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Dirac neutrinos in dense matter

James Pantaleone
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
and Department of Physics, University of California, Riverside, California 92521
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The evolution of a supernova core can be dramatically different if the neutrinos trapped in the core mix and/or flip helicity and escape as "sterile, right-handed" neutrinos. Thus the observation of neutrinos from supernova SN1987A constrains the mass and mixings of neutrinos. Here we develop the general description of neutrino mixing and spin flip in a background of matter when the nonforward scattering rate is important. The constraints are estimated.

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I. INTRODUCTION

Many astrophysical and cosmological quantities are very sensitive to small neutrino masses. Two well-known examples of this are the solar-neutrino flux and the ratio of neutrino-to-baryon mass densities in the Universe. In addition, the dynamics of stellar collapse is also sensitive to neutrino properties [1–4]. The aim of this paper is to develop the description of Dirac neutrinos in the core of a supernova.

Current popular prejudice favors that, if neutrinos have a small mass, the mass is of the Majorana type. This is because small neutrino masses are natural in "seesaw" models and the simplest such model yields Majorana neutrinos. However with a slight increase in complexity [5] the "seesaw" mechanism can yield light Dirac neutrinos without fine tuning. Here we shall usually (but not always) assume Dirac neutrinos with sterile right-handed components.

In the Sun, the neutrinos propagate through a background of matter as they freely stream out from the core. Forward scattering off of this matter background can produce a large change in the neutrino's flavor. This is known as the Mikheyev-Smirnov-Wolfenstein (MSW) [6, 7] effect and it has been extensively discussed in the literature (for a review, see [8]). However these discussions have always assumed that the neutrino's spin is a conserved quantity and can be neglected. Then the description for Majorana and Dirac neutrinos is identical. However when nonforward scattering occurs the helicity may change and then the descriptions of the two types are different. Here we take standard-model neutrinos with small Dirac masses and develop the relativistic quantum theory including simultaneously the matter background and the spin dependence.

Nonforward scattering of neutrinos occurs during the early Universe and during stellar collapse. In particular, the dynamics of the hot neutron star produced by stellar collapse have yielded useful limits on the mass of Dirac neutrinos. Massless standard-model neutrinos are trapped in a hot neutron star, however a massive Dirac neutrino can escape by flipping its spin during scattering to become a sterile, "right-handed" neutrino. Recent detailed calculations have yielded a limit on the mass of a Dirac muon or tau neutrino (i.e., no mixing) of approximately [2, 1, 9]

$$m < 28 \text{ keV}. \tag{1}$$

If mixing of the massive neutrino with the electron neutrino occurs, the bound on the mass becomes more severe [9]. This is because in the hot neutron star the electron-neutrino density is far greater than the muon- or tau-neutrino density, hence mixing increases the emission rate. In this paper the changes in the bound, Eq. (1), from neutrino mixing are estimated.

This limit has come under intense scrutiny of late since several nuclear decay experiments [10] indicate that there may be a neutrino with mass of 17 keV and 1% mixing with the electron neutrino. Cosmology constrains such a neutrino to decay much faster than the age of the Universe. More importantly, double-beta decay experiments limit the component of the electron neutrino's Majorana-type mass to be less than a few eV's (see, e.g., [11]), and so the simplest interpretation of the nuclear-decay results is that the 17-keV neutrino is of the Dirac type. Thus there is a conflict between these experiments and the supernova limit [12].

At the core of a hot neutron star, the background matter induces an effective electron-neutrino mass-squared difference of

$$A_e = (\,40\text{ keV})^2\left(\frac{Y_e}{0.35}\right)\left(\frac{E}{60\text{ MeV}}\right)\times\left(\frac{\rho}{5\times 10^{14}\text{ g/cm}^3}\right) \tag{2}$$

(N.B. the neutrino density is *not* included above, see Appendix). This is comparable to the mass limit found from spin flip, Eq. (1), and thus it is to be expected (from experience with the solar-neutrino flux) that the background matter will have a large effect on the neutrino mixing. In order to accurately calculate the rate for spin flip under these conditions we derive in Sec. II the fields for two mixed Dirac neutrinos in a matter background.

In Sec. III our expressions for the neutrino fields are applied to calculate scattering reactions relevant for neutrino spin flip in a hot neutron star. The rate for spin flip in neutrino-nucleus scattering is calculated for an arbitrary initial neutrino distribution. In addition, the rates for neutrino-electron and neutrino-neutrino scattering are calculated and a new contribution is identified. These results are then applied to calculate the generalization of the bound given in Eq. (1) to nonzero mixing angles.

In a hot neutron star, and at times in the early Universe, neutrinos make up a significant fraction of the total number density. Under such conditions, neutrinos forward scatter off of other neutrinos and the flavor evolution is in principle nonlinear. This topic has been discussed many times in the literature, however previous authors mistakenly neglected important terms in the induced mass. In an Appendix we give these terms and discuss their interpretation.

The emphasis here is on a Dirac neutrino with standard-model interactions. However many of the results are also valid if the neutrino is of the Majorana type. The difference between the two types is not discernable when only the negative helicities of a relativistic neutrino are involved. Accordingly, the expressions for the neutrino fields derived in Secs. II A and in the beginning of II B, and also the spin-flip rates derived in Sec. III C, are specific to Dirac neutrinos. All other discussions herein apply equally to Dirac or Majorana neutrinos.

II. THE FIELDS FOR DIRAC NEUTRINOS IN A BACKGROUND OF MATTER

This section addresses the basic issues of the form of the neutrino field in a matter background. In Sec. II A general kinematic issues are examined by calculating the field exactly for one neutrino flavor. In Sec. II B the issue of spin versus flavor mixing is examined by studying the more realistic case of two relativistic neutrinos in a matter background. In Sec. II C the physical situation of three neutrino flavors is discussed.

A. One neutrino flavor

In the standard model of weak interactions, neutrinos interact with other particles through exchange of W and Z bosons. This interaction is coherent for neutrinos forward scattering off of a background of normal matter. Assuming this background matter is unpolarized, and working in its rest frame, the fermion field bilinear of the background matter in the weak interactions can be replaced by the number density. Then the weak-interaction terms in the Lagrangian act like a potential for the evolution of a neutrino. For a Dirac neutrino in an unpolarized constant-density matter background the equations of motion are

$$\begin{aligned}&(\partial_\mu\gamma^\mu - V\gamma^0)\nu_L - m\nu_R = 0,\\ &\partial_\mu\gamma^\mu\nu_R - m\nu_L = 0,\end{aligned} \tag{3}$$

where ν_L and ν_R are the left-handed and right-handed chiral components of the Dirac neutrino field and m is the vacuum mass. V is the potential term and acts only on the left-handed field because of the chiral nature of the weak interactions. For a background of only electrons, and including only the charged-current interaction, $V = \sqrt{2}G_F N_e$ with N_e the number density of electrons and the weak-interaction constant $G_F = 1.16637 \times 10^{-5}$ GeV^{-2}. For a discussion of all the different contributions to the potential, see, e.g., Ref. [8] (and also the Appendix). Here we examine the exact solutions of Eq. (3).

Motivated by the chiral potential, we choose to work in the chiral representation of the γ matrices [13] where

$$\gamma^0 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \quad \boldsymbol{\gamma} = \begin{bmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{bmatrix}, \quad \gamma^5 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{4}$$

In this representation the neutrino field can be written as

$$\nu_L = \begin{bmatrix} 0 \\ \Phi \end{bmatrix}, \quad \nu_R = \begin{bmatrix} -\chi \\ 0 \end{bmatrix}, \tag{5}$$

where Φ and χ are two component fields. Because the medium is assumed to be uniform and unpolarized, linear and angular momentum will be conserved quantities. Thus to derive expressions for Φ and χ we expand them in terms of the helicity eigenstates for the positive- and negative-energy states [14, 15]. The helicity operator is $\boldsymbol{\sigma}\cdot\boldsymbol{k}$ where $\boldsymbol{\sigma}$ is the spin and $\mathbf{k}$ the three-momentum. In vacuum the helicity eigenstates are degenerate; however, this is not the case in a matter background since the weak interactions are chiral. The positive- and negative-helicity eigenstates for the particle and antiparticle must each be considered explicitly. The equations of motion yield relations among the coefficients of these states. The overall normalization of the coefficients is determined by constructing a bilinear of the fermion fields, for example, the Hamiltonian, and requiring it to have the canonical form. This procedure completely determines the expression for the fermion fields up to some overall phases.

The expressions for Φ and χ are found to be, in the usual particle-antiparticle formalism [16]

$$\begin{aligned}\Phi = \sum_{\mathbf{k}} \Bigg(&\frac{(E_+ - k)}{\sqrt{m^2 + (E_+ - k)^2}} a_+ \alpha(\mathbf{k}) e^{(-iK_+\cdot x)} - \frac{(\bar{E}_+ + k)}{\sqrt{m^2 + (\bar{E}_+ + k)^2}} d_+^\dagger \beta(\mathbf{k}) e^{(i\bar{K}_+\cdot x)} \\ &+ \frac{(E_- + k)}{\sqrt{m^2 + (E_- + k)^2}} a_- \beta(\mathbf{k}) e^{(-iK_-\cdot x)} - \frac{(\bar{E}_- - k)}{\sqrt{m^2 + (\bar{E}_- - k)^2}} d_-^\dagger \alpha(\mathbf{k}) e^{(i\bar{K}_-\cdot x)} \Bigg),\end{aligned} \tag{6}$$

$$\chi = \sum_{\mathbf{k}} \left(\frac{m}{\sqrt{m^2 + (E_+ - k)^2}} a_+ \alpha(\mathbf{k}) e^{(-iK_+ \cdot x)} + \frac{m}{\sqrt{m^2 + (\bar{E}_+ + k)^2}} d_+^\dagger \beta(\mathbf{k}) e^{(i\bar{K}_+ \cdot x)} + \frac{m}{\sqrt{m^2 + (E_- + k)^2}} a_- \beta(\mathbf{k}) e^{(-iK_- \cdot x)} - \frac{m}{\sqrt{m^2 + (\bar{E}_- - k)^2}} d_-^\dagger \alpha(\mathbf{k}) e^{(i\bar{K}_- \cdot x)} \right),$$

where $a_\pm$ ($a_\pm^\dagger$) and $d_\pm$ ($d_\pm^\dagger$) are the usual annihilation (creation) operators for the $\pm$ helicity of the neutrino and antineutrino, respectively. $\alpha(\mathbf{k})$ and $\beta(\mathbf{k})$ are two-component helicity-eigenstate spinors defined such that

$$\boldsymbol{\sigma} \cdot \boldsymbol{k}\alpha(\mathbf{k}) = k\alpha(\mathbf{k}), \quad \alpha^\dagger(\mathbf{k})\alpha(\mathbf{k}) = 1,$$
$$\boldsymbol{\sigma} \cdot \boldsymbol{k}\beta(\mathbf{k}) = -k\beta(\mathbf{k}), \quad \beta^\dagger(\mathbf{k})\beta(\mathbf{k}) = 1, \tag{7}$$

or, explicitly for $\mathbf{k} = k(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ (caution: do not confuse these spatial rotations with later neutrino-mixing angles)

$$\alpha(\mathbf{k}) = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2)e^{i\phi} \end{bmatrix},$$
$$\beta(\mathbf{k}) = \begin{bmatrix} \sin(\theta/2) \\ -\cos(\theta/2)e^{i\phi} \end{bmatrix}. \tag{8}$$

Note that for neutrinos, $\mathbf{S} = \boldsymbol{\sigma}/2$ and $\alpha(\mathbf{k})$ and $\beta(\mathbf{k})$ are the positive- and negative-helicity eigenstates, respectively, however for antineutrinos the spin operator is $\mathbf{S} = -\boldsymbol{\sigma}/2$ so that then $\alpha(\mathbf{k})$ and $\beta(\mathbf{k})$ are the negative- and positive-helicity eigenstates, respectively. The neutrino and antineutrino four-momentum of the $\pm$ helicity are defined to be $K_\pm = (E_\pm, \mathbf{k})$ and $\bar{K}_\pm = (\bar{E}_\pm, \mathbf{k})$, respectively. The energy-momentum relations are found to be

$$E_+ = V/2 + \sqrt{(k - V/2)^2 + m^2},$$
$$E_- = V/2 + \sqrt{(k + V/2)^2 + m^2},$$
$$\bar{E}_+ = -V/2 + \sqrt{(k - V/2)^2 + m^2},$$
$$\bar{E}_- = -V/2 + \sqrt{(k + V/2)^2 + m^2}, \tag{9}$$

where $E_\pm(\bar{E}_\pm)$ denote the energy of the $\pm$ helicity neutrino (antineutrino) and k is the magnitude of the three-momentum.

Equation (6) agrees with expectations for the relativistic limit. Then the left-handed chiral field, Φ, is dominated by the negative-helicity neutrino and the positive-helicity antineutrino (and the reverse holds for the right-handed chiral field, χ). In fact, using Eqs. (9) in calculating the relativistic limit, it turns out that Eq. (6) becomes independent of the potential, V, to leading order. In part, this is because it is the vacuum mass term which connects the different degrees of freedom together through Eq. (3). This observation has an important physical implication—the potential is irrelevant for the kinematics of relativistic neutrinos. In particular, in the dense matter of a supernova core, the neutrino spin-flip cross section is independent of the potential, V, to leading order in the relativistic limit [16].

It is amusing to note that some unusual behavior occurs for nonrelativistic neutrinos [17]. Equations (9) show that the minimum energy corresponds to a nonzero value for the momentum. This can be easily understood qualitatively since a small momentum allows definition of the helicity and hence then the energy can be lowered considerably by the potential V. One implication of this is that velocity and momentum are no longer strictly proportional to each other. The flux density of a negative-helicity neutrino, $\mathbf{v} = \bar{\nu}_- \gamma\, \nu_-$, can be calculated using Eqs. (6) to yield

$$|\mathbf{v}| = \frac{(k + V/2)}{\sqrt{m^2 + (k + V/2)^2}}. \tag{10}$$

Thus for a neutrino in a medium with $V > 0$, if the momentum is small compared to $V/2$ the neutrino will move in the direction opposite to its spin with a velocity independent of the momentum. Similarly, for a neutrino to be at rest in a medium it must have a nonzero momentum.

B. Two neutrino flavors, relativistic approximation

In the previous section it was demonstrated that a matter background has only small effects on the kinematics of a relativistic neutrino. However it is well known that a matter background has a large influence on the mixing (see, e.g., [8]). Briefly summarizing the previous findings on mixing, a relativistic, negative-helicity electron neutrino has a large "induced mass squared," A, given in Eq. (2). However the electron neutrino is a flavor eigenstate and this is generally different than a mass eigenstate when the neutrinos have vacuum masses. The mixing between the flavor and mass eigenstates is sensitive to the induced mass term. When the induced mass is comparable to the difference between the vacuum masses squared, a resonance occurs. At the resonance the mixing is maximal, while for much larger A the mixing is suppressed. Previously, these results have always been derived by assuming that the neutrinos' spin is fixed and using an effective Klein-Gordon equation of motion [6, 4, 18, 14]. Here we shall keep the spin dependence and study mixing using the Dirac equations of motion.

To simplify the discussion of two Dirac neutrinos in a background of matter, we shall assume that only one neutrino is massive. By neglecting the mass of the lighter neutrino it is then described by only a single Weyl field since the right-handed component of this neutrino decouples. This approximation is probably realistic since known fermion masses exhibit a strong hierarchy. Then the equations of motion, in the chiral representation of the *vacuum* mass basis, are

$$\begin{aligned}
&(i\partial_0 - i\sigma\cdot\nabla - V_n)\Phi_{1v} - V\cos\theta[\cos\theta\Phi_{1v} + \sin\theta\Phi_{2v}] = 0,\\
&(i\partial_0 - i\sigma\cdot\nabla - V_n)\Phi_{2v} - m\chi - V\sin\theta[\cos\theta\Phi_{1v} + \sin\theta\Phi_{2v}] = 0,\\
&(i\partial_0 + i\sigma\cdot\nabla)\chi - m\Phi_{2v} = 0,
\end{aligned} \tag{11}$$

where Φ_{1v} andΦ_{2v} are the left-handed two-component neutrino spinors and χ is the right-handed two-component neutrino spinor (see Eq. (5) and also Ref. [14]). m is the vacuum mass term which connects the Φ_{2v} and χ degrees of freedom to form the massive Dirac neutrino. Φ_{1v} has no vacuum mass but is mixed with Φ_{2v} by the weak interaction with the background matter. The potentials V_n and V are the neutral-current and electron-neutrino charged-current potentials, respectively. θ is the vacuum mixing angle

$$\begin{bmatrix}\Phi_{1v}\\ \Phi_{2v}\end{bmatrix} = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\Phi_e\\ \Phi_\mu\end{bmatrix} \tag{12}$$

between the mass eigenstates and the flavor eigenstates, where the latter are denoted by subscripts e and μ. These equations of motion are the two-flavor generalization of Eq. (3).

The procedure for calculating the fields is analogous to that described in Sec. II A. Assuming the background matter density is constant (so that the potentials are constants), the solution to the equations of motion is written as a sum over plane waves. The physical mass eigenstates will be the helicity eigenstates. For one massive Dirac neutrino and one massless neutrino there are two negative-helicity and one positive-helicity particle degrees of freedom (in vacuum *and* in a matter background). The equations of motion yield relations among the coefficients of these degrees of freedom. The normalization of the coefficients is determined by constructing the Hamiltonian. To further simplify the situation I assume that the neutrinos are relativistic. This is a very realistic assumption for applications to the supernova or the early Universe. Then it turns out that the expression for the fields in the flavor basis is

$$\begin{aligned}
\Phi_e = \sum_{\mathbf{k}}\Bigg(&\sin\theta\frac{m}{2k}[a_+\alpha(\mathbf{k})e^{(-iK_+\cdot x)} - d_-^\dagger\alpha(\mathbf{k})e^{(\,i\bar{K}_-\cdot x)}] + (\sin\bar{\theta}_m d_{2+}^\dagger e^{(\,i\bar{K}_{2+}\cdot x)} + \cos\bar{\theta}_m d_{1+}^\dagger e^{(\,i\bar{K}_{1+}\cdot x)})\beta(\mathbf{k})\\
&+(\sin\theta_m a_{2-}e^{(-iK_{2-}\cdot x)} - \cos\theta_m a_{1-}e^{(-iK_{1-}\cdot x)})\beta(\mathbf{k})\Bigg),\\
\Phi_\mu = \sum_{\mathbf{k}}\Bigg(&\cos\theta\frac{m}{2k}[a_+\alpha(\mathbf{k})e^{(-iK_+\cdot x)} - d_-^\dagger\alpha(\mathbf{k})e^{(\,i\bar{K}_-\cdot x)}] + (\cos\bar{\theta}_m d_{2+}^\dagger e^{(\,i\bar{K}_{2+}\cdot x)} - \sin\bar{\theta}_m d_{1+}^\dagger e^{(\,i\bar{K}_{1+}\cdot x)})\beta(\mathbf{k})\\
&+(\cos\theta_m a_{2-}e^{(-iK_{2-}\cdot x)} + \sin\theta_m a_{1-}e^{(-iK_{1-}\cdot x)})\beta(\mathbf{k})\Bigg).
\end{aligned} \tag{13}$$

The expression for the χ field is not given since it is not relevant for neutrino scattering. Here the notation is the same as in the one-flavor case but with the subscripts 1 and 2 denoting the lower and upper mass eigenstates in the *matter* background, respectively. θ_m is the mixing angle in matter for neutrinos, which is well known from earlier studies of neutrino propagation through matter

$$\sin^2(2\theta_m) = \frac{(m^2\sin 2\theta)^2}{[(A - m^2\cos 2\theta)^2 + (m^2\sin 2\theta)^2]}. \tag{14}$$

For $A \ll m^2$ vacuum parameters dominate and $\theta_m \approx \theta_{\text{vacuum}}$; for $A \approx m^2$ a resonance occurs and $\theta_m \approx \pi/4$; and for $A \gg m^2$ flavor effects dominate so $\theta_m \approx \pi/2$ and mixing is suppressed. The $\bar{\theta}_m$ is the mixing angle in matter for antineutrinos and is also given by Eq. (14), but with $A = 2Vk \to -A$. For antineutrinos, $\theta_{\text{vacuum}} \geq \bar{\theta}_m > 0$.

The kinematics of the neutrino are independent of the matter background, to leading order in the relativistic limit, but the second-order terms are important. While solving the equations of motion, the energy and momentum relations are determined

$$\begin{aligned} E_+ &= k + m^2/2k + \cdots , \\ \bar{E}_- &= k + m^2/2k + \cdots , \\ E_{-;1,2} &= k + V_n + M^2_{1,2}(A)/2k + \cdots , \\ \bar{E}_{+;1,2} &= k - V_n + M^2_{1,2}(-A)/2k + \cdots , \end{aligned} \tag{15}$$

where we keep the next-to-leading order terms in an expansion of V_n/k, V/k, or $(m/k)^2$. The quantity M_i^2 is the mass eigenvalue in matter of the ith state and it has the same form found in previous analysis (see, e.g., [8]) of matter-dependent mixing

$$M^2_{1,2}(A) = [m^2 + A \mp \sqrt{(A - m^2\cos 2\theta)^2 + (m^2 \sin 2\theta)^2}\ ]/2, \tag{16}$$

where $A \equiv 2Vk$. The minus and plus signs correspond to the 1 and 2 states, respectively, so that $M_2^2 > M_1^2$ for positive and negative A.

The next-to-leading-order terms given in Eq. (15) are typically not important when integrating over phase space in calculating neutrino scattering rates. However they are important in determining the proper mixing states for discussing scattering. The nonforward-scattering length scale is the mean-free path, $L_{\text{scatt}} = 1/(\sigma N)$, where in a supernova core, N is the nucleon number density and σ is the cross section, of order $G_F^2 E_\nu^2$. The length scale which determines when two mixed states are separable is the oscillation wavelength, $L_{\text{osc}} = 4\pi k/[M_2^2 - M_1^2]$. However this length scale is *always* far shorter than the neutrino scattering length

$$L_{\text{osc}} <<< L_{\text{scatt}}. \tag{17}$$

This is because typically L_{osc} is less than or of order the forward scattering length scale $4\pi k/A$ and hence Eq. (17) is equivalent to $G_F E^2 \ll 1$ which is well satisfied for E's typical to supernovae (N.B. in early universe applications $|N_e - \bar{N}_e| \ll N_e$ and this may not hold). Equation (17) has an important implication. The relative phase between the mass eigenstates that is acquired during propagation between nonforward scatterings is very, very large. Hence with many neutrinos this large phase averages out and the mass eigenstates in matter can be taken to be incoherent when they scatter. Thus when discussing scattering in a uniform matter background the rates are calculated using the neutrino mass eigenstates as the physical initial and/or final states.

For three neutrino flavors, the situation is only slightly different. The separation of the mass eigenstates which couple dominantly to ν_μ and ν_τ can be much smaller than A, the electron-neutrino induced mass-squared difference. If there is no net muon density, then the relative forward scattering potential between these neutrinos is generated by radiative corrections and, as calculated by Botella, Lim, and Marciano [19], is

$$V_\tau - V_\mu = \frac{3G_F^2 m_\tau^2}{2\pi^2}\left[(N_p + N_n)\ln\left(\frac{M_W^2}{m_\tau^2}\right) - (N_p + \tfrac{2}{3}N_n)\right]. \tag{18}$$

In evaluating Eq. (18) for the conditions found in a supernova core, we find that Eq. (17) still typically holds because m_τ is much larger than neutrino energies therein. If there is a net muon density, it will typically generate an even larger difference between these mass eigenstates in matter. Thus it is proper to take all three of the neutrino mass eigenstates in matter to be the incoherent, asymptotic, physical states when calculating scattering rates.

The main result of this section is Eq. (13). These equations are easy to justify, *a posteriori*. For the negative-helicity neutrinos, the mixing between the flavor basis and the mass-eigenstates basis is the same function of the matter background, Eq. (14), as found in previous analyses of the MSW effect. For the relativistic, positive-helicity neutrinos, the matter background does not directly affect these particles so the mixing between the mass eigenstates and the flavor eigenstates remains the vacuum mixing. Thus, for example, the probability of an electron-neutrino interaction producing a positive helicity neutrino is just $(\sin\theta\ m/2k)^2$, independent of the matter background.

The derived neutrino fields in matter, Eqs. (13), can be directly applied to calculate scattering rates in matter. One interesting, general result emerges when they are applied to neutrino scattering via the weak neutral current. The form of this interaction is

$$\begin{aligned} L_n &\propto Z_\mu \left(\sum_a \bar{\nu}_{aL}\gamma^\mu \nu_{aL}\right) \\ &\propto Z_\mu \left[\sum_a \Phi_a^\dagger G^\mu \Phi_a\right], \end{aligned} \tag{19}$$

where G^μ are two-by-two matrices and the sum above is over the *flavor* or *vacuum mass* eigenstates. However when expressed in the background matter mass-eigenstate basis, the physical basis for neutrinos in uniform matter, the interaction *does not* generally have the form of Eq. (19). This is because the mixing between the flavor and mass eigenstates of the negative helicity neutrinos is momentum dependent. If the two neutrino fields in Eq. (19) have different momentum, then each must be rotated by a different amount to become mass eigenstates in matter. The two rotations will in general leave off diagonal neutral-current terms. Using Eq. (13), the mixing amplitudes for scattering from one neutrino mass eigenstate into another via the neutral current off of some particle $\mathbf{Q}$, $\nu_{x-}(k) + Q \to \nu_{y-}(k') + Q$, is

$$U^\dagger(k')U(k) = \begin{bmatrix} \cos[\theta_m(k) - \theta_m(k')] & \sin[\theta_m(k) - \theta_m(k')] \\ -\sin[\theta_m(k) - \theta_m(k')] & \cos[\theta_m(k) - \theta_m(k')] \end{bmatrix}. \tag{20}$$

This has been conjectured previously [20] and now with the derivation of the fields herein it is proved.

C. Three neutrino flavors

It is known that there are three flavors of light neutrinos. In the absence of direct experimental evidence, it will be assumed herein that there is a hierarchy of neutrino masses with small mixing angles, in analogy with that observed for the charged fermions. Assuming that only two neutrino flavors mix, as done in the previous section, is an approximation that may not always be accurate. The mixing of three negative-helicity neutrino flavors in a background of normal matter has been discussed in the literature on solar neutrinos (for a list of references see [8]). However in the core of a supernova, new effects not covered in previous discussions are relevant.

For three flavors, the mixing between the flavor and mass eigenstates in matter of the negative helicity neutrinos is determined by the effective Klein-Gordon mass matrix. In its most general form,

$$M^2 = U \begin{bmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{bmatrix} U^\dagger + \begin{bmatrix} A_e & & \\ & A_\mu & \\ & & A_\tau \end{bmatrix} . \qquad (21)$$

The notation used here is that of Refs. [21, 8]. U is a 3×3 vacuum mixing matrix analogous to the Cabibbo-Kobayashi-Maskawa matrix, m_i denotes the vacuum masses, and the $A_\alpha = 2kV_\alpha$'s are mass-squared terms induced by the matter background. $A_e = 2k\sqrt{2}G_F N_e$ is as given in Eq. (2) and comes from charged-current forward scattering of neutrinos off of electrons in the matter background. $A_\mu = 2k\sqrt{2}G_F N_\mu$ denotes the similar induced mass term from charged current forward scattering off of the muon excess in the matter background. A_τ denotes the contribution from radiative corrections given in Eq. (18). Diagonal contributions to the mass matrix, as from neutral-current scattering, are irrelevant so only differences in the A_α's are important. For the general case, Eq. (21) must be diagonalized to find the masses and mixings in a background of matter.

In discussions of solar neutrinos, only A_e is relevant in Eq. (21). In the Sun, A_μ vanishes since there is no muon excess there. A_τ is present for solar neutrinos but negligible since it corresponds to a length scale far larger than the Sun's radius. However for neutrinos in dense matter, A_μ and A_τ may be relevant. A_τ is smaller than A_e by roughly five orders of magnitude but, as discussed in the two-flavor section, the oscillation wavelength from this term is still small compared to the nonforward scattering length. A_μ may become comparable to A_e in a supernova core since typical lepton chemical potentials are larger than the muon mass (and a large muon excess can be produced through neutrino mixing). In general, all three of the contributions to the induced mass; the electron background, the muon background, and the radiative μ-τ corrections, can be relevant for discussions of neutrino mixing in dense matter.

A general discussion of three-flavor effects for arbitrary neutrino masses is quite involved and beyond the scope of the present work. Herein we shall make the assumption that $m_2^2 - m_1^2 \ll A_\mu - A_\tau$ and so it is a good approximation to take $m_1^2 = m_2^2 = 0$. Then the general 3×3 mixing matrix may be written as

$$U = \begin{bmatrix} 0 & C_\phi & S_\phi \\ -C_\psi & -S_\psi S_\phi & S_\psi C_\phi \\ S_\psi & -C_\psi S_\phi & C_\psi C_\phi \end{bmatrix} , \qquad (22)$$

where ϕ and ψ are two mixing angles, and S_a and C_a denote $\sin a$ and $\cos a$, respectively. From Eq. (21), U is defined to rotate the mass eigenstates into the flavor eigenstates, $|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle$, where $\alpha = e, \mu, \tau$ denote the flavor eigenstates and i =1, 2, 3 denote the mass eigenstates. The ordering of mass eigenstates is chosen such that $m_3^2 > m_2^2 \geq m_1^2$ for all $A_e > A_\mu$. With $m_1^2 = m_2^2 = 0$, one linear combination of these states may be chosen orthogonal to the electron neutrino, hence the zero element in Eq. (22).

In matter with $A_\mu \ll A_e$, it is possible to obtain simple, approximate expressions for ϕ_m and ψ_m. The problem is then formally equivalent to the case of two nonzero vacuum masses with only one induced mass, which has been solved in [21]. Using this previous solution it is easy to see that there are two possible two-flavor resonances, an "e-τ" resonance when $m_3^2 \approx A_e$ and a "μ-τ" resonance when $m_3^2 \approx A_\mu$. The expression for ϕ_m is given by Eq. (14) with $\theta \to \phi, A \to A_e$, and $\Delta \to m_3^2$. The expression for ψ_m is given by Eq. (14) with $\theta \to \psi, A \to A_\mu$, and $\Delta \to m_3^2 C_\phi^2$. Thus when there is only one relevant neutrino vacuum mass, there are two mixing angles which are important, describing "e-τ" and "μ-τ" mixings, and in matter they depend on the background electron and muon densities, respectively.

III. SCATTERING IN A SUPERNOVA CORE

Here the results of Sec. II, especially Eq. (13), are applied to neutrinos in the core of a supernova. The techniques necessary to develop a quantitative description of core dynamics with neutrino mixing are illustrated. The constraint on Dirac neutrino masses from spin flip is estimated for various neutrino masses and mixings.

A. A supernova core without neutrino mixing

After stellar collapse, the core of supernova SN1987A is thought to have formed a young, hot neutron star. The properties of such objects are discussed in the astrophysical literature (see, e.g., [22, 23]). Briefly, the densities therein are typical nuclear densities over most of the core, of order 5×10^{14} g/cm^3. However neutrinos are trapped in matter for densities down to 3×10^{11} g/cm^3 because at lower densities the nucleons are still in large nuclei which have large elastic-scattering cross sections. The initial core temperature is expected to be somewhere in the range from 10 to 40 MeV. The nucleons are semidegenerate with their chemical potentials minus the rest mass comparable to the temperature. The ratio of leptons to nucleons, Y_l, is initially about 0.35 and (for Dirac neutrinos) this only changes as neutrinos carry away lepton number by leaving the core. In the standard picture, neutrino emission is a very slow process—the neutrinos diffuse out of the hot neutron star over several seconds. This is consistent with the observed neutrino pulse from SN1987A [24]. Thus the electron chemical potential is

$$\mu_e = 290 \text{ MeV}\left[\left(\frac{Y_e}{0.35}\right)\left(\frac{\rho}{5\times 10^{14}\text{ g/cm}^3}\right)\right]^{1/3} \tag{23}$$

much larger than the temperature and so the electrons, and other leptons which are in "chemical" equilibrium with the electron, form degenerate Fermi-Dirac gases.

When there is no neutrino mixing, only the electron and the electron neutrino share the core's lepton number. If the number densities were equal, the chemical potentials of the two particles would be about 230 MeV. However the electrons have electric charge and balance the protons' charge while the neutral neutrinos cannot do this. Thus whether complete transfer of lepton number is achieved or not depends on details of nuclear statistical equilibrium (see, e.g., [22]). In numerical simulations of the hot neutron star [23], the electron-neutrino chemical potential in the core is somewhat less than this value.

In the calculations below it is assumed that the core has constant and uniform density and temperature. Such an approximation may be quite unrealistic—especially for considerations of neutrino mixing in matter. Diffusion (and convection) carries neutrinos through varying density and greatly enhances neutrino species mixing. However the estimates below using constant density should give conservative values for when neutrino species share the lepton number in a supernova core. It is intended that this discussion clarifies how to handle neutrino mixing in dense matter for subsequent numerical treatments of a supernova core.

B. Distribution of lepton number for mixed neutrinos

In general, for Dirac neutrinos, the total lepton number is the only conserved quantity. Thus it is in principle (and often in practice) possible for the electron, all three neutrinos, and the muon to share the lepton number in the core of a neutron star. If and how this comes about is determined by the reactions in the core which change one lepton into another. A particular lepton must be produced on a time scale faster than the neutrino diffusion time for it to share the core's lepton number. In this section some of the relevant reactions are examined.

When neutrinos mix, how we must describe the neutrinos changes. As discussed in Sec. II, the physical basis for describing neutrino scattering in dense matter is the mass-eigenstate basis. For vanishing vacuum mixing angles, the flavor eigenstates are equivalent to the mass eigenstates. However this equivalence is a little strange for massive neutrinos—a flavor eigenstate does not always correspond to the same mass eigenstate. At a resonance, the approximate identification between the mass eigenstates in matter and the interaction eigenstates flips. Hence the reaction rates of the mass eigenstates are strongly energy dependent—approaching step functions in energy at a resonance. They are also quite different between neutrinos and antineutrinos. Because of this, *it is no longer generally possible to make a distinction between chemical and thermal equilibrium* when describing how mixed neutrinos approach equilibrium. Typically, a particular mass eigenstate in matter comes into equilibrium at many different rates, and the rates for different mass eigenstates are analogous.

This means that it is incorrect to take different chemical potentials for different neutrino mass eigenstates and then watch how they evolve in time. The phase-space distribution will generally be quite far from Fermi-Dirac, and in calculations where the relative lepton number of different neutrino species changes in time, the energy dependence of the neutrino number densities must be kept explicitly (at each matter density). Only after a long enough time, such that all of the neutrino phase-space densities are identical, will the common distribution be Fermi-Dirac. There may be special limiting cases, when the neutrino resonance energy is much larger or much smaller than the chemical potential, where an approximate intermediate situation exists and a particular partial mass eigenstate can be described by a single chemical potential. However one must be careful since the resonance energy depends on the electron density and so this approximation may not hold over the whole supernova core. In general, the energy dependence of the neutrino number densities should be kept explicitly (at each matter density).

1. $e+p\to\nu_{i-}+n$

The fastest initial reaction that creates new lepton flavors is typically [9, 22] the neutronization process, $e+p\to\nu_i+n$. This is a purely charged-current reaction involving only the electron-neutrino field so the mixing factor for producing a neutrino is just $|U_{ei}|^2$ where the subscript i denotes the ith mass eigenstate in matter. For free nucleons and unblocked neutrinos, the rate equation is

$$\frac{d^2Y_i}{dE_i dt} = (10^9/\text{sec})\frac{E_i^4}{\mu_e^5/5}|U_{ei}|^2(E_i). \tag{24}$$

The neutrino's energy dependence is shown explicitly and μ_e is as given in Eq. (23).

From the three-flavor mixing matrix given in Eq. (22), we see that only the neutrino mass eigenstates 2 and 3 can be produced in this reaction. This is because we took the lightest two vacuum masses to vanish so one linear combination of these states can be chosen orthogonal to the electron neutrino. For $m_3^2 \gg A_e$, the ϕ_m reduces to the vacuum angle, ϕ. The mass-eigenstate-2 neutrino is dominantly the electron neutrino and is produced quickly while the mass-eigenstate-3 neutrinos will share the core's lepton number for

$$\sin^2\phi > 2\times 10^{-10}. \tag{25}$$

However since A_e given by Eq. (2) is generally larger than the mass given in Eq. (1), the more relevant limit

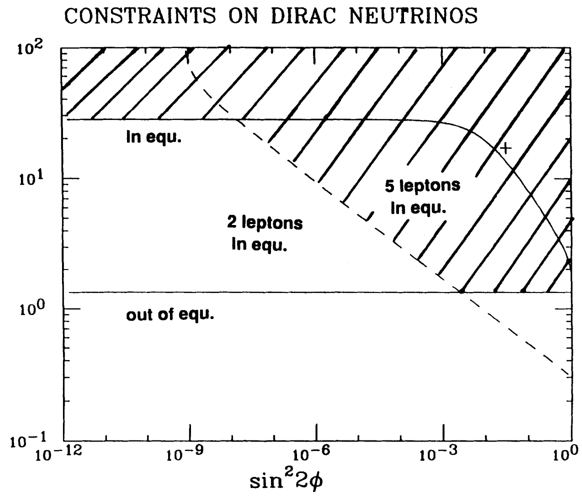

FIG. 1. Plot of vacuum mass versus vacuum "e-τ" mixing for a Dirac neutrino in a supernova core. The dashed contour is an estimate of when all leptons share the core's lepton number. The solid lines indicate bounds on the mass from neutrino energy loss for various assumptions about equilibrium. The cross denotes a 17-keV neutrino with 1% mixing. The shaded region is estimated to be excluded.

for finding the lower bounds on Dirac masses from the spin-flip process is $A_e \gg m^2$. In this limit, the i=1 mass eigenstate is the dominantly nonelectron neutrino, and using that $\cos^2\phi_m \approx [(m_3^2 \sin 2\phi)/(2A_e)]^2$ from Eq. (14), the equilibrium condition is

$$\left(\frac{m_3^2}{40 \text{ keV}}\right)^4 \sin^2 2\phi > 3 \times 10^{-9}. \quad (26)$$

The conditions assume a neutrino diffusion time scale of 1 sec. These equations determine the vacuum mass and mixing-angle parameters for which the electron and two neutrinos will be in equilibrium and share the core's lepton number. These constraints are plotted in Fig. 1 as a dashed line.

2. $\nu_{i-} + n \to p + \mu$

The neutronization reaction transfers lepton number from the electrons to the 2 and 3 neutrino mass eigenstates. The reaction discussed here will subsequently transfer lepton number to the muon. The typical chemical potential, Eq. (23), is larger than the muon mass and when the electron and muon chemical potentials are equal the muon number density will be about $\frac{1}{2}$ the electron number density.

For free nucleons and unblocked muons, the rate equation is

$$\frac{d^2Y_\mu}{dE_\nu dt} = (5 \times 10^8/\text{sec}) \frac{E_\nu^3 \sqrt{E_\nu^2 - m_\mu^2}}{2.1\mu_\mu^5} |U_{\mu i}|^2 (E_\nu), \quad (27)$$

where Y_μ is the muon fraction, m_μ the muon mass, and $U_{\mu i}$ the mixing matrix element between the muon neutrino flavor state and the ith neutrino mass eigenstate in matter, as given in Eq. (22). μ_μ represents the chemical potential of the initial neutrino and is taken to be the same as μ_e given in Eq. (23). We shall assume that the neutronization reactions have been fast enough, Eqs. (25) and (26), to bring the 2 and 3 neutrino mass eigenstates in matter into equilibrium so that they have equal, large lepton numbers. Then $\nu_{i-} + n \to p + \mu$ will increase the net lepton numbers of muons when

$$\sin^2 \psi > 2 \times 10^{-10} \quad (28)$$

for $A_\mu \ll m_3^2$. When the muon density becomes large such that $A_\mu \gg m_3^2$ then this mixing factor is suppressed. Assuming that the muon chemical potential becomes comparable to the electron chemical potential, ignoring muon blocking of the final state, but including the mixing matrix element suppression, the equilibrium condition becomes

$$\left(\frac{m_3^2}{40 \text{ keV}}\right)^4 \sin^2 2\psi > 1 \times 10^{-8}. \quad (29)$$

Initially there is no muon background so the rate is unsuppressed and Eq. (28) is applicable. However as the muon number density becomes large and comparable to the electron number density the rate is eventually suppressed and then Eq. (29) is comparable to that for neutronization, Eq. (26).

As the muon lepton number grows, the lepton number of the 1 neutrino mass eigenstate in matter grows with it. This state is dominantly muon neutrino with no electron-neutrino component. Thus there is no small mixing angle suppression at all for converting muons into this neutrino via the neutronization reaction $\mu + p \to \nu_{i-} + n$.

The large electron number density suppresses the rate for producing nonelectron neutrinos, Eq. (26). However once these neutrinos are produced, they can in turn produce muons without this suppression and the muons in turn quickly produce the missing neutrino state. Thus the first step is the bottleneck in the process of transferring lepton number from the electron to the other leptons. It will typically be the case that either only two leptons share the lepton number, the electron and the dominantly electron neutrino, or all five leptons share the lepton number, the electron, the muon, and all three neutrinos. The situation is summarized in Fig. 2.

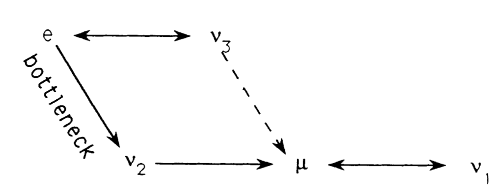

FIG. 2. Diagram describing how lepton number is transferred from electrons to the other leptons from charged-current interactions with nucleons. The neutrino-number assignment assumes that $A_e \gg m^2$. The dashed rate is a possible reaction that is extra suppressed.

3. $\nu_{i-} + e \to \nu_{j-} + e$

Neutrino-electron scattering also contributes to bringing neutrinos into equilibrium. The nominal rate for this process is about 2–3 orders of magnitude smaller than the nominal neutrino-nucleon charged-current reaction rate. In part, this is because only a small amount of phase space is available to the final-state electron. However the neutronization rate may eventually be blocked by the scarcity of free protons and by the neutron shell blocking of electron capture on heavy mass nuclei [22]. Thus this reaction must be included in a detailed treatment of how neutrinos are brought into equilibrium.

The mixing matrix elements, and the neutrino energy dependence, of this reaction are different from those used in the neutronization process. Both of these effects may make neutrino-electron scattering important for neutrino equilibrization. When calculating the rates it is vital to notice that because the neutrino energy changes substantially in this reaction, the neutral-current amplitude can change the neutrino's species [see Sec. II, Eq. (20)] in addition to the species changing from the charged current. To illustrate the nondiagonal neutral current, we assume only two neutrino flavors and find the matrix element for $\nu_{2-} + e \to \nu_{1-} + e$ scattering to be proportional to

$$(\{\sin\theta_m(k)\cos\theta_m(k') + (-\tfrac{1}{2}+x)\sin[\theta_m(k)-\theta_m(k')]\}^2(\mathbf{k}\cdot\mathbf{q})^2 + x^2\sin^2[\theta_m(k)-\theta_m(k')](\mathbf{k}\cdot\mathbf{q}')^2), \tag{30}$$

where $x \equiv \sin^2\theta_{\rm weak} = 0.22$, $\mathbf{k}$ and $\mathbf{k}'$ are the initial and final neutrino momenta, and $\mathbf{q}$ and $\mathbf{q}'$ are the initial and final electron momenta. For $A_e = 2V_e k \gg m^2$, $\theta_m(k) \approx \pi/2 - [(m^2\sin 2\theta)/(2A_e)]$, and assuming both A_e and $A'_e \gg m^2$, then this matrix element can be approximated as

$$\left(\frac{m^2\sin 2\theta}{2A'_e}\right)^2\left\{\left[1+\left(-\frac{1}{2}+x\right)\left(1-\frac{k'}{k}\right)\right]^2(\mathbf{k}\cdot\mathbf{q})^2 + x^2\left(1-\frac{k'}{k}\right)^2(\mathbf{k}\cdot\mathbf{q}')^2\right\}. \tag{31}$$

The terms which vanish when $k = k'$ come from the neutral current and the remaining term comes from the charged current. In general both currents contribute to bringing neutrinos into full equilibrium.

The species-changing nature of the neutral-current scattering is only relevant when the flavor of the final neutrino is to be measured after the scattering. If one sums the cross section over all possible final-state neutrino species, the cross section becomes independent of the final neutrino basis.

4. $\nu_{i-} + \nu_{i-} \to \nu_{j-} + \nu_{k-}$

The large densities of negative helicity neutrinos means that neutrinos will also scatter off of other neutrinos. This scattering is mediated by the neutral-current weak interaction however [see Sec. II, Eq. (20)] the mass eigenstate can change because the scattering is in a background of matter. The matrix element squared for $\nu_{2-} + \nu_{2-} \to \nu_{1-} + \nu_{1-}$ is given by

$$|M_{2,2\to1,1}|^2 = G_F^2 32(\mathbf{p}\cdot\mathbf{r})(\mathbf{q}\cdot\mathbf{k})\{\sin[\theta_m(p)-\theta_m(q)]\sin[\theta_m(r)-\theta_m(k)] + \sin[\theta_m(p)-\theta_m(k)]\sin[\theta_m(r)-\theta_m(q)]\}^2, \tag{32}$$

where $\mathbf{q}$ and $\mathbf{k}$ ($\mathbf{p}$ and $\mathbf{r}$) are the initial (final) neutrino four-momentum.

In vacuum, $\theta_m = \theta$ and this process vanishes as expected since then the neutral current is diagonal. The matrix element is a function of differences of mixing angles because of the symmetries of the neutral current. The neutral current is covariant under a *constant* rotation by angle ξ and hence the matrix element is invariant under $\theta_m \to \theta_m + \xi$.

The matrix element squared for $\nu_{2-}+\nu_{2-} \to \nu_{2-}+\nu_{1-}$ with the final ν_{2-} having four-momentum p can be obtained from Eq. (32) by replacing the first two sin factors with cos's in the two terms in the curly brackets. The matrix element squared for $\nu_{2-}+\nu_{2-} \to \nu_{2-}+\nu_{2-}$ can be obtained from Eq. (32) by replacing all sin's with cos's.

5. $\nu_{i-} + N \to \nu_{j-} + N$

Neutrino-nucleus scattering is the principle reaction responsible for trapping neutrinos in the hot neutron star. This scattering is purely via the neutral current weak interaction. Because the nucleus is much more massive than the temperature, the scattering conserves neutrino energy. Thus this reaction will also typically conserve a neutrino's species, even in a background of matter.

However there may be certain exceptions to this general behavior. For example, neutrino-nucleus neutral current scattering which leaves the nucleus in an excited state will be nonconservative and hence can change neutrino species [see Sec. II, Eq. (20)]. Also, in the densest part of the core, collective effects can reduce the effective mass of a nucleon such that neutrino-nucleon scattering may be nonconservative. Thus this reaction may be relevant for transferring lepton number between neutrino species.

C. Energy loss by "right-handed" neutrino emission

Negative helicity neutrinos are trapped in the hot neutron star. However positive helicity neutrinos can be produced during scattering and, since they are mostly sterile, freely leave the high-density core. The cross section is proportional to mass squared so a too massive Dirac neutrino would cause the core to lose energy faster than observed by neutrino detectors here on Earth [24]. The bound on the Dirac neutrino mass, Eq. (1), follows approximately from a bound on the rate of energy loss [1, 2].

1. $\nu_{i-} + N \rightarrow \nu_+ + N$

This reaction is usually taken to be the dominant process for emitting positive helicity neutrinos. Assuming two mixed, neutrino species, the rate of energy loss is given by

$$\frac{d\epsilon}{dt} = \int \frac{d^3\mathbf{k}}{(2\pi)^3 2k^0}\frac{d^3\mathbf{p}}{(2\pi)^3 2p^0}\frac{d^3\mathbf{k}'}{(2\pi)^3 2k'^0}\frac{d^3\mathbf{p}'}{(2\pi)^3 2p'^0}|M|^2(2\pi)^4\delta^4(k'+p'-k-p)Ff_N(1-f'_N)k'^0 \tag{33}$$

with

$$F \equiv (\cos^2[\theta_m(k)-\theta]f_2 + \sin^2[\theta_m(k)-\theta]f_1 + \cos^2[\bar{\theta}_m(k)-\theta]\bar{f}_2 + \sin^2[\bar{\theta}_m(k)-\theta]\bar{f}_1) \tag{34}$$

and

$$|M|^2 = 8m^2G_F^2M_N^2(C_V^2 + 3C_A^2), \tag{35}$$

where k and k' (p and p') are the initial and final neutrino (nucleon) momentum, respectively, $|M|^2$ is the squared scattering matrix element [2, 9, 16], and F is the initial neutrino phase-space density factor including the relevant mixing matrix element. f_a denotes the phase-space density for particle a and the $\bar{f}$'s denote antineutrino phase-space densities. The neutrino and nucleon masses are m and M_N, respectively, and C_V and C_A are the appropriate weak-interaction vertex factors. The angular dependence has been dropped from the scattering matrix element.

The mixing matrix elements are taken from Eq. (13), however a heuristic justification of them is possible. The positive helicity states couple through the Dirac mass to the linear combination of the negative helicity states which form the vacuum mass eigenstates. The unitarity matrix which connects the negative helicity vacuum mass eigenstates to the negative helicity mass eigenstates in matter can be written as

$$UU_m^\dagger(k) = \begin{bmatrix} \cos(\theta_m(k)-\theta) & \sin(\theta_m(k)-\theta) \\ -\sin(\theta_m(k)-\theta) & \cos(\theta_m(k)-\theta) \end{bmatrix}. \tag{36}$$

First the vacuum mixing matrix rotates from the vacuum mass basis to the flavor basis and then we rotate from the flavor basis to the mass-eigenstate-in-matter basis. This agrees with the results of Eq. (13). From this heuristic argument, it is easy to use Eq. (22) to generalize to the appropriate transformation matrix for three flavors.

As explained previously in Sec. III B, the f's for neutrino mass eigenstates in matter are only described by Fermi-Dirac distributions when all three of the neutrinos are in full ("chemical") equilibrium, or in special limiting situations. In order to evaluate Eq. (34) here, we shall make some assumptions about whether or not the neutrinos are in equilibrium.

First let us assume that all possible leptons are in full equilibrium and equally share the core's lepton number with the electron, $f_e \approx f_1 = f_2$, $\bar{f}_1 = \bar{f}_2 = 0$. In this case all dependence on neutrino mixing cancels, by unitarity, and Eq. (34) simply reduces to just $F = f_e$. Then the large chemical potential for neutrinos in equilibrium with the electron enhances [9] the rate of energy loss by a factor of $4.4 \times 10^2(\mu/200\text{ MeV})^4(20\text{ MeV}/T)^4$ over the μ=0 case used to derive Eq. (1). The bound on the Dirac neutrino mass in Eq. (1) is improved to

$$m > 1\text{ keV}. \tag{37}$$

This is plotted in Fig. 1 and is the lower solid contour there. It is apparent that this sometimes lies below the region where the neutrino comes into full equilibrium. Thus the excluded region for the Dirac mass follows the conditions for equilibrium, as shown in Fig. 1.

Let us now assume that only the "electron neutrino" is degenerate. The appropriate limit is $A_e \gg m^2$ since A_e in Eq. (2) is larger than m^2 in Eq. (1). Then the electron-neutrino and the electron-antineutrino are dominantly the mass-eigenstates-2 and -1 in matter, respectively. Thus the phase-space densities are $f_e = f_2$, $\bar{f}_1 = 0$, and $f_1 = \bar{f}_2 =$ a Fermi-Dirac distribution with vanishing chemical potential. Enhanced neutrino emission will also occur for this case because the neutrino mass eigenstates in matter are different from the mass eigenstates in vacuum. Using that $\theta_m = \pi/2$ and $\bar{\theta}_m = 0$ for $A_e \gg m^2$, Eq. (34) becomes

$$F = (\cos^2\theta\, 2f_1 + \sin^2\theta\, f_2). \tag{38}$$

The first and second phase-space densities lead to the bounds in Eqs. (1) and (37), respectively. Thus the bound on the Dirac mass for out-of-equilibrium neutrinos is

$$m^2[\cos^2\theta + 4.4\times 10^2\sin^2\theta] < (28\text{ keV})^2. \tag{39}$$

This constraint interpolates between Eq. (1) and Eq. (37). It is plotted in Fig. 1, and is the upper solid line there.

2. $e + p \rightarrow \nu_+ + n$

This reaction is very similar to neutrino-nucleus neutral-current scattering as discussed in the previous section. Equation (33) is also applicable here with some small modifications. The squared matrix element, $|M|^2$, is a factor of 2 smaller than as given in Eq. (35) since the electron is unpolarized. The factor F is now

$$F \equiv f_e \sin^2\theta, \tag{40}$$

where f_e is the electron phase-space density. The mixing matrix element follows from Eq. (13), it describes the rotation from the interaction basis to the vacuum mass eigenstate basis for negative helicity neutrinos. Hence this mixing is independent of the matter background.

The rate does not depend on neutrino phase space factors, so it can be evaluated analytically. Neglecting the nucleon blocking in the final state,

$$\frac{d\epsilon}{dt} = \frac{G_F^2(C_V^2 + 3C_A^2)}{32\pi^3} m^2 \mu_e^4 N_p \sin^2\theta, \tag{41}$$

where N_p is the proton number density. The rate of energy loss by this process, Eq. (41), vanishes for zero neutrino mixing ($\theta = 0$) but when neutrinos do mix then it may dominate over Eq. (33) since the electron density is larger than the negative helicity neutrino density in numerical calculations.

3. $\pi^- + p \rightarrow n + \nu\bar{\nu}$

If the core contains a large pion density, than this is the dominant process for producing "wrong-helicity" neutrinos [9]. However the equation of state of high-density nuclear matter is not well known so the density of pions is uncertain. In addition, "line broadening" of the virtual nuclear state will suppress this process somewhat [25] and must be included in calculations.

In the two-flavor approximation, there are four possible final states: $\nu_{2-} + \bar{\nu}_-$, $\nu_{1-} + \bar{\nu}_-$, $\nu_+ + \bar{\nu}_{2+}$, $\nu_+ + \bar{\nu}_{1+}$. Defining k to be the magnitude of the momentum of the ν_-'s or $\bar{\nu}_+$'s, then the expression for the rate of energy loss by this reaction has a factor of

$$(1-F) \equiv (\cos^2[\theta_m(k)-\theta](1-f_2) + \sin^2[\theta_m(k)-\theta](1-f_1) + \cos^2[\bar{\theta}_m(k)-\theta](1-\bar{f}_2) + \sin^2[\bar{\theta}_m(k)-\theta](1-\bar{f}_1)). \tag{42}$$

This expression is very similar to that of Eq. (34) except that here the phase space densities are changed to blocking factors for the final-state trapped neutrino.

If the phase-space distributions of all the trapped neutrinos are identical, $f_1 = f_2$, $\bar{f}_1 = \bar{f}_2$, then all mixing dependence vanishes, by unitarity. In particular, if the neutrinos are nondegenerate so that the blocking factors approach unity, then $(1-F) = 2$ as expected since there are only two possible final states in vacuum. Assuming that $A_e \gg m^2$ is a good approximation for all k, then

$$(1-F) = (\sin^2\theta(1-f_2) + \cos^2\theta(1-f_1) + \cos^2\theta(1-\bar{f}_2) + \sin^2\theta(1-\bar{f}_1)). \tag{43}$$

Note the different mixing behavior between the neutrino and antineutrino states.

The discussion given here also applies to the reaction $N + N \rightarrow N + N + \nu\bar{\nu}$.

IV. SUMMARY

In this paper we have examined particle-physics issues relevant to describing massive, mixed, Dirac neutrinos in the core of a supernova.

In dense matter, neutrinos comprise a sizable fraction of the total number density. Then neutrinos forward scatter off of other neutrinos and this may contribute to neutrino mixing effects. New, important contributions to this scattering have been described in the Appendix. This nonlinear effect may be important, but has not been included in the present analysis.

In the first section, the full Dirac neutrino field in a constant matter background is derived. The interaction with the matter background is helicity dependent so the vacuum pairing between positive and negative helicity states is broken. However the field for a relativistic Dirac neutrino in a background of matter has a relatively simple form when expressed in the interaction basis. Then the negative helicity mass eigenstates exhibit the matter-dependent mixing of the MSW effect while the positive helicity neutrino mixing is unaffected by a matter background. The expression for the neutrino field is necessary for calculations of neutrino decay or scattering in a matter background.

For the scattering of relativistic neutrinos, the matter background has little effect on the kinematics—but the effect on the flavor content of the neutrino states is profound. The matter background ensures that the mass eigenstates in matter always become incoherent between nonforward scatterings and hence are the physical basis for describing neutrinos in a supernova core. However since the weak-interaction content of a negative helicity neutrino mass eigenstate in matter varies strongly with energy (and density), this complicates discussions of how neutrinos scatter. For one thing, the neutral current is not diagonal in the mass-eigenstate-in-matter basis if the initial and final neutrinos have different energies. In addition, the phase-space distributions of the negative helicity neutrinos are typically far from Fermi-Dirac and so energy and density dependence must be accounted for explicitly when describing how neutrinos approach an equilibrium distribution of lepton number.

Mixing of all three neutrino species must be taken into account when in the core of a supernova. All leptons—the electron, the muon, and the three neutrinos—quickly come into equilibrium and share the core's lepton number, unless *all* vacuum neutrino masses are small. Typically, the bottleneck in the chain of reactions which distribute the lepton number is the step which brings the first "nonelectron neutrino" into full equilibrium. An estimate of when neutronization does this, valid for Majorana or Dirac neutrinos, is plotted as the dashed line in Fig. 1.

A supernova core can easily lose energy by emission of sterile, positive helicity Dirac neutrinos. An estimate of the mass and mixing-angle range excluded by SN1987A neutrino observations is shown as the shaded region in Fig. 1. This region is particularly interesting because a 17-keV neutrino with 1% mixing with the electron neutrino lies well inside the region excluded for Dirac neutrinos. Thus if the recent beta-decay results are indeed indications of a neutrino mass, that mass is probably not Dirac.

This conclusion may be weakened by effects not included in existing calculations. Sharing the lepton number between all the five possible leptons increases entropy and tends to strengthen the emission of all neutrino types. Also, the back reaction of lepton-number loss on the core must be included. In addition, neutrino diffusion and/or convection should be accounted for, given the strong density dependence of neutrino mixing. Thus detailed modeling of neutrino emission from a supernova core is necessary to establish the precise bounds. The emphasis here has been on developing the framework for describing mixed neutrinos in dense matter in order to enable such calculations.

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APPENDIX A: NEUTRINO-NEUTRINO FORWARD SCATTERING

In the dense media of a supernova core, and in the early Universe, neutrinos comprise a sizable fraction of the total number density. Under such conditions there will be a contribution to the induced mass of a neutrino from its forward scattering off of other neutrinos. Physically, the flavor evolution must be solved when it is necessary to describe how these neutrinos approach or depart from equilibrium. This evolution has been discussed many times in the literature (see, e.g., [26] and references therein), unfortunately the starting point [27, 28] used for these discussions is incomplete. Here we point out some extra terms, valid for Majorana or Dirac neutrinos, which earlier analyses omitted.

The equation describing the flavor evolution of one neutrino of energy E can be written as (see, e.g., [8], Eq. (2.29))

$$i\frac{d}{dt}\begin{bmatrix}\nu_e\\ \nu_\mu\end{bmatrix} = \frac{M^2}{2E}\begin{bmatrix}\nu_e\\ \nu_\mu\end{bmatrix}. \tag{A1}$$

Here M^2 is the effective mass squared and it is assumed that there are only two neutrino generations. The mass squared consists of a vacuum term plus background induced terms. In the standard model, a background of neutrinos induces a contribution to M^2 from the neutrino-neutrino interaction mediated by Z^0 exchange. For neutrino energies much less than the mass of the Z^0, the effective interaction is

$$H_{\text{int}} = \frac{G_F}{\sqrt{2}}\left(\sum_i \bar{\nu}_{iL}\gamma^\mu\nu_{iL}\right)\left(\sum_j \bar{\nu}_{jL}\gamma_\mu\nu_{jL}\right), \tag{A2}$$

where the sum is over all neutrino species, the subscript L denotes the left-handed chirality, and the U(2) flavor symmetry is manifest. However before one can write down these induced mass terms, information on the relative coherence of the neutrinos is necessary.

To begin, let us assume we have a system of massless neutrinos described by infinite plane waves. Then the effective mass squared term for one of these equations describing a neutrino of energy E is, in the "charged lepton" basis,

$$M^2 = 2\sqrt{2}G_F E\sum_j \begin{bmatrix} 2|\nu_e^j|^2 + |\nu_\mu^j|^2 & \nu_e^j\nu_\mu^{j*}\\ \nu_e^{j*}\nu_\mu^j & |\nu_e^j|^2 + 2|\nu_\mu^j|^2\end{bmatrix}. \tag{A3}$$

The sum is over all neutrinos other than that one whose propagation equation we are considering and ν_α^j is the α flavor component of the jth neutrino wave function. It has been assumed that the angular-dependent terms cancel out in the sum. References [27, 28] correctly calculated the diagonal terms in the induced mass, including the additional factor of 2 for identical final-state particles, however the off-diagonal terms were mistakenly omitted therein.

The off-diagonal terms are crucial for preserving the symmetries of the Lagrangian. Using the unitarity relation $|\nu_e|^2 + |\nu_\mu|^2 = 1/V$, Eq. (A3) can be rewritten as

$$M^2 = 2\sqrt{2}G_F E\left\{N_\nu + \sum_j \begin{bmatrix}\nu_e^j\\ \nu_\mu^j\end{bmatrix}[\nu_e^{j*}\ \ \nu_\mu^{j*}]\right\}, \tag{A4}$$

where N_ν is the neutrino number density. In this formulation, it is apparent that basis rotations of the "propagating" neutrino cancel with those of the "background" neutrinos. Thus the U(2) flavor symmetry is maintained. *To neglect the off-diagonal terms in every basis is obviously incorrect since it breaks this symmetry and then the result of the flavor evolution of a given state would be different in each basis.* The U(2) symmetry maintains the net flavor content.

Off-diagonal induced mass terms can be interpreted as an exchange of flavor between the "background" neutrino and the "propagating" neutrino, Fig. 3. Total flavor is conserved, but the flavor associated with a given single neutrino is not conserved. The fact that there are off-diagonal induced mass terms is nothing new, the induced mass for forward scattering off of a charged lepton has off-diagonal terms when expressed in any basis other than that which diagonalizes the charged current interaction. What is unusual is that because the flavor is exchanged between neutrinos with different momenta, a one-particle propagation formalism may no longer be appropriate for describing the situation.

To illustrate the effects of the off-diagonal terms in the flavor basis, let us consider the simple example of a box containing massive, relativistic, nondegenerate, negative

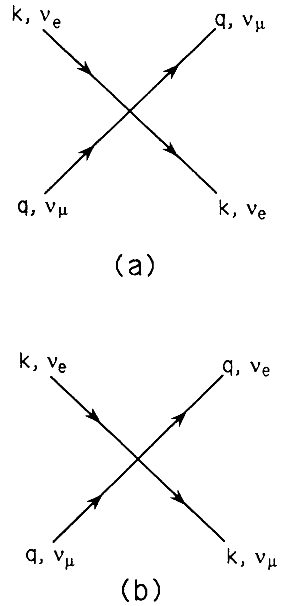

FIG. 3. Feynman diagrams associated with neutrino-neutrino forward scattering, (a) diagonal and (b) off-diagonal terms.

helicity neutrinos, and no charged leptons. We make no assumptions at all as to coherence between the neutrinos. There are constraints on the dynamics of such a system since the total effective neutrino Lagrangian is invariant under separate, global U(1) rotations of each vacuum mass eigenstate (at tree level). Thus it is clear that there are conserved quantities, L_i, one for each neutrino species, where

$$L_i = \sum_j |\nu_i^j(t)|^2. \tag{A5}$$

ν_i denotes the neutrino wave function for the ith *vacuum* mass eigenstate and the sum is over all neutrinos in the box. L_i is the amount of *vacuum* mass eigenstate i in the box. However, the amount of electron-neutrino number in the box is in general time dependent

$$L_e(t) = \sum_j |\nu_e^j(t)|^2 \tag{A6}$$

but it can be expressed in terms of the conserved quantities. Assuming only two flavors and using Eq. (12) for the mixing matrix,

$$L_e(t) = \cos^2\theta L_1 + \sin^2\theta L_2 + \sin 2\theta\sqrt{L_1L_2}\cos[\phi(t)], \tag{A7}$$

where L_e and ϕ are the only time-dependent quantities. Equation (A7) shows that the maximum change in L_e is restricted by the vacuum mixing angle θ *and* by the relative amounts of vacuum mass eigenstate. A change in the volume of the box as a function of time could not substantially change L_e if either of these latter quantities are small. This is very different than the more familiar case of a neutrino in a charged-lepton background where a density change causes a resonant transition, the MSW effect. In fact, Eq. (A7) also applies when $G_F \to 0$ and the flavor oscillations are purely due to vacuum masses. Here, neutrino-neutrino forward scattering may affect the phase between the mass eigenstates but not the flavor mixing. In this example, neutrino-neutrino forward scattering does not contribute to an MSW type resonance!

The microphysical description of the case of large vacuum neutrino masses is straightforward. Energy and momentum conservation can prevent relativistic neutrinos from exchanging their mass identity in forward scattering. We estimate that this is the case if the two-neutrino energy change

$$E_f - E_i \approx \tfrac{1}{2}(m_2^2 - m_1^2)\left(\frac{1}{E} - \frac{1}{E'}\right) \tag{A8}$$

is large compared to the potential from neutrino-neutrino scattering, $O(G_F N_\nu)$. Here E and E' are the energies of the two scattering neutrinos. When the above condition is satisfied we expect that the off-diagonal terms should average out *in the vacuum mass eigenstates basis* and then the effective mass squared for the neutrino is

$$M'^2 = \frac{1}{2}\begin{bmatrix} -\Delta & 0 \\ 0 & \Delta \end{bmatrix} + 2\sqrt{2}G_F E\left(N_1 + N_2 + \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix}\right), \tag{A9}$$

where $\Delta = m_2^2 - m_1^2$ and N_i are the number densities of the ith vacuum mass eigenstate for the neutrino. Now the multineutrino system can be described in terms of one-particle equations. The propagation equation manifests the U(1)×U(1) flavor symmetry which is as expected from the massive-neutrino Lagrangian. Explicitly, since there are no off-diagonal terms anywhere in this matrix, it is clear that there is no contribution to the relative mixing of the neutrinos—only the relative phase of the oscillations is affected by neutrino-neutrino forward scattering. This is consistent with Eq. (A7). An MSW-type resonance does not occur.

Now let us consider the situation of massless neutrinos in a background of charged fermions. The Lagrangian has a U(1)×U(1) flavor symmetry and no flavor mixing. At the microphysical level, energy and momentum conservation are always satisfied because $E_f - E_i = 0$ since the constant potentials from the charged lepton background just cancel out. Thus Eqs. (A3) and (A4) are still valid.

When there is a charged-fermion background and also the vacuum neutrino masses are relevant, then the Lagrangian no longer has any global flavor symmetries. Neutrino mixing can and does depend on the charged lepton background, of course. This complicates the discussion of neutrino-neutrino forward scattering. At the microphysical level, the different energy dependences of the vacuum mass term and the charged lepton induced mass term means that there is not a common mass eigenstate basis where energy and momentum conservation imply that the off-diagonal terms from neutrino-neutrino forward scattering can be neglected. It is clear that the off-diagonal neutrino-neutrino terms will play an important role, but a full, general analysis of this case is beyond the scope of this paper.

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