

Heavy-Higgs-boson decays to $W^+ W^- Z$ in the standard model

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Parity conservation forbids the standard model Higgs boson H from decaying to three longitudinally polarized gauge bosons ($W^+ W^- Z$) at tree level to leading order in $(M_H/M_W)^2$. As a result, the decay $H \rightarrow W^+ W^- Z$ is dominated by states with two longitudinal and one transverse gauge bosons for a range of (heavy) Higgs-boson masses. An approximate analytic result that shows this important feature is derived, and the full lowest-order calculation is carried out for all Higgs masses of practical interest. The branching ratio is about 1% for $M_H \sim 1$ TeV. With an integrated luminosity of 10^{40} cm², the number of events can be as large as 600 at the Superconducting Super Collider, depending on the Higgs-boson mass. The connection of these results to the equivalence theorem is also discussed.

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I. INTRODUCTION

If the Higgs boson H of the standard model has a mass greater than 250 GeV, it will decay to, among other things [1], the three-gauge-boson state $W^+ W^- Z$. This decay is sensitive to the gauge structure of the underlying theory because the lowest-order Feynman graphs (Fig. 1)

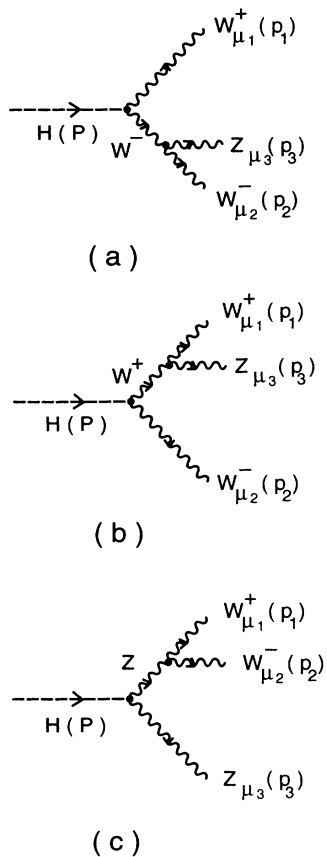


FIG. 1. Lowest-order Feynman diagrams for the decay $H \rightarrow W^+ W^- Z$. In the Feynman-'t Hooft gauge, there are additional diagrams containing Higgs-Goldstone bosons.

have a trilinear gauge interaction vertex. Thus the study of the decay $H \rightarrow W^+ W^- Z$ may provide us an interesting test of the gauge theory, provided its branching fraction and the production rate of H are sufficiently large.

The study of the three-gauge-boson Higgs-boson decay is also important for the direct production of $W^+ W^- Z$, which has also been suggested for exploring the gauge structure of the fundamental theory. While extensive studies [2-4] have shown interesting prospects for studying $W^+ W^- Z$ production at future high-energy colliders, a non-negligible background due to $H \rightarrow W^+ W^- Z$ may arise unless the difference between these two different processes is well understood.

The three-gauge-boson Higgs-boson decay has been examined before [5], but only for a very heavy Higgs boson ($M_H \sim 1$ TeV) and, more crucially, assuming that the three gauge bosons are longitudinally polarized. It incorrectly (1) gave a nonzero result for its leading terms and (2) suggested that the decay is insignificant in physical processes of interest.

As shown below, the leading term of the decay $H \rightarrow W_L^+ W_L^- Z_L$ violates parity and hence is not allowed by the lowest-order weak interactions in the standard model. Here the subscript L refers to the longitudinal polarization. It turns out that for an ~ 1 -TeV Higgs boson the decay final states are dominated by two longitudinal and one transverse gauge bosons (in the rest frame of H). For a range of Higgs-boson masses, we find that the three-gauge-boson Higgs-boson decay has a sizable branching ratio $\sim 1\%$ and its effect is not negligible.

The rest of this paper is organized as follows. In Sec. II we provide an approximate analytic calculation in the limit in which the Higgs boson is very heavy and the weak isospin is conserved (i.e., $\sin^2 \theta_W = 0$). We obtain a simple asymptotic formula, showing that decays with only two longitudinally polarized gauge bosons provide the dominant contribution for $M_H \sim 1$ TeV. In Sec. III a complete lowest-order numerical calculation is carried out. There, weak-isospin-breaking effects (i.e., $\sin^2 \theta_W \neq 0$) are taken into account and the Higgs-boson mass is allowed to take any value of practical interest.

The numerical result shows that the expected asymptotic behavior agrees with the approximate analytical result within $2 \sin^2 \theta_w \sim 50\%$ for $M_H \gtrsim 700$ GeV. The prospects for detecting $H \rightarrow W^+ W^- Z$ at high-energy colliders such as the Superconducting Super Collider (SSC), the Large Hadron Collider (LHC), and future TeV $e^+ e^-$ machines are also briefly discussed in this section. Our results and concluding remarks are summarized in Sec. IV.

II. HEAVY-HIGGS-BOSON LIMIT

Consider the three-gauge-boson Higgs-boson decay in the heavy-Higgs-boson limit. It is well known that in this energy range longitudinally polarized W 's and Z 's have strong interactions [6–8]. This follows because the polarization vector $\epsilon_\mu(k)$ of a longitudinal vector boson in the limit $k \gg M_{W,Z}$ is proportional to k_μ ,

$$\epsilon_\mu(k) = \frac{k_\mu}{M_{W,Z}} + O\left(\frac{M_{W,Z}}{k}\right). \quad (2.1)$$

In the present situation, k is of the order of M_H . Because of Eq. (2.1), many high-energy interactions are dominated by configurations in which all of the external gauge bosons are longitudinally polarized. Consider the process $H \rightarrow W^+ W^-$ as an example. The leading amplitude of the decay is proportional to $igM_W(M_H/M_W)^2$, in which each (M_H/M_W) factor corresponds to a longitudinally polarized W . The decay can be approximated by $H \rightarrow W_L^+ W_L^-$, and its rate behaves like

$$g^2 M_W^2 (M_H/M_W)^4 / M_H = g^2 M_H^3 / M_W^2.$$

On the basis of Eq. (2.1), one might expect that, in the heavy-Higgs-boson limit,

$$\begin{aligned} \Gamma(H \rightarrow W^+ W^- Z) &\approx \Gamma(H \rightarrow W_L^+ W_L^- Z_L) \\ &\sim g^4 \left[\frac{M_H^3}{M_W^2} \right] \left[\frac{M_H^2}{M_W^2} \right], \end{aligned} \quad (2.2)$$

with an additional enhancement factor $(M_H/M_W)^2$ due to the extra longitudinal vector boson in the final state. Such a naive expectation is however incorrect: To leading order the decay $H \rightarrow W_L^+ W_L^- Z_L$ is not allowed be-

cause the Higgs and three longitudinal gauge bosons have the opposite parity and the boson sector of the standard model conserves parity [9].

It should be pointed out that the suppression of the decay $H \rightarrow W_L^+ W_L^- Z_L$ is not due to G parity. If G parity were the reason for the suppression, this decay could still occur via weak mixing, which violates G parity, with a rate which is $\sin^4 \theta_w$ times Eq. (2.2), as given by Ref. [5].

Parity conservation is respected in the decay $H \rightarrow W^+ W^- Z$ if one of the gauge bosons has a transverse polarization. In that case, Eq. (2.1) implies that the rate is proportional to $g^4 M_H^3 / M_W^2$ for $M_H \gg M_W$ and its branching ratio has an asymptotic form

$$B(H \rightarrow W^+ W^- Z) \sim O(\alpha/\pi). \quad (2.3)$$

Thus, in contrast to previous expectation, the lowest-order interaction for $H \rightarrow W^+ W^- Z$ does not introduce an additional (M_H^2/M_W^2) enhancement factor.

A modest enhancement depending on $\ln^2(M_H^2/M_W^2)$ is expected, however. This can be understood easily if one regards the transverse gauge boson as massless (compared to the heavy-Higgs-boson mass scale). In this extreme case, one can think of the decay $H \rightarrow W^+ W^- Z$ as a decay $H \rightarrow W_L W_L, Z_L Z_L$, or $W_L Z_L$ followed by a bremsstrahlung. If indeed the transverse gauge boson is massless, a slight acceleration of an ‘‘isocharged’’ particle such as W_L^\pm or Z_L would result in an emission of an infinite number of ‘‘soft’’ transverse gauge bosons. As a result, both infrared [10] and collinear singularities would emerge [11–13] just as what happens to the photon in QED and to the gluons in QCD [14,15]. In the real world, the transverse gauge boson is not massless, but the above argument suggests that the rate for the decay with a transverse gauge boson must have logarithmic factors such as $[\ln(M_H^2/M_W^2)]^2$ and $\ln(M_H^2/M_W^2)$, which may further enhance the contribution [16].

To have a feeling for the order of magnitude, let us now estimate the decay rate generated from the Feynman graphs (Fig. 1) in the limit in which weak isospin is conserved (i.e., $\sin^2 \theta_w = 0$) and the Higgs-boson mass is very heavy. The square of the spin-summed matrix element of the decay is

$$\begin{aligned} \sum_{\text{spin}} |M_{\text{iso}}|^2 &= -g^4 M_W^2 \left[\frac{m^2(1,2,3)}{[(p_1+p_2)^2 - M_W^2]^2} + \frac{2m'^2(1,2,3)}{[(p_1+p_2)^2 - M_W^2][(p_2+p_3)^2 - M_W^2]} \right] \\ &\quad + [\text{permutations of } (1,2,3)], \end{aligned} \quad (2.4)$$

where

$$\begin{aligned} m^2(1,2,3) &= -\frac{1}{M_W^6} (p_1 \cdot p_2)^2 [p_3 \cdot (p_1 - p_2)]^2 - \frac{2}{M_W^4} [2(p_1 \cdot p_2)(p_1 \cdot p_3 + p_2 \cdot p_3)^2 + (p_1 \cdot p_2)^3] \\ &\quad - \frac{2}{M_W^2} [7(p_1 \cdot p_2)^2 - 4(p_1 \cdot p_3)(p_2 \cdot p_3) - (p_1 \cdot p_3 + p_2 \cdot p_3)^2] - 4(p_1 \cdot p_2) + 20M_W^2, \end{aligned} \quad (2.5)$$

and

$$\begin{aligned}
m'^2(1,2,3) = & \frac{1}{M_W^6} (p_1 \cdot p_2)(p_2 \cdot p_3)(p_1 \cdot p_2 - p_1 \cdot p_3)(p_3 \cdot p_2 - p_3 \cdot p_1) \\
& + \frac{1}{M_W^4} [(p_1 \cdot p_2)^2(2p_2 \cdot p_3 + p_1 \cdot p_2 + p_1 \cdot p_3) + (p_2 \cdot p_3)^2(2p_1 \cdot p_2 + p_1 \cdot p_3 + p_2 \cdot p_3) \\
& \quad - (p_1 \cdot p_3)^2(p_1 \cdot p_2 + p_2 \cdot p_3 + p_3 \cdot p_1) - 5(p_1 \cdot p_2)(p_2 \cdot p_3)(p_3 \cdot p_1)] \\
& + \frac{1}{M_W^2} [9(p_1 \cdot p_2)(p_2 \cdot p_3) + 6(p_2 \cdot p_3)(p_1 \cdot p_3) + 6(p_1 \cdot p_2)(p_1 \cdot p_3) - 2(p_1 \cdot p_2)^2 - 2(p_2 \cdot p_3)^2] \\
& - 7(p_1 \cdot p_2 + p_2 \cdot p_3 + p_1 \cdot p_3) - 4(p_1 \cdot p_3) + 8M_W^2. \tag{2.6}
\end{aligned}$$

The terms in Eqs. (2.5) and (2.6) proportional to $(M_H/M_W)^6$ correspond to final states with no transverse gauge boson, and terms proportional to $(M_H/M_W)^4$ are dominated by states with one transverse gauge boson. Other terms can be neglected if $M_H \gtrsim 1$ TeV.

Neglecting terms smaller than and of order $(M_H/M_W)^2$, we find that the leading longitudinal terms cancel (as they must), and in the rest frame of H ,

$$\begin{aligned}
\Gamma(H \rightarrow W^+W^-Z)_{\text{iso}} &= \frac{3G_F^2 M_W^2 M_H^3}{64\pi^3} \left[\left(\ln \frac{M_H^2}{M_W^2} \right)^2 \right. \\
& \quad \left. - 4 \ln \frac{M_H^2}{M_W^2} - \frac{7\pi^2}{9} + \frac{\pi}{3\sqrt{3}} + \frac{59}{6} \right]. \tag{2.7}
\end{aligned}$$

Equation (2.7) shows the expected asymptotic behavior. This approximate result is valid only if $M_H \gg M_W$ and is useful in the TeV Higgs-boson-mass range. Since $\sin^2\theta_W$ is not a very small number, we expect that Eq. (2.7) may receive corrections of the order $2\sin^2\theta_W \sim 50\%$. For $M_H \lesssim 500$ GeV, large corrections from the three-transverse-gauge-boson states will also arise. If the Higgs-boson mass is much heavier than 1 TeV, then, among other things, perturbation theory breaks down and nothing can be concluded at this stage.

At this point it is instructive to apply the equivalence theorem [17–23] to this process, in which one identifies a longitudinally polarized vector boson by its Higgs-Goldstone boson [24]. From the equivalence theorem, one immediately sees from parity conservation that the leading terms of the decay $H \rightarrow W_L^+ W_L^- Z_L$ are zero, as given by the exact calculation.

The equivalence theorem is useful only in situations in which longitudinal-gauge-boson interactions are dominant. Precautions for the applicability of the equivalence theorem have been advised [25]. For the next-to-leading term, we generally expect that the equivalence theorem cannot produce the complete answer. In the present situation, the correction to the equivalence theorem result for $H \rightarrow W_L^+ W_L^- Z_L$ turns out to be

$$\Delta\Gamma(H \rightarrow W_L^+ W_L^- Z_L) = \frac{3G_F^2 M_W^2 M_H^3}{64\pi^3} \left[\frac{11}{6} - \frac{\pi}{\sqrt{3}} \right] \tag{2.8}$$

and is formally (in terms of the factor M_H^2/M_W^2) of the same order as the other next-to-leading terms. These terms are given by states in which only two vector bosons are longitudinally polarized. In terms of the equivalence theorem, there are a total of six tree-level diagrams. In the limit $\sin^2\theta_W = 0$, we find

$$\begin{aligned}
\Gamma(H \rightarrow W^+W^-Z)_{\text{ET}} &= \frac{3G_F^2 M_W^2 M_H^3}{64\pi^3} \left[\left(\ln \frac{M_H^2}{M_W^2} \right)^2 \right. \\
& \quad \left. - 4 \ln \frac{M_H^2}{M_W^2} - \frac{7\pi^2}{9} + 8 + \frac{4\pi}{3\sqrt{3}} \right]. \tag{2.9}
\end{aligned}$$

As we expected from the argument based on the infrared and collinear singularities, Eq. (2.9) gives all the expected logarithmic factors, and the sum of (2.8) and (2.9) reproduces the complete result (2.7). Because of a partial cancellation between the two constant terms in (2.8), $\Delta\Gamma(H \rightarrow W_L^+ W_L^- Z_L)$ turns out to be numerically insignificant. As a result, the decay is dominated by the two-longitudinal-and-one-transverse states, as shown explicitly in (2.9).

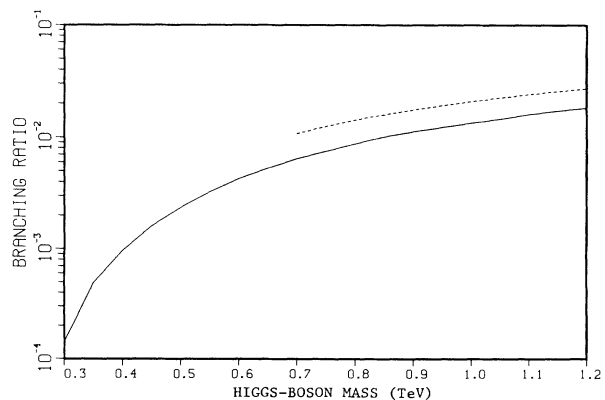


FIG. 2. Branching ratio (solid line) compared with the approximate analytic result (dashed line). $m_t = 150$ GeV is assumed.

TABLE I. Event rate for the decay $H \rightarrow W^+ W^- Z$ at the SSC with an integrated luminosity of 10^{40} cm^2 . σ_{gg} is the gluon-fusion cross section, and $\sigma_{\nu\nu}$ represents the intermediate-vector-boson-fusion cross section. The $l^+ l^- + 4$ jet events are estimated by assuming that τ is not identified.

M_H (TeV)	σ_{gg} (nb)	$\sigma_{\nu\nu}$ (nb)	$B(H \rightarrow W^+ W^- Z)$	Γ_H (GeV)	Events (total)	Events ($l^+ l^- + 4$ jets)
0.3	4.6×10^{-2}	7.5×10^{-3}	1.4×10^{-4}	8	75	2
0.4	4.6×10^{-2}	5.3×10^{-3}	9.6×10^{-4}	30	492	15
0.5	2.2×10^{-2}	3.9×10^{-3}	2.4×10^{-3}	64	621	17
0.6	1.2×10^{-2}	3.0×10^{-3}	4.3×10^{-3}	113	645	20
0.7	0.6×10^{-2}	2.3×10^{-3}	6.4×10^{-3}	180	531	16
0.8	0.3×10^{-2}	1.9×10^{-3}	8.7×10^{-3}	268	426	13
0.9	0.2×10^{-2}	1.5×10^{-3}	1.1×10^{-2}	380	385	11
1.0	0.1×10^{-2}	1.3×10^{-3}	1.3×10^{-2}	520	299	9

III. HIGGS DECAYS TO $W^+ W^- Z$ IN GENERAL SITUATIONS

With the understanding of the asymptotic behavior and the underlying physics, we now extend the investigation to general situations. Taking into account the weak-isospin-violation effect, i.e., $\sin^2 \theta_W \neq 0$, the lowest-order amplitude is generated from Fig. 1. At this point one can either employ the helicity amplitude technique [26] or the conventional method, in which the kinematics for $H \rightarrow W^+ W^- Z$ are basically the same as for the $K_{\pi 3}$ decay, and the remaining numerical calculation is straightforward. The derivation of the square of spin-summed matrix elements is tedious but otherwise straightforward, and the results can be obtained from the authors upon request.¹

Subject to the phase-space constraints, we perform the integration numerically. The branching ratio is plotted in Fig. 2. It shows the expected asymptotic behavior given by the analytic formula. The agreement between these two calculations is about 50% for $M_H > 700$ GeV.

The prospects for observing the decay $H \rightarrow W^+ W^- Z$ at future high-energy supercolliders such as the SSC and LHC depend on (1) the Higgs-boson production rate and

(2) the branching fraction of the decay. The event rate of the decay at the SSC (LHC) is summarized in Table I (Table II), in which we take $m_t = 150$ GeV as an illustration. For comparison, we also show the event rate at a $\sqrt{s} = 2$ TeV $e^+ e^-$ collider in Table III. The most recognizable signal at the SSC and LHC corresponds to the situation in which the Z decays leptonically ($Z \rightarrow e^+ e^-$, $\mu^+ \mu^-$) and the W^\pm to four jets. The major source of background comes from the QCD continuum such as $pp \rightarrow Z + 4$ jets, etc. Other processes such as $pp \rightarrow ZW + \text{jets}$, $pp \rightarrow ZZ + \text{jets}$, and $pp \rightarrow \bar{t}tZ \rightarrow W^+ W^- Z b \bar{b}$ are also likely to produce serious backgrounds. A detailed background study lies beyond the scope of the present paper.

The sizable rate for the decay $H \rightarrow W^+ W^- Z$ also provides a non-negligible background to the $W^+ W^- Z$ production. In the most serious situation, corresponding to $M_H \sim 600$ GeV, the effect is about 15%. It is always bigger than about 10% if $400 \lesssim M_H \lesssim 800$ GeV.

IV. CONCLUDING REMARKS

We have reexamined the decay $H \rightarrow W^+ W^- Z$ in the standard model. For a range of interesting Higgs-boson

TABLE II. Same as Table I, only for the LHC with an integrated luminosity of 10^{41} cm^2 .

M_H (TeV)	σ_{gg} (pb)	$\sigma_{\nu\nu}$ (pb)	$B(H \rightarrow W^+ W^- Z)$	Γ_H (GeV)	Events (total)	Events ($l^+ l^- + 4$ jet)
0.3	10.0	1.8	1.4×10^{-4}	8	165	5
0.4	8.0	1.0	9.6×10^{-4}	30	864	26
0.5	3.4	0.8	2.4×10^{-3}	64	1008	31
0.6	1.6	0.5	4.3×10^{-3}	113	903	28
0.7	0.8	0.4	6.4×10^{-3}	180	768	24
0.8	0.4	0.3	8.7×10^{-3}	268	609	19
0.9	0.2	0.2	1.1×10^{-2}	380	440	13
1.0	0.1	0.2	1.3×10^{-2}	520	390	13

¹See AIP document no. PAPS PRUDA-46-5069-11 for 11 pages of formulae, derivations, etc. Order by PAPS number and journal reference from American Institute of Physics, Physics Auxiliary Publication Service, 335 East 45th Street, New York, N.Y. 10017. The prepaid price is \$1.50 for a microfiche, or \$5.00 for a photocopy. Airmail additional. Make checks payable to the American Institute of Physics.

TABLE III. Event rate for the decay $H \rightarrow W^+W^-Z$ at a 2 TeV e^+e^- collider, assuming an integrated luminosity of 10^{40} cm^2 . The number of events does not include the vector-boson-decay branching fraction.

M_H (TeV)	$\sigma(e^+e^- \rightarrow He^+e^-)$ (pb)	$\sigma(e^+e^- \rightarrow H\nu\nu)$ (pb)	Events (total)
0.3	2.2×10^{-2}	2.3×10^{-1}	0
0.4	1.7×10^{-2}	1.7×10^{-1}	1
0.5	1.3×10^{-2}	1.3×10^{-1}	3
0.6	9.8×10^{-3}	1.0×10^{-1}	4
0.7	7.4×10^{-3}	7.7×10^{-2}	5
0.8	5.5×10^{-3}	5.7×10^{-2}	5
0.9	4.1×10^{-3}	4.2×10^{-2}	4
1.0	2.9×10^{-3}	3.0×10^{-2}	4

masses, we have found that the number of events of the decay is significant at the SSC and LHC. However, an assessment on the observability of the decay can be made only after detailed background studies have been carried out.

Theoretically, the present study provides an interesting example of the applicability of the equivalence theorem. As shown in the text, the equivalence theorem is not useful if its result does not dominate. Because of parity symmetry of the gauge and Higgs sector of the standard model, the would-be-dominant decay $H \rightarrow W_L^+ W_L^- Z_L$ is not allowed to leading order. The decay is dominated by states with one transverse and two longitudinal gauge bo-

sons for an interesting range of heavy Higgs-boson masses. Nonleading corrections to $H \rightarrow W_L^+ W_L^- Z_L$ are formally of the same order, but are numerically small. Asymptotically, the branching ratio of the decay $H \rightarrow W^+ W^- Z$ only has a logarithmic dependence on (M_H^2/M_W^2) .

The results discussed in this paper are valid only if higher-order radiative corrections are negligible. This presumably is the case if M_H is not too much heavier than 600 GeV. The exact location of the boundary can only be determined by an explicit higher-order calculation. The validity of the lowest-order calculation seems to be more questionable here than in situations in which only two gauge bosons are involved [27]. The reason has implicitly been pointed out in the text: Although parity is conserved in the gauge and Higgs sector, it is maximally violated by the weak currents. Therefore its effect could be significant for a sufficiently large (but still within the range of practical interest) Higgs-boson mass, even though it can only show up from loop diagrams. This is in contrast to situations in which there are two (or any number of) gauge bosons, where leading terms obtainable from the equivalence theorem are not suppressed [28].

ACKNOWLEDGMENTS

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