supersymmetry leave fermions invariant.*[3]

The positivity bound for Einstein-Maxwell theory, derived from $N = 2$ supergravity in [1] is in fact different from the Bogomolny bound for monopoles, as discussed in [2]. In general, *the identification of the central charges in the supersymmetry algebra, which is necessary to derive the positivity bound (7), is not universal.* It depends both on the dynamics and on the properties of the solutions.

To see the relation between the $N = 2$ supersymmetry bound and the cosmic censorship conjecture, consider the charged Reissner-Nordström black hole with electric and magnetic charges $P$ and $Q$. The quantity $q$ appearing in the metric (3) is $q = \sqrt{Q^2 + P^2}$. In order that the singularity at $r = 0$ be hidden by an event horizon, we have to require that

$$M \geq q \ , \tag{11}$$

just as in the purely electric case considered in the beginning of this section. This condition coincides with the requirement following from supersymmetry [1]. Therefore, from the point of view of $N = 2$ supersymmetry there exist special solutions of the Einstein-Maxwell equations, which happen to coincide with extreme Reissner-Nordström black holes and which solve not only the second-order Einstein-Maxwell differential equations, but also the *first-order differential equations* for spinors (10). This does not happen for nonextreme charged black holes.

In this paper we will investigate the corresponding issues for the dilaton black holes [5–8].

The positivity bound (7) for an asymptotically flat space consists of two equations:

$$\begin{aligned} M &\geq |z_1| \ , \\ M &\geq |z_2| \ , \end{aligned} \tag{12}$$

since for $N = 4$ there are two central charges, according to [16]. We will identify those two central charges by considering the local supersymmetry algebra. It will also be shown that extreme black holes saturate the supersymmetry bound (either one of them for the solutions with electric and magnetic charge or both for the solutions with only electric or only magnetic charge).[4]

The fundamental *first-order differential equations* of the $N = 4$ theory, which will be solved to produce extreme dilaton black holes, generalize those of the $N = 2$ theory, given in Eq. (10). The four gravitinos and four dilatinos will be required to have vanishing local supersymmetry transformations in presence of gravity $g_{\mu\nu}$, dilaton $\phi$, electric $A_\mu$, and magnetic $B_\nu$ fields:

$$\begin{aligned} \delta\Psi_{\mu I} &= (\hat{\nabla}_\mu(\phi, g, A, B)\,\epsilon)_I = 0 \ , \\ \delta\Lambda_I &= -(\hat{\not\partial}\phi(\phi, g, A, B)\,\epsilon)_I = 0 \ , \qquad I = 1,2,3,4. \end{aligned} \tag{13}$$

## III. DILATON BLACK HOLES

A dimensionally reduced superstring theory in $d = 4$ can be described in terms of $N = 4$ supergravity. The latter exists in two versions. One usually refers to the original one as the SO(4) version [19], and to the second one as the SU(4) version [20]. For the latter the action is invariant under a rigid SU(4)⊗SU(1,1) symmetry, which makes that theory simpler. In both versions, the vector fields transform under an SO(4)≡SU(2)⊗SU(2) group. In the SO(4) version both factors contain three vector fields, while in the SU(4) version one factor consists of three vector fields and the other has three axial-vector fields. We will consider U(1)⊗U(1) solutions, i.e., solutions with one nontrivial vector in each subgroup. There is also a complex scalar. Its real part is the dilaton and its imaginary part is the axion. We will look for solutions which depend only on the dilaton $\phi$; the axion field will be put to a constant. The remaining bosonic part of the action in these two cases is given by[5]

$$\begin{aligned} I_{\mathrm{SO}(4)} &= \int d^4x\,\sqrt{-g}[-R + 2\partial^\mu\phi\cdot\partial_\mu\phi - (e^{-2\phi}F_{\mu\nu}F^{\mu\nu} + e^{2\phi}\tilde{G}_{\mu\nu}\tilde{G}^{\mu\nu})] \ , \\ I_{\mathrm{SU}(4)} &= \int d^4x\,\sqrt{-g}\,[-R + 2\partial^\mu\phi\cdot\partial_\mu\phi - e^{-2\phi}\,(F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu})] \ , \end{aligned} \tag{14}$$

where

[3]The supersymmetry variation of the bosons contains only fermions and so is zero trivially for a bosonic solution.

[4]To make the analysis complete, one would also like to prove the positivity bound for $N = 4$ using a Witten-Nester-Israel–type analysis. We leave this investigation for future work.

[5]The parameter $a$ (or $g$), which governs the strength of the coupling of the dilaton to vector fields we keep always equal to 1, as required by $N = 4$, $d = 4$ supersymmetry, and as in superstring theory; i.e., we consider only the case $a = 1$ which has been qualified in [8] as enigmatic. However, for the $a = 1$ case, the difference with other investigations of charged dilaton black holes [7, 8] is the presence of two charges, electric and magnetic, simultaneously, in the absence of the axion.

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ ,$$
$$\tilde{G}_{\mu\nu} = \partial_\mu \tilde{B}_\nu - \partial_\nu \tilde{B}_\mu \ , \tag{15}$$
$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \ .$$

The actions are almost the same, except for the terms depending on the vector $B$ or $\tilde{B}$. The equations of motion of the two theories are equivalent. In fact, those of the SO(4) version are

$$\begin{aligned} \nabla_\mu(e^{-2\phi}F^{\mu\nu}) &= 0 \ , \\ \nabla_\mu(e^{2\phi}\tilde{G}^{\mu\nu}) &= 0 \ , \\ \nabla^2\phi - \tfrac{1}{2}e^{-2\phi}F^2 + \tfrac{1}{2}e^{2\phi}\tilde{G}^2 &= 0 \ , \\ R_{\mu\nu} + 2\nabla_\mu\phi\cdot\nabla_\nu\phi - e^{-2\phi}(2F_{\mu\lambda}F_{\nu\delta}g^{\lambda\delta} - \tfrac{1}{2}g_{\mu\nu}F^2) - e^{2\phi}(2\tilde{G}_{\mu\lambda}\tilde{G}_{\nu\delta}g^{\lambda\delta} - \tfrac{1}{2}g_{\mu\nu}\tilde{G}^2) &= 0 \ , \end{aligned} \tag{16}$$

while from the SU(4) version we obtain

$$\begin{aligned} \nabla_\mu(e^{-2\phi}F^{\mu\nu}) &= 0 \ , \\ \nabla_\mu(e^{-2\phi}G^{\mu\nu}) &= 0 \ , \\ \nabla^2\phi - \tfrac{1}{2}e^{-2\phi}F^2 - \tfrac{1}{2}e^{-2\phi}G^2 &= 0 \ , \\ R_{\mu\nu} + 2\nabla_\mu\phi\cdot\nabla_\nu\phi - e^{-2\phi}(2F_{\mu\lambda}F_{\nu\delta}g^{\lambda\delta} - \tfrac{1}{2}g_{\mu\nu}F^2) - e^{-2\phi}(2G_{\mu\lambda}G_{\nu\delta}g^{\lambda\delta} - \tfrac{1}{2}g_{\mu\nu}G^2) &= 0 \ . \end{aligned} \tag{17}$$

The equivalence of these equations [20] can be demonstrated through the duality rotation

$$\tilde{G}^{\mu\nu} = \tfrac{1}{2}i(-g)^{-\frac{1}{2}}e^{-2\phi}\epsilon^{\mu\nu\lambda\delta}G_{\lambda\delta} \ . \tag{18}$$

Both $G$ and $\tilde{G}$ are real with our conventions. Such a duality rotation [21] transforms the equation of motion of $\tilde{B}$ (the second line) to the Bianchi identity of $B$, while the field equation of $B$ is the Bianchi identity of $\tilde{B}$. The other field equations are mapped into each other. Note that this transformation does not transform one action in the other, a minus sign difference occurs for the $G$ and $\tilde{G}$ terms in a space-time of Lorentzian signature.[6]

The solution of this system of equations has been given by Gibbons [2], and discussed in detail later by Gibbons and Maeda [6]. Each vector field $A_\mu$ and $B_\mu$ (or $\tilde{B}_\mu$) was taken to be either electric or magnetic, to satisfy (in the simplest way) the axion field equation for a constant axion field, which reduces to

$$F_{\mu\nu}{}^\star F^{\mu\nu} + G_{\mu\nu}{}^\star G^{\mu\nu} = 0 \ . \tag{19}$$

The purely magnetic (electric) dilaton black holes have been studied in [7, 8]. The solution generalizes the one given in [2] by including asymptotically nonvanishing dilaton field $\phi_0$ and the ones given in [7, 8] by keeping both electric and magnetic charge. We will in fact take $A_\mu$ to be purely electric, and $B_\mu$ to be magnetic. This implies that $\tilde{B}_\mu$ is also electric, and the calculations are often simpler when using the electric solution $\tilde{B}$, rather than the magnetic $B$.

[6]In Euclidean signature the two actions are connected by duality, though.

The solution depends on four independent parameters: $M, Q, P, \phi_0$. The mass of a black hole is $M$, the asymptotic value of the dilaton field is $\phi_0$. The electric charge of the $F$ field is $Q_{\rm elec} = e^{\phi_0}Q$ and the magnetic charge of the $G$ field $P_{\rm magn} = e^{\phi_0}P$, or equivalently, the electric charge of the $\tilde{G}$ field is $P_{\rm elec} = e^{-\phi_0}P$.[7]

There are few combinations of these parameters which will appear in the solutions.

(1) The dilaton charge, which is not an independent variable, is given by [7]

$$\Sigma = \frac{P^2 - Q^2}{2M} \ , \tag{20}$$

where $\Sigma$ is defined by the equation $\phi \sim \phi_0 + \Sigma/r$ at $r \to \infty$.

(2) The parameter $r_0$, which vanishes when the black hole becomes extremal, is given by

$$\begin{aligned} r_0^2 &= M^2 + \Sigma^2 - P^2 - Q^2 \\ &= M^2 + \Sigma^2 - e^{-2\phi_0}P^2_{\rm magn} - e^{-2\phi_0}Q^2_{\rm elec} \ . \end{aligned} \tag{21}$$

(3) The outer and the inner horizons are defined in terms of the mass and $r_0$:

$$r_\pm = M \pm r_0 \ . \tag{22}$$

The solution of Eqs. (16) can be given in the form

[7]Only when the asymptotic value of the dilaton is zero does the electric charge equal $Q$ and the magnetic one equal $P$. We have chosen the definition of charges in presence of $\phi_0$ in a way which simplifies equations, since it is the parameters $P, Q$ which appear in all equations rather than $Q_{\rm elec}, P_{\rm magn}$.

$$\begin{aligned} ds^2 &= e^{2U}dt^2 - e^{-2U}dr^2 - R^2 d\Omega \ , \\ e^{2\phi} &= e^{2\phi_0}\,\frac{r+\Sigma}{r-\Sigma} \ , \\ F &= \frac{Q\,e^{\phi_0}}{(r-\Sigma)^2}\,dt\wedge dr \ , \\ \tilde{G} &= \frac{P\,e^{-\phi_0}}{(r+\Sigma)^2}\,dt\wedge dr \ , \end{aligned} \tag{23}$$

where

$$e^{2U} = \frac{(r-r_+)(r-r_-)}{R^2} \tag{24}$$

and

$$R^2 = r^2 - \Sigma^2 \ . \tag{25}$$

The curvature singularity occurs at $r = |\Sigma|$.

The solution has manifest dual symmetry:

$$\begin{aligned} &Q \leftrightarrow P \ , \\ &\Sigma \leftrightarrow -\Sigma \ , \\ &F \leftrightarrow \tilde{G} \ , \\ &\phi \leftrightarrow -\phi \ . \end{aligned} \tag{26}$$

To write the solution in a form which corresponds to the solution of Eqs. (17), we have to add to Eqs. (23) the result for the nondually rotated field $G_{\mu\nu}$:

$$G = P\,e^{\phi_0}\sin\theta\,d\theta\wedge d\phi = P_{\rm magn}\sin\theta\,d\theta\wedge d\phi \ . \tag{27}$$

Notice that the solution also yields a solution of a theory with a smaller field content $(g_{\mu\nu}, \phi, \mathcal{F}_{\mu\nu})$ and action

$$I_{\mathcal{F}} = \int d^4x\,\sqrt{-g}\left(-R + 2\partial^\mu\phi\cdot\partial_\mu\phi - e^{-2\phi}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\right) . \tag{28}$$

To see this, take $\mathcal{F}$ to be both electric and magnetic,

$$\mathcal{F} = F + G = \frac{Q\,e^{\phi_0}}{(r-\Sigma)^2}\,dt\wedge dr + P\,e^{\phi_0}\,\sin\theta\,d\theta\wedge d\phi \ , \tag{29}$$

and note that the energy-momentum tensor of $\mathcal{F}$ is just that of the two fields $F$ and $G$, because the electric and magnetic fields are parallel. This implies that the equations of motion for this theory are consistent with those of the original one. All thermodynamic properties, which are controlled by the metric, will be indifferent to whether we use this arrangement of fields or our original one.

Let us also introduce the notation

$$z_1 = \frac{Q-P}{\sqrt{2}} \ , \qquad z_2 = \frac{Q+P}{\sqrt{2}} \ , \tag{30}$$

so that

$$Q_{\rm elec} = e^{\phi_0}\,\frac{z_1+z_2}{\sqrt{2}} \ , \qquad P_{\rm magn} = e^{\phi_0}\,\frac{z_2-z_1}{\sqrt{2}} \ . \tag{31}$$

Later on, we will identify these combinations of electric and magnetic charges as central charges in the supersymmetry algebra. The dilaton charge in this notation is given by

$$\Sigma = -\frac{z_1 z_2}{M} \ , \tag{32}$$

and the parameter $r_0$ showing the deviation from extremality is

$$r_0^2 = \frac{1}{M^2}\,(M^2 - z_1^2)(M^2 - z_2^2) \ . \tag{33}$$

In Sec. VI the supersymmetric properties of the dilaton black holes will be studied and it will be shown that supersymmetry leads to the positivity bound (1), which implies that

$$\begin{aligned} M &\geq |z_1| \ , \\ M &\geq |z_2| \ . \end{aligned} \tag{34}$$

Either of these inequalities can be saturated only if at least $N = 1$ supersymmetry is unbroken, see Sec. VI. In this case $r_0$ vanishes and we deal with extreme black holes.

Equation (34) implies that the parameters of the dilaton black hole can vary only inside the square:

$$\frac{|Q| + |P|}{\sqrt{2}} \equiv M_{\rm extr} \leq M \ . \tag{35}$$

It is instructive to consider various special cases of the dilaton black hole (23) for a given mass $M$, see Fig. 1.

(1) The Schwarzschild solution is given by Eqs. (23) at $P = Q = \Sigma = \phi_0 = 0$, $r_+ = 2M$, $r_- = 0$. This solution corresponds to the point at the center of coordinates in Fig. 1.

(2) Classical Reissner-Nordström black hole with equal electric and magnetic charges. $|P| = |Q|$, $\Sigma = \phi_0 = 0$. This solution corresponds to the lines crossing the center of coordinates which are parallel to the boundaries of the square in Fig. 1.

(3) Purely magnetic dilaton black hole described in [7]. $Q = 0$, $-z_1 = z_2 = P/\sqrt{2}$, $\Sigma = P^2/2M$, and $r_0 = M - \Sigma$, $r_- = \Sigma$, $r_+ = 2M - \Sigma$. By performing the change of variables $r' = r + \Sigma$, we recover the metric as given in [7]. This solution corresponds to the $P$ axis in Fig. 1.

(4) Purely electric dilaton black hole described in [8]. Change $Q$ to $P$, $\Sigma$ to $-\Sigma$ in the previous case. The solution corresponds to the $Q$ axis in Fig. 1.

(5) Extreme black holes with electric and magnetic charges. $M = (|Q| + |P|)/(\sqrt{2})$, $\Sigma = (|P| - |Q|)/(\sqrt{2})$, $M > |\Sigma|$ $r_0 = 0$, $r_+ = r_- = M$. For $PQ > 0$, $M = |z_2| > |z_1|$, while for $PQ < 0$, $M = |z_1| > |z_2|$. These solutions will be discussed later. In Fig. 1 they correspond to the boundary of the square excluding the four vertices.

(6) Extreme black holes with either electric or magnetic charge. $r_+ = r_- = M$, $r_0 = 0$, $M = |z_1| = |z_2| = |\Sigma|$. In Fig. 1 these solutions correspond to the four vertices of the square and are the stringy extreme charged electric or magnetic dilaton black holes of [7, 8].

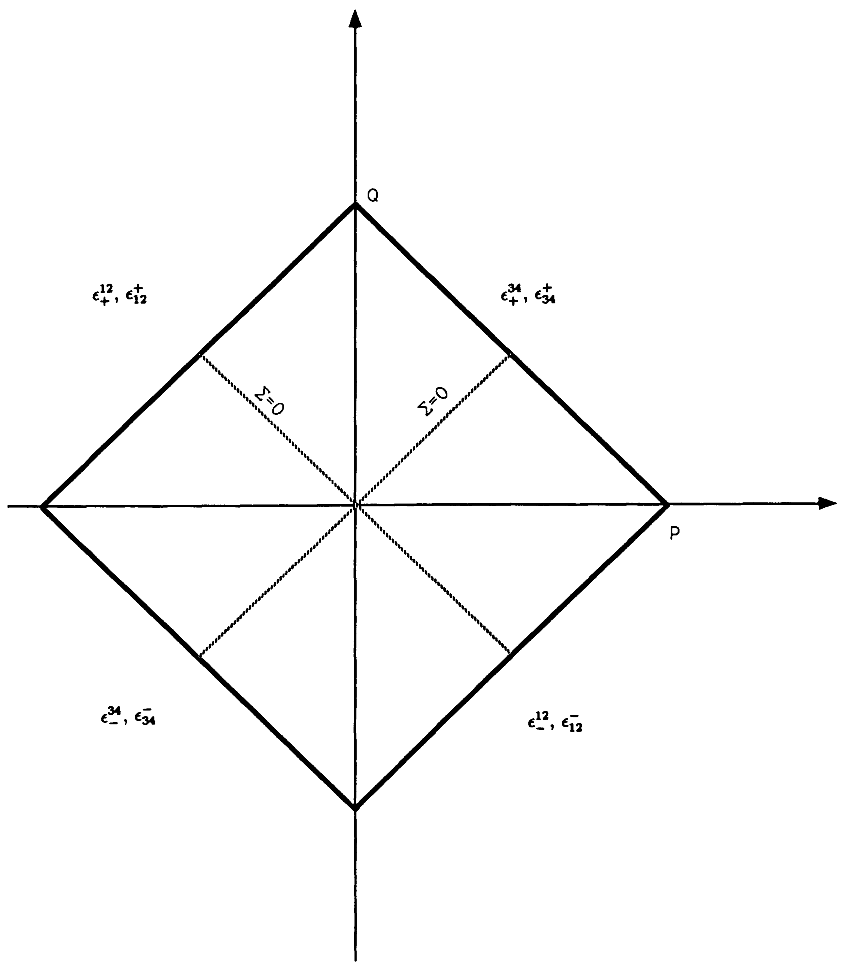


FIG. 1. The space of electrically and magnetically charged dilaton black holes with charges $P$ and $Q$ and a fixed mass $M$. $Q_{\max} = P_{\max} = \sqrt{2}M$. Every point inside the square corresponds to a regular black hole. The points outside the square (which are forbidden by supersymmetry) correspond to metrics with naked singularities. The points *on* the square correspond to extreme black holes. The unbroken $N = 1$ supersymmetries for the extreme black holes on each of the four sides (I, II, III, IV) of the square are shown. In the corners, we have unbroken $N = 2$ supersymmetry.

There are many ways to generalize the black-hole solutions which we have presented above. We considered a simple solution depending on two charges $P$ and $Q$; i.e., we assumed that each of the fields $F$ and $G$ has either electric or magnetic charge, but not both. However, these solutions can be easily generalized to solutions in which both fields $F$ and $G$ have electric and magnetic charges. The general form of these solutions is the same as that of our solutions, with $P_F^2 + P_G^2$ replacing $P^2$ in all our equations, and similarly for $Q$. (Only products of two $F$'s or two $G$'s appear in the equations of motion.) These solutions are consistent with a constant axion if $Q_F P_F + Q_G P_G = 0$. Thus, instead of solutions characterized by two parameters $P$ and $Q$, we essentially have a set of solutions depending on three independent parameters. All the properties that hold for the solutions we have studied and depend only on the metric, remain true for the new set of solutions.

## IV. EXTREME DILATON BLACK HOLES

We will look for a static solution of equations of motions of the theory (14), not necessarily spherically symmetric, with the following ansatz for the metric in isotropic coordinates (conformastatic metric):

$$ds^2 = e^{2U} dt^2 - e^{-2U} (dx^i)^2 \ . \tag{36}$$

The nonzero components of the vierbein ${e_\mu}^a$ are

$$e_{\hat{0}}{}^0 = e^U \ , \qquad e_{\hat{i}}{}^j = e^{-U} \delta_i{}^j \ , \tag{37}$$

where $g_{\mu\nu} = {e_\mu}^a {e_\nu}^b \eta_{ab}$, and the function $U$ is time independent. For the determinant of the metric we have $\sqrt{-g} = e^{-2U}$. The nonzero elements of the spin connection $\omega_\mu^{ab}$ and Ricci tensor $R_{\mu\nu}$ are given by

$$\begin{aligned} \omega_0^{i0} &= e^{2U}\partial_i U \ , \\ \omega_i^{jk} &= 2\delta_{i[j}\partial_{k]}U \ , \\ R_{00} &= -e^{4U}\partial_i\partial_i U \ , \\ R_{ij} &= 2\partial_i U\cdot\partial_j U - \delta_{ij}\partial_k\partial_k U \ . \end{aligned} \tag{38}$$

We look for solutions where $A_\mu$ is electric and $B_\mu$ is magnetic (or $\tilde{B}_\mu$ is again electric):

$$A_\mu = \delta^0_\mu \psi \ , \qquad \tilde{B}_\mu = \delta^0_\mu \chi \ . \tag{39}$$

For $G_{\mu\nu}$ this implies that it has only spacelike components given by

$$\tfrac{1}{2}\epsilon^{ijk}G_{jk} = -e^{2\phi-2U}\partial_i\chi \ . \tag{40}$$

Then the field equations (16) in this metric are

$$\begin{aligned} \partial_i e^{-2U-2\phi}\partial_i\psi &= 0 \ , \\ \partial_i e^{-2U+2\phi}\partial_i\chi &= 0 \ , \\ -\partial_i\partial_i\phi + e^{-2U-2\phi}(\partial_i\psi)^2 - e^{-2U+2\phi}(\partial_i\chi)^2 &= 0 \ , \\ -\partial_i\partial_i U + e^{-2U-2\phi}(\partial_i\psi)^2 + e^{-2U+2\phi}(\partial_i\chi)^2 &= 0 \ , \end{aligned} \tag{41}$$

$$\partial_i U\cdot\partial_j U + \partial_i\phi\cdot\partial_j\phi$$

$$-e^{-2U-2\phi}\partial_i\psi\cdot\partial_j\psi - e^{-2U+2\phi}\partial_i\chi\cdot\partial_j\chi = 0 \ .$$

We define

$$H_1 = e^{-U-\phi} \ , \qquad H_2 = e^{-U+\phi} \ . \tag{42}$$

These equations can be solved as follows:

$$\begin{aligned} & ds^2 = e^{2U}dt^2 - e^{-2U}d\mathbf{x}^2 \ , \\ & A = \psi dt, \qquad \tilde{B} = \chi dt \ , \\ & F = d\psi\wedge dt, \qquad \tilde{G} = d\chi\wedge dt \ , \\ & e^{-2U} = H_1 H_2, \qquad e^{2\phi} = H_2/H_1 \ , \\ & \sqrt{2}\,\psi = \pm H_1^{-1}, \qquad \sqrt{2}\,\chi = \pm H_2^{-1} \ , \\ & \partial_i\partial_i H_1 = 0, \qquad \partial_i\partial_i H_2 = 0 \ . \end{aligned} \tag{43}$$

Thus, two arbitrary harmonic functions $H_1, H_2$ can be used to build the metric, dilaton, and vector fields according to Eqs. (43). Specific examples are given below.

(i) The *extreme multi-black-hole* solution is the solution of the equations given above with

$$\begin{aligned} H_1 &= e^{-\phi_0}\left(1 + \sum_{s=1}^{n}\frac{\sqrt{2}|Q_s|}{|x-x_s|}\right) \ , \\ H_2 &= e^{+\phi_0}\left(1 + \sum_{s=1}^{n}\frac{\sqrt{2}|P_s|}{|x-x_s|}\right) \ , \end{aligned} \tag{44}$$

where there is the following relation between the parameters of each black hole:

$$\begin{aligned} M_s &= \frac{|P_s| + |Q_s|}{\sqrt{2}} \ , \\ \Sigma_s &= \frac{|P_s| - |Q_s|}{\sqrt{2}}. \end{aligned} \tag{45}$$

It follows that

$$M_s^2 + \Sigma_s^2 = P_s^2 + Q_s^2 \ . \tag{46}$$

This allows a static equilibrium due to the balance of gravitational, scalar, and electromagnetic forces. The total mass and charges of the full configuration are given by

$$\begin{aligned} & M = \sum_{s=1}^{n} M_s \ , \qquad \Sigma = \sum_{s=1}^{n}\Sigma_s \ , \\ & P = \pm\sum_{s=1}^{n}|P_s| \ , \qquad Q = \pm\sum_{s=1}^{n}|Q_s| \ , \\ & \operatorname{sgn}(P) = \operatorname{sgn}(P_s) \ , \qquad \operatorname{sgn}(Q) = \operatorname{sgn}(Q_s) \ . \end{aligned} \tag{47}$$

They also satisfy the condition

$$M^2 + \Sigma^2 = P^2 + Q^2 \ . \tag{48}$$

To see the force balance explicitly, let us consider Newtonian, Coulomb, and dilatonic forces. The force between two distant objects of masses and charges $(M_1, Q_1, P_1, \Sigma_1)$ and $(M_2, Q_2, P_2, \Sigma_2)$ is

$$F_{12} = -\frac{M_1 M_2}{r_{12}^2} + \frac{Q_1 Q_2}{r_{12}^2} + \frac{P_1 P_2}{r_{12}^2} - \frac{\Sigma_1\Sigma_2}{r_{12}^2} \ . \tag{49}$$

The dilatonic force is attractive for charges of the same sign and repulsive for charges of opposite sign. Using the relations (45) for the masses and dilaton charges in terms of the magnetic and electric charges, we see that $F_{12}$ vanishes. In particular, it follows that a purely magnetic and a purely electric extreme black hole can be in equilibrium, as the attractive gravitational force is balanced by the repulsive dilatonic force.

The extreme electrically (or magnetically) charged multi-black-hole solutions are solutions of the type (44) with $P_s = 0$ (or $Q_s = 0$). They can be formally identified with a special case of a metric of Bonnor [10] describing charged dust in equilibrium.[8] The corresponding equations are

$$\begin{aligned} \nabla_\mu F^{\mu\nu} &= J^\nu \ , \\ R_{\mu\nu} - (2F_{\mu\lambda}F_{\nu\delta}g^{\lambda\delta} - \tfrac{1}{2}g_{\mu\nu}F^2) &= T^{\mu\nu} \ , \end{aligned} \tag{50}$$

where

$$\begin{aligned} T^{\mu\nu} &= \varepsilon u^\mu u^\nu \ , \\ J^\mu &= \sigma u^\mu \ , \end{aligned} \tag{51}$$

and $u^\mu$ is the four velocity of dust with normalization $g_{\mu\nu}u^\mu u^\nu = 1$. The charged dust in equilibrium is characterized, according to Bonnor [10], by the condition $\varepsilon = \pm\sigma$. We have found that if the density of charged dust in equilibrium is

$$\varepsilon(x) = \pm\sigma(x) = -2(\nabla\phi)^2(x) \ , \tag{52}$$

or, equivalently, the trace of the energy-momentum ten-

[8]The relevance of Bonnor metrics with charged dust in equilibrium to metrics admitting supercovariant constant spinors in the context of $N = 2$ supergravity was discovered by Tod [22].

sor of dust is proportional to the scalar curvature of the space due to the presence of the dilaton,

$$T = R = -2(\nabla\phi)^2 \ , \tag{53}$$

then the Bonnor solution of the system of Eqs. (50)–(52) coincides with the set of extreme electrically (or magnetically, after duality transformation) charged dilaton black holes. The reason behind the formal identification of extreme dilaton black holes with charged dust is the following: The energy-momentum tensor in our Eqs. (16) and (17) is covariant; however, on solutions it coincides with the noncovariant energy momentum tensor of the charged dust in the Bonnor equation (50).

(ii) As a specific example of extreme black holes, let us now consider an extreme electrically and magnetically charged *spherically symmetric* black hole with dilaton field not vanishing at infinity:

$$ds^2 = \left(1 + \frac{\sqrt{2}(|Q|+|P|)}{\rho} + \frac{2|PQ|}{\rho^2}\right)^{-1} \times dt^2 - \left(1 + \frac{\sqrt{2}(|Q|+|P|)}{\rho} + \frac{2|PQ|}{\rho^2}\right) d\mathbf{x}^2 \ , \tag{54}$$

where $\rho = |\mathbf{x}|$ and

$$e^{2\phi} = e^{2\phi_0}\left(\frac{\rho + \sqrt{2}|P|}{\rho + \sqrt{2}|Q|}\right) ; \tag{55}$$

i.e., we choose

$$H_1 = e^{-\phi_0}\left(1 + \frac{\sqrt{2}|Q|}{\rho}\right) , \qquad H_2 = e^{\phi_0}\left(1 + \frac{\sqrt{2}|P|}{\rho}\right) . \tag{56}$$

This solution can be compared with the one described in the previous section (case 5 in the list) under a suitable change of variables:

$$r = \rho + M \ . \tag{57}$$

The electrically and magnetically charged spherically symmetric dilaton black hole (23) with

$$r_+ = r_- = M \ , \qquad r_0 = 0 \ , \tag{58}$$

is the extreme dilaton black hole. This is in accordance with Fig. 1 where the boundary of the square (excluding the vertices) corresponds to the extreme black hole with both electric and magnetic charge. The mass and charges of the extreme dilaton black hole with nonvanishing electric and magnetic charge satisfy the bounds $M = \frac{|P|+|Q|}{\sqrt{2}}$ , $M > \Sigma = \frac{|P|-|Q|}{\sqrt{2}}$ . Thus for the generic extreme black hole with electric and magnetic charges only one of the positivity bounds (34) required for cosmic censorship is saturated. For $PQ > 0$ (sides I and III of the square in Fig. 1) $M = |z_2|$ and the second one is still a positivity bound since $M > |\Sigma| = |z_1|$. For sides II and IV with $PQ < 0$ we have $M = |z_1|$, and $M > |\Sigma| = |z_2|$. Thus, all over the boundary of the square in Fig. 1, except for the vertices, the absolute value of the dilaton charge is smaller than the mass. This property of the extreme black hole with both electric and magnetic charge means that the singularity $r = |\Sigma|$ is inside the horizon $r_+ = r_- = M$. The purely magnetic (electric) extreme black holes are the solutions given in Eq. (56) with $Q = 0$ ($P = 0$). These solutions in Fig. 1 correspond to the four vertices of the square. For these solutions $M = |z_1| = |z_2| = |\Sigma|$. For purely magnetic extreme black holes, the singularity at $r = |\Sigma| = M$ coincides with the horizon $r_+ = r_- = M$. It is argued, however, that for the metric which the string sees, $ds^2_{\rm str} = e^{2\phi} ds^2$, the horizon moves infinitely far away and the curvature tensor becomes nonsingular [7]. For purely electric extreme black holes this kind of argument is absent.

It is important that for all these solutions the positivity bounds (34) imply that the singularity $r = |\Sigma|$ is either inside the horizon $r_+$ or coincides with it. One can easily check that $r_+$ is, indeed, an event horizon, i.e., the integral in (6) diverges for $r_1 = r_+$. This happens independently of our choice of normal metric versus stringy one. This means that, in agreement with the cosmic censorship conjecture, supersymmetry saves an outside observer from seeing the singularity.

## V. THERMAL PROPERTIES OF THE DILATON BLACK HOLE

The explicit expression for the metric of the charged dilaton black hole allows us to calculate its thermal properties. A detailed analysis of such properties has been performed in [7, 8] for the electric or magnetic stringy black holes (the solutions on the $Q$ and $P$ axes of Fig. 1) or for nonstringy black holes with $e^{-2a\phi}$, $a \neq 1$ in the action. Discussion of some of the thermal properties and singularities of electrically and magnetically charged black holes can be found in [6].

In calculating the temperature and entropy of black holes, we must keep in mind that the interpretation of the results as *physical* temperature and entropy may not be reliable in some limits. In fact, the thermal description of purely electric or magnetic dilaton black holes breaks down near extremality [8]. At the end of this section we will analyze the breakdown of the thermal description for our class of charged dilaton black holes. However, purely geometric quantities such as the area $A$ and the surface gravity $\kappa$ of the black-hole horizon always make sense and the "thermodynamic" relationship between them will be seen to hold in any case when a black hole has a nonsingular horizon.

The Hawking temperature of the black hole (23) can be calculated by a variety of standard methods. In terms of the surface gravity $\kappa$, it is given by $T = \frac{\kappa}{2\pi}$. The surface gravity can be calculated from the Killing vector $\zeta^\mu$, which for any static metric

$$ds^2 = g_{tt}(x)\, dt^2 - h_{ij}(x)\, dx^i dx^j \tag{59}$$

is simply

$$\zeta_\mu = \delta_{\mu t}\, g_{tt} \ . \tag{60}$$

Thus, the surface gravity is

$$\kappa = \left[-\tfrac{1}{2}(\nabla^\mu\zeta^\nu)(\nabla_\mu\zeta_\nu)\right]^{\frac{1}{2}}_{r=r_{\text{horizon}}} = \frac{1}{2}\left(\frac{dg_{tt}}{dr}\right)_{r=r_+} = \frac{1}{2}\,\frac{r_+-r_-}{r_+^2-\Sigma^2}\,, \quad (61)$$

where again

$$\Sigma = \frac{P^2-Q^2}{2M}\,, \qquad r_\pm = M \pm \sqrt{M^2+\Sigma^2-P^2-Q^2}\,. \quad (62)$$

Then the temperature of our black hole (23) is given by

$$T = \frac{1}{4\pi}\,\frac{r_+-r_-}{r_+^2-\Sigma^2}\,. \quad (63)$$

The isothermals are drawn on the $P, Q$ plane in Fig. 2 for a fixed value of the mass $M$. From Eq. (63) a fixed-mass surface $T = T(z_1, z_2)$ is plotted in Fig. 3. The temperature falls very sharply to zero temperature near extremality at the borders of the regular black holes. In Fig. 4, the value of the temperature as a function of the mass is plotted for different values of $P$ and $Q$. By inspection of the figures, we can see the following: *Extreme black holes with both electric and magnetic charges have zero temperature. At the corners there is a discontinuity.* Consider as an example the purely magnetic extreme dilaton black hole. We may either first take the limit $Q \to 0$ in our expression for the temperature in Eq. (63) and after that take the limit to extreme $(r_+ - r_-) \to 0$, or vice versa; the limiting temperature depends on which choice we make:

$$\lim_{(r_+-r_-)\to 0}\ \lim_{Q\to 0} T(P,Q,M) = \frac{1}{8\pi M}\,, \quad (64)$$

$$\lim_{Q\to 0}\ \lim_{(r_+-r_-)\to 0} T(P,Q,M) = 0\,. \quad (65)$$

We can see this also from Figs. 3 and 4. Note that different results can be obtained by calculating the limit along different isothermals shown in Fig. 2. The limit can be anywhere between 0 and $\frac{1}{8\pi M}$. The fact that the temperature at the corners is not well defined explains why there are apparently contradictory statements about it

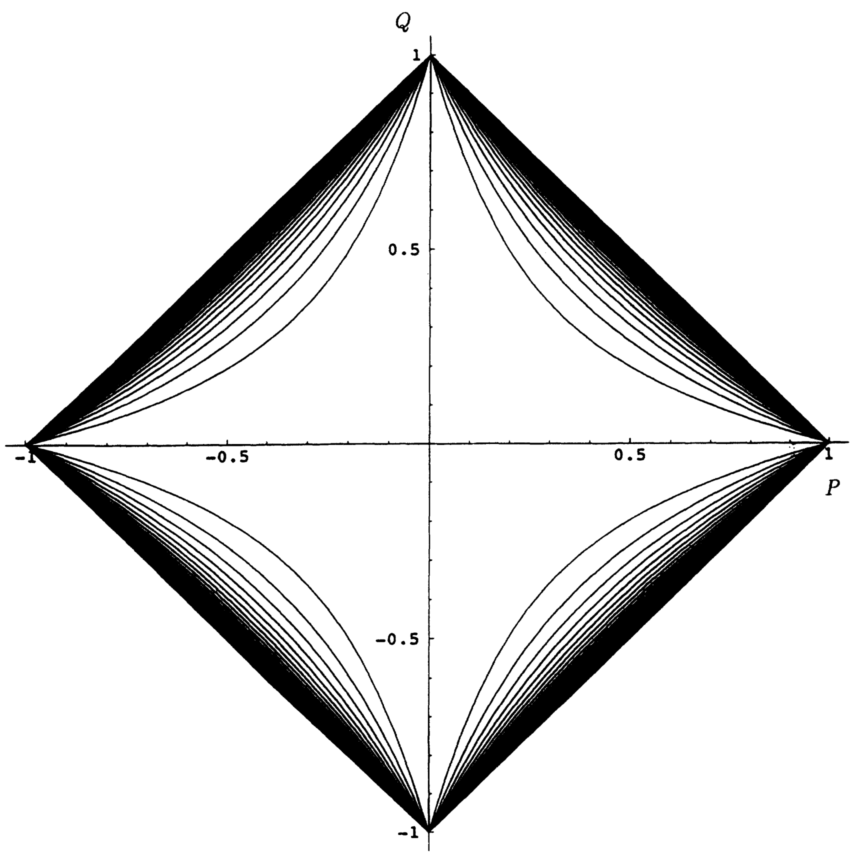


FIG. 2. Isothermals in the space of charged dilaton black holes of constant mass. The interval of temperature between two contiguous isothermals is $\frac{1}{50}T_{\max}$, where $T_{\max} = \frac{1}{8\pi M}$ is a temperature of the Schwarzschild black hole with a mass $M$. The two axes of coordinates are isothermals corresponding to $T = \frac{1}{8\pi M}$. The four sides of the square (excluding the corners) are isothermals corresponding to $T = 0$. The corners are very special: All the isothermals (for all the different allowed temperatures) converge to the corners. This can be better seen in Fig. 3.

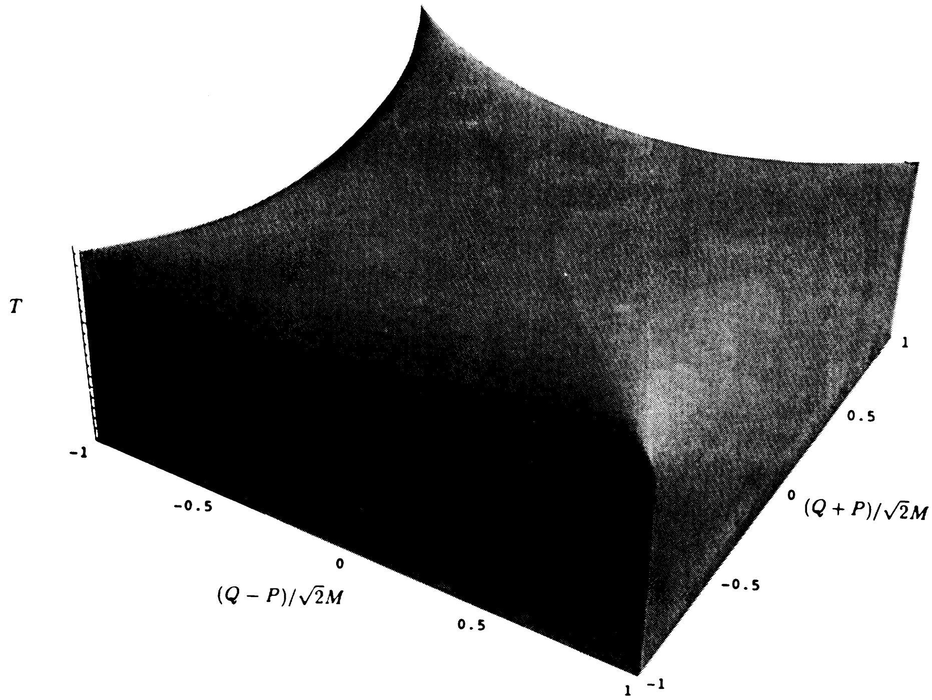


FIG. 3. The temperature of charged dilaton black holes of a given mass $M$ as a function of $z_1/M = (Q-P)/\sqrt{2}M$ and $z_2/M = (Q+P)/\sqrt{2}M$. The extreme black holes correspond to the sides $|z_1|/M = 1$ and $|z_2|/M = 1$ of the square.

in the literature [7, 8].[9]

To investigate this problem further, let us examine carefully the limits of applicability of the thermal description of black holes. There are several conditions which must be satisfied. One of them was obtained and extensively discussed in Ref. [8]. A thermal description of a system is possible only if, after an emission of a single quantum of a typical energy $T$, the temperature of the system changes by $|\Delta T| \ll T$. Applying this to a black hole of a mass $M$ gives, according to [8],

$$\left|\left(\frac{\partial T}{\partial M}\right)_{P,Q}\right| \ll 1 \,. \tag{66}$$

This inequality may be rewritten as

$$T\left(\frac{\partial S}{\partial T}\right)_{P,Q} \gg 1 \,. \tag{67}$$

According to [8], this condition has a profound physical interpretation; it says that a thermal description is possible when the *available entropy* of the black hole, i.e., the number of states available within its thermal-energy interval, is very large.

The general belief expressed in Ref. [8] was that the thermal description breaks down for extreme black holes. However, there remained some confusion, since the criteria (66) and (67), applied to purely electric (or magnetic) black holes, did not show any signal of danger even arbitrarily close to the extreme point, when $M \to Q/\sqrt{2}$ (or $P/\sqrt{2}$) [8]. Our dilaton black holes correspond to the case $a = 1$ in terminology of Ref. [8], where this case was labeled "enigmatic." To clarify what is going on, let us go back to the derivation of (66).

When a black hole emits a particle of typical energy $T$ (from the point of view of a static observer at infinity), its mass decreases, $M \to (M - T)$. Its temperature $T(M)$ becomes $T(M - T)$, i.e., $T$ as a function of $(M - T)$. The condition $\Delta T \ll T$ can be written as

$$|T(M-T) - T(M)| \ll T \,. \tag{68}$$

If (and only if) the function $T(M)$ has a derivative which remains almost constant in the interval between $M$ and $M - T$, Eq. (68) can be rewritten in the form $T\,|\partial T/\partial M| \ll T$, which is equivalent to (66). In most theories studied in [8] all the conditions necessary for the derivation of (66) are satisfied. However, for purely electric (or magnetic) dilaton black holes, we see violation when the mass of the black hole approaches its extreme value $M_{\rm extr} = \frac{|P|+|Q|}{\sqrt{2}}$. For example, (68) is violated near extremality, where any emission of a quantum of typical energy $T \approx \frac{1}{8\pi M_{\rm extr}}$ would reduce the mass below the lower bound coming from supersymmetry, and this is absolutely forbidden. Thus, failure of the thermal description occurs for $PQ = 0$ holes when

[9] One possible indicator as to which temperature we should take is provided by supersymmetry, which is usually related to zero rather than finite temperature. We will show in Sec. VI that the extreme purely magnetic or electric solutions possess two out of four possible supersymmetries, so one may prefer to take $T = 0$.

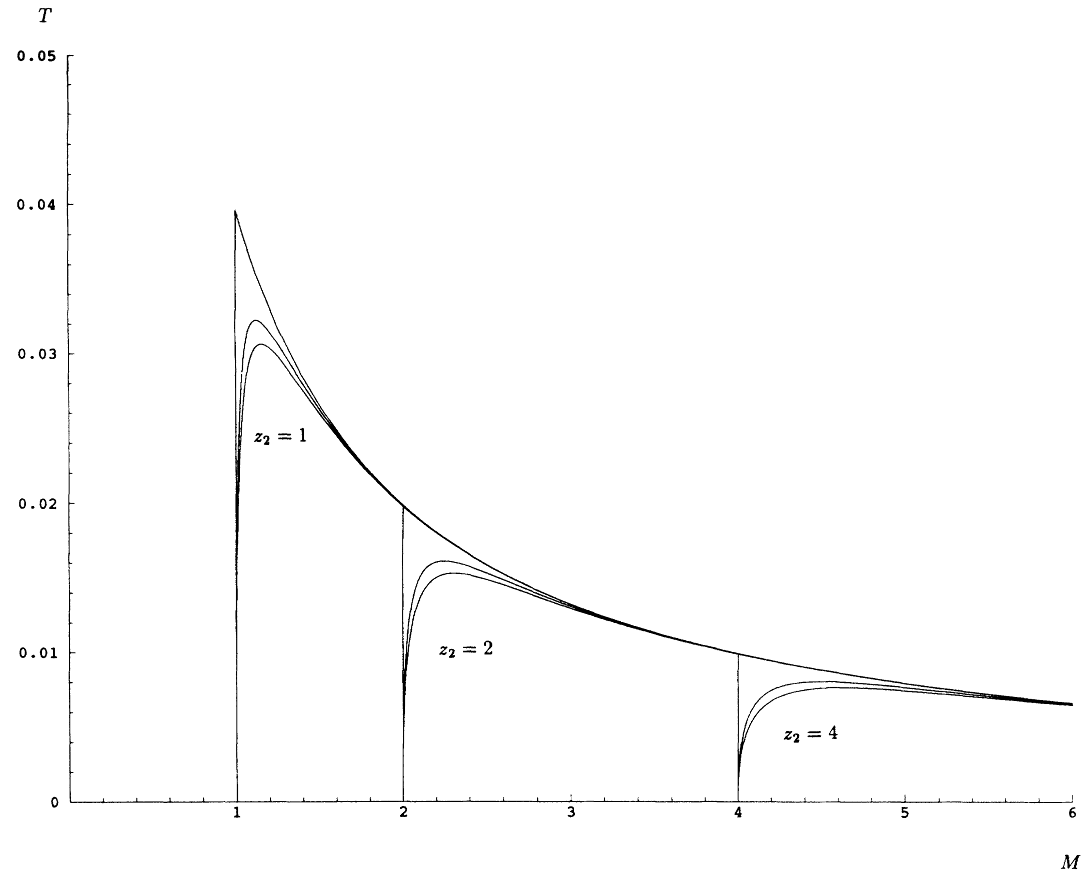


FIG. 4. The temperature vs the mass for different electric and magnetic charges. For definiteness, we take $Q > P > 0$. The black hole evaporates until its mass approaches the limiting value $M_{\rm extr} = z_2 = (Q + P)/\sqrt{2}$. The three families of curves correspond to $z_2 = 1$, $z_2 = 2$, and $z_2 = 4$. For each of these values of $z_2 = (Q + P)/\sqrt{2}$ we choose three different $P/Q$ ratios: $P/Q = 1, \frac{1}{4}, 0$. The smoothest curves are the ones with $P = Q$ (classical Reissner-Nordström). The sharpest correspond to the limit $P/Q \to 0$, which reproduces purely electric dilaton black holes. There is always a maximum for the temperature (a point where the specific heat diverges and reverses sign), and always the temperature falls sharply to zero in the vicinity of the bound. This implies the breakdown of the thermal description when we approach extremality *for all values of P and Q*.

$$\Delta M = M - M_{\rm extr} \geq \frac{1}{8\pi M_{\rm extr}} \,. \tag{69}$$

To obtain further insight, let us approach this question in a more general context, when both $Q$ and $P$ do not vanish. Assume, for example, that $Q > P > 0$. In this case $z_2 > z_1 > 0$, and the black hole becomes extreme when its mass decreases down to $M_{\rm extr} = z_2 = (Q + P)/\sqrt{2}$. One can easily verify by using Eq. (63), that when the mass of the black hole approaches $z_2$, i.e., when $\Delta M = (M - M_{\rm extr}) \to 0$, the temperature of the black hole vanishes as $\sqrt{\Delta M}$:

$$T \sim \frac{1}{2\pi}\left(\frac{2\,\Delta M}{z_2(z_2^2 - z_1^2)}\right)^{1/2} \,. \tag{70}$$

Therefore, the expression for $\left(\frac{\partial T}{\partial M}\right)_{P,Q}$ diverges in this limit,

$$\begin{aligned}\left(\frac{\partial T}{\partial M}\right)_{P,Q} &\sim \frac{1}{2\pi}\,\frac{1}{\sqrt{\Delta M}\sqrt{2\,z_2(z_2^2 - z_1^2)}}\\ &\sim \frac{1}{2\pi Q}\,\frac{1}{\sqrt{\Delta M}\sqrt{2\sqrt{2}\;P}} \,.\end{aligned} \tag{71}$$

This result is illustrated by Fig. 4. Equation (71) implies that the condition (66) is always violated when the black hole approaches its extreme limit. It is just more difficult to see it working in the limit $P = 0$. We see, in particular, that in the limit $P \to 0$ the thermal description breaks down along the whole slope from $T_{\rm extr} \sim 1/8\pi M$ to $T = 0$. Thus, the discrepancy between different ways of calculating the temperature of extreme electric (or magnetic) black holes is just one manifestation of the breakdown of the thermal description in this limit.

However, from our previous arguments it follows that thermal description breaks down even earlier. Indeed, one can easily check that for small $P$ the assumptions used in the derivation of Eq. (66) break down, and that the thermal description becomes inapplicable even before the temperature reaches its maximum, just as in the case $P = 0$, see Eq. (69).

Additional information can be obtained by studying the behavior of the entropy. The expression in Eq. (67) can be interpreted as the available entropy only if the temperature of the extreme black hole is equal to zero. Indeed, only in this case $\Delta T \equiv T - T_{\rm extr} = T$ and one may write that $\Delta S = (\partial S/\partial T)\, T$. As we noted already, the value of $T$ for extreme purely electric (or magnetic) black holes is ambiguous, due to the failure of thermal description of extreme black holes. Fortunately, the entropy can be calculated by several other methods, and the results do not depend on this ambiguity.

The entropy of a black hole can most easily be calculated as one-fourth of the area of the horizon $A$. The physical radial coordinate is $R$, so that the area of a sphere of radius $R$ is simply $4\pi R^2$. This gives

$$S = \pi R^2|_{r=\Sigma}^{r=r_+} = \pi(r_+^2 - \Sigma^2) \ . \tag{72}$$

The thermodynamic relation $T = \left(\frac{\partial S}{\partial M}\right)^{-1}_{P,Q}$ may be readily checked to be obeyed. There is also a nice relation between the temperature and the entropy of the charged dilaton black hole:

$$ST = \tfrac{1}{4}(r_+ - r_-) \ . \tag{73}$$

One can check that Eq. (72) correctly describes all the particular cases listed in the end of Sec. III, for which the entropy was already known. For example, it is easy to see from (72) that the entropy unambiguously vanishes at the corners of our square, when the horizon $r_+$ coincides with the value of the dilaton charge $|\Sigma|$. However, on the sides of the square (for $PQ \neq 0$ extremal black holes) the entropy does not vanish. Its value is

$$S_{\rm extr} = \pi(M^2 - \Sigma^2) = 2\pi|PQ| = \pi\left|z_1^2 - z_2^2\right| \ . \tag{74}$$

All these properties can be seen in Fig. 5, where the surface $S = S(z_1, z_2)$ (for fixed mass $M$) is plotted. Another example of extreme dilaton black holes with nonzero entropy has recently been studied by Horne and Horowitz [23]. The angular momentum $J$ plays there the same role as the mixing of electric and magnetic charge plays here.

To calculate the *available* entropy $S_a$, one should subtract $S_{\rm extr}$ from $S$:

$$S_a = S - S_{\rm extr} = \pi(r_+^2 - \Sigma^2) - 2\pi|PQ| \ . \tag{75}$$

This quantity becomes much larger than 1 only far away from the extreme regime, as we can see by considering two particular cases.

(i) If the black hole has only electric charge $Q > 0$, then we obtain, for the regime $\Delta M \equiv M - M_{\rm extr} \ll M_{\rm extr}$,

$$S = S_a = 8\pi\ \Delta M\, M_{\rm extr} \ , \tag{76}$$

while, for $\Delta M \gg M_{\rm extr}$,

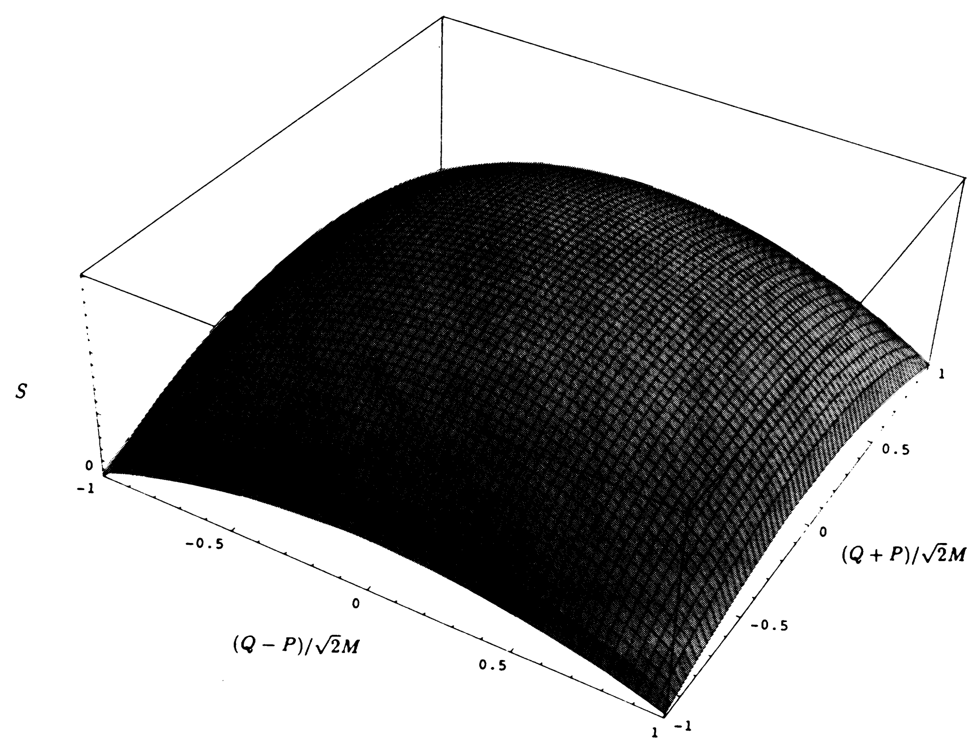


FIG. 5. The entropy $S$ of the charged dilaton black holes as a function of $z_1/M$ and $z_2/M$. It has a maximum for the Schwarzschild black hole, which corresponds to the origin of coordinates ($P = Q = \Sigma = 0$). For purely electric or magnetic extreme dilaton black holes (in the corners) it is zero. On the sides of the square the temperature vanishes, but the total entropy (Euclidean action) remains nonzero, $S = 2\pi\,|PQ|$.

$$S = S_a = 4\pi \left(M^2 - \tfrac{3}{4}M^2_{\rm extr}\right) . \qquad (77)$$

(ii) If the black hole has $Q = P > 0$, then in the case $\Delta M \ll M_{\rm extr}$ we have

$$S_a = 2\sqrt{2}\pi\sqrt{\Delta M}\, M^{3/2}_{\rm extr} . \qquad (78)$$

In the opposite limit, $\Delta M \gg M_{\rm extr}$,

$$S = S_a = 4\pi \left(M^2 - \tfrac{1}{2}M^2_{\rm extr}\right) . \qquad (79)$$

The thermal description is valid only if $S_a \gg 1$. Note that the entropy $S = S_a$ of the purely electric black hole is always *smaller* than the entropy of the black hole with both electric and magnetic charge corresponding to the same $M_{\rm extr}$; i.e., breakdown of the thermal description occurs earlier during the evaporation process for $PQ = 0$ holes. This result is rather surprising, since the (uncritical) use of Eq. (66) would have led us to the opposite conclusion.

To get a more quantitative estimate, let us assume that the thermal description is applicable when $S_a > N$, where $N$ is some large constant, $N \gg 1$. Let us assume also that the thermal description breaks down (with a decrease of $\Delta M$) in the domain where $\Delta M \ll M_{\rm extr}$. This is possible only if $8\pi M^2_{\rm extr} \gg N$. In the purely electric case the thermal description breaks down at

$$\Delta M < \frac{N}{8\pi M_{\rm extr}} . \qquad (80)$$

This condition is in an agreement with our earlier estimate (69), but is even stronger. For large $M_{\rm extr}$, the temperature of the black hole at that time is very close to $(8\pi M_{\rm extr})^{-1}$, but it never reaches this limit in the region where a thermal description is possible.

In the case $P = Q$ the thermal description breaks down later, at

$$\Delta M < \frac{N^2}{8\pi^2 M^3_{\rm extr}} . \qquad (81)$$

At this stage the temperature of the black hole is given by

$$T \sim \frac{1}{4\pi^2}\,\frac{N}{M^3_{\rm extr}} . \qquad (82)$$

The larger the body, the smoother the function $T(M)$, and the smaller the temperatures at which thermodynamics describes successfully the black hole near its extremal state.

In Fig. 4 we also see that the temperature has a maximum for every combination of the charges. $\left(\frac{\partial T}{\partial M}\right)_{P,Q}$ vanishes there, has different sign at both sides, and goes to zero when we approach extremality. This means that the specific heat blows up at the temperature maximum, changes sign there and goes smoothly to zero when we approach extremality. A direct calculation gives

$$C^{-1} = \left(\frac{\partial T}{\partial M}\right)_{P,Q} = \frac{T}{M}\frac{1}{r_0^2}\left[M^2 - \Sigma^2 - 2Mr_0\right] . \qquad (83)$$

In Fig. 6 the specific heat $C$ is plotted in terms of $M/M_{\rm extr}$ for the classical Reissner-Nordström family of black holes ($P = Q$). Note that the specific heat is positive near the extreme value of $M$ for all black holes with $PQ \neq 0$. Thus, as distinct from ordinary Schwarzschild black holes, the charged dilaton black holes with $PQ \neq 0$ (as well as the Reissner-Nordström ones) can be in a state of stable thermal equilibrium with "hot" matter with a temperature smaller than $T_{\rm max} \sim \frac{1}{8\pi M_{\rm extr}}$.

The values of $\mathcal{P} = P/\sqrt{2}M$ and $\mathcal{Q} = Q/\sqrt{2}M$ for which $C^{-1}$ vanishes obey the equation

$$[\mathcal{P} - \mathcal{Q}]^4 - 6[\mathcal{P} - \mathcal{Q}]^2 + 8[\mathcal{P} + \mathcal{Q}] - 3 = 0 . \qquad (84)$$

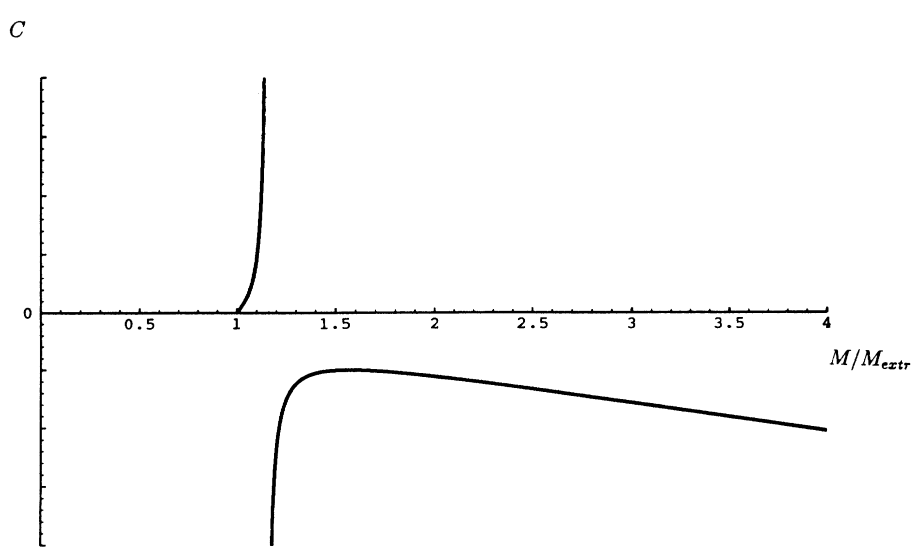


FIG. 6. The specific heat of a Reissner-Nordström black hole with $P = Q > 0$ as a function of $M/M_{\rm extr} = \sqrt{2}M/(Q + P)$.

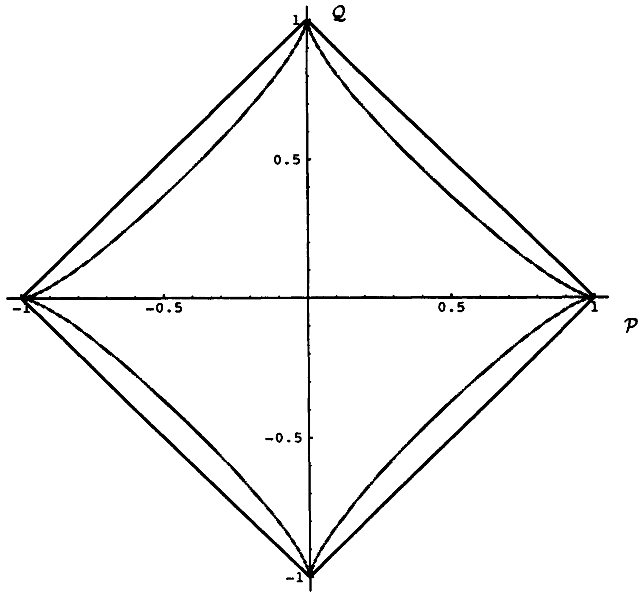


FIG. 7. The locus of points in the $(\mathcal{P},\mathcal{Q})$ plane where the specific heat of a fixed-mass charged dilaton black hole diverges. Inside the curve, the specific heat is negative; outside, it is positive.

In the $(\mathcal{P},\mathcal{Q})$ plane, the line of the zeros of (83) divides the square into two regions of different sign of $C$. This line is plotted in Fig. 7. There is also a line of zeroes of $C$ in the $(J,Q)$ plane in the case of Kerr-Newman black holes. The fact that our black holes have both electric and magnetic charges is analogous to the fact that Kerr-Newman black holes have angular momentum.

It is clearly very important to understand the intrinsic properties of extreme black holes as candidates for the final state after the process of evaporation. The extremal limit $(r_+ - r_-) \to 0$ is also the limit in which the supersymmetry is restored since $r_0 = (r_+ - r_-)/2$ will be shown to be the parameter of supersymmetry breaking. The understanding of the supersymmetric properties of extreme black holes may shed new light on the final stages of the evaporation of charged black holes.

## VI. SUPERSYMMETRIC PROPERTIES OF DILATON BLACK HOLES

First we will prove that nonextreme black holes always break supersymmetry. After that we will consider extreme solutions and find unbroken supersymmetries.

In a bosonic background, the unbroken supersymmetries are determined by the terms depending on the bosons in the transformation laws of the fermions. Deleting there again the axion field, the relevant transformation laws of the $N=4$ theory are[10] [19, 20, 24] (in chiral notation, see Appendix A)

[10]We use the SO(4) version for convenience, but the duality transformation (18) can be used to translate everything to the SU(4) version.

$$
\begin{aligned}
\tfrac{1}{2}\delta\Psi_{\mu I} &= \nabla_\mu \epsilon_I - \tfrac{1}{8}\sigma^{\rho\sigma}T^{+}_{\rho\sigma,IJ}\gamma_\mu\epsilon^J \,, \\
\tfrac{1}{2}\delta\Lambda_I &= -\gamma^\mu\epsilon_I\partial_\mu\phi \\
&\quad + \frac{1}{\sqrt{2}}\sigma^{\rho\sigma}\left(e^{-\phi}F_{\rho\sigma}\alpha_{IJ} - e^{\phi}\tilde{G}_{\rho\sigma}\beta_{IJ}\right)^{-}\epsilon^J \,,
\end{aligned}
\qquad (85)
$$

where the covariant derivative contains the spin connection (see Appendix A), and would also contain a U(1) connection if the axion had been included. $T_{\mu\nu,IJ}$ is an auxiliary field. Its algebraic field equation, which was used to obtain the action (14), has put it equal to

$$
T^{+}_{\rho\sigma,IJ} = 2\sqrt{2}e^{-\phi}\left(F_{\rho\sigma}\alpha_{IJ} + e^{2\phi}\tilde{G}_{\rho\sigma}\beta_{IJ}\right)^{+} . \qquad (86)
$$

The local supersymmetry algebra of $N=4$ [24] contains the following terms which are relevant for the solutions which we consider:

$$
\begin{aligned}
[\delta_Q(\epsilon),\delta_Q(\epsilon')] = &-2\delta_{\rm GC}\left(\bar{\epsilon}'^{I}\gamma^a\epsilon_I + \text{H.c.}\right) \\
&+ \delta_{\rm Lor}(\bar{\epsilon}^I\epsilon'^J T^{+ab}_{IJ} + \text{H.c.}) + \cdots \,,
\end{aligned}
\qquad (87)
$$

where $\delta_{\rm GC}$ and $\delta_{\rm Lor}$ are the general coordinate and Lorentz transformations which act on the vierbeins in the following way:

$$
\delta_{\rm GC}(\xi)e^a_\mu = \partial_\mu\xi^a - \omega^{ab}_\mu\xi_b \,, \qquad \delta_{\rm Lor}(\Lambda)e^a_\mu = \Lambda^{ab}e_{\mu b} \,. \qquad (88)
$$

The nonvanishing value of the auxiliary field $T$ in our solution will imply that the last term of (87) produces central charges.

To simplify the analysis of supersymmetry, we will use a system of coordinates which has a conformally flat three-dimensional space. Defining

$$
r = \rho + \frac{r_0^2}{4\rho} + M \qquad (89)
$$

and $\rho^2 = (x^i)^2$, we have

$$
ds^2 = e^{2U}dt^2 - e^{-2U+2C}(dx^i)^2 \,, \qquad (90)
$$

where

$$
e^C = \frac{\partial r}{\partial\rho} = 1 - \frac{r_0^2}{4\rho^2} \,. \qquad (91)
$$

From (86) we see that the only nonzero components of $T_{\mu\nu,IJ}$ for purely electric $F$ and $\tilde{G}$ (and no axion) are

$$
T_{\hat{i}\hat{0},IJ} = 2\sqrt{2}\left(\alpha_{IJ}e^{-\phi}\partial_i\psi + \beta_{IJ}e^{\phi}\partial_i\chi\right) . \qquad (92)
$$

We now take the dilaton field going to a constant $\phi_0$ at infinity, and the electric field strengths going to zero as

$$
\begin{aligned}
\frac{x^i}{\rho}F_{\hat{i}\hat{0}} &= \partial_\rho\psi = -\frac{Q_{\rm elec}}{\rho^2} + O(\rho^{-3}) \,, \\
\frac{x^i}{\rho}\tilde{G}_{\hat{i}\hat{0}} &= \partial_\rho\chi = -\frac{P_{\rm elec}}{\rho^2} + O(\rho^{-3}) \,.
\end{aligned}
\qquad (93)
$$

Then, defining as before,

$$Q = e^{-\phi_0} Q_{\rm elec} \;, \qquad P = e^{\phi_0} P_{\rm elec} \;, \tag{94}$$

we have at large $\rho$ the behavior

$$T_{\hat\rho\hat0,IJ} \equiv \frac{x^i}{\rho} T_{\hat i\hat0,IJ} = -2\sqrt{2}\,\rho^{-2}\,(\alpha_{IJ}Q + \beta_{IJ}P) + O(\rho^{-3}) \,. \tag{95}$$

We will now take $T$ to have this asymptotic value for large $\rho$.

We choose the antisymmetric matrices $\alpha_{IJ}$ and $\beta_{IJ}$ as $\alpha^3$ and $\beta^3$ in the notation of [25]:

$$\alpha_{IJ} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} , \qquad \beta_{IJ} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} . \tag{96}$$

Thus, they are block diagonal in the pairs (1,2) and (3,4). In each of the pairs $T_{IJ}$ is proportional to $\epsilon_{IJ}$, the two-index antisymmetric symbol with $\epsilon_{12} = 1$ (or $\epsilon_{34} = 1$). So we put

$$T_{\hat\rho\hat0,IJ} = -4\,\frac{Z}{\rho^2}\,\epsilon_{IJ} + O(\rho^{-3}) \;, \tag{97}$$

and the values of $Z$ in the $(1,2)$ and the $(3,4)$ sector are, respectively,

$$z_1 = \frac{Q-P}{\sqrt{2}} \;, \qquad z_2 = \frac{Q+P}{\sqrt{2}} \;. \tag{98}$$

We see now that $Z$ is a central charge operation in the sense that

$$\begin{aligned}[\delta_Q(\epsilon), \delta_Q(\epsilon')] = & -2\delta_{\rm GC}\left(\bar\epsilon'^I\gamma^a\epsilon_I + {\rm H.c.}\right) \\ & -4\frac{Z}{\rho^2}\left[(\bar\epsilon^I\epsilon'^J\epsilon_{IJ})\delta^+_{\rm Lor} + (\bar\epsilon_I\epsilon'_J\epsilon^{IJ})\delta^-_{\rm Lor}\right] \\ & + \cdots \;, \end{aligned} \tag{99}$$

where $\delta^+_{\rm Lor}$ is a self-dual version of a Lorentz transformation, and $\delta^-_{\rm Lor}$ an anti-self-dual one:

$$\delta^\pm_{\rm Lor} = \delta_{\rm Lor}\left(\Lambda^{ab} = \frac{x^i}{2r}\left(e^{\hat i[a}e^{b]\hat 0} \pm \tfrac12\epsilon^{abcd}e_c^{\hat i}e_d^{\hat 0}\right)\right) \;. \tag{100}$$

Let us prove that *nonextreme charged dilaton black holes break all supersymmetries.*

In the supersymmetry transformation law we will use now the metric (90), and the assumption that $F_{\mu\nu}$ and $\tilde G_{\mu\nu}$ are electric solutions, i.e., that $F_{i0} = \partial_i\psi$ and $\tilde G_{i0} = \partial_i\chi$ are the only nonzero components. The covariant derivatives on spinors are

$$\nabla_0 = \partial_0 - e^{2U-C}\partial_i U\sigma_{i0} \;, \qquad \nabla_i = \partial_i - \sigma_{ij}\,\partial_j(U-C) \;, \tag{101}$$

where indices on derivatives are curved, while those on $\gamma$ matrices are flat. The resulting transformation laws of the fermions are

$$\begin{aligned} \tfrac12\delta\Lambda_I &= \gamma_i e^{-C}\left(e^U\,\partial_i\phi\cdot\epsilon_I - \frac{1}{\sqrt2}\left(\alpha_{IJ}\,e^{-\phi}\,\partial_i\psi - \beta_{IJ}\,e^{\phi}\,\partial_i\chi\right)\gamma_0\,\epsilon^J\right) \;, \\ \tfrac12\delta\Psi_{0I} &= \tfrac12\sigma_{0i}\,e^{U-C}\left(e^U\,\partial_i U\cdot\epsilon_I - \frac{1}{\sqrt2}\left(\alpha_{IJ}\,e^{-\phi}\partial_i\psi + \beta_{IJ}\,e^{\phi}\,\partial_i\chi\right)\gamma_0\epsilon^J\right) \;, \\ \tfrac12\delta\Psi_{iI} &= \partial_i\epsilon_I - \sigma_{ij}\,\partial_j(U-C)\cdot\epsilon_I - \frac{1}{2\sqrt2}\gamma_j\gamma_i\left[\alpha_{IJ}\,e^{-\phi}\,\partial_j\psi + \beta_{IJ}\,e^{\phi}\,\partial_j\chi\right]\gamma_0\,e^{-U}\epsilon^J \;. \end{aligned} \tag{102}$$

For preserved supersymmetries, the first two relations lead to

$$\begin{aligned} \epsilon_I\,\partial_i\,e^{U+\phi} &= \sqrt2\,\gamma_0\,\alpha_{IJ}\,\epsilon^J\,\partial_i\psi \;, \\ \epsilon_I\,\partial_i\,e^{U-\phi} &= \sqrt2\,\gamma_0\,\beta_{IJ}\,\epsilon^J\,\partial_i\chi \;, \end{aligned} \tag{103}$$

while the remaining equation $\delta\Psi_{iI} = 0$ reduces to

$$\partial_i\epsilon_I - \tfrac12\epsilon_I\,\partial_i U + \sigma_{ij}\,\epsilon_I\,\partial_j C = 0 \;. \tag{104}$$

Acting with $\partial_j$ on this equation and antisymmetrizing with respect to $ij$ (the integrability condition) gives zero only if $C$ is a constant, which proves that we can only have supersymmetry for $r_0 = 0$, the "extreme case."

Let us find now the *unbroken supersymmetries of extreme charged dilaton black holes* with $C = r_0 = 0$. In that case we find

$$\epsilon_I = e^{U/2}\epsilon_I^{(0)} \;, \tag{105}$$

where $\epsilon_I^{(0)}$ are constant spinors.

The relations (103) (and consistency with their complex conjugates in view of $\alpha_{IJ}\alpha^{JK} = -\delta_I^K$) imply that

$$\sqrt2(\psi - \psi_0) = a\,e^{U+\phi} \;, \qquad \sqrt2(\chi - \chi_0) = b\,e^{U-\phi} \;, \tag{106}$$

where $\psi_0$ and $\chi_0$ are undetermined constants, and $a = -{\rm sgn}(Q)$ and $b = -{\rm sgn}(P)$. We now get two conditions from Eqs. (103):

$$\begin{aligned} \left(\epsilon_I - a\,\alpha_{IJ}\gamma_0\epsilon^J\right)\partial_i(U+\phi) &= 0 \;, \\ \left(\epsilon_I - b\,\beta_{IJ}\gamma_0\epsilon^J\right)\partial_i(U-\phi) &= 0 \;. \end{aligned} \tag{107}$$

Consider first the case in which both $PQ \neq 0$, which means that neither $\partial_i(U+\phi)$ nor $\partial_i(U-\phi)$ vanishes.

We get in each of the four quadrants of Fig. 1 one unbroken supersymmetry. To see this, let us introduce a new basis for the supersymmetries:

$$\begin{aligned} \epsilon^{12}_{\pm} &= \epsilon_2 \pm \gamma_0\epsilon^1 , \qquad \epsilon^{\pm}_{12} = \epsilon^2 \pm \gamma_0\epsilon_1 , \\ \epsilon^{34}_{\pm} &= \epsilon_4 \pm \gamma_0\epsilon^3 , \qquad \epsilon^{\pm}_{34} = \epsilon^4 \pm \gamma_0\epsilon_3 . \end{aligned} \tag{108}$$

Note that these spinors are still chiral.

The unbroken supersymmetries in each quadrant are

$$\begin{aligned} &\epsilon^{34}_{+} \text{ and } \epsilon^{+}_{34} \text{ for } Q>0, P>0 , \\ &\epsilon^{12}_{+} \text{ and } \epsilon^{+}_{12} \text{ for } Q>0, P<0 , \\ &\epsilon^{34}_{-} \text{ and } \epsilon^{-}_{34} \text{ for } Q<0, P<0 , \\ &\epsilon^{12}_{-} \text{ and } \epsilon^{-}_{12} \text{ for } Q<0, P>0 . \end{aligned} \tag{109}$$

Thus, for each side of the square in Fig. 1 we have one unbroken $N=1$ supersymmetry, each time a different part of the original $N=4$ supersymmetry.

If $Q$ or $P$ are zero, then in the first case $\partial_i(U+\phi)=0$, and in the second case $\partial_i(U-\phi)=0$. So only one of the two conditions in (107) for spinors applies, and we have two remaining supersymmetries:

$$\begin{aligned} &\epsilon^{34}_{+},\ \epsilon^{+}_{34},\ \epsilon^{12}_{-},\ \epsilon^{-}_{12} \text{ for } Q=0,\ P>0 , \\ &\epsilon^{34}_{-},\ \epsilon^{-}_{34},\ \epsilon^{12}_{+},\ \epsilon^{+}_{12} \text{ for } Q=0,\ P<0 , \\ &\epsilon^{34}_{+},\ \epsilon^{+}_{34},\ \epsilon^{12}_{+},\ \epsilon^{+}_{12} \text{ for } P=0,\ Q>0 , \\ &\epsilon^{34}_{-},\ \epsilon^{-}_{34},\ \epsilon^{12}_{-},\ \epsilon^{-}_{12} \text{ for } P=0,\ Q<0 . \end{aligned} \tag{110}$$

Thus in each vertex in Fig. 1 there is an unbroken $N=2$ supersymmetry, each time a different part of the original $N=4$ supersymmetry. And since the vertex of the square is the intersection of two sides, the supersymmetries which are unbroken in the vertex are those which are unbroken on both sides adjoining the given vertex.

The bound $M \geq |Z|$ was derived in [16] using this basis for the spinors. Indeed, one may check that [with a sum over the pairs (1,2) and (3,4)]

$$\begin{aligned} \bar\epsilon'^{I}\gamma_0\epsilon_I &= \tfrac{1}{2}\left(\bar\epsilon'^{+}\gamma_0\epsilon_+ + \bar\epsilon'^{-}\gamma_0\epsilon_-\right) , \\ \bar\epsilon'^{I}\gamma_i\epsilon_I &= \tfrac{1}{2}\left(\bar\epsilon'^{+}\gamma_i\epsilon_- + \bar\epsilon'^{-}\gamma_i\epsilon_+\right) , \\ \bar\epsilon'^{I}\bar\epsilon^{J}\epsilon_{IJ} &= \tfrac{1}{4}\left(\bar\epsilon'^{+}\gamma_0\epsilon_+ - \bar\epsilon'^{-}\gamma_0\epsilon_- + \bar\epsilon'^{-}\gamma_0\epsilon_+ - \bar\epsilon'^{+}\gamma_0\epsilon_-\right) \\ &\quad -(\epsilon \leftrightarrow \epsilon') , \\ \bar\epsilon'_I\bar\epsilon_J\epsilon^{IJ} &= \tfrac{1}{4}\left(\bar\epsilon'^{+}\gamma_0\epsilon_+ - \bar\epsilon'^{-}\gamma_0\epsilon_- - \bar\epsilon'^{-}\gamma_0\epsilon_+ + \bar\epsilon'^{+}\gamma_0\epsilon_-\right) \\ &\quad -(\epsilon \leftrightarrow \epsilon'). \end{aligned} \tag{111}$$

The complex conjugates of the first two equations are minus these expressions with $\epsilon$ and $\epsilon'$ interchanged.

This implies that when one takes the commutator as in (99) between two supersymmetries $\epsilon_+$ and $\epsilon'_+$ then only the space translation does not enter. The time translation is proportional to the mass, and there is a central charge (acting as $\delta^{+}_{\rm Lor}+\delta^{-}_{\rm Lor}$) proportional to $Z$. In this basis both terms are proportional to $\bar\epsilon'^{+}\gamma_0\epsilon_+$. The Hermiticity properties then imply that $M+Z$ should be non-negative, and becomes zero only if $\epsilon_+$ is an unbroken supersymmetry [16]. When one uses $\epsilon_-$, one can see in (111) that the sign of the $Z$ contribution changes, and one finds $M-Z\geq 0$ and zero only for unbroken $\epsilon_-$ supersymmetry. In this way one obtains $M \geq |Z|$. Using the pair (1,2) leads to $M \geq z_1$, while the pair (3,4) leads to the bound $M \geq z_2$. From these arguments it is also clear that supersymmetries are unbroken if and only if the charges are equal to their extreme values as in Fig. 1.

There is nothing in our analysis which depends on spherical symmetry; thus, the multi-black-hole case is included automatically. We must, however, remember that $P$ and $Q$ refer to the total charge. Only purely magnetic or purely electric multi black holes possess $N=2$ supersymmetry; a configuration with mixing between $P$ and $Q$, such as the one with charges $(P,0)$ and $(0,Q)$, possesses only $N=1$ supersymmetry.

Thus we have shown that the extreme dilaton multi-black-hole solutions of $N=4$ supergravity, given in Eqs. (42)–(44), have some $N=1$ or $N=2$ supersymmetries unbroken.

Note that there exists a particular conformal transformation of the original canonical geometry which brings the parameters of unbroken supersymmetry to constant spinors, according to Eq. (105):

$$\begin{aligned} \epsilon_I^{(0)} &= e^{-U/2}\epsilon_I^{\rm can} , \quad g_{\mu\nu}^{(0)} = e^{-2U}g_{\mu\nu}^{\rm can} , \\ \Lambda_I^{(0)} &= e^{+U/2}\Lambda_I^{\rm can}, \quad \text{etc} . \end{aligned} \tag{112}$$

After such a conformal transformation, the metric takes the form

$$ds^{2\,(0)} = dt^2 - e^{-4U}(dx^i)^2 . \tag{113}$$

In such a geometry, supersymmetry with parameter $\epsilon^{(0)}$ exists globally on the space, in contrast with the canonical geometry where the parameter of unbroken supersymmetry is $\epsilon_I^{\rm can} = e^{U/2}\epsilon_I^{(0)}$ and goes to zero near the horizon. In addition, the time component of the spin connection vanishes and the time derivative coincides with the covariant time derivative. The conformal transformation (112) defines a choice of time coordinate for the supersymmetric state for which the commutator of two supersymmetries is a translation in time. For one of the solutions discussed above, namely, for purely magnetic multi black holes with $U=-\phi$, the corresponding conformal transformation is

$$g_{\mu\nu}^{(0)} = e^{-2U}g_{\mu\nu}^{\rm can} = e^{2\phi}g_{\mu\nu}^{\rm can} = g_{\mu\nu}^{\rm string} . \tag{114}$$

So we see that in the purely magnetic case the unbroken supersymmetries in the stringy geometry are realized in terms of *constant* spinors.

## VII. NONRENORMALIZATION THEOREM FOR THE PARTITION FUNCTION OF THE EXTREME CHARGED DILATON BLACK HOLE

The advantage of establishing the supersymmetric properties of extreme black holes is that of obtaining the possibility to prove a powerful nonrenormalization theorem for quantum corrections. For classical Reissner-Nordström extreme black holes, the corresponding theorem has been established in [3]; the analogous theorem exists for extreme dilaton black holes, as we will now show.

An extensive and complementary treatment of quantum corrections in $N = 2$ supergravity, in the context of quantization of the collective coordinates of the black-hole solution, may be found in Aichelburg and Embacher [26].

The language we will use for formulation of the theorem is that used by Gibbons and Hawking [27] for calculation of actions and partition functions in the context of black holes and de Sitter space. In our previous publication [3] we used the language of the "effective on-shell action," which is often used when considering supersymmetric theories. It will be clear from the following equations that we will discuss the calculation of the same path integral as before, but for the black holes, in addition, the thermodynamic interpretation of the path integral as the partition function will be available.

The fundamental path integral in quantum gravity is

$$Z = \int d[g]d[\Phi]\exp\{iI[g,\Phi]\}\ , \tag{115}$$

where $d[g]$ is the measure on the space of metrics, $d[\Phi]$ is the measure on the space of matter fields and $I[g,\Phi]$ is the action. We assume that the path integral is well defined; i.e., an appropriate background invariant gauge-fixing of all local symmetries is performed. Let $g_0, \Phi_0$ be extremals of the classical action, i.e., solutions of classical equations of motion. One can then represent our integration variables as

$$g = g_0 + \tilde{g}\ , \tag{116}$$

$$\Phi = \Phi_0 + \tilde{\Phi}\ , \tag{117}$$

and expand the action around this background,

$$I[g,\Phi] = I[g_0,\Phi_0] + I_2[\tilde{g},\tilde{\Phi}] + I_3[\tilde{g},\tilde{\Phi}] + \cdots, \tag{118}$$

where $I_2$ contains terms quadratic in fluctuations, $I_3$ contains terms cubic in fluctuations, etc.

In other words, we are calculating the background functional by expanding it near the classical extremal (saddle point):

$$\ln Z = iI[g_0,\Phi_0] + \ln\int d[\tilde{g}]d[\tilde{\Phi}]\exp\{i(I_2[\tilde{g},\tilde{\Phi}] + \cdots)\}. \tag{119}$$

At finite temperature, this path integral in Euclidean space can also be interpreted as a thermal partition function with the properties

$$\ln Z = -I_{\rm Eucl} = -T^{-1}F\ , \tag{120}$$

where $F = M - TS - \sum_i \mu_i C_i$ is the free energy and $\mu_i$ are chemical potentials associated with conserved charges $C_i$, $S$ being the entropy of the system. Gibbons and Hawking have calculated the classical action on the black-hole solution in [1] [the first term in Eq. (119)] and in this way have established, for all black holes known at that time, that

$$-\ln Z^{\rm cl} = I_{\rm Eucl} = S = \tfrac{1}{4}A\ ; \tag{121}$$

i.e., the Euclidean action is $\frac{1}{4}$ of the area of the horizon. Our investigation of electrically and magnetically charged dilaton black holes (23) also confirms the rule (121); the logarithm of the partition function in the classical approximation is given by the expression

$$-\ln Z^{\rm cl} = S = \pi(r_+^2 - \Sigma^2) = \pi\{[M + \tfrac{1}{2}(r_+ - r_-)]^2 - \Sigma^2\}\ . \tag{122}$$

Our calculation of the value of the on-shell action required a careful treatment of all terms in the action, including the extrinsic curvature term, as was done in the calculation of the entropy of the Reissner-Nordström black hole in [27].

The calculation of the partition function of one extreme spherically symmetric dilaton black hole can be performed, for example, by taking the limit $r_+ \to r_-$ in Eq. (122). The result is

$$\begin{aligned} I^{(1)}_{\rm extr} &= -\ln Z^{\rm cl}_{\rm extr} = S_{\rm extr} = \tfrac{1}{2}\pi|z_2^2 - z_1^2| \\ &= \pi(M^2 - \Sigma^2) = 2\pi|PQ|\ . \end{aligned} \tag{123}$$

Rather than taking the limit, we can calculate the action for extreme purely magnetic or purely electric black holes directly, even though the temperature is not well defined. After we express the volume integral of the Lagrangian as a surface integral, this term is exactly canceled by the surface integral of the extrinsic curvature. This vanishing of the Lagrangian, due to supersymmetry, confirms the previous result (123) at $PQ = 0$.

In the extreme case, the force balance condition is satisfied, so we may have multi-black-hole solutions which are not spherically symmetric. However, we can still calculate the partition function of such a configuration, by following [28]. Using the equation of motion for the vector fields, we can write the action, including extrinsic curvature term, as a single surface integral. Near the horizon of the $r$th black hole, the metric, dilaton, and electromagnetic fields are all dominated by terms involving only the charges of the $r$th black hole, so that the action ends up being just the sum of $\frac{1}{4}$ of the areas of the individual holes:

$$I^{(n)}_{\rm extr} = -\ln Z^{\rm cl}_{\rm extr} = 2\pi\sum_{r=1}^{n}|P_rQ_r| = \pi\sum_{r=1}^{n}({M_r}^2 - {\Sigma_r}^2)\ . \tag{124}$$

Notice that the action $I^n_{\rm extr}$ of the multi-black-hole configuration is always less than that of the action $I^{(1)}_{\rm extr}$ of a single black hole with total charges $P = \sum_{r=1}^n |P_r| \neq 0$ and $Q = \sum_{r=1}^n |Q_r| \neq 0$:

$$I^{(1)}_{\rm extr} = 2\pi|PQ| = 2\pi\sum_{r=1}^{n}|P_r|\sum_{s=1}^{n}|Q_s| = \pi(M^2 - \Sigma^2)\ . \tag{125}$$

Thus the total area of the horizons of the dilaton multi-black-hole configuration at a given $P$ and $Q$ is smaller than that of one extreme spherically symmetric black hole. All these solutions with any number of black holes, but different Euclidean actions, have unbroken $N = 1$ supersymmetry.

The extreme dilaton multi black holes have a special solution with only electric or only magnetic charge of each hole. All these solutions have zero Euclidean action, independently of the number of holes. This vanishing of the action is the consequence of the higher unbroken supersymmetry, $N = 2$, as opposite to the ones with mixing of $P$ and $Q$, which have only $N = 1$ supersymmetry unbroken.

A nonrenormalization theorem can be derived in complete analogy with that in Ref. [3] for the classical extreme Reissner-Nordström case.

For the extreme dilaton multi black holes the theorem can be formulated as follows: The exact partition function of the extreme dilaton multi black hole is

$$Z^n_{\text{extr}} = \exp\left(-\sum_{r=1}^{n} \tfrac{1}{4} A_r\right) , \tag{126}$$

where $A_r$ is the area of the horizon of the $r$th individual hole, i.e., *the partition function, calculated in the semiclassical approximation, acquires no quantum corrections.*

The absence of quantum corrections to the path integral $Z = \exp(-I_{\text{Eucl}}[g_0, \Phi_0])$ (119) takes place under the following conditions. One should perform the calculations of the path integral within $N = 4$ supergravity or in superstring theory. This means that perturbations near the extreme dilaton black hole have to include the graviton, four gravitino, four dilatino, six vectors, a dilaton, and an axion. Also an $N = 4$ vector multiplet may be included. This means that the matter fields $\Phi$ in Eq. (115) are all fields which are superpartners of the graviton in the $N = 4$ supermultiplet, and a matter $N = 4$ supermultiplet, including a gluon and gluino. There exists a choice of the representation of the fields in the $N = 4$ multiplet which leads to the absence of one-loop conformal-axial anomalies [4]. Also it is known that the one-loop anomalies are absent in extended supergravities for $N \geq 3$ in the case that the loop calculations are performed in terms of $N = 1$ superfields. Starting from $N = 3$ supergravity, the net number of chiral $N = 1$ matter and ghost multiplets is zero, and therefore there are no anomalies. In particular, there is no divergent one-loop correction to the supersymmetric form of the Euler number [4].

The proof of the nonrenormalization theorem for the on-shell effective action (119) consists of the following steps.

(i) Establishing that in $N = 4$ supergravity or superstring theory $Z[g_0, \Phi_0]$ has to be a locally supersymmetric functional of supergravity fields for an arbitrary on-shell background.

(ii) Realizing the property (i) in the form of manifestly supersymmetric on-shell $N = 4$ superinvariants, which are given by the integrals over the superspace.

(iii) Observing that manifestly supersymmetric on-shell $N = 4$ superinvariants vanish in a bosonic background which has some unbroken supersymmetries. In those backgrounds the dependence on some combinations of Grassmann coordinates of the superspace vanishes and the corresponding superinvariants vanish due to the properties of Berezin integration over the anticommuting variables. For example, all local gauge-independent counterterms in this background acquire the form

$$\int d^4x d^{4N}\theta \,\text{Ber}\, E\, L(x,\theta) = \int d^4x\, D_{\text{unbr}}\Psi(x,\theta)|_{\theta=0} , \tag{127}$$

where $\Psi$ is some spinorial superfield [the result of the action of $(4N-1)$ fermionic derivatives on the superfield Lagrangian]. The last fermionic derivative, denoted by $D_{\text{unbr}}$, is chosen to correspond to one of the unbroken supersymmetries of the background. Using the fact that the standard definition of the supersymmetry variation of the superfield is

$$\delta_\epsilon \Psi(x,\theta) = \sum_{\alpha=1}^{4N} \epsilon^\alpha D_\alpha \Psi(x,\theta) , \tag{128}$$

we conclude that

$$D_{\text{unbr}}\Psi(x,\theta)|_{\theta=0} = 0 , \tag{129}$$

and that the supersymmetric invariant (127) is vanishing in the bosonic background with an unbroken supersymmetry.

Thus, the existence of unbroken supersymmetries in the purely bosonic background means that quantum corrections to the partition function cannot change the semiclassical value of $\ln Z$ if the corrections satisfy generalized Ward identities following from local supersymmetry, i.e., if the theory has no anomalies.

The nonrenormalization theorem was derived above in the context of the Euclidean action. However, our derivation of unbroken supersymmetries in Sec. VI was in the context of a space-time with Lorentzian signature. One may wonder whether the theorem and the supersymmetries apply in both signatures. The only previously known example of absence of quantum supergravity corrections was in case of super-self-dual instanton backgrounds [29]. These backgrounds exist only in Euclidean space; the unbroken supersymmetry and corresponding nonrenormalization theorem rely on properties of Euclidean space-time, where right-handed spinors can be set to zero while left-handed spinors may be nonvanishing. This does not extend to Lorentzian space-time, where right and left spinors are complex conjugates of each other and cannot be set to zero separately. In contrast to this, the unbroken supersymmetries for extreme dilaton black holes exist in both signatures, since in deriving the constraints on the parameters of unbroken supersymmetry we never used chirality properties specific to Euclidean signatures.

Therefore our nonrenormalization theorem for extreme dilaton black holes holds for both Euclidean and Lorentzian signatures.

## VIII. DISCUSSION

The main goal of this paper was to study the relationship between black-hole physics and supersymmetry. We were especially interested in both electrically and magnetically charged dilaton black holes. Such black holes may appear in supergravity and in string theory. They interpolate between purely electric and purely magnetic dilaton black holes, which exhibit very interesting but somewhat confusing properties discussed by many authors.

We found that supersymmetry indeed provides us with powerful tools for investigation of black holes. First, we were able to find a supersymmetric theory which contains electrically and magnetically charged dilaton black holes. We have shown that the mass of each black hole satisfies two inequalities, $M \geq |z_1|$ and $M \geq |z_2|$, where $z_i$ are the central charges of the supersymmetry algebra, and are related to the electric and magnetic charges as follows: $z_1 = \frac{Q-P}{\sqrt{2}}$ and $z_2 = \frac{Q+P}{\sqrt{2}}$. When neither of these inequalities is saturated (i.e., when $M > |z_1|$, $M > |z_2|$), supersymmetry is broken; when one is saturated, we have extreme $N = 1$ supersymmetric dilaton black holes. The two inequalities can be saturated only for purely magnetic or purely electric extreme black holes, in which case $N = 2$ supersymmetry becomes restored.

Thus, with the help of supersymmetry one can justify the very notion of an *extreme* black hole as a body which has a minimal mass for given values of charges. This implies that extreme black holes cannot evaporate by emitting (uncharged) elementary particles. This is consistent with the vanishing of the Hawking temperature and/or breakdown of the thermal description for extreme black holes. It is consistent also with the vanishing of the imaginary part of the effective action in the extreme black-hole background.

But the most unusual result, which we obtained with the aid of supersymmetry, is that the higher-order (perturbative) quantum supergravity or superstring corrections to the effective action vanish in both Lorentzian and Euclidean extreme dilaton black-hole backgrounds. Previously, the only example of such a background (with Lorentzian signature) was flat Minkowski space. As a consequence of the Euclidean result, we were able to obtain an exact expression for the entropy of the extreme dilaton black hole.

This allowed us to describe properties of dilaton black holes near the extreme limit with greater confidence. We calculated the temperature, entropy, and specific heat of dilaton black holes as a function of $M$, $P$, and $Q$. We have shown that even though the Euclidean action (entropy) for extreme black hole can be calculated exactly, the usual thermal description of dilaton black holes breaks down near the extreme limit for all possible values of $P$ and $Q$.

Another interesting observation is the relation between supersymmetry and the cosmic censorship conjecture. Indeed, the supersymmetric bound on the black-hole mass ensures that the black-hole singularity is hidden by the event horizon. It approaches the event horizon only in the extreme case when $N = 2$ supersymmetry becomes restored, which is possible if only one of the charges is present. It is not clear whether an evaporating black hole can ever reach its extreme limit (due to breakdown of the thermal description), but even if it can do so, the singularity can never appear *outside* the event horizon. This means that an outside observer will never see the singularity.

It is interesting that this relationship between supersymmetry and the cosmic censorship hypothesis in an asymptotically flat space-time is valid for all static black-hole solutions which are known to us. Indeed, this relationship is valid for the ordinary Schwarzschild black holes, for the Reissner-Nordström ones and, as we have shown in this paper, for a large class of electrically and magnetically charged dilaton black holes.

We expect that the coincidence of the bounds from supersymmetry and cosmic censorship may occur for more general situations as well. In addition, it is plausible that any complicated dynamics occurring in an asymptotically flat space will still respect the positivity conditions obtained from the global extended supersymmetry algebra and expressed in Eqs. (7). This makes it very tempting to propose the following super cosmic censorship conjecture.

(1) Supersymmetry does not like naked singularities. It either hides a singularity under the horizon, or keeps it at the event horizon, where it still cannot be seen by an outside observer.

(2) Broken supersymmetry dislikes singularities even more. When supersymmetry is broken (which is the case in our Universe), a singularity always remains hidden under the horizon.

At present we are investigating the possibility of whether this conjecture could be violated when black holes are not static (for example, when black holes are rotating), or when the singularity is not of a (multi-) black-hole type, but, for example, is linear [30]. An investigation of global supersymmetry in such situations is much more complicated. In any case, it seems that an investigation of static black holes is more relevant to the study of the last stages of gravitational collapse. Indeed, it is very difficult for us to imagine any physical process which would lead to creation of a linear singularity; for example, collapsing cosmic strings typically produce either rotating or static black holes. On the other hand, with an account taken of quantum effects, rotating black holes usually lose their angular momentum much faster than their mass and eventually become static. We believe, therefore, that the coincidence of the bounds from supersymmetry and cosmic censorship for static black-hole solutions is most interesting, and so far we have not found any exceptions to this rule.[11] In particular, we have verified recently that this rule remains true for the

[11] The solution with a naked singularity found in [31] is not relevant to the discussion of the weak cosmic censorship conjecture, since it is a solution which is asymptotically anti–de Sitter, rather than flat.

dilaton black holes with three independent charges mentioned in Sec. III and for the solutions with two electric, two magnetic, and an axion charge obtained in [32].

Throughout the paper, we have been studying the supersymmetry of extreme black holes mainly in the canonical geometry, i.e., with the metric of standard four-dimensional Einstein theory. It is, however, possible to address the question: what will happen with unbroken supersymmetries after a conformal transformation, for example, to the stringy geometry? We have found a very nice feature of purely magnetic solutions: the string sees a geometry which possesses unbroken supersymmetry with *constant* spinors. It would be interesting to understand why the magnetic dilaton black hole is special in this respect.

Another interesting problem is to establish some relation between our study of dilaton four-dimensional (4D) dilaton black holes and the recent studies of 2D black holes. In particular, there is a discussion of whether a naked singularity can be formed in the process of evolution of a two-dimensional black hole, see e.g., [33]. Our results suggest that if a two-dimensional black hole is obtained as a result of a consistent dimensional reduction of a four-dimensional supersymmetric theory or superstring theory, then no naked singularities should appear.

Note that even though we embedded our black-hole solutions in a supersymmetric theory, all our solutions are purely bosonic. It is possible, therefore, that the positivity bounds which we obtained are valid for such bosonic solutions not only in the context of the supersymmetric theories where they were derived, but in other theories which have the same bosonic sector. A similar statement is known to be true in $N = 1$ supergravity, where it was shown that the positivity bound on energy in supergravity implies the positivity of energy in ordinary gravity [17, 18]. As it was formulated by Grisaru [17], it is enough that Einstein theory "knows" that it can be successfully coupled to gravitinos.

In this paper we discussed not only single extreme black holes, but also extreme multi-black-hole solutions. An important feature of these solutions is that extreme black holes can be in an equilibrium state due to cancellation of Coulomb-like forces between their electric, magnetic, gravitational, *and* dilaton charges. The multi black holes also have unbroken supersymmetry: $N = 1$ for $PQ \neq 0$ and $N = 2$ for $PQ = 0$, where $P, Q$ are the total charges of the multi-black-hole configuration.

The existence of many equilibrium configurations of black holes with the same total mass and charge, independent of the position of each black hole, raises many interesting questions. Is there any possibility of quantum tunneling between these degenerate configurations, similar to the tunneling between vacua with different topological charges in QCD? Does this degeneracy mean that the final result of a charged black-hole evaporation will be not a single black hole but a quantum superposition of states corresponding to an arbitrary number of charged black holes with a given total mass and charge? Is it possible that at the last stage of the evaporation of a charged black hole it splits into many smaller black holes? Our understanding of these problems is rather limited. In order to stimulate their investigation, we will discuss some relevant issues in Appendix B.

The solutions which we presented depended on only two charges. It is natural to consider solutions with more parameters, for example those discussed at the end of Sec. III. The study of the supersymmetry properties of those solutions is in progress. Also to be studied in this context are dual dilaton dyons and rotating black holes with or without axion. Investigation of these solutions would give us an extra opportunity to study the relation between supersymmetry and the cosmic censorship conjecture.

In fact, our investigations suggest the following challenge. Is it possible to classify and to find *all solutions of Einstein theory, interacting with matter, which are supersymmetric?* There exists a partial answer to this problem, given by Tod [22], for $N = 2$ supergravity. He has found *all metrics admitting supercovariantly constant spinors* of this theory. Currently, only part of his results are understood from the point of view of field theory, where the Einstein and Maxwell equations together with their right-hand side are derived from some Lagrangian. Tod has solved first-order differential equations for unbroken supersymmetry, but only some of his solutions have been identified with solutions of some covariant Lagrangian theory. For other supersymmetric theories including gravity, such a complete analysis has not yet been performed, though many interesting results are already known; see, for example, the review on supersymmetric string solitons in [34], supersymmetric domain walls in $N = 1$ supergravity [35], and the results of the present paper for $N = 4$ supergravity.

Thus we expect on the basis of Tod's results that a rich family of covariant Lagrangians and their solutions (not only asymptotically flat spaces as studied here) might have a supersymmetric embedding in the sense explained in our paper. Those theories will include plane waves, Israel-Wilson-Perjes metrics, etc. For all of these solutions we may expect that the unbroken supersymmetry will take quantum gravity corrections under control.

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## APPENDIX A: NOTATION AND CONVENTIONS

We use the metric signature $(+---)$. The curved indices are denoted by $\mu, \nu, \ldots = 0, \ldots, 3$ and the flat ones by $a, b, \ldots$. If restricted to space, we use $i, j, \ldots = 1, 2, 3$ for both types of indices. From the context it is usually clear which type they represent, e.g., on derivatives $\partial$ the indices are curved, while on $\gamma$ matrices they are flat. Where confusion can arise, we use $\hat{0}$ or $\hat{\imath}$ to indicate that indices are curved ones.

We define

$$\epsilon^{\mu\nu\rho\sigma} = \sqrt{-g}\, e_a^\mu\, e_b^\nu\, e_c^\rho\, e_d^\sigma\, \epsilon^{abcd}\;, \qquad \epsilon^{0123} = i = -\epsilon_{0123} \tag{A1}$$

where the former implies that the latter is true for flat as as well for curved indices. For spherical coordinates we have $\epsilon^{tr\theta\phi} = i$. The dual of an antisymmetric tensor is defined as

$$^\star F^{ab} = \tfrac{1}{2}\epsilon^{abcd} F_{cd}\;. \tag{A2}$$

We introduce the self-dual and anti-self-dual tensors

$$F^\pm_{ab} = \tfrac{1}{2}\left(F_{ab} \pm {}^\star F_{ab}\right)\;. \tag{A3}$$

If we write antisymmetric tensors as forms, there is the correspondence

$$F = \tfrac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu\;. \tag{A4}$$

Antisymmetrization is done with weight one: $[ab] = \frac{1}{2}(ab - ba)$. A centered dot is used to indicate that derivatives do not act further to the right. Without such a symbol all $\partial$ operations are assumed to act on all fields to the right in the same terms, unless it is enclosed in brackets.

The $\gamma$ and $\sigma$ matrices are defined by

$$\gamma_a\gamma_b = -\eta_{ab} + 2\sigma_{ab}\;, \qquad \gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3\;, \tag{A5}$$

which implies that $\frac{1}{2}\epsilon^{abcd}\sigma_{cd} = -\gamma^5\sigma^{ab}$. The matrices $\gamma_i$ and $\gamma_5$ are Hermitian, while $\gamma_0$ is anti-Hermitian.

For the spinors, we use a chiral notation, where the chirality is indicated by the position of the $I, J$ [SO(4)] index, or $\pm$ after the redefinitions (108). Which position corresponds to which chirality is not the same for all spinors. We have

$$\begin{aligned}
&\Psi^I_\mu = \tfrac{1}{2}(1+\gamma_5)\Psi^I_\mu\;, \Psi_{\mu I} = \tfrac{1}{2}(1-\gamma_5)\Psi_{\mu I}\;,\\
&\epsilon^I = \tfrac{1}{2}(1+\gamma_5)\epsilon^I\;, \quad \epsilon_I = \tfrac{1}{2}(1-\gamma_5)\epsilon_I\;,\\
&\lambda^I = \tfrac{1}{2}(1-\gamma_5)\lambda^I\;, \quad \lambda_I = \tfrac{1}{2}(1+\gamma_5)\lambda_I\;.
\end{aligned} \tag{A6}$$

The conjugates for any spinor $\chi$ are

$$\bar{\chi}^I = -i(\chi_I)^\dagger\gamma_0 = (\chi^I)^T\mathcal{C}\;, \tag{A7}$$

where $\mathcal{C}$ is the charge conjugation matrix

$$\mathcal{C}^T = -\mathcal{C}\;, \qquad \mathcal{C}\gamma_a\mathcal{C}^{-1} = -\gamma_a^T\;, \tag{A8}$$

such that, e.g., $\bar{\epsilon}^I\gamma_5 = \bar{\epsilon}^I$. In chiral notation the antisymmetric tensors are often automatically (anti)self-dual. For example,

$$\sigma^{ab}F_{ab}\epsilon^I = \sigma^{ab}F^-_{ab}\epsilon^I\;. \tag{A9}$$

For the spin connections and curvatures we have

$$\begin{aligned}
\omega_\mu{}^{ab} &= 2e^{\nu[a}\partial_{[\mu}e_{\nu]}{}^{b]} - e^{a\rho}e^{b\sigma}e_{\mu c}\partial_{[\rho}e^c_{\sigma]}\;,\\
R^{ab}_{\mu\nu}(\omega) &= 2\partial_{[\mu}\omega_{\nu]}{}^{ab} + 2\omega^{c[a}_\mu\omega_\nu{}^{b]}{}_c\;,\\
R_{\mu\nu} &= e_a{}^\rho e_{\mu b}R_{\nu\rho}{}^{ab}\;, \qquad R = -R_{\mu\nu}g^{\mu\nu}\;.
\end{aligned} \tag{A10}$$

This implies that

$$\frac{\delta}{\delta g^{\mu\nu}}\int d^4x\sqrt{-g}R = -\sqrt{-g}\left(R_{\mu\nu} + \tfrac{1}{2}g_{\mu\nu}R\right)\;. \tag{A11}$$

The covariant derivative on spinors is

$$\nabla_\mu\epsilon = \left(\partial_\mu - \tfrac{1}{2}\omega_\mu{}^{ab}\sigma_{ab}\right)\epsilon\;. \tag{A12}$$

For the translation from [19, 20] (the notation is in the first article) we replaced their $\gamma_a$ by $i\gamma_a$ and $\gamma_5$ gets a minus sign. We replace their $\epsilon^{\mu\nu\rho\sigma}$ by $-i\epsilon^{\mu\nu\rho\sigma}$. For the spinors, their $\Psi_\mu$ becomes $\frac{1}{\sqrt{2}}\Psi_\mu$ and $\epsilon$ becomes $\sqrt{2}\epsilon$, and we changed the sign of $A$.

For translation from [24] we changed the metric, thus $g_{\mu\nu}$ gets a minus sign, at any implicit appearance, e.g., in $\partial^\mu$. The vierbein $e^a_\mu$ is unchanged, but then, e.g., $e_{\mu a}$ gets a minus sign. Note that with the translation given above, $\omega$ is the same as in [24] which is opposite to the conventions of [36]. But we define $R_{\mu\nu}$ and $R$ such that they are the same [the minus sign in the last equation in (A10) is thus due to the metric].

## APPENDIX B: SPLITTING OF EXTREME BLACK HOLES

A very interesting situation appears when we consider multi-black-hole solutions. As we already noted, they describe an equilibrium configuration of black holes which have the same mass and total charge independent of the position of each individual black hole. Let us now try to understand the behavior of such a configuration at the quantum level. One may expect that quantum fluctuations of the metric, as well as those of the dilaton and vector fields, may lead to quantum jumps of the positions of black holes. In a normal situation, when the total energy of a system depends on the positions of its constituents, this would lead to some change $\Delta E$ of the energy of the system which would violate energy conservation. Such a system then returns to its original state within a time $(\Delta E)^{-1}$, in accordance with the uncertainty principle. However, in our case $\Delta E = 0$ for any change of the extreme black hole configuration. This suggests that extreme black holes behave as Brownian particles at a flat surface. If originally they are localized in some place, later on the distance between them grows and eventually become indefinitely large. This behavior has a simple quantum mechanical interpretation as a spreading of the wave packet describing several noninteracting particles.

Now we can make a second step and ask the question: what will happen if an extreme black hole splits quantum mechanically into two extreme black holes of the same total mass and charge? This process is not forbidden by energy and charge conservation. Usually, it is forbidden by the second law of black-hole physics, since in such a process the total area of the horizons of the black holes (and the total entropy) would decrease. However, the thermodynamic interpretation of the law suggests that this process may be possible due to fluctuations of the entropy $\Delta S < 0$, even though the probability of such fluctuations will be exponentially suppressed. Moreover, as distinct from the ordinary Reissner-Nordström black holes, the area of the horizon (and the entropy) of purely magnetic and purely electric extreme *dilaton* black holes vanishes. Thus, the second law of black-hole physics does not forbid their splitting.

To obtain an intuitive (though, admittedly, vague) understanding of the process of splitting, let us consider a purely electric or magnetic dilaton black hole near its horizon. The horizon is singular (for the moment, we will not consider the stringy version of a magnetic black hole), and the black hole can be completely described by a conformastatic metric (36), with the singularity at $x = 0$. Since the area of the horizon vanishes, one may imagine that quantum fluctuations of the metric on the Planck scale can easily split the singularity into two, i.e., we will get a conformastatic metric with two black holes very close to each other. (After all, we cannot actually interpret processes at the Planck scale in terms of classical space time with a fixed number of classical singularities.) In a normal situation, such an event would not have any interesting consequences, since the baby black hole would immediately recombine with its parent. However, extreme black holes described by the conformastatic solution (36), (43) do not attract each other. If our picture of Brownian motion of extreme black holes is correct, then the average distance between the baby black holes and their parent can only grow. This is very similar to the standard picture of black-hole evaporation: If the black hole is not surrounded by ultrarelativistic particles with a large temperature, the particles emitted by the black hole move away and its mass decreases. Similarly, if the Universe is not filled by a dense gas of black holes, then the new black holes, produced by the black-hole splitting, typically will move away due to Brownian motion. Perhaps a more adequate way to say it is to remember that splitting of the black hole (which changes the number of singularities) occurs without any energy release. Thus, the products of splitting have vanishing relative momenta, which means that the distances between them become indefinitely large. In this sense, the theory of the black-hole splitting resembles the theory of baby Universe formation, where the baby Universe is produced with vanishing energy and momenta, hence the place where it is created cannot be localized. Another useful analogy is the tunneling between different vacua in QCD which have the same energy but are characterized by different topological numbers.

In a more general case, when the black hole has both electric and magnetic charges, its singularity is hidden under the horizon. Then the transition between one black hole, with one singularity, and two black holes, with two singularities separated by their two horizons, is discontinuous. This is why we expect the probability of such processes to be exponentially suppressed. To get an estimate of the probability of splitting, one may use standard thermodynamic arguments, which suggest that it should be proportional to $\exp(\Delta S)$, where $\Delta S$ is the change of entropy [28].[12] If this interpretation is correct, one may expect that the probability of splitting of one Reissner-Nordström black hole into many, with a total mass $M = \sum_r M_r$, is given by

$$\Gamma \sim \exp\left\{-\pi\left[\left(\sum_r M_r\right)^2 - \sum_r M_r^2\right]\right\} . \tag{B1}$$

One can easily see that the probability of splitting of large black holes is exponentially suppressed. However, this suppression may be not very strong for small black holes with masses of the order of the Planck mass $M_P = 1$.

An analogous expression for dilaton black holes, which follows from Eqs. (124) and (125) in Sec. VII, is

$$\Gamma \sim \exp\left\{-\pi\left[\left(\sum_r M_r\right)^2 - \left(\sum_r \Sigma_r\right)^2 - \sum_r ({M_r}^2 - \Sigma_r^2)\right]\right\}$$
$$= \exp\left[-2\pi\left(\sum_r |P_r| \sum_s |Q_s| - \sum_r |P_r Q_r|\right)\right] = \exp\left[-2\pi\left(\sum_{r\neq s} |P_r||Q_s|\right)\right] . \tag{B2}$$

This expression, unlike Eq. (B1), shows that there is no exponential suppression of splitting of purely electric or purely magnetic dilaton black holes. This agrees with our qualitative discussion of quantum fluctuations and Brownian motion of extreme black holes. It would be very desirable to confirm (or disprove) the validity of Eqs. (B1) and (B2) by finding an explicit instanton solution which is responsible for the black-hole splitting. It may well happen that the suppression of splitting is given by a

[12] This argument was used in [28] applied to splitting of Bertotti-Robinson Universes, which have the same geometry as the geometry near the horizon of the Reissner-Nordström black hole. It is interesting that this simple argument sometimes gives correct results even in some situations where the description of tunneling in terms of instantons is ambiguous; for example, it gives a correct expression for the probability of tunneling in an inflationary Universe, in terms of the entropy of de Sitter space.

more complicated expression than $\exp(\Delta S)$, especially in the situation where the thermal description of black holes breaks down. However, at this stage it would be most important to understand whether this probability is finite at all. If this is the case (and at least for purely electric black holes it seems to be a reasonable possibility), then the physics of black holes may prove to be even more interesting than we thought.

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[1] G. Gibbons, in *Unified Theories of Elementary Particles. Critical Assessment and Prospects*, Proceedings of the Heisenberg Symposium, München, West Germany, 1981, edited by P. Breitenlohner and H. P. Dürr, Lecture Notes in Physics Vol. 160 (Springer-Verlag, Berlin, 1982), G. W. Gibbons and C. M. Hull, Phys. Lett. **109B**, 190 (1982).

[2] G. W. Gibbons, in *Supersymmetry, Supergravity and Related Topics*, Proceedings of the XVth GIFT International Seminar on Theoretical Physics, Girona, Spain, 1984, edited by F. del Aguila, J. de Azcárraga, and L. Ibáñez (World Scientific, Singapore, 1985), p. 147.

[3] R. Kallosh, Phys. Lett. B **282**, 80 (1992).

[4] M. Duff, in *Supergravity '81*, edited by S. Ferrara and J. G. Taylor (Cambridge University Press, Cambridge, England, 1982); S. Gates, M. Grisaru, M. Roček, and W. Siegel, *Superspace* (Benjamin-Cummings, London, 1983), p. 451.

[5] G. W. Gibbons, Nucl. Phys. **B207**, 337 (1982).

[6] G. W. Gibbons and K. Maeda, Nucl. Phys. **B298**, 741 (1988).

[7] D. Garfinkle, G. T. Horowitz, and A. Strominger, Phys. Rev. D **43**, 3140 (1991).

[8] J. Preskill, P. Schwarz, A. Shapere, S. Trivedi, and F. Wilczek, Mod. Phys. Lett. A **6**, 2353 (1991); C. F. E. Holzhey and F. Wilczek, Nucl. Phys. **B380**, 447 (1992).

[9] A. Shapere, S. Trivedi, and F. Wilczek, Mod. Phys. Lett. A **6**, 2677 (1991).

[10] W. B. Bonnor, Gen. Relativ. Gravit. **12**, 453 (1980).

[11] R. Penrose, in *Structure of Space-Time*, Battelle Rencontres, 1967, edited by C. M. DeWitt and J. A. Wheeler, Lectures in Mathematics and Physics (Benjamin, New York, 1968); S. W. Hawking and G. F. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, Cambridge, England, 1973).

[12] R. Penrose, Rev. Nuovo Cimento **1**, 252 (1969).

[13] R. Penrose, in *General Relativity, an Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979).

[14] R. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984).

[15] E. Witten and D. Olive, Phys. Lett. **78B**, 97 (1978).

[16] S. Ferrara, C. A. Savoy, and B. Zumino, Phys. Lett. **100B**, 393 (1981).

[17] S. Deser and C. Teitelboim, Phys. Rev. Lett. **39**, 249 (1977); M. Grisaru, Phys. Lett. **73B**, 207 (1978).

[18] E. Witten, Commun. Math. Phys. **80**, 381 (1981); J. M. Nester, Phys. Lett. **83A**, 241 (1981); W. Israel and J. M. Nester, *ibid.* **85A**, 259 (1981); C. M. Hull, Commun. Math. Phys. **90**, 545 (1983).

[19] A. Das, Phys. Rev. D **15**, 2805 (1977); E. Cremmer, J. Scherk, and S. Ferrara, Phys. Lett. **68B**, 234 (1977); E. Cremmer and J. Scherk, Nucl. Phys. **B127**, 259 (1977).

[20] E. Cremmer, J. Scherk, and S. Ferrara, Phys. Lett. **74B**, 61 (1978).

[21] S. Ferrara, J. Scherk, and B. Zumino, Nucl. Phys. **B121**, 393 (1977); E. Cremmer, J. Scherk, and S. Ferrara, Phys. Lett. **74B**, 61 (1978); B. de Wit, Nucl. Phys. **B158**, 189 (1979); E. Cremmer and B. Julia, *ibid.* **B159**, 141 (1979); M. K. Gaillard and B. Zumino, *ibid.* **B193**, 221 (1981).

[22] K. P. Tod, Phys. Lett. **121B**, 241 (1983).

[23] J. H. Horne and G. T. Horowitz, Phys. Rev. D **46**, 1340 (1992).

[24] E. Bergshoeff, M. de Roo, and B. de Wit, Nucl. Phys. **B182**, 173 (1981); M. de Roo, Phys. Lett. **156B**, 331 (1985); Nucl. Phys. **B255**, 515 (1985).

[25] F. Gliozzi, J. Scherk, and D. Olive, Nucl. Phys. **B122**, 253 (1977).

[26] P. C. Aichelburg and F. Embacher, Phys. Rev. D **37**, 338 (1988); **37**, 911 (1988); **37**, 1436 (1988); **37**, 2132 (1988).

[27] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2752 (1977).

[28] D. Brill, Phys. Rev. D **46**, 1560 (1992).

[29] R. E. Kallosh, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 192 (1979) [JETP Lett. **29**, 172 (1979)]; **29**, 493 (1979) [**29**, 449 (1979)]; Nucl. Phys. **B165**, 119 (1980); S. Christensen, S. Deser, M. Duff, and M. Grisaru, Phys. Lett. **84B**, 411 (1979).

[30] J. B. Hartle and S. W. Hawking, Commun. Math. Phys. **26**, 87 (1972); L. Witten (private communication).

[31] L. Romans, Nucl. Phys. **B393**, 395 (1992).

[32] T. Ortín, Stanford University Report No. SU-ITP-92-24, 1992 (unpublished).

[33] J. Russo, L. Susskind, and L. Thorlacius, Phys. Rev. D **46**, 3444 (1992); S. W. Hawking and J. M. Stewart, "Naked and Thunderbolt Singularities in Black Hole Evaporation," Cambridge University report, 1992 (unpublished).

[34] C. G. Callan, J. A. Harvey, and A. Strominger, "Supersymmetric String Solitons," Report No. EFI-91-66, 1991 (unpublished).

[35] M. Cvetič, S.-J. Rey, and S. Griffies, "Nonperturbative Stability of Supergravity and Superstring Vacua," Report No. NSF-ITP-92-84, 1992 (unpublished).

[36] P. van Nieuwenhuizen, Phys. Rep. **68**, 189 (1981).

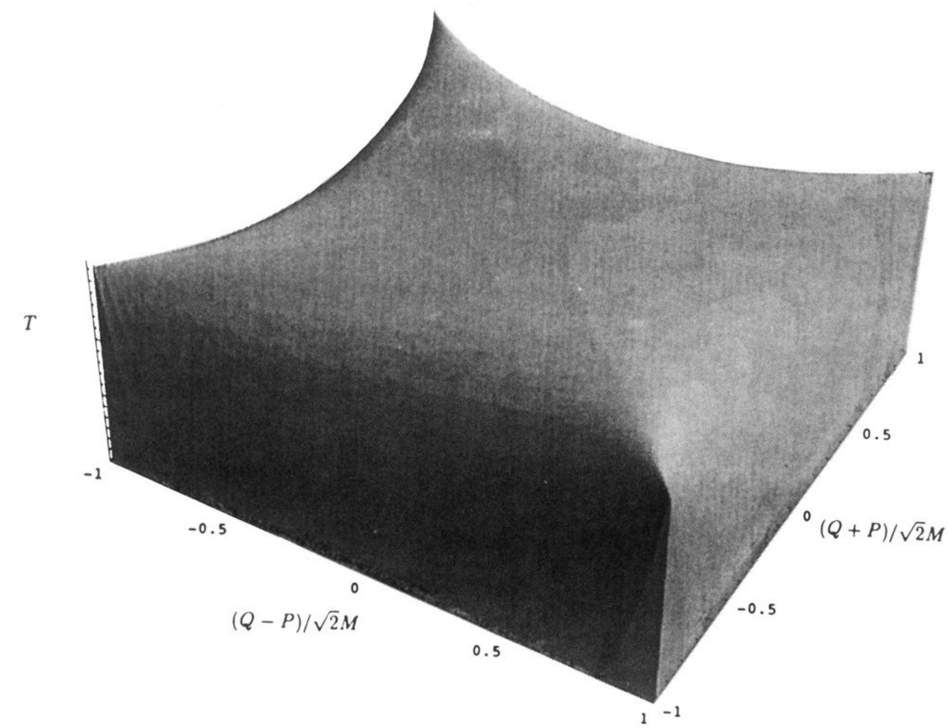


FIG. 3. The temperature of charged dilaton black holes of a given mass $M$ as a function of $z_1/M = (Q-P)/\sqrt{2}M$ and $z_2/M = (Q+P)/\sqrt{2}M$. The extreme black holes correspond to the sides $|z_1|/M = 1$ and $|z_2|/M = 1$ of the square.

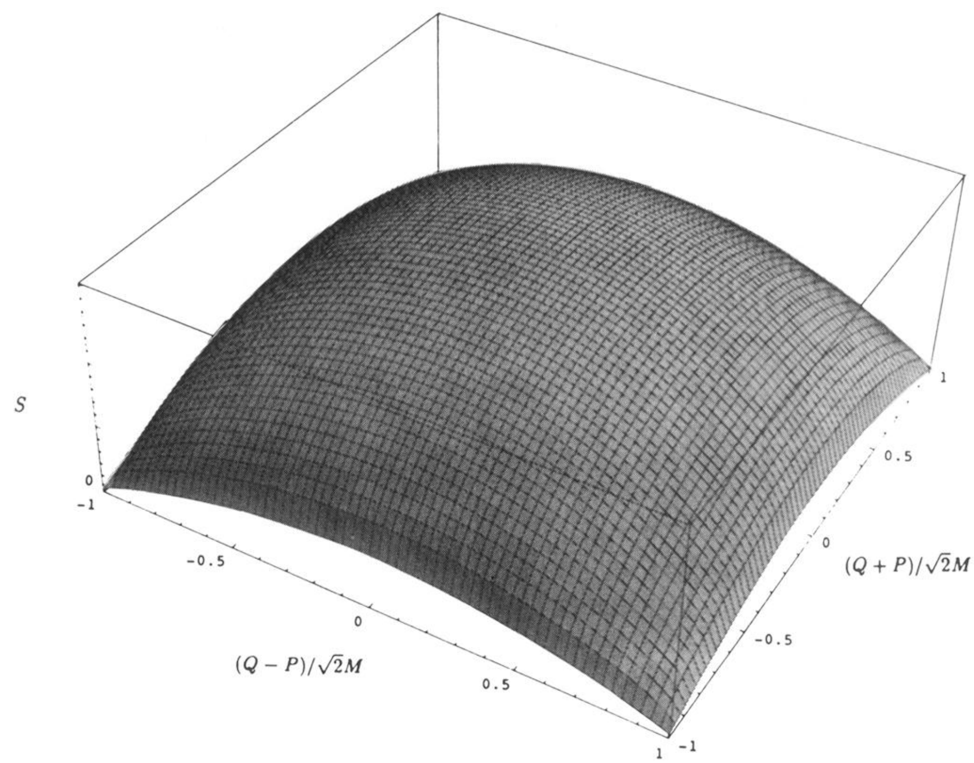


FIG. 5. The entropy $S$ of the charged dilaton black holes as a function of $z_1/M$ and $z_2/M$. It has a maximum for the Schwarzschild black hole, which corresponds to the origin of coordinates ($P = Q = \Sigma = 0$). For purely electric or magnetic extreme dilaton black holes (in the corners) it is zero. On the sides of the square the temperature vanishes, but the total entropy (Euclidean action) remains nonzero, $S = 2\pi\,|PQ|$.