Heavy-flavor-conserving nonleptonic weak decays of heavy baryons

Hai-Yang Cheng, Chi-Yee Cheung, and Guey-Lin Lin
Institute of Physics, Academia Sinica, Taipei, Taiwan 11529, Republic of China

Y. C. Lin
Physics Department, National Central University, Chung-li, Taiwan 32054, Republic of China

Tung-Mow Yan
Institute of Physics, Academia Sinica, Taipei, Taiwan 11529, Republic of China
*and Floyd R. Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853**

Hoi-Lai Yu
Institute of Physics, Academia Sinica, Taipei, Taiwan 11529, Republic of China
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The heavy-flavor-conserving nonleptonic weak decays of heavy baryons are studied in a formalism that incorporates both heavy-quark symmetry and chiral symmetry. The phenomenological $\Delta S=1$ nonleptonic weak chiral Lagrangian for these transitions contains two independent coupling constants that describe the transitions between two flavor-SU(3) antitriplet heavy baryons, and the transitions between two flavor-SU(3) sextet heavy baryons. In the MIT bag model and the diquark model, only transitions between antitriplets are allowed. The coupling constants for these transitions are calculated in both models. The result is applied to specific nonleptonic decays such as $\Xi_c \to \Lambda_c \pi$, and the branching ratios are found to be of the order of 10^{-4}. An example of a nonleptonic decay due to symmetry breaking is provided by $\Omega_c \to \Xi_c' \pi$, which is estimated to have a smaller branching ratio, of the order of 10^{-5}.

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I. INTRODUCTION

Theoretical progress on the nonleptonic weak decays of heavy baryons has been very slow over the last ten years or so; a rigorous and reliable approach suitable for analyzing the heavy-baryon decays does not exist thus far. The well-known factorization approach, which has been applied successfully to heavy-meson decays, in general does not work for the weak decays of baryons. While the hyperon decay can be tackled with the help of current algebra, such a technique is in principle no longer applicable to the heavy-baryon case as the emitted meson is not necessarily "soft" and a pseudoscalar.

In spite of the absence of a general framework for describing the nonleptonic decays of heavy baryons, there is a special class of weak decays that can be studied in a reliable way, namely, heavy-flavor-conserving nonleptonic decays. Some examples are $\Xi_Q \to \Lambda_Q \pi$ and $\Omega_Q \to \Xi_Q \pi$. The idea is simple: In these decays only the light quarks inside the heavy baryon will participate in weak interactions; that is, while the two light quarks undergo weak transitions, the heavy quark behaves as a "spectator." As the emitted light mesons are soft, the $\Delta S=1$ weak transitions among light quarks can be handled by the well-known technique, such as the short-distance effective Hamiltonian and the current algebra or, equivalently, the nonlinear chiral Lagrangians. The recent development [1–4] of combining heavy-quark symmetries [5] and chiral symmetry provides a natural setting for investigating these reactions. More specifically, in the present paper, we will apply the formalism of Ref. [1] to these processes and study the heavy-flavor-conserving nonleptonic decays among the charmed baryons. The framework setup in this study can be easily generalized to the heavy baryons containing a b quark.

The heavy baryon of interest is that constructed from a heavy quark and two light quarks, which we often refer to as a "diquark." The two light quarks form a symmetric sextet **6** or an antisymmetric antitriplet $\bar{\mathbf{3}}$ in flavor-SU(3) space. We will denote these baryons as B_6 and $B_{\bar{3}}$, respectively. For the ground-state baryons in the quark model, the symmetries in the flavor and spin of the diquarks are correlated. Hence SU(3)-symmetric sextet diquarks have spin 1, whereas the SU(3)-antisymmetric antitriplet diquarks have spin 0. In the heavy-quark limit, there are two independent coupling constants describing the $B_{\bar{3}}$-$B_{\bar{3}}$ and B_6-B_6 transitions, respectively, while any transition between a B_6 and a $B_{\bar{3}}$ is forbidden.

Two methods will be used to evaluate these two coupling constants. The first employs the MIT bag model [6,7], and the second utilizes the recently developed quark-quark correction mechanism, namely, the diquark model [8]. Both models predict a vanishing coupling constant for B_6-B_6 transitions. The results from both models for the remaining coupling constant agree within a factor of 2. It should be noted that since these decays involve light quarks alone, the parameters needed have

*Permanent address.

been determined from the analysis of light-hadron processes. Furthermore, according to Ref. [8], the combination of parameters that enters the decay amplitudes here is scale invariant. As a result, their values obtained from the light-hadron sector can be directly used for charmed-baryon decays without modifications.

When we apply the model calculations to singly Cabibbo-suppressed charmed-baryon decays such as $\Xi_c^0 \to \Lambda_c^+ \pi^-$, $\Xi_c^+ \to \Lambda_c^+ \pi^0$, and $\Omega_c \to \Xi_c' \pi$, we also include a factorizable contribution which is responsible for the last decay and affects the first two decays somewhat. The branching ratios for these decays depend on the Ξ_c-Ξ_c' mixing angle, but are found to be typically in the range of 10^{-4}–10^{-5}.

II. GENERAL FORMALISM

For the processes we wish to study, the relevant QCD-corrected $\Delta S = 1$ effective weak Hamiltonian is given by (for a review, see Ref. [9])

$$\mathcal{H}_{\text{eff}}^{\Delta S=1} = \frac{G_F}{2\sqrt{2}} \sin\theta_C \cos\theta_C \sum_i c_i(\mu) O_i(\mu) + \text{H.c.} \ , \tag{1}$$

with the four-quark operators

$$\begin{aligned} O_1 &= (\bar{d}s)(\bar{u}u) - (\bar{u}s)(\bar{d}u) \ , \\ O_2 &= (\bar{d}s)(\bar{u}u) + (\bar{u}s)(\bar{d}u) + 2(\bar{d}s)(\bar{d}d) + 2(\bar{d}s)(\bar{s}s) \ , \\ O_3 &= (\bar{d}s)(\bar{u}u) + (\bar{u}s)(\bar{d}u) + 2(\bar{d}s)(\bar{d}d) - 3(\bar{d}s)(\bar{s}s) \ , \\ O_4 &= (\bar{d}s)(\bar{u}u) + (\bar{u}s)(\bar{d}u) - (\bar{d}s)(\bar{d}d) \ , \\ O_5 &= (\bar{d}\lambda^a s)_L [(\bar{u}\lambda^a u)_R + (\bar{d}\lambda^a d)_R + (\bar{s}\lambda^a s)_R] \ , \\ O_6 &= (\bar{d}s)[(\bar{u}u)' + (\bar{d}d)' + (\bar{s}s)'] \ , \end{aligned} \tag{2}$$

where $(\bar{q}_1 q_2) = \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2$, $(\bar{q}_1 q_2)' = \bar{q}_1 \gamma_\mu (1+\gamma_5) q_2$, $(\bar{q}_1 \lambda^a q_2)_{L,R} = \bar{q}_1 \gamma_\mu (1 \mp \gamma_5) \lambda^a q_2$, and θ_C is the Cabibbo angle. The operators O_5 and O_6 are induced from the so-called "penguin" diagram. The Wilson coefficient functions $c_i(\mu)$ that appear in Eq. (1) have the values [9]

$$\begin{aligned} &c_1 = -2.11 \ , \quad c_2 = 0.12 \ , \quad c_3 = 0.09 \ , \\ &c_4 = 0.45 \ , \quad c_5 = -0.045 \ , \quad c_6 = -0.008 \ , \end{aligned} \tag{3}$$

at the renormalization scale $\mu \approx 1$ GeV. Since the baryon-color wave function is totally antisymmetric, it is evident that among the six four-quark operators O_1–O_6, only O_1 contributes to the baryon-baryon transition matrix element since it is the only operator antisymmetric in color indices.

We would like to transcribe the four-quark interactions (2) in terms of the phenomenological fields for the heavy baryons introduced in Ref. [1]. From the discussion above, we only have to consider those terms in (2) which transform as octets in $SU(3)_L$ under chiral $SU(3)_L \times SU(3)_R$ transformations. Construction of the phenomenological weak Lagrangian follows the standard procedure of using a spurion which has the desired properties. This is provided by the SU(3) generator λ_6 which connects $\Delta S = \pm 1$ states and transforms under $SU(3)_L \times SU(3)_R$:

$$\lambda_6 \to L \lambda_6 L^\dagger \ , \tag{4}$$

where L is a linear $SU(3)_L$ transformation. A general $|\Delta S| = 1$ effective weak chiral Lagrangian responsible for heavy-flavor-conserving nonleptonic weak decays of heavy baryons is then given by [10]

$$\begin{aligned} \mathcal{L}^{\Delta S=1} = &h_1 \text{tr}(\bar{B}_{\bar{3}} \xi^\dagger \lambda_6 \xi B_{\bar{3}}) + h_2 \text{tr}(\bar{B}_6 \xi^\dagger \lambda_6 \xi B_6) \\ &+ h_3 \text{tr}(\bar{B}_6 \xi^\dagger \lambda_6 \xi B_{\bar{3}}) + \text{H.c.} \\ &+ h_4 \text{tr}(\bar{B}_6^{*\mu} \xi^\dagger \lambda_6 \xi B_{6\mu}^*) \ , \end{aligned} \tag{5}$$

which are the leading terms in the double expansion of light-meson momenta and inverse heavy-baryon masses. Following the notation of Ref. [1], the spin-$\frac{1}{2}$ heavy-baryon fields B_6 and $B_{\bar{3}}$ and the meson field ξ transform under an SU(3) chiral transformation as

$$\begin{aligned} &B_6 \to B_6' = U B_6 U^T \ , \\ &B_{\bar{3}} \to B_{\bar{3}}' = U B_{\bar{3}} U^T \ , \\ &\xi \to \xi' = L \xi U^\dagger = U \xi R^\dagger \ , \end{aligned} \tag{6}$$

and the spin-$\frac{3}{2}$ heavy-baryon fields $B_{6\mu}^*$ transform similarly as B_6 and $B_{\bar{3}}$. Explicitly, the symmetric sextets B_6 and the antisymmetric triplet $B_{\bar{3}}$ are given by the matrices [1]

$$B_6 = \begin{pmatrix} \Sigma_c^{++} & \frac{1}{\sqrt{2}}\Sigma_c^+ & \frac{1}{\sqrt{2}}\Xi_c'^+ \\ \frac{1}{\sqrt{2}}\Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}}\Xi_c'^0 \\ \frac{1}{\sqrt{2}}\Xi_c'^+ & \frac{1}{\sqrt{2}}\Xi_c'^0 & \Omega_c^0 \end{pmatrix} , \tag{7}$$

$$B_{\bar{3}} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix} , \tag{8}$$

and the meson field ξ is

$$\xi = \exp\left[i \frac{M}{\sqrt{2} f_\pi} \right] , \tag{9}$$

with

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -(\frac{2}{3})^{1/2}\eta \end{pmatrix} , \tag{10}$$

and $f_\pi = 93$ MeV is the pion's decay constant. The corresponding terms of (5) with a γ_5 inserted do not exist to leading order as a consequence of the heavy-quark symmetry [5]:

$$\begin{aligned} \bar{B}_{\bar{3}}(v) \gamma_5 B_{\bar{3}}(v) &= \bar{B}_6(v) \gamma_5 B_6(v) \\ &= \bar{B}_6^{*\mu}(v) \gamma_5 B_{6\mu}^*(v) = 0 \ . \end{aligned} \tag{11}$$

To leading order there are no couplings between B_6^* and

B_6 or between B_6^* and $B_{\bar{3}}$ because of the fact that

$$v^\mu B^*_{6\mu}(v)=\gamma^\mu B^*_{6\mu}(v)=0 \ . \tag{12}$$

It should be stressed that, contrary to the case of hyperon decays [11], there are no analogous D and F terms for nonleptonic weak decays of heavy baryons because of the definite symmetry of the matrices B_6 and $B_{\bar{3}}$. For example, one can show that

$$\mathrm{tr}(\bar{B}_6\xi^\dagger\lambda_6\xi B_6)=\mathrm{tr}(\bar{B}_6 B_6\xi^T\lambda_6\xi^*) \ , \tag{13}$$

and so these two structures are not independent.

The coupling constants $h_1,\ldots,h_4$ are constrained by the heavy-quark symmetry. To see what these constraints are, let us set $\xi=\xi^\dagger=1$ and pick an initial and final state connected by the matrix λ_6. Then, from (5), we have

$$\langle B_{\bar{3},f}|\mathcal{L}^{\Delta S=1}|B_{\bar{3},i}\rangle=h_1\bar{u}_f u_i \ , \tag{14a}$$

$$\langle B_{6,f}|\mathcal{L}^{\Delta S=1}|B_{6,i}\rangle=h_2\bar{u}_f u_i \ , \tag{14b}$$

$$\langle B_{6,f}|\mathcal{L}^{\Delta S=1}|B_{\bar{3},i}\rangle=h_3\bar{u}_f u_i \ , \tag{14c}$$

$$\langle B^*_{6,f}|\mathcal{L}^{\Delta S=1}|B^*_{6,i}\rangle=h_4\bar{u}^\mu_f u_{i\mu} \ . \tag{14d}$$

We now make use of the interpolating fields introduced in Ref. [1]:

$$B_{\bar{3}}(v,s)=\bar{u}(v,s)\phi_v h_v \ , \tag{15a}$$

$$B_6(v,s,\kappa)=\bar{B}_\mu(v,s,\kappa)\phi^\mu_v h_v \ , \tag{15b}$$

where ϕ_v and ϕ^μ_v are the 0^+ and 1^+ diquarks, respectively, which combine with the heavy quark h_v of velocity v to form the appropriate heavy baryon. The argument κ indicates the spin of the baryon: $\kappa=1$ for spin-$\frac{1}{2}$ baryons (B_6) and $\kappa=2$ for spin-$\frac{3}{2}$ baryons (B_6^*). The wave function $\bar{B}_\mu$ is given by [12]

$$\bar{B}_\mu(v,s,\kappa=1)=\frac{1}{\sqrt{3}}\bar{u}(v,s)\gamma_5(v_\mu+\gamma_\mu) \ , \tag{16a}$$

$$\bar{B}_\mu(v,s,\kappa=2)=\bar{u}_\mu(v,s) \ . \tag{16b}$$

We now evaluate the same matrix elements in (14) from the four-quark interactions (1). Consider

$$\begin{aligned}\langle B_6(v,s',\kappa')|\mathcal{H}^{\Delta S=1}_{\rm eff}|B_{\bar{3}}(v,s)\rangle&=\bar{B}_\mu(v,s',\kappa')\langle 0|h_v\phi^\mu_v\mathcal{H}^{\Delta S=1}_{\rm eff}\bar{h}_v\phi^\dagger_v|0\rangle u(v,s)\\&=\bar{B}_\mu(v,s',\kappa')\langle 0|h_v\bar{h}_v|0\rangle u(v,s)\langle 0|\phi^\mu_v\mathcal{H}^{\Delta S=1}_{\rm eff}\phi^\dagger_v|0\rangle \ .\end{aligned} \tag{17}$$

The heavy-quark "propagator" has the simple form

$$\langle 0|h_v\bar{h}_v|0\rangle=\frac{\not{v}+1}{2} \ , \tag{18}$$

and the "vacuum expectation value" of the light-quark operators is dictated by Lorentz covariance to be

$$\langle 0|\phi^\mu_v\mathcal{H}^{\Delta S=1}_{\rm eff}\phi^\dagger_v|0\rangle=av^\mu \ . \tag{19}$$

In principle, an axial-vector term can also appear in (19), but none is available. Now the wave function $\bar{B}_\mu$ satisfies

$$\bar{B}_\mu(v,s',\kappa')v^\mu=0 \ . \tag{20}$$

Consequently, we obtain

$$\langle B_6(v,s',\kappa')|\mathcal{H}^{\Delta S=1}_{\rm eff}|B_{\bar{3}}(v,s)\rangle=0 \ . \tag{21}$$

Equation (21) implies that

$$h_3=0 \tag{22}$$

in Eq. (5). A similar analysis gives

$$\langle B_6(v,s',\kappa')|\mathcal{H}^{\Delta S=1}_{\rm eff}|B_6(v,s,\kappa)\rangle=\bar{B}_\mu(v,s',\kappa')B_\nu(v,s,\kappa)M^{\mu\nu} \ ,$$

where

$$M^{\mu\nu}=\langle 0|\phi^\mu_v\mathcal{H}^{\Delta S=1}_{\rm eff}\phi^{\nu\dagger}_v|0\rangle=bg^{\mu\nu}+cv^\mu v^\nu \ . \tag{23}$$

Because of (20), only the first term will contribute. It follows that

$$\langle B^*_{6,f}|\mathcal{H}^{\Delta S=1}_{\rm eff}|B^*_{6,i}\rangle=b\bar{u}^\mu_f u_{i\mu} \ , \tag{24a}$$

$$\langle B_{6,f}|\mathcal{H}^{\Delta S=1}_{\rm eff}|B_{6,i}\rangle=-b\bar{u}_f u_i \ , \tag{24b}$$

$$\langle B_6^*|\mathcal{H}^{\Delta S=1}_{\rm eff}|B_6\rangle\equiv 0 \ . \tag{24c}$$

For the coupling constants appearing in Eq. (5), Eqs. (24a) and (24b) yield the relation

$$h_2=-h_4\equiv h' \ , \tag{25}$$

and Eq. (24c) simply confirms the nonexistence of couplings between B_6^* and B_6. Finally, the interaction (5) becomes

$$\begin{aligned}\mathcal{L}^{\Delta S=1}=h\,\mathrm{tr}(\bar{B}_{\bar{3}}\xi^\dagger\lambda_6\xi B_{\bar{3}})+h'\,\mathrm{tr}(\bar{B}_6\xi^\dagger\lambda_6\xi B_6)\\-h'\,\mathrm{tr}(\bar{B}^{*\mu}_6\xi^\dagger\lambda_6\xi B^*_{6\mu}) \ ,\end{aligned} \tag{26}$$

where $h\equiv h_1$. The heavy-quark symmetry predicts the coupling constants h and h' to be independent of heavy-quark masses. Later, we will show that h' vanishes in the MIT-bag-model [6] and diquark-model [8] calculations.

As an application, let us first discuss the weak-decay mode $\Xi_c\to\Lambda_c\pi$, which is kinematically allowed as $m_{\Xi_c}-m_{\Lambda_c^+}\simeq 170$ MeV [13]. (In the present paper, Ξ_c denotes an antitriplet charmed baryon.) The general expression for the baron decay amplitude of $B_i\to B_f+P$ (P=pseudoscalar meson) reads

$$M(B_i\to B_f+P)=i\bar{u}_f(A-B\gamma_5)u_i \ , \tag{27}$$

where A and B are the s- and p-wave amplitudes, respectively. It follows from Eq. (26) that the s-wave amplitude for $\Xi_c \to \Lambda_c^+ \pi$ is given by

$$\begin{aligned} M(\Xi_c^+ \to \Lambda_c^+ \pi^0)_{s\ \text{wave}} &= \langle \Lambda_c^+ \pi^0 | \mathcal{H}_{\text{eff}}^{\Delta S=1} | \Xi_c^+ \rangle \\ &= -\langle \Lambda_c^+ \pi^0 | \mathcal{L}^{\Delta S=1} | \Xi_c^+ \rangle \\ &= -i\frac{h}{2f_\pi}\bar{u}_{\Lambda_c} u_{\Xi_c} . \end{aligned} \tag{28}$$

So we identify

$$\begin{aligned} A(\Xi_c^+ \to \Lambda_c^+ \pi^0) &= -\frac{h}{2f_\pi} \\ &= -\frac{1}{2f_\pi}\langle \Lambda_c^+ \uparrow | \mathcal{L}^{\Delta S=1} | \Xi_c^+ \uparrow \rangle , \end{aligned} \tag{29}$$

where the arrows denote spin eigenvalues. Isospin invariance then gives the relation

$$A(\Xi_c^0 \to \Lambda_c^+ \pi^-) = \sqrt{2} A(\Xi_c^+ \to \Lambda_c^+ \pi^0) . \tag{30}$$

Incidentally, Eq. (29) shows that the coupling constant h is related to the matrix element

$$h = \langle \Lambda_c^+ \uparrow | \mathcal{L}^{\Delta S=1} | \Xi_c^+ \uparrow \rangle = -\langle \Lambda_c^+ \uparrow | \mathcal{H}_{\text{eff}}^{\Delta S=1} | \Xi_c^+ \uparrow \rangle , \tag{31}$$

which will be used later in our model calculation of the coupling constant.

In the framework of current algebra, the s-wave amplitude arises from the parity-violating commutator term

$$A_{\text{com}} = -\frac{1}{f_{P^a}}\langle B_f | [Q_5^a, \mathcal{H}^{\text{PV}}] | B_i \rangle , \tag{32}$$

where f_P is the decay constant of the pseudoscalar meson P. For the charmed-baryon decay $\Xi_c^+ \to \Lambda_c^+ \pi^0$, we have

$$\begin{aligned} A_{\text{com}}(\Xi_c^+ \to \Lambda_c^+ \pi^0) &= -\frac{1}{f_\pi}\langle \Lambda_c^+ \uparrow | [Q_5^3, \mathcal{H}_{\text{eff}}^{\text{PV}}] | \Xi_c^+ \uparrow \rangle \\ &= -\frac{1}{2f_\pi}\langle \Lambda_c^+ \uparrow | \mathcal{H}_{\text{eff}}^{\text{PC}} | \Xi_c^+ \uparrow \rangle , \end{aligned} \tag{33}$$

which is in agreement with Eq. (29) up to an inconsequential minus sign. This difference is due to the different sign convention commonly used for f_π in the chiral Lagrangian and current algebra. The s-wave amplitude is in general dominated by the low-lying negative-parity baryon poles. In the soft-pion limit, the sum of the $\frac{1}{2}^-$ pole terms reduces to a commutator term [7]. This explains the subscript "com" for the amplitudes in (32) and (33).

We next turn to the p-wave amplitude of $\Xi_c \to \Lambda_c^+ \pi$. In the soft-pion limit, the dominant contributions to the parity-conserving amplitude come from the ground-state baryon poles as depicted in Fig. 1. The ground-state $\frac{1}{2}^+ \Xi_c^+$ pole does not contribute since the strong-coupling constant $g_{\Xi_c \Xi_c \pi}$ vanishes in the heavy-quark limit, as we have stressed in Ref. [1]. By the same token, $g_{\Lambda_c \Lambda_c \pi}$ also vanishes, a fact reinforced by isospin conservation arguments. Therefore the Λ_c^+ pole contribution can be disregarded. Furthermore, sextet Σ_c and Ξ_c' poles also make no contributions because of vanishing B_6-$B_{\bar{3}}$ weak transitions. It thus appears that baryon poles do not cause any effects on the p-wave amplitude of $\Xi_c \to \Lambda_c^+ \pi$. Nevertheless, a contribution to the parity-conserving transition can arise from a possible $\Xi_c - \Xi_c'$ mixing. Let us denote the mass eigenstates by Ξ_{c1} and Ξ_{c2}:

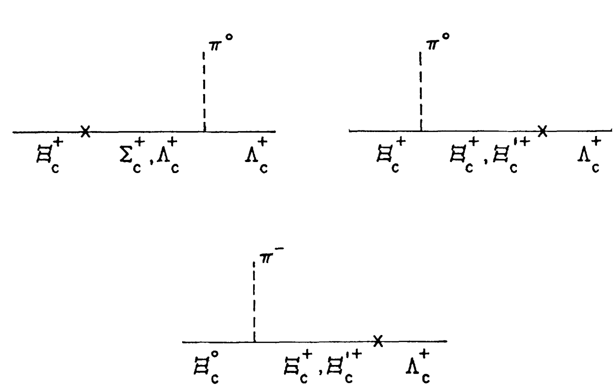

FIG. 1. Possible Feynman diagrams for the p-wave nonleptonic decay $\Xi_c \to \Lambda_c \pi$.

$$\begin{aligned} \Xi_{c1} &= \cos\phi\, \Xi_c + \sin\phi\, \Xi_c' , \\ \Xi_{c2} &= -\sin\phi\, \Xi_c + \cos\phi\, \Xi_c' , \end{aligned} \tag{34}$$

with ϕ being the mixing angle of Ξ_c and Ξ_c'. Then the decay $\Xi_{c1} \to \Lambda_c^+ \pi$ can proceed through the pole diagrams as exhibited in Fig. 2. However, the pole diagram due to the Σ_c intermediate state does not contribute because of vanishing weak transitions between two sextet heavy baryons (see next section). Consequently, we are led to the p-wave amplitude

$$\begin{aligned} B(\Xi_{c1}^+ \to \Lambda_c^+ \pi^0) = \frac{g_2}{2\sqrt{2} f_\pi} \frac{m_{\Xi_c} + m_{\Xi_c'}}{m_{\Lambda_c} - m_{\Xi_{c2}}} \\ \times \langle \Lambda_c^+ \uparrow | \mathcal{H}_{\text{eff}}^{\text{PC}} | \Xi_c^+ \uparrow \rangle \sin\phi \end{aligned} \tag{35}$$

and

$$B(\Xi_{c1}^0 \to \Lambda_c^+ \pi^-) = \sqrt{2} B(\Xi_{c1}^+ \to \Lambda_c^+ \pi^0) , \tag{36}$$

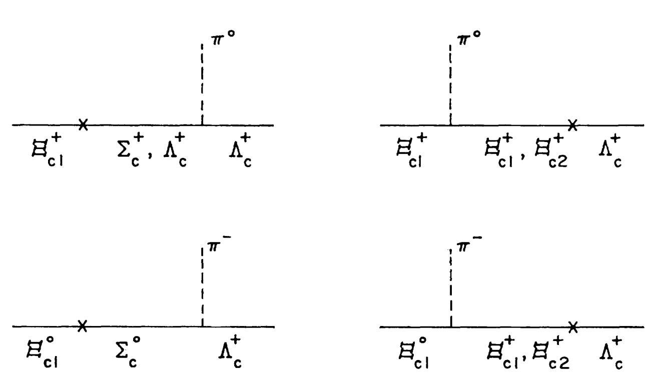

FIG. 2. Possible Feynman diagrams for the p-wave nonleptonic decay $\Xi_{c1} \to \Lambda_c \pi$ when $\Xi_c - \Xi_c'$ mixing is taken into account.

where g_2 is the coupling constant of the $B_6 B_{\bar{3}} \pi$ interaction defined in Eq. (3.12) of Ref. [1]. Because of the mixing effect, the s-wave amplitude (33) is modified to

$$A_{\rm com}(\Xi_{c1}^+ \to \Lambda_c^+ \pi^0) = -\frac{1}{2f_\pi} \langle \Lambda_c^+ \uparrow | \mathcal{H}_{\rm eff}^{\rm PC} | \Xi_c^+ \uparrow \rangle \cos\phi \ . \tag{37}$$

From Eqs. (35)–(37), it is clear that our next main task is to evaluate the parity-conserving baryon matrix element $\langle \Lambda_c^+ | \mathcal{H}_{\rm eff}^{\rm PC} | \Xi_c^+ \rangle$. This will be done in the next section.

III. MODEL CALCULATIONS

In this section we will first evaluate the matrix element $\langle \Lambda_c^+ | \mathcal{H}_{\rm eff}^{\rm PC} | \Xi_c^+ \rangle$ by using the effective Hamiltonian (1) in conjunction with the MIT bag model [6] and diquark fields [8]. The two methods give similar answers. These results are then applied to specific heavy-flavor-conserving nonleptonic decays of charmed baryons.

As already mentioned, only O_1 in (2) will contribute. Consequently,

$$\langle \Lambda_c^+ | \mathcal{H}_{\rm eff}^{\rm PC} | \Xi_c^+ \rangle = \frac{G_F}{\sqrt{2}} \sin\theta_C \cos\theta_C c_1 \times \langle \Lambda_c^+ | (\bar{d}s)(\bar{u}u)^{\rm PC} | \Xi_c^+ \rangle \ . \tag{38}$$

For the bag-model calculation, we will make use of a result that relates the matrix element of a local operator between two zero-momentum eigenstates to a matrix element of an integrated operator between two localized bag states. The relation is [14]

$$\langle A(\mathbf{p}=0) | O(0) | B(\mathbf{p}=0) \rangle = \left\langle A \left| \int d^3x \, O(\mathbf{x}) \right| B \right\rangle_{\rm bag} , \tag{39}$$

where the momentum eigenstates and bag states satisfy the normalizations

$$\langle B(p,\lambda) | B(p',\lambda') \rangle = (2\pi)^3 \frac{E}{M} \delta_{\lambda\lambda'} \delta^3(p-p') \quad \text{(baryons)} \tag{40}$$

and

$$\langle A | A \rangle_{\rm bag} = 1 \ . \tag{41}$$

Following the method of Ref. [7], we obtain, from Eq. (38),

$$\langle \Lambda_c^+ | (\bar{d}s)(\bar{u}u)^{\rm PC} | \Xi_c^+ \rangle = 2(4\pi) X \langle \Lambda_c^+ | b_{1u}^\dagger b_{1u} b_{2d}^\dagger b_{2s} (1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) | \Xi_c^+ \rangle , \tag{42}$$

where the subscript i indicates that the operator acts on the ith light quark in the baryon wave function, and the four-quark overlap integral X is given by [7]

$$X = \int_0^R r^2 dr (u_d u_u + v_d v_u)(u_s u_u + v_s v_u) \ . \tag{43}$$

Here R is the radius of the MIT bag and $u(r)$ as well as $v(r)$ are the large and small components of the quark wave function, respectively, defined by

$$\psi = \begin{bmatrix} iu(r)\chi \\ v(r) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} \chi \end{bmatrix} , \tag{44}$$

for the ground states. As mentioned before, in the antitriplet charmed-baryon wave functions

$$\begin{aligned} |\Lambda_c^+ \uparrow\rangle &= \tfrac{1}{2}[|u^\uparrow d^\downarrow c^\uparrow\rangle - |u^\downarrow d^\uparrow c^\uparrow\rangle - |d^\uparrow u^\downarrow c^\uparrow\rangle + |d^\downarrow u^\uparrow c^\uparrow\rangle] , \\ |\Xi_c^+ \uparrow\rangle &= \tfrac{1}{2}[|u^\uparrow s^\downarrow c^\uparrow\rangle - |u^\downarrow s^\uparrow c^\uparrow\rangle - |s^\uparrow u^\downarrow c^\uparrow\rangle + |s^\downarrow u^\uparrow c^\uparrow\rangle] , \end{aligned} \tag{45}$$

the two light quarks form spin singlets, so that

$$1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 = 4 \ . \tag{46}$$

It follows from Eqs. (38), (42), and (46) that

$$\langle \Lambda_c^+ \uparrow | \mathcal{H}_{\rm eff}^{\rm PC} | \Xi_c^+ \uparrow \rangle = \frac{G_F}{\sqrt{2}} (16\pi) \sin\theta_C \cos\theta_C c_1 X \ . \tag{47}$$

A straightforward bag-model calculation gives

$$X = 1.66 \times 10^{-4} \ \text{GeV}^3 , \tag{48}$$

which leads to

$$\langle \Lambda_c^+ \uparrow | \mathcal{H}_{\rm eff}^{\rm PC} | \Xi_c^+ \uparrow \rangle = -3.1 \times 10^{-8} \ \text{GeV} \ . \tag{49}$$

To obtain the numerical result (48), we have used the values of the bag parameters [6]

$$\begin{aligned} &m_u = m_d = 0 \ , \quad m_s = 0.279 \ \text{GeV} \ , \\ &m_c = 1.551 \ \text{GeV} \ , \quad R = 5 \ \text{GeV}^{-1} \ . \end{aligned} \tag{50}$$

We next turn to the diquark model for the evaluation of the baryon matrix element. To implement the diquark idea, we note that the effective Hamiltonian given by Eq. (1) can be recast in the form

$$\mathcal{H}_{\rm eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \sin\theta_C \cos\theta_C \{ b_1 (\bar{d}u)(\bar{u}s) + b_2 (\bar{u}u)(\bar{d}s) + \text{penguin operators} \} + \text{H.c.} , \tag{51}$$

where the Wilson coefficients b_i are related to c_i by the relations [9]

$$\begin{aligned} b_1 &= \tfrac{1}{2}(-c_1 + c_2 + c_3 + c_4) , \\ b_2 &= \tfrac{1}{2}(c_1 + c_2 + c_3 + c_4) . \end{aligned} \tag{52}$$

By performing a Fierz transformation, one can reexpress the effective Hamiltonian in an explicit local diquark-current form [8]

$$\mathcal{H}_{\rm eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \sin\theta_C \cos\theta_C \{ b_- (du)_{\bar{3}}^\dagger (us)_{\bar{3}} + b_+ (du)_6^\dagger (us)_6 + \text{penguin operators} \} + \text{H.c.} , \tag{53}$$

where

$$(du)_{\bar{3}}=\epsilon_{ijk}\bar{d}_i^c(1-\gamma_5)u_j\ ,$$

$$(us)_{\bar{3}}=\epsilon_{lmk}\bar{u}_l^c(1-\gamma_5)s_m\ , \tag{54}$$

$$b_\pm=b_1\pm b_2\ ,\quad b_-=-c_1\ ,$$

and i,j,k,l,m are color indices. In (53) and (54), we follow the same notation used in Ref. [8]. Since the two quarks in a baryon must be in a color-antisymmetric state (i.e., color-antitriplet state), the color sextet currents in (53) cannot contribute. The effective Hamiltonian (53) thus bears a simple interpretation in the constituent quark model: It annihilates a scalar or pseudoscalar antitriplet (us) diquark in the initial baryon and then creates a scalar or pseudoscalar antitriplet (du) diquark in the final baryon, leaving the spectator heavy quark unchanged. The measure of the annihilation and creation of diquarks through the diquark current in (53) is governed by the "diquark decay constant" $g_{qq'}$ defined by [8]

$$\langle 0|\epsilon_{ijk}\bar{q}_j^c\gamma_5 q_k'|(qq')_l^{0^+}\rangle=(\tfrac{2}{3})^{1/2}\delta_{il}g_{qq'} \tag{55}$$

for a 0^+ scalar diquark, where $(\frac{2}{3})^{1/2}$ is a color factor. The diquark states are normalized according to

$$\langle (qq')_l^{0^+}(\mathbf{k})|(qq')_m^{0^+}(\mathbf{k}')\rangle=(2\pi)^3 2E\,\delta^3(\mathbf{k}-\mathbf{k}')\delta_{lm}\ . \tag{56}$$

It has been shown in the literature [8] that the combination of the diquark decay constant and the corresponding Wilson coefficient is practically scale independent. In this paper we will adopt the value found in Ref. [8]:

$$b_-g_{du}g_{us}=0.075\pm0.015\ \text{GeV}^4\ . \tag{57}$$

Before proceeding to compute baryon matrix elements, it is worth mentioning that Eq. (22) is consistent with the effective Hamiltonian expressed in the diquark form. It comes from the fact that $\mathcal{H}_{\text{eff}}^{\Delta S=1}$ contains no products of one sextet and one antitriplet diquark currents. In the nonrelativistic quark model, the wave function of the $B_{\bar{3}}$ heavy-baryon state takes the form

$$|B_{\bar{3}}\rangle=(2\pi)^{3/2}\left[\frac{p_0}{m_B}\right]^{1/2}\int d^3\mathbf{k}\,d^3\mathbf{K}\,\delta(\mathbf{p}-\mathbf{k}-\mathbf{K})f(\mathbf{k},\mathbf{K})Q_i^\dagger(\mathbf{K})D_i^\dagger(\mathbf{k})|0\rangle\ , \tag{58}$$

where $Q_i^\dagger(\mathbf{K})$ is a creation operator for the heavy quark Q with color i and three-momentum $\mathbf{K}$, and $D_i^\dagger(\mathbf{k})$ creates a (qq') diquark with momentum $\mathbf{k}$ and obeys the commutation relation

$$\langle 0|[D_i(\mathbf{k}),D_j^\dagger(\mathbf{k}')]|0\rangle=\delta_{ij}\delta(\mathbf{k}-\mathbf{k}')\ . \tag{59}$$

The momentum-space wave function $f(\mathbf{k},\mathbf{K})$ describes a heavy baryon constructed from a heavy quark Q with momentum $\mathbf{K}$ plus a (qq') diquark with momentum $\mathbf{k}$. As far as the ground-state baryon-pole contributions to nonleptonic decays are concerned, we can neglect the 0^- pseudoscalar diquark and keep only the 0^+ scalar diquark contribution as the former is considerably heavier than the latter. This amounts to neglecting the parity-violating baryon matrix elements, which is known to be a good approximation in the treatment of hyperon nonleptonic weak decays. From Eqs. (53)–(59), it is easily shown that

$$\langle \Lambda_c^+\uparrow|\mathcal{H}_{\text{eff}}^{\text{PC}}|\Xi_c^+\uparrow\rangle=-\frac{G_F}{\sqrt{2}}\sin\theta_C\cos\theta_C\frac{1}{3m_D}b_-g_{du}g_{us}\ , \tag{60}$$

in the SU(3) limit, where m_D is the diquark mass and can be taken as

$$m_D=m_{\Lambda_c^+}-m_c\simeq785\ \text{MeV}\ . \tag{61}$$

This together with Eqs. (57) and (61) leads to

$$\langle \Lambda_c^+\uparrow|\mathcal{H}_{\text{eff}}^{\text{PC}}|\Xi_c^+\uparrow\rangle=-(5.8\pm1.2)\times10^{-8}\ \text{GeV}\ . \tag{62}$$

The diquark result (62) and bag result (49) predict the values of the coupling constant h to be

$$h=3.1\times10^{-8}\ \text{GeV}\quad(\text{MIT bag model})\ , \tag{63a}$$

$$h=(5.8\pm1.2)\times10^{-8}\ \text{GeV}\quad(\text{diquark model})\ . \tag{63b}$$

Here is a good place to show that the other coupling constant h' vanishes in the MIT bag model or diquark model. In the MIT-bag-model calculation, the matrix element $\langle B_6|\mathcal{H}_{\text{eff}}^{\Delta S=1}|B_6\rangle$ will contain the operator $(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$ as in (42). But $(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$ vanishes when it acts on a spin-1 diquark state such as in a B_6. In the diquark picture, with the help of the interpolating fields, we have

$$\langle B_6(v,s',\kappa')|\mathcal{H}_{\text{eff}}^{\Delta S=1}|B_6(v,s,\kappa)\rangle =\bar{B}_\mu(v,s',\kappa')B_\nu(v,s,\kappa)\langle 0|\phi_v^\mu(du)_{\bar{3}}^\dagger(us)_{\bar{3}}\phi_v^{\nu\dagger}|0\rangle\ . \tag{64}$$

The independent particle calculation used in the diquark model is equivalent to the vacuum saturation

$$\langle 0|\phi_v^\mu(du)_{\bar{3}}^\dagger(us)_{\bar{3}}\phi_v^{\nu\dagger}|0\rangle=\langle 0|\phi_v^\mu(du)_{\bar{3}}^\dagger|0\rangle\langle 0|(us)_{\bar{3}}\phi_v^{\nu\dagger}|0\rangle =dv^\mu v^\nu\ . \tag{65}$$

The last step follows from the fact that each factor must be proportional to v^μ. Equation (20) implies that the matrix element $\langle B_6|\mathcal{H}_{\text{eff}}^{\Delta S=1}|B_6\rangle$ vanishes. Thus we find both in the MIT bag and diquark models that

$$h'=0\ . \tag{66}$$

There is an additional factorizable contribution to the decay mode $\Xi_c^0 \to \Lambda_c^+ \pi^-$, which is absent in the framework of current algebra or chiral perturbation theory since such a contribution vanishes in the soft-pion limit. We will use the factorization scheme incorporating the large-N_c expansion, which is known to be a better framework to use for describing nonleptonic decays of heavy mesons [15]. This amounts to dropping Fierz-transformed terms. We note that the factorizable contribution induced by the operator O_1, for example, is

$$\begin{aligned}\langle \pi^- \Lambda_c^+ | O_1 | \Xi_c^0 \rangle &= -\langle \pi^-(q) | (\bar{d}u) | 0 \rangle \langle \Lambda_c^+ | (\bar{u}s) | \Xi_c^0 \rangle \\ &= -i\sqrt{2} f_\pi \bar{u}_{\Lambda_c} [(m_{\Xi_c} - m_{\Lambda_c}) f_1^{\Lambda_c \Xi_c} \\ &\quad + (m_{\Xi_c} + m_{\Lambda_c}) \tilde{f}_1^{\Lambda_c \Xi_c} \gamma_5] u_{\Xi_c} ,\end{aligned} \tag{67}$$

where f_1 and $\tilde{f}_1$ are the form factors defined by

$$\begin{aligned}\langle \Lambda_c^+ | (\bar{u}s) | \Xi_c^0 \rangle = \bar{u}_{\Lambda_c} [& f_1^{\Lambda_c \Xi_c} \gamma_\mu + f_2^{\Lambda_c \Xi_c} i\sigma_{\mu\nu} q^\nu \\ & + f_3^{\Lambda_c \Xi_c} q_\mu - \tilde{f}_1^{\Lambda_c \Xi_c} \gamma_\mu \gamma_5 \\ & - \tilde{f}_2^{\Lambda_c \Xi_c} i\sigma_{\mu\nu} q^\nu \gamma_5 - \tilde{f}_3^{\Lambda_c \Xi_c} q_\mu \gamma_5] u_{\Xi_c} .\end{aligned} \tag{68}$$

Performing a Fierz transformation on the penguin operators O_5, O_6 gives

$$O_5 = \tfrac{16}{3} \sum_q (\bar{d}_L^\alpha q_R^\beta)(\bar{q}_R^\beta s_L^\alpha) - 16 \sum_q (\bar{d}_L^\alpha q_R^\alpha)(\bar{q}_R^\beta s_L^\beta) , \tag{69a}$$

$$O_6 = -8 \sum_q (\bar{d}_L^\alpha q_R^\beta)(\bar{q}_R^\beta s_L^\alpha) , \tag{69b}$$

where α and β are color indices, $q = u,d,s$, and use has been made of

$$\sum_a \lambda_{\alpha\beta}^a \lambda_{\gamma\delta}^a = -\tfrac{2}{3} \delta_{\alpha\beta} \delta_{\gamma\delta} + 2 \delta_{\alpha\delta} \delta_{\beta\gamma} . \tag{70}$$

Applying the equations of motion,

$$-i\partial^\mu (\bar{q}_1 \gamma_\mu \gamma_5 q_2) = (m_1 + m_2) \bar{q}_1 \gamma_5 q_2 , \tag{71a}$$

$$-i\partial^\mu (\bar{q}_1 \gamma_\mu q_2) = (m_1 - m_2) \bar{q}_1 q_2 , \tag{71b}$$

we obtain in the large-N_c limit that

$$\begin{aligned}\langle \pi^- \Lambda_c^+ | O_5 | \Xi_c^0 \rangle &= -4 \frac{m_\pi^2}{(m_u + m_d)(m_s - m_d)} \langle \pi^- \Lambda_c^+ | O_1 | \Xi_c^0 \rangle , \\ \langle \pi^- \Lambda_c^+ | O_6 | \Xi_c^0 \rangle &= 0 .\end{aligned} \tag{72}$$

Since the form factor $\tilde{f}_1^{\Lambda_c \Xi_c}$ vanishes in the heavy-quark limit [see Eq. (3.26) of Ref. [1]], it is evident that only the s-wave amplitude receives a factorizable contribution

$$A_{\text{fac}}(\Xi_{c1}^0 \to \Lambda_c^+ \pi^-) = \frac{G_F}{2} \sin\theta_C \cos\theta_C \cos\phi f_\pi (m_{\Xi_c} - m_{\Lambda_c^+}) f_1^{\Lambda_c \Xi_c} \left[-c_1 + c_2 + c_3 + c_4 + 4 \frac{m_\pi^2}{(m_u + m_d)(m_s - m_d)} c_5 \right] . \tag{73}$$

Note that the penguin contribution is destructive. The form factor $f_1^{\Lambda_c \Xi_c}$ is evaluated in the bag model to be

$$\begin{aligned}f_1^{\Lambda_c \Xi_c} &= 4\pi \int_0^R r^2 dr (u_u u_s + v_u v_s) \langle \Lambda_c^+ \uparrow | b_u^\dagger b_s | \Xi_c^0 \uparrow \rangle \\ &= -4\pi \int_0^R r^2 dr (u_u u_s + v_u v_s) .\end{aligned} \tag{74}$$

We find, numerically,

$$f_1^{\Lambda_c \Xi_c} = -0.985 . \tag{75}$$

Since the mass difference of Ω_c and Ξ_c (Ξ_c') is about 280 (180) MeV [13], the weak decays $\Omega_c^0 \to \Xi_c' \pi$ and $\Xi_c \pi$ are also kinematically allowed. Because of Eqs. (22) and (66), it is easily seen that charm-flavor-conserving decays of Ω_c cannot proceed except for $\Omega_c^0 \to \Xi_c'^+ \pi^-$, which receives a factorizable contribution given by

$$\begin{aligned}M_{\text{fac}}(\Omega_c^0 \to \Xi_c'^+ \pi^-) = i \frac{G_F}{2} \sin\theta_C \cos\theta_C f_\pi & [(m_{\Omega_c} - m_{\Xi_c'}) f_1^{\Xi_c' \Omega_c} + (m_{\Omega_c} + m_{\Xi_c'}) \tilde{f}_1^{\Xi_c' \Omega_c} \gamma_5] \\ & \times \left[-c_1 + c_2 + c_3 + c_4 + 4 \frac{m_\pi^2}{m_s (m_u + m_d)} c_5 \right] ,\end{aligned} \tag{76}$$

with

$$\begin{aligned}f_1^{\Xi_c' \Omega_c} &= 4\pi \int_0^R r^2 dr (u_u u_s + v_u v_s) \langle \Xi_c'^+ \uparrow | b_u^\dagger b_s | \Omega_c^0 \uparrow \rangle \\ &= -\sqrt{2} f_1^{\Lambda_c \Xi_c}\end{aligned} \tag{77a}$$

and

$$\begin{aligned}\tilde{f}_1^{\Xi_c' \Omega_c} &= 4\pi \int_0^R r^2 dr (u_u u_s - \tfrac{1}{3} v_u v_s) \langle \Xi_c'^+ \uparrow | b_u^\dagger b_s \sigma_z | \Omega_c^0 \uparrow \rangle \\ &= 0.709 \times \frac{2\sqrt{2}}{3} .\end{aligned} \tag{77b}$$

In (76) we have neglected the down-quark mass relative to that of the strange quark, and so the terms dependent on the Wilson coefficients become a common factor of both the s- and p-wave amplitudes.

We are now ready to present numerical results. Collecting the results obtained so far, we find the s- and p-wave amplitudes

$$\begin{aligned} A(\Xi_{c1}^0 \to \Lambda_c^+ \pi^-) &= A_{\rm com} + A_{\rm fac} \\ &= (3.4\times 10^{-7} - 3.4\times 10^{-8})\cos\phi\ , \\ A(\Xi_{c1}^+ \to \Lambda_c^+ \pi^0) &= \frac{1}{\sqrt{2}} 3.4\times 10^{-7}\cos\phi\ , \\ A(\Omega_c^0 \to \Xi_{c2}^+ \pi^-) &= A_{\rm fac} = 5\times 10^{-8}\cos\phi\ , \\ B(\Xi_{c1}^0 \to \Lambda_c^+ \pi^-) &= \sqrt{2} B(\Xi_{c1}^+ \to \Lambda_c^+ \pi^0) \\ &= 2.7\times 10^{-6}\sin\phi\ , \\ B(\Omega_c^0 \to \Xi_{c2}^+ \pi^-) &= B_{\rm fac} = -7.4\times 10^{-7}\cos\phi\ , \end{aligned} \tag{78}$$

where use of $g_2 = -(\frac{2}{3})^{1/2}$ (0.75) [1] and Eqs. (3), (75), and (77) has been made. To arrive at (78), we have used the average value $\sim -4.5\times 10^{-8}$ GeV [cf. Eq. (63)] for the matrix element $\langle \Lambda_c^+ | \mathcal{H}_{\rm eff}^{\rm PC} | \Xi_c^+ \rangle$ and the current quark masses $m_u \approx 5.6$ MeV, $m_d \approx 9.9$ MeV, and $m_s \approx 199$ MeV [16]. Note that the factorizable amplitude is sensitive to the choice of the quark masses. It is easily seen that its magnitude in Eq. (78) will be reduced by a factor of 2 if the Weinberg's estimate [17] $m_u \approx 4.2$ MeV, $m_d \approx 7.5$ MeV, and $m_s \approx 150$ MeV is used. The decay rate for $B_i \to B_f + P$ is given by

$$\Gamma = \frac{p}{8\pi} \left\{ \frac{(m_i + m_f)^2 - m_P^2}{m_i^2} |A|^2 + \frac{(m_i - m_f)^2 - m_P^2}{m_i^2} |B|^2 \right\} , \tag{79}$$

where p is the momentum of the meson in the rest frame of B_i. Evidently, the p-wave effect is badly suppressed because $(m_i - m_f)^2 \ll (m_i + m_f)^2$. We conclude that the decay rate is dominated by the s-wave channel. Assuming $\Xi_{c1} \approx \Xi_c$ and hence $\Xi_{c2} \approx \Xi_c'$, we obtain

$$\begin{aligned} \Gamma(\Xi_c^0 \to \Lambda_c^+ \pi^-) &= 1.7\times 10^{-15}\ \text{GeV}\ , \\ \Gamma(\Xi_c^+ \to \Lambda_c^+ \pi^0) &= 1.0\times 10^{-15}\ \text{GeV}\ , \\ \Gamma(\Omega_c^0 \to \Xi_c'^+ \pi^-) &= 4.3\times 10^{-17}\ \text{GeV}\ . \end{aligned} \tag{80}$$

Using the theoretical values of the charmed-baryon lifetimes [18,19],

$$\begin{aligned} &\tau(\Xi_c^0) = 1.5\times 10^{-13}\ \text{s}\ , \quad \tau(\Xi_c^+) = 3.3\times 10^{-13}\ \text{s}\ , \\ &\tau(\Omega_c^0) = 1.3\times 10^{-13}\ \text{s}\ , \end{aligned} \tag{81}$$

we finally get the branching ratios

$$\begin{aligned} B(\Xi_c^0 \to \Lambda_c^+ \pi^-) &= 3.8\times 10^{-4}\ , \\ B(\Xi_c^+ \to \Lambda_c^+ \pi^0) &= 5.0\times 10^{-4}\ , \\ B(\Omega_c^0 \to \Xi_c'^+ \pi^-) &= 0.9\times 10^{-5}\ . \end{aligned} \tag{82}$$

Recall that the branching ratio of Cabibbo-allowed decays of charmed baryons, e.g., $\Lambda_c^+ \to \Lambda\pi^+$, is typically of order 1%. Therefore the predicted branching ratios for the charm-flavor-conserving decays $\Xi_c^0 \to \Lambda_c^+ \pi^-$, $\Xi_c^+ \to \Lambda_c^+ \pi^0$ are of the same order of magnitude as singly Cabibbo-suppressed decay modes, e.g., $\Xi_c^+ \to \Sigma\pi^+$, $\Xi_c^0 \to \Sigma^+ \pi^-$.

Up to now, we have been mostly concerned with the evaluation of the parity-conserving weak matrix element $\langle \Lambda_c^+ \uparrow | \mathcal{H}_{\rm eff}^{\rm PC} | \Xi_c^+ \uparrow \rangle$. As we have shown in Sec. II, the corresponding matrix element for the parity-violating part of the weak Hamiltonian $\mathcal{H}_{\rm eff}^{\rm PV}$ vanishes in the heavy-quark limit. It also vanishes in the bag model when the momentum transfer $\mathbf{q} = 0$ between the initial and final states. A method for extracting the p-wave amplitude from this matrix element has been proposed [20]. We find that this (nonleading) matrix element only connects a heavy baryon in the symmetric sextet in flavor SU(3) to a heavy baryon in the antisymmetric antitriplet. Furthermore, whether or not the matrix element vanishes depends on the quark contents of the states. For example,

$$\langle \Lambda_c^+ \uparrow | \mathcal{H}_{\rm eff}^{\rm PV} | \Xi_c'^+ \uparrow \rangle \neq 0\ , \tag{83a}$$

but

$$\langle \Xi_c^+ \uparrow | \mathcal{H}_{\rm eff}^{\rm PV} | \Sigma_c^+ \uparrow \rangle = 0\ . \tag{83b}$$

This is a most unexpected result. It is not clear to us whether the result holds beyond the MIT bag model. It certainly deserves further study.

IV. CONCLUSIONS

The synthesis of the heavy-quark and chiral symmetries offers a new framework for studying strong and weak interactions of heavy hadrons with Goldstone bosons. A crucial requirement is that the Goldstone bosons involved must be soft. Consequently, this formalism is only applicable to certain classes of physical processes. In addition to the strong decays and semileptonic decays of the heavy hadrons studied in Ref. [1], heavy-flavor-conserving nonleptonic decays of heavy baryons studied in this paper also belong to the class.

The combined symmetries of heavy and light quarks severely restrict the weak transitions allowed. In the symmetry limit, we find that there cannot be $B_{\bar{3}}$-B_6 and B_6^*-B_6 nonleptonic weak transitions. Symmetries alone permit three types of transitions: $B_{\bar{3}}$-$B_{\bar{3}}$, B_6-B_6, and B_6^*-B_6^* transitions. However, in both the MIT bag and diquark models, only $B_{\bar{3}}$-$B_{\bar{3}}$ transitions have nonzero amplitudes. These transitions, such as $\Xi_c^0 \to \Lambda_c^+ \pi^-$, $\Xi_c^+ \to \Lambda_c^+ \pi^0$, have a branching ratio of order 10^{-4}.

The B_6-B_6 transition $\Omega_c^0 \to \Xi_c'^+ \pi^-$, which vanishes in the chiral limit, receives a finite factorizable contribution as a result of symmetry-breaking effects. Its branching ratio is estimated to be about 10^{-5}.

We urge the experimentalists to check carefully our predictions: (1) the rates and branching ratios of the allowed $B_{\bar{3}}$-$B_{\bar{3}}$ transitions obtained in Sec. III, (2) the absence of $B_{\bar{3}}$-B_6 and B_6^*-B_6 transitions in the limit of heavy-quark symmetry, and (3) the weaker predictions by the MIT bag and diquark models that in the symmetry limit B_6-B_6 and B_6^*-B_6^* nonleptonic weak transitions should not occur. These transitions are, therefore, suppressed relative to the allowed $B_{\bar{3}}$-$B_{\bar{3}}$ transitions. We already see that a transition of this kind, $\Omega_c^0 \to \Xi_c'^+ \pi^-$, can proceed via factorizable processes, but with a branching ratio smaller by one order of magnitude.

Finally, we would like to stress that symmetry-breaking effects be systematically investigated. In the limit of heavy quarks and soft pions, the theory is a double expansion in the pion momenta and inverse heavy-quark masses. It is important to ascertain what the corrections are to the results obtained in this paper. We would also like to clarify the results on the matrix elements $\langle B_6 | \mathcal{H}_{\rm eff}^{\rm PV} | B_{\bar{3}} \rangle$ in the MIT bag model which seem to depend on the quark contents of the states.

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