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Mass corrections to "forbidden" charmonium decays: $\eta_c, \chi_{c0} \rightarrow p\bar{p}$

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Some exclusive decays of charmonium states are forbidden, in the framework of perturbative QCD quark models, by the helicity-conservation rule, related to the vector coupling of gluons to (almost) massless quarks. Among them, we consider here η_c and χ_{c0} decays into $p\bar{p}$; after discussing possible nonperturbative corrections, we show that, assigning to the quarks a constituent rather than a current mass, one obtains nonzero values for such processes. The values found for $\Gamma(\chi_{c0} \rightarrow p\bar{p})$ are almost as large as those obtained for similar, nonforbidden decays in the massless quark scheme, $\chi_{c1,c2} \rightarrow p\bar{p}$; $\Gamma(\eta_c \rightarrow p\bar{p})$, however, turns out to be much smaller than the experimental data, enhancing, once more, the peculiarity of many η_c decays. Mass corrections to some other allowed decays, $\chi_{c0} \rightarrow \pi\pi$ and χ_{c0} into longitudinally polarized vector mesons, are computed and shown to be relevant.

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I. INTRODUCTION

Exclusive heavy-meson decays into hadrons are supposed to be a good testing ground for perturbative QCD: The heavy quarks in the initial state can be safely treated in the nonrelativistic, zero-binding approximation, and the decays are mediated by the exchange of hard (large-Q^2) gluons, which create $q\bar{q}$ pairs, which, in turn, hadronize into the final observed hadrons. Factorization is supposed to hold, in that the elementary constituent interaction, the creation of $q\bar{q}$ pairs starting from a heavy-quark pair, can be computed separately according to perturbative QCD rules and then convoluted with the final hadron wave functions. Such a scheme is expected to work in the large-Q^2 limit, as advocated by many authors [1–4].

In practice, however, experimental information is available only for charmonium decays [5–10], that is, for a Q^2 region of few GeV^2, where it is not yet clear whether perturbative QCD alone should account for a correct description or other nonperturbative effects should still be non-negligible. The comparison between the theoretical predictions and the experimental data shows some failure as well as some successes of the perturbative QCD scheme, suggesting that, indeed, at least in some cases, higher-order or nonperturbative corrections should still be carefully considered.

The computations of the decay rates of J/ψ and $\chi_{c0,c2} \rightarrow p\bar{p}$ are among the perturbative QCD successes [11–14]. The numerical values depend very strongly on the choice of the distribution amplitudes of the quark momenta inside the proton: The choice suggested by QCD sum rules [14–16] is the one which consistently best reproduces the experimental data. Also, $\chi_{c0,c2} \rightarrow \pi\pi$ and $\rho\rho$ have been computed [17]: again, the QCD sum-rule wave functions allow a very good agreement with the data on $\Gamma(\chi_{c0,c2} \rightarrow \pi\pi)$. Similar values are found for $\Gamma(\chi_{c0,c2} \rightarrow \rho\rho)$, but no data are yet available on such processes.

In other charmonium decays, instead, the perturbative QCD scheme for exclusive processes seems to fail. The reason is due to the vector coupling of gluons and quarks, which, in the limit of massless quarks, conserves the quark helicity: This simple fact leads to the "helicity-conservation rule" in exclusive processes [11], which forbids many two-body heavy-meson decays [17]. The $q\bar{q}$ pairs emitted from a gluon with high virtuality must have opposite helicities; the quark (antiquark) helicities sum up to the final particle (antiparticle) helicity; then, also, the particle-antiparticle pairs created, via hard gluon exchanges, in the decay of heavy mesons, must be in opposite-helicity states. This immediately forbids, for example, the decays $\eta_c, \chi_{c0} \rightarrow p\bar{p}$: A spin-0 particle cannot decay into two fermions with opposite helicities. Similarly forbidden decays are $\eta_c \rightarrow VV$ (V=vector meson) and $J/\psi \rightarrow \pi\rho$ (in general, $J/\psi \rightarrow$any pseudoscalar–vector-meson pair). However, most of these decays have been observed [10].

The intermediate-Q^2 region of charmonium decays may offer many possible solutions to the above problems, some of which have been investigated in the literature. The contribution of higher-order Fock states (such as $q\bar{q}g$ in a meson), depressed by powers of α_s/Q^2, might still be important in some decays such as $J/\psi \rightarrow \rho\pi$ [4] and $\eta_c, \chi_{c0,c2} \rightarrow \omega\phi$ [18]. Also, the intrinsic transverse momentum k_T of quarks inside the final hadrons might help, implying that the quark helicity does not coincide exactly with its spin projection onto the direction of the hadron momentum, so that pairs of opposite-helicity quarks do not necessarily hadronize into opposite-helicity hadrons. This correction should be proportional to k_T/m_c, where m_c is the charmed-quark mass; its full evaluation, however, is not simple and has never been performed. More radical solutions have also been proposed, concerning the $J/\psi \to \rho\pi$, $K^*\bar{K}$, and the η_c decays, by advocating strong gluonic components inside the J/ψ [19] and the η_c [20]; in such cases, the analogous decays for the J' and η_c' should be strongly suppressed, as required by the helicity-conservation rule: This is actually observed for the J', while no data are yet available for the η_c'.

In another attempt to overcome the problems which massless perturbative QCD has to face in the description of charmonium decays into $p\bar{p}$ (and in many other spin effects in exclusive reactions), a quark-diquark model of the nucleon has been introduced and widely applied [21–23]. Two quark correlations, induced by QCD color forces, must exist inside baryons [24]: In the intermediate-Q^2 region of the charmonium decays, such correlations behave as effective single particles, scalar or (pseudo)vector diquarks. The coupling of vector diquarks to gluons allows helicity flips, thus avoiding the decay selection rules imposed by helicity conservation. The quark-diquark model has been applied to the description of $\eta_c, \chi_{c0,c1,c2} \to p\bar{p}$ [21,22] and $J/\psi \to \gamma p\bar{p}$ [23] decays: It agrees, as well as perturbative QCD, with the data on $\Gamma(\chi_{c1,c2} \to p\bar{p})$, and it also gives a reasonable account of the data on $J/\psi \to \gamma p\bar{p}$. Concerning the decays forbidden in perturbative QCD, it yields a value for $\Gamma(\chi_{c0} \to p\bar{p})$ similar to, or greater than, those measured for $\chi_{c1,c2}$ and in agreement with an existing large upper bound; however, the value found for $\Gamma(\eta_c \to p\bar{p})$, although different from zero, turns out to be much smaller than the experimental data.

In this paper we consider yet another class of nonperturbative corrections to the original perturbative QCD scheme, namely, mass corrections. According to the model of Refs. [1–4], the elementary interactions among quarks and gluons are computed, following the perturbative QCD Feynman rules, assigning the light quarks their current mass of few MeV. In the small-Q^2 region, however, one might think that the constituent quarks, that is, the current quarks surrounded by their cloud of $q\bar{q}$ pairs and gluons, still act as a single particle; moreover, as shown by Weinberg [25], these constituent quarks can be treated as bare Dirac particles, with the same couplings as for current quarks in the standard SU(3)⊗SU(2)⊗U(1) Lagrangian. It is then natural, in small-Q^2 regions, to assign the quark an effective mass xm_H, like in the naive parton model, where x is the fraction of the four-momentum of the hadron H (with mass m_H) carried by the quark. The different values of x will be weighted by the hadron wave function. Massive quarks will allow helicity flips in the elementary amplitudes, proportional to m_H/m_c: We expect then to obtain nonzero values for the charmonium decays forbidden in the perturbative QCD scheme with massless quarks. In particular, we discuss in this paper the decays $\eta_c \to p\bar{p}$ and $\chi_{c0} \to p\bar{p}$. Quark mass effects were previously considered in $J/\psi \to B\bar{B}$ [26] and $\eta_c \to VV$ [27] decays.

The plan of the work is as follows. In Sec. II we recall the general formalism for the computation of the decay rate of heavy mesons in the QCD model of Refs. [1–4] and we first apply it, with massive quarks, to the computation of $\Gamma(\chi_{c0} \to \pi\pi)$ and $\Gamma(\chi_{c0} \to V_L V_L)$, where V_L is a longitudinally polarized vector meson (ρ, K^*, ϕ); such decays have already been computed in the massless case [4], with results which we rederive in the limit $m_q \to 0$. We also show explicitly that mass corrections are relevant. In Sec. III we compute $\Gamma(\chi_{c0} \to p\bar{p})$ and $\Gamma(\eta_c \to p\bar{p})$ with different choices of the proton wave function and compare the results with the experimental data. While the results for χ_{c0} are almost as large as those measured for $\Gamma(\chi_{c1,c2} \to p\bar{p})$, the results for η_c are still much smaller than the data. In Sec. IV we give some further comments on the peculiarities of the η_c decays, together with the conclusions.

II. GENERAL FORMALISM AND MASS CORRECTIONS TO $\chi_{c0} \to \pi\pi, V_L V_L$ DECAYS

In the perturbative QCD scheme of Refs. [1–4], the amplitudes for the decay of a heavy ($c\bar{c}$) meson at rest with quantum numbers J,M,L,S into a pair of hadrons with helicities $\lambda_H, \lambda_{\bar{H}}$ are given, in the constituent helicity basis, by [21]

$$A_{JMLS}^{\lambda_H \lambda_{\bar{H}}}(\theta) = \sum_{\lambda_c \lambda_{\bar{c}}} \left[\frac{2L+1}{4\pi} \right]^{1/2} C^{1/2\,1/2\,S}_{\lambda_c\,-\lambda_{\bar{c}}\,\lambda} C^{LSJ}_{0\lambda\lambda} \int d^3k \, M_{\lambda_H \lambda_{\bar{H}};\lambda_c \lambda_{\bar{c}}}(\theta;\mathbf{k}) D^{J*}_{M\lambda}(\beta,\alpha,0) \psi_c(k) \,, \tag{2.1}$$

where

$$\mathbf{k} = (k \sin\alpha \cos\beta,\ k \sin\alpha \sin\beta,\ k \cos\alpha) \,,$$

λ_c and $\lambda_{\bar{c}}$ are, respectively, the c and $\bar{c}$ relative momentum and helicities; $\lambda = \lambda_c - \lambda_{\bar{c}}$; the matrix rotation $D(\beta,\alpha,0)$ and $\psi(k)$ originate from the charmonium wave function in momentum space, and the C are the usual Clebsch-Gordan coefficients for the c and $\bar{c}$ quarks to combine into a state with spin S, which, in turn, combines with an orbital angular momentum L to make up a charmonium state with total spin J. Finally, θ is the angle between the final hadron H and the spin quantization axis for the initial state, always chosen as the z axis, and the helicity amplitudes M for the process $c\bar{c} \to H\bar{H}$ are given by

$$M_{\lambda_H \lambda_{\bar{H}};\lambda_c \lambda_{\bar{c}}}(\theta;\mathbf{k}) = \sum_{\{\lambda_h\},\{\lambda_{\bar{h}}\}} \int [dx_h][dx_{\bar{h}}] \psi^*_{H,\lambda_H}(x_h,\{\lambda_h\}) \psi^*_{\bar{H},\lambda_{\bar{H}}}(x_{\bar{h}},\{\lambda_{\bar{h}}\}) T_{\{\lambda_h\},\{\lambda_{\bar{h}}\};\lambda_c \lambda_{\bar{c}}}(\mathbf{k};x_h,x_{\bar{h}},\theta) \,, \tag{2.2}$$

where by x_i and $\{\lambda_i\}$ we denote, respectively, the whole set of momentum fractions and helicities carried by the quarks inside the hadron I ($I=H,\bar{H}$). As usual, $[dx_i]=dx_1dx_2\cdots dx_{n_I}\delta(1-x_1-x_2-\cdots-x_{n_I})$, with n_I the number of valence quarks inside hadron I. The ψ are the (flavor, color, spin, and momentum) final hadron wave functions, and the T are the helicity amplitudes for the elementary interaction which annihilates the initial $c\bar{c}$ pair and creates new $q\bar{q}$ pairs. All final quarks are supposed to be collinear, moving parallel to their parent hadron, and their helicities sum up to the hadron helicity. In Eq. (2.2) we have not explicitly shown the Q^2 dependence of the hadronic wave function induced by QCD evolution [2].

Equations (2.1) and (2.2) apply, in principle, to the two-body decays of mesons with a very large mass, such that the virtual gluons exchanged between $q\bar{q}$ pairs all have a large value of Q^2 and lowest-order perturbative QCD gives the leading contribution. In such a case, all masses, including the final hadron masses, can safely be neglected. However, charmonium masses, typically between 3 and 4 GeV, might not yet be, as we said in the Introduction, in the above asymptotic region; certainly, in most cases, the final hadron masses cannot be neglected relatively to the c-quark mass. It is the purpose of this paper, as we explained in the previous section, to leave aside other possible corrections and to explore the consequences of assigning the quarks, throughout the paper, not their tiny current masses, but their constituent ones.

Our computation of charmonium decay rates will still be performed in the theoretical scheme of Eqs. (2.1) and (2.2); the only difference is that each quark is now carrying a fraction x of the hadron *four-momentum*, so that its mass, like in the naive quark model, is $m_q=xm_H$; we also properly keep into account, in all the kinematics and phase-space integrations, the hadron masses m_H. The elementary quark interactions are computed according to the usual perturbative QCD rules, which amounts to considering the constituent quarks as effective Dirac particles with the usual pointlike couplings [25]. The only ambiguity in such a scheme, as we shall see in Sec. III, might be related to the value of the mass of the quark which should appear in its propagator.

In this section we start by computing, with massive quarks, the decay rates $\Gamma(\chi_{c0}\to\pi\pi, V_LV_L)$, where V_L is a longitudinally polarized ($\lambda_V=0$) vector meson. Such decays have also been computed in the massless case [17], in good agreement, although strongly dependent on the choice of the meson wave functions, with the existing experimental data. We can then check that our results reproduce, in the $m_q\to 0$ limit, the previous ones and show that the mass corrections can be very sizable.

A. Decay $\chi_{c0}\to\pi^+\pi^-$

Inserting the quantum numbers of the χ_{c0} charmonium state ($L=S=1$, $J=0$) into Eq. (2.1), we have (dropping all helicity indices which are trivially zero)

$$A(\theta)=-\frac{1}{2\sqrt{2\pi}}\int d^3k\,\psi_{\chi_{c0}}(k)\{M_{++}(\theta,\mathbf{k})+M_{--}(\theta,\mathbf{k})\}\ . \tag{2.3}$$

The pion wave functions are given by

$$\begin{aligned}\psi_{\pi^+}(x)&=\frac{f_\pi}{4\sqrt{3}}(u_-\bar{d}_+-u_+\bar{d}_-)\varphi(x)\ ,\\ \psi_{\pi^-}(y)&=\frac{f_\pi}{4\sqrt{3}}(d_+\bar{u}_--d_-\bar{u}_+)\varphi(y)\ ,\end{aligned} \tag{2.4}$$

where f_π is the pion decay constant whose value (according to the conventions here adopted) is $f_\pi\simeq 133$ MeV. In Eq. (2.4) we have omitted, for simplicity of notation, the color indices and the φ are the distribution amplitudes of the quarks inside the pions. By using Eqs. (2.4) in Eq. (2.2), one obtains

$$M_{\lambda_c\lambda_{\bar{c}}}=\frac{f_\pi^2}{48}\int dx\,dy\,\varphi(x)\varphi(y)\{T_{+--+;\lambda_c\lambda_{\bar{c}}}+T_{-++-;\lambda_c\lambda_{\bar{c}}}-T_{+-+-;\lambda_c\lambda_{\bar{c}}}-T_{-+-+;\lambda_c\lambda_{\bar{c}}}\}\ . \tag{2.5}$$

One has now to compute the elementary amplitudes T contributing to Eq. (2.5); the corresponding Feynman diagrams are shown in Fig. 1, where we also define the kinematics.

The two-body decay of the spinless χ_{c0} is isotropic: Thus it suffices to compute the elementary amplitudes T in the simplest configuration of the π^+ moving along the z axis ($\theta=0$). We do not repeat here the details of the calculation which can be found in Ref. [28]; one finds

$$\begin{aligned}T_{\lambda_{q_1},-\lambda_{q_1},-\lambda_{\bar{q}_2},\lambda_{\bar{q}_2};\lambda_c\lambda_{\bar{c}}}=-i8C_d\Big\{&km_\pi^2\cos^2\alpha\,\delta_{\lambda_{q_1}\lambda_{\bar{q}_2}}\delta_{\lambda_c\lambda_{\bar{c}}}+4\lambda_c m_\pi^2 k\frac{E}{M_\chi}\sin\alpha\cos\alpha\,\delta_{\lambda_{q_1}\lambda_{\bar{q}_2}}\delta_{\lambda_c,-\lambda_{\bar{c}}}\\ &-E^2[k\sin^2\alpha+8\lambda_{q_1}\lambda_c(x-y)p]\delta_{\lambda_{q_1},-\lambda_{\bar{q}_2}}\delta_{\lambda_c\lambda_{\bar{c}}}\\ &+4k\frac{E^3}{M_\chi}[\lambda_{q_1}(x+y-1)\sin\alpha+\lambda_c\sin\alpha\cos\alpha]\delta_{\lambda_{q_1},-\lambda_{\bar{q}_2}}\delta_{\lambda_c,-\lambda_{\bar{c}}}\Big\}\\ -i8C_f(x-y)p\Big\{&m_\pi^2\cos\alpha\,\delta_{\lambda_{q_1}\lambda_{\bar{q}_2}}\delta_{\lambda_c\lambda_{\bar{c}}}+4\lambda_c m_\pi^2\frac{E}{M_\chi}\sin\alpha\,\delta_{\lambda_{q_1}\lambda_{\bar{q}_2}}\delta_{\lambda_c,-\lambda_{\bar{c}}}\\ &+2E^2\cos\alpha\,\delta_{\lambda_{q_1},-\lambda_{\bar{q}_2}}\delta_{\lambda_c\lambda_{\bar{c}}}+8\lambda_c\frac{E^3}{M_\chi}\sin\alpha\,\delta_{\lambda_{q_1},-\lambda_{\bar{q}_2}}\delta_{\lambda_c,-\lambda_{\bar{c}}}\Big\}\ ,\end{aligned} \tag{2.6}$$

where

$$C_d = \frac{Cd^2}{d^4-(x-y)^2k^2p^2\cos^2\alpha} \ , \tag{2.7a}$$

$$C_f = (x-y)\frac{Cf^2}{d^4-(x-y)^2k^2p^2\cos^2\alpha} \ , \tag{2.7b}$$

with

$$C = \frac{-ic_F g_s^4}{g_1^2 g_2^2} \ , \tag{2.7c}$$

$$g_1^2 = (x-y)^2 m_\pi^2 + 4xyE^2 \ , \tag{2.7d}$$

$$g_2^2 = (x-y)^2 m_\pi^2 + 4(1-x)(1-y)E^2 \ , \tag{2.7e}$$

$$d^2 = (x-y)^2 m_\pi^2 + 2(2xy-x-y)E^2 \ , \tag{2.7f}$$

$$f^2 = \mathbf{k}\cdot\mathbf{p} = kp\cos\alpha \ . \tag{2.7g}$$

In Eq. (2.7c), c_F is the color factor, which, when convoluted with the final meson wave functions, yields a factor $2/(3\sqrt{3})$.

We proceed by using Eqs. (2.7) and (2.6) in Eqs. (2.5) and (2.3) and perform the d^3k integration by exploiting the explicit form of the χ_{c0} wave function $\psi(k)$ in the nonrelativistic limit

$$\psi_{\chi_{c0}}(k) = i3\sqrt{2\pi}|R'(0)|\frac{1}{k^3}\delta(k) \ . \tag{2.8}$$

It results in

$$A(\chi_{c0}\to\pi^+\pi^-) = -i\frac{4096}{9\sqrt{3}}\pi^3\alpha_s^2\frac{f_\pi^2|R'(0)|}{M_\chi^4}I_{\chi_{c0}}(\epsilon) \ , \tag{2.9}$$

where $\epsilon = m_\pi/M_\chi$ and the integral $I_{\chi_{c0}}(\epsilon)$ is given by

$$I_{\chi_{c0}}(\epsilon) = -\frac{1}{32}\int_0^1 dx\,dy\,\varphi(x)\varphi(y)\frac{1}{xy+(x-y)^2\epsilon^2}\,\frac{1}{(1-x)(1-y)+(x-y)^2\epsilon^2}\,\frac{1}{2xy-x-y-2(x-y)^2\epsilon^2}$$
$$\times\left\{1-\frac{1}{2}\,\frac{(x-y)^2(1-4\epsilon^2)}{2xy-x-y+2(x-y)^2\epsilon^2}+2\epsilon^2\left[1+\frac{1}{2}\,\frac{(x-y)^2(1-4\epsilon^2)}{2xy-x-y+2(x-y)^2\epsilon^2}\right]\right\} . \tag{2.10}$$

In the limit $\epsilon\to 0$, the integral $I_{\chi_{c0}}(\epsilon)$ reduces to the corresponding integral found, with massless quarks, by Chernyak and Zhitnitsky [4]:

$$\lim_{\epsilon\to 0} I_{\chi_{c0}}(\epsilon) = I_{\chi_{c0}}^{\mathrm{CZ}} \ . \tag{2.11}$$

Finally, from

$$\Gamma(\chi_{c0}\to\pi^+\pi^-) = \frac{1}{8(2\pi)^5}(1-4\epsilon^2)^{1/2}\int |A|^2 d\Omega \ , \tag{2.12}$$

we derive

$$\Gamma(\chi_{c0}\to\pi^+\pi^-) = (\tfrac{2}{3})^5 2^{13}\pi^2\alpha_s^4(1-4\epsilon^2)^{1/2}f_\pi^4\frac{|R'(0)|^2}{M_\chi^8}I_{\chi_{c0}}^2(\epsilon) \ , \tag{2.13}$$

which, in the $\epsilon\to 0$ limit, agrees with the result of Ref. [4].

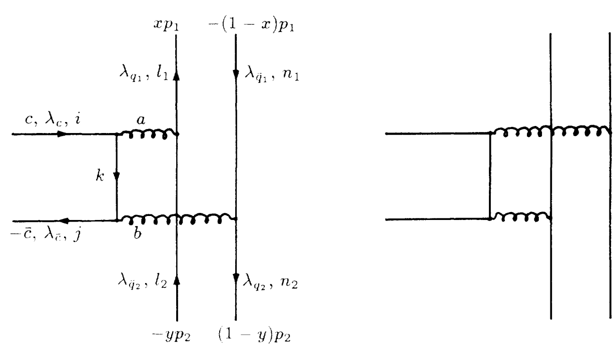

FIG. 1. Feynman diagrams contributing, to lowest order in α_s, to the elementary process $Q\bar{Q}\to q_1\bar{q}_1 q_2\bar{q}_2$, for a quarkonium state with charge conjugation $C=+1$. In the $Q\bar{Q}$ center-of-mass frame, $c^\mu=(E,\mathbf{k}/2)$, $\bar{c}^\mu=(E,-\mathbf{k}/2)$, with $\mathbf{k}=(k\sin\alpha\cos\beta, k\sin\alpha\sin\beta, k\cos\alpha)$; $p_1^\mu=(E,\mathbf{p})$, $p_2^\mu=(E,-\mathbf{p})$, with $\mathbf{p}=(p\sin\theta, 0, p\cos\theta)$. $a,b,i,j,k,l_{1,2},n_{1,2}$ are color indices; the λ's label helicities.

B. Numerical estimates of mass corrections for the decays $\chi_{c0} \rightarrow \pi\pi$ and $\chi_{c0} \rightarrow V_L V_L$

The above results can easily be modified to the case of the decay $\chi_{c0} \rightarrow V_L V_L$, where V_L is a vector meson longitudinally polarized, $\lambda_V = 0$; such decay is the only one allowed by the helicity-conservation rule induced by the perturbative coupling of gluons to massless quarks and has already been computed [1,4]. Again, by computing it with massive quarks, we shall evaluate the relative importance of mass corrections, and as a consistency check, we recover, in the limit $m_q \rightarrow 0$, the existing results.

The main difference between a pion and a longitudinally polarized vector meson is obviously the spin wave function, which now reads

$$\psi_{\text{spin}}(S=1,\lambda=0) = \frac{1}{\sqrt{2}}(|-+\rangle + |+-\rangle) \ . \qquad (2.14)$$

The sign difference with respect to the pion wave function (2.4) is reflected in a few changes of some signs; it is easy to realize that Eqs. (2.9), (2.12), and (2.13) still hold true, with the obvious replacements $m_\pi \rightarrow m_V$, $f_\pi \rightarrow f_V$, and the only change in Eq. (2.10) is a different sign in front of the square bracket (mass correction terms). The φ are the distribution amplitudes of the proper final mesons.

We are now in the position of estimating the size of the mass corrections to $\Gamma(\chi_{c0} \rightarrow \text{mesons})$ decays. The results in the two cases, $m_q = 0$ and $m_q \neq 0$, can be read from Eqs. (2.13) and (2.10) (and the analogous ones for vector mesons), setting, respectively, $\epsilon = m_M / M_\chi$ (where m_M is the mass of the meson) and $\epsilon = 0$. Apart from the factor $(1-4\epsilon^2)^{1/2}$, the difference comes from the different values, in the two cases, of the integral $I_{\chi_{c0}}(\epsilon)$. To give a first estimate of the relative importance of the mass corrections, we evaluate $I_{\chi_{c0}}(\epsilon)$ for different choices of the momentum distribution amplitudes φ, taken to be of the kind

$$\varphi(x) = N(\alpha,\beta) x^\alpha (1-x)^\beta \ , \qquad (2.15)$$

where N is the normalization factor such that $\int_0^1 dx\, \varphi = 1$.

In Table I we show the values of $I_{\chi_{c0}}(\epsilon = m_M / M_\chi)$ for different final mesons (with helicity 0) and several choices of α and β. For particles with uneven quark contents, such as K and K^*, we allow α to differ from β. We also give the values of $I_{\chi_{c0}}(\epsilon = 0)$, which only depend on α and β and not on the final mesons, together with the relative variation of the decay width

$$[\Gamma(m_q \neq 0) - \Gamma(m_q = 0)]/\Gamma(m_q = 0) \ .$$

In Table II we show the mass corrections to the results, in agreement with the data (when the comparison is possible), obtained by Chernyak and Zhitnitsky; that is, we compute the decay rates using the same distribution amplitudes as in Ref. [4], but with massive quarks.

Few comments are now in order.

The mass corrections to $\Gamma(\chi_{c0} \rightarrow \pi\pi)$, given the smallness of the ratio m_π / m_c, are indeed negligible; with increasing masses, the corrections become larger, up to 40–60% for ρ, K^*, and ϕ. The qualitative results of the models, however, are not changed by the introduction of massive quarks: Changing the meson wave functions still induces variations in the results much bigger than those induced by mass corrections.

The Chernyak-Zhitnitsky wave functions give, in most cases, much larger results than the wave functions (2.15): The same is true for the absolute value of the mass corrections, whereas their relative values show little dependence on the choice of the distribution amplitudes.

An analogous study of mass corrections has been performed also for the $\chi_{c2} \rightarrow \pi\pi, V_L V_L$ decays, with results similar to those shown for the χ_{c0}. More details can be found in Ref. [28].

The fact that massive quarks give sizable corrections, almost as big as the zero-mass results, for decays into particles with large masses (~ 1 GeV), suggests that mass terms alone might be able to account for the forbidden decays $\eta_c, \chi_{c0} \rightarrow p\bar{p}$. We then turn to the computation of $\Gamma(\eta_c, \chi_{c0} \rightarrow p\bar{p})$ with massive quarks.

III. χ_{c0} AND η_c DECAYS INTO $p\bar{p}$

Let us start from $\chi_{c0} \rightarrow p\bar{p}$ and repeat the same procedure followed for the decay $\chi_{c0} \rightarrow \pi\pi$ in Sec. II A. From Eq. (2.1) we have that the only nonzero, independent helicity amplitude is given by

TABLE I. Evaluations of mass corrections to the decays $\chi_{c0} \rightarrow M\bar{M}$ for different final mesons. For each value of α and β [see Eq. (2.15)], the first line gives the value of $I_{\chi_{c0}}(\epsilon)$, and the second gives, in percentage, the relative variation of the decay width, $[\Gamma(m_q \neq 0) - \Gamma(m_q = 0)]/\Gamma(m_q = 0)$.

α	β	0	m_π	m_K	m_ρ	m_{K^*}	m_ϕ
1	1	3.039	3.002	2.853	2.324	2.163	1.992
			−3	−16	−48	−57	−66
2	2	1.764	1.764	1.771	1.497	1.416	1.324
			0	−4	−36	−45	−55
3	3	1.472	1.475	1.501	1.282	1.220	1.148
			0	0	−32	−41	−51
1	2	3.528		3.370		2.579	
				−13		−54	
1	3	5.579		5.317		4.046	
				−13		−55	
2	3	1.831		1.849		1.485	
				−2		−44	

$$A_{++}(\chi_{c0}\to p\bar{p})=-\frac{1}{2\sqrt{2\pi}}\int d^3k\,\psi_{\chi_{c0}}(k)\{M_{++;++}(\theta,\mathbf{k})+M_{++;--}(\theta,\mathbf{k})\}\ . \tag{3.1}$$

All other amplitudes are either forbidden by total angular momentum conservation ($A_{+-}=A_{-+}=0$) or are related by parity to A_{++}:

$$A_{--}(\chi_{c0}\to p\bar{p})=A_{++}(\chi_{c0}\to p\bar{p})\ . \tag{3.2}$$

The most general proton wave function, proposed by Chernyak, Ogloblin, and Zhitnitsky [14,15], can be explicitly written as [29]

$$\begin{aligned}\psi_{p,\lambda_p}(x_1,x_2,x_3)=2\lambda_p\frac{F_N}{4\sqrt{6}}\{&\varphi(123)u_{\lambda_p}(1)u_{-\lambda_p}(2)d_{\lambda_p}(3)+\varphi(213)u_{-\lambda_p}(1)u_{\lambda_p}(2)d_{\lambda_p}(3)\\ &+\varphi(132)u_{\lambda_p}(1)d_{\lambda_p}(2)u_{-\lambda_p}(3)+\varphi(231)d_{\lambda_p}(1)u_{\lambda_p}(2)u_{-\lambda_p}(3)\\ &+\varphi(312)u_{-\lambda_p}(1)d_{\lambda_p}(2)u_{\lambda_p}(3)+\varphi(321)d_{\lambda_p}(1)u_{-\lambda_p}(2)u_{\lambda_p}(3)\\ &-[\varphi(213)+\varphi(312)]d_{-\lambda_p}(1)u_{\lambda_p}(2)u_{\lambda_p}(3)-[\varphi(321)+\varphi(123)]u_{\lambda_p}(1)d_{-\lambda_p}(2)u_{\lambda_p}(3)\\ &-[\varphi(132)+\varphi(231)]u_{\lambda_p}(1)u_{\lambda_p}(2)d_{-\lambda_p}(3)\}\ ,\end{aligned} \tag{3.3}$$

where we have omitted the color indices and $\varphi(i,j,k)$ is a short notation for $\varphi(x_i,x_j,x_k)$ $(i,j,k=1,2,3)$. F_N is a dimensional constant, analogous to the pion decay constant and related to the value of the nucleon wave function at the origin; QCD sum rules yield [14,16]

$$|F_N|=(0.5\pm0.03)\times10^{-2}\ \mathrm{GeV}^2\ , \tag{3.4}$$

in qualitative agreement with a lattice calculation [30].

By using Eq. (3.3) in Eq. (2.2), we have

$$\begin{aligned}M_{++;\pm\pm}=\frac{F_N^2}{96}\int[dx_i][dy_i](&\{\varphi^2(132)+\varphi^2(231)+[\varphi(132)+\varphi(231)]^2\}T_{++-,++-;\pm\pm}\\ &+\{\varphi(231)\varphi(321)-\varphi(132)[\varphi(321)+\varphi(123)]-[\varphi(132)+\varphi(231)]\varphi(123)\}T_{++-,+-+;\pm\pm}\\ &+\{\varphi(132)\varphi(312)-\varphi(231)[\varphi(213)+\varphi(312)]-[\varphi(132)+\varphi(231)]\varphi(213)\}\\ &\quad\times T_{++-,-++;\pm\pm}\\ &+\{\varphi(321)\varphi(231)-\varphi(123)[\varphi(132)+\varphi(231)]-[\varphi(321)+\varphi(123)]\varphi(132)\}\\ &\quad\times T_{+-+,++-;\pm\pm}\\ &+\{\varphi^2(123)+\varphi^2(321)+[\varphi(123)+\varphi(321)]^2\}T_{+-+,+-+;\pm\pm}\\ &+\{\varphi(123)\varphi(213)-\varphi(321)[\varphi(213)+\varphi(312)]-[\varphi(321)+\varphi(123)]\varphi(312)\}\\ &\quad\times T_{+-+,-++;\pm\pm}\\ &+\{\varphi(312)\varphi(132)-\varphi(213)[\varphi(132)+\varphi(231)]-[\varphi(213)+\varphi(312)]\varphi(231)\}\\ &\quad\times T_{-++,++-;\pm\pm}\\ &+\{\varphi(213)\varphi(123)-\varphi(312)[\varphi(321)+\varphi(123)]-[\varphi(213)+\varphi(312)]\varphi(321)\}\\ &\quad\times T_{-++,+-+;\pm\pm}\\ &+\{\varphi^2(213)+\varphi^2(312)+[\varphi(213)+\varphi(312)]^2\}T_{-++,-++;\pm\pm})\ ,\end{aligned} \tag{3.5}$$

where, as usual,

$$[dz_i]\equiv dz_1dz_2dz_3\delta(1-z_1-z_2-z_3)\ ;$$

by the product of two distribution amplitudes $\varphi(i,j,k)\varphi(l,m,n)$ we mean $\varphi(x_i,x_j,x_k)\varphi(y_l,y_m,y_n)$ $(i,j,k,l,m,n=1,2,3)$.

The helicity amplitudes for the elementary process $T_{\lambda_{q_1}\lambda_{q_2}\lambda_{q_3},\lambda_{\bar{q}_1}\lambda_{\bar{q}_2}\lambda_{\bar{q}_3};\lambda_c\lambda_{\bar{c}}}$ can be computed from the Feynman diagrams of Fig. 2, where we also define the kinematical variables (details of the computation can be found in Ref. [28]).

By inserting their expressions into Eqs. (3.5) and (3.1) and performing the d^3k integration with the help of Eq. (2.8), we obtain

$$\begin{aligned}
A_{++}(\chi_{c0}\to p\bar{p}) = {} & \frac{256}{27\sqrt{3}}\pi^4\alpha_s^3 F_N^2 |R'(0)| m_p (M_\chi^2-4m_p^2)^{1/2} \\
& \times \int_0^1 dx_2 \int_0^{1-x_2} dx_3 \int_0^1 dy_2 \int_0^{1-y_2} dy_3 \frac{1}{(x_2y_2-x_2-y_2+1)M_\chi^2+(x_2-y_2)^2m_p^2} \\
& \times \frac{1}{[x_2y_2M_\chi^2+(x_2-y_2)^2m_p^2][x_3y_3M_\chi^2+(x_3-y_3)^2m_p^2]} \\
& \times \frac{1}{(1-x_2)y_3M_\chi^2+(1-x_2-y_3)^2m_p^2-\epsilon^2m_p^2} \\
& \times \frac{1}{\frac{1}{2}(2x_2y_2-x_2-y_2)M_\chi^2+(x_2-y_2)^2m_p^2} \\
& \times \Bigg\{ m_p^2[\varphi^2_{++-;++-}+\varphi^2_{+-+;+-+}+\varphi^2_{-++;-++}] \\
& \quad \times \Bigg[-\frac{1}{4}\frac{(x_2^2-y_2^2)(1-x_2+y_3-\epsilon)M_\chi^2}{\frac{1}{2}(2x_2y_2-x_2-y_2)M_\chi^2+(x_2-y_2)^2m_p^2} \\
& \quad\quad + \left[\frac{1}{4}\frac{(x_2-y_2)^2(M_\chi^2-4m_p^2)}{\frac{1}{2}(2x_2y_2-x_2-y_2)M_\chi^2+(x_2-y_2)^2m_p^2}+1\right](1-x_2-y_3+\epsilon)\Bigg] \\
& \quad -\varphi^2_{++-;+-+}M_\chi^2\left[\frac{1}{4}\frac{(x_2-y_2)^2(M_\chi^2-4m_p^2)}{\frac{1}{2}(2x_2y_2-x_2-y_2)M_\chi^2+(x_2-y_2)^2m_p^2}-1\right](1-x_2-y_3-\epsilon) \\
& \quad +M_\chi^2[\varphi^2_{++-;-++}+\varphi^2_{-++;++-}] \\
& \quad\quad \times \Bigg[\frac{1}{4}\frac{(x_2^2-y_2^2)(M_\chi^2y_3+2(1-x_2-y_3-\epsilon)m_p^2)}{\frac{1}{2}(2x_2y_2-x_2-y_2)M_\chi^2+(x_2-y_2)^2m_p^2} \\
& \quad\quad\quad + \left[\frac{1}{4}\frac{(x_2-y_2)^2(M_\chi^2-4m_p^2)}{\frac{1}{2}(2x_2y_2-x_2-y_2)M_\chi^2+(x_2-y_2)^2m_p^2}+1\right]y_3\Bigg] \\
& \quad +M_\chi^2[\varphi^2_{+-+;-++}+\varphi^2_{-++;+-+}] \\
& \quad\quad \times \left[\frac{1}{4}\frac{(x_2-y_2)^2(M_\chi^2-4m_p^2)}{\frac{1}{2}(2x_2y_2-x_2-y_2)M_\chi^2+(x_2-y_2)^2m_p^2}-1\right](1-x_2)\Bigg\}\,,
\end{aligned} \tag{3.6}$$

where by $\varphi^2_{\lambda_{q_1}\lambda_{q_2}\lambda_{q_3};\lambda_{\bar{q}_1}\lambda_{\bar{q}_2}\lambda_{\bar{q}_3}}$ we mean the bilinear combinations of φ's appearing in the squared brackets multiplying $T_{\lambda_{q_1}\lambda_{q_2}\lambda_{q_3};\lambda_{\bar{q}_1}\lambda_{\bar{q}_2}\lambda_{\bar{q}_3};\pm\pm}$ in Eq. (3.5).

The above amplitude, as expected, is proportional to the proton mass m_p so that it gives zero in the limit in which all masses are neglected. The parameter ϵ appearing in Eq. (3.6) requires some explanation. In each of the Feynman diagrams of Fig. 2, there is a quark propagator which brings a factor $(\not{q}+m)/(q^2-m^2)$, if q is the quark four-momentum and m its mass. In our scheme, where the (constituent) quarks are considered as Dirac particles with an effective mass xm_p or ym_p, it is not clear what value to use for the quark mass m in the propagator; we have kept trace of this ambiguity by defining $\epsilon=m/m_p$. However, in all subsequent numerical evaluations, we shall fix $\epsilon=0$; that is, we properly take into account the quark masses in the kinematics and external Feynman diagram legs, but stick to the usual perturbative QCD Feynman rules for gluon and quark couplings and propagators.

The value of $|R'(0)|$, the first derivative of the radial wave function at the origin, can be fixed from experiment. The two gluon decays of $\chi_{c0,c2}$ are computed to be [31]

TABLE II. Mass corrections to the results of Chernyak and Zhitnitsky [4]. For each meson the first column gives the value of $I^{\rm CZ}_{\chi_{c0}}$ with $m_q=0$. The second column gives $I^{\rm CZ}_{\chi_{c0}}$ with mass corrections, and the third one gives, in percentage, the value of $[\Gamma(m_q\neq 0)-\Gamma(m_q=0)]/\Gamma(m_q=0)$.

Meson M	$I^{\rm CZ}_{\chi_{c0}}(0)$	$I^{\rm CZ}_{\chi_{c0}}(m_M/M_\chi)$	$\Delta\Gamma^{\rm CZ}$ (%)
π	14.114	13.830	−4
K	8.896	8.078	−21
ρ_L	5.357	3.902	−53
K_L^*	4.820	3.468	−56
ϕ_L	2.736	1.828	−64

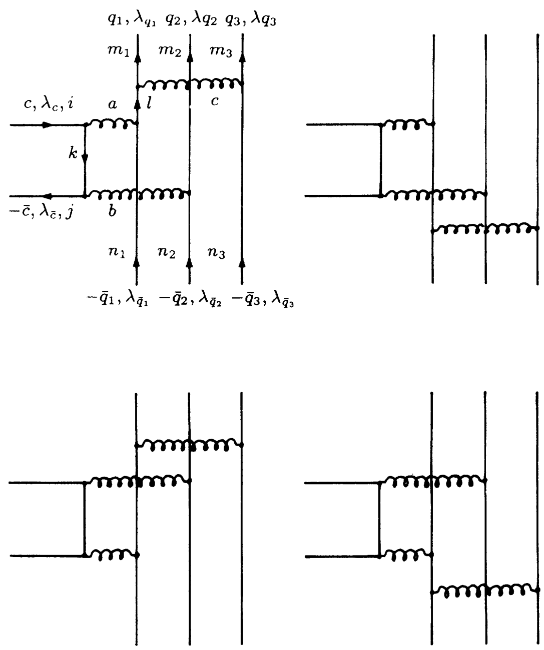

FIG. 2. Feynman diagrams contributing, to lowest order in α_s, to the elementary process $Q\bar{Q}\to q_1q_2q_3\bar{q}_1\bar{q}_2\bar{q}_3$, for a quarkonium state with charge conjugation $C=+1$. In the $Q\bar{Q}$ center-of-mass frame, $c^\mu=(E,\mathbf{k}/2)$, $\bar{c}^\mu=(E,-\mathbf{k}/2)$, with $\mathbf{k}=(k\sin\alpha\cos\beta,\ k\sin\alpha\sin\beta,\ k\cos\alpha)$; $q_i=x_ip$, $\bar{q}_i=y_i\bar{p}$ $(i=1,2,3)$, with $p^\mu=(E,\mathbf{p})$, $\bar{p}^\mu=(E,-\mathbf{p})$, and $\mathbf{p}=(p\sin\theta,\ 0,\ p\cos\theta)$. $a,b,c,i,j,k,l,m_{1,2,3},n_{1,2,3}$ are color indices; the λ's label helicities.

$$\Gamma(\chi_{c0}\to gg)=6\frac{\alpha_s^2}{m_c^4}|R'(0)|^2\ , \tag{3.7a}$$

$$\Gamma(\chi_{c2}\to gg)=\frac{8}{5}\frac{\alpha_s^2}{m_c^4}|R'(0)|^2\ , \tag{3.7b}$$

where we have given, consistently with our calculation, only the lowest-order results. The data tell us [9,32]

$$\Gamma(\chi_{c0}\to\text{hadrons})=13.5\pm5.3\ \text{MeV}\ , \tag{3.8a}$$

$$\Gamma(\chi_{c2}\to\text{hadrons})=1.71\pm0.21\ \text{MeV}\ . \tag{3.8b}$$

Assuming, as usual, $\Gamma(\chi_c\to\text{hadrons})\simeq\Gamma(\chi_c\to gg)$ and comparing Eqs. (3.7) and (3.8), we obtain

$$|R'_{\chi_{c0}}(0)|=0.52\pm0.10\ \text{GeV}^{5/2}\ , \tag{3.9a}$$

$$|R'_{\chi_{c2}}(0)|=0.39\pm0.03\ \text{GeV}^{5/2}\ . \tag{3.9b}$$

The two above values, as they should, agree with each other, at least within errors. In our numerical computations, we shall use their average value

$$|R'(0)|=0.46\pm0.10\ \text{GeV}^{5/2}\ . \tag{3.10}$$

From the knowledge of the decay amplitude, we have the decay width

$$\Gamma(\chi_{c0}\to p\bar{p})=\frac{1}{32\pi^4M_\chi}(M_\chi^2-4m_p^2)^{1/2}|A_{++}(\chi_{c0}\to p\bar{p})|^2\ , \tag{3.11}$$

where the remaining $dx_{2,3}$ and $dy_{2,3}$ integrations in A_{++} can be performed numerically.

The results given by Eqs. (3.11) and (3.6) depend on the choice of the proton momentum distribution amplitudes φ. In our computation we shall use the following different wave functions, which can be found in the literature.

Asymptotic QCD predicts [2]

$$\varphi^{\text{as}}(x_i)=120x_1x_2x_3\ . \tag{3.12a}$$

Chernyak and Zhitnitsky [15] first modified the above expression by exploiting QCD sum rules to obtain

$$\varphi^{\text{CZ}}(x_i)=\varphi^{\text{as}}(x_i)[18.06x_1^2+4.62x_2^2+8.82x_3^2-1.68x_3-2.94]\ . \tag{3.12b}$$

More refined versions of the previous QCD sum-rule wave function were subsequently proposed by King and Sachrajda [16],

$$\varphi^{\text{KS}}(x_i)=\varphi^{\text{as}}(x_i)[20.16x_1^2+15.12x_2^2+22.682x_3^2-6.72x_3+1.68(x_1-x_2)-5.04]\ , \tag{3.12c}$$

and by Chernyak, Ogloblin, and Zhitnitsky [14]:

$$\varphi^{\text{COZ}}(x_i)=\varphi^{\text{as}}(x_i)[23.814x_1^2+12.978x_2^2+6.174x_3^2+5.88x_3-7.098]\ . \tag{3.12d}$$

Another possible choice is offered by Gari and Stefanis [33],

$$\varphi^{\text{GS}}(x_i)=\varphi^{\text{as}}(x_i)[-1.027x_1^2+12.307x_3^2+111.32x_1x_3+25.88x_2+9.105(x_1-x_3)-19.84]\ , \tag{3.12e}$$

and, finally, we mention the nonrelativistic distribution

$$\varphi^{\text{NR}}(x_i)=\prod_{i=1}^{3}\delta(x_i-\tfrac{1}{3})\ . \tag{3.12f}$$

In Table III we show the values obtained for $\Gamma(\chi_{c0}\to p\bar{p})$ with the different distribution amplitudes (DA's) described above. We have used, in agreement with Ref. [9], $\alpha_s(M_\chi)\simeq0.27$.

The same procedure can be repeated for the process $\eta_c\to p\bar{p}$. In such a case $(L=S=0)$, instead of Eq. (3.1), we have

TABLE III. Values of $\Gamma(\chi_{c0}\to p\bar{p})$ for the different distribution amplitudes considered in the text. All results are in eV. Experimentally, $\Gamma(\chi_{c0}\to p\bar{p})<12\,000$ eV.

DA	NR	as	CZ	COZ	KS	GS
Γ	$(2\pm1)\times10^{-3}$	0.46±0.23	45±22	26±13	10±5	23±11

$$A_{++}(\eta_c\to p\bar{p})=\frac{1}{2\sqrt{2\pi}}\int d^3k\,\psi_{\eta_c}(k)\{M_{++;++}(\theta,\mathbf{k})-M_{++;--}(\theta,\mathbf{k})\}\ , \tag{3.13}$$

with

$$A_{+-}(\eta_c\to p\bar{p})=A_{-+}(\eta_c\to p\bar{p})=0\ ,$$
$$A_{--}(\eta_c\to p\bar{p})=-A_{++}(\eta_c\to p\bar{p})\ . \tag{3.14}$$

We can now exploit the nonrelativistic η_c wave function

$$\psi_{\eta_c}(k)=\left[\frac{\pi}{2}\right]^{1/2}|R(0)|\frac{1}{k^2}\delta(k)\ , \tag{3.15}$$

to write

$$A_{++}(\eta_c\to p\bar{p})=\pi|R(0)|\{M_{++;++}(k=0)-M_{++;--}(k=0)\}\ , \tag{3.16}$$

where the $M_{++;\pm\pm}$ can be computed from Eq. (3.5) and the Feynman diagrams of Fig. 2. The final result is

$$\begin{aligned}A_{++}(\eta_c\to p\bar{p})=&\,i\frac{64}{27\sqrt{3}}\pi^4\alpha_s^3F_N^2|R(0)|m_pM_\eta^2(M_\eta^2-4m_p^2)\\&\times\int_0^1dx_2\int_0^{1-x_2}dx_3\int_0^1dy_2\int_0^{1-y_2}dy_3\frac{1}{(x_2y_2-x_2-y_2+1)M_\eta^2+(x_2-y_2)^2m_p^2}\\&\times\frac{1}{[x_2y_2M_\eta^2+(x_2-y_2)^2m_p^2][x_3y_3M_\eta^2+(x_3-y_3)^2m_p^2]}\\&\times\frac{1}{(1-x_2)y_3M_\eta^2+(1-x_2-y_3)^2m_p^2-\epsilon^2m_p^2}\\&\times\frac{(x_2-y_2)}{\frac{1}{2}(2x_2y_2-x_2-y_2)M_\eta^2+(x_2-y_2)^2m_p^2}\\&\times\{\varphi^2_{++-;+-+}(1-x_2-y_3-\epsilon)\\&\quad+[\varphi^2_{+-+;-++}-\varphi^2_{-++;+-+}](1-x_2)\}\ .\end{aligned} \tag{3.17}$$

The value of $|R(0)|$ can be evaluated by comparing the computed expression [31] of

$$\Gamma(\eta_c\to\gamma\gamma)=\frac{16}{27}\frac{\alpha^2}{m_c^2}|R(0)|^2\ , \tag{3.18}$$

with the experimental value [34]

$$\Gamma(\eta_c\to\gamma\gamma)=5.9^{+2.1}_{-1.8}\ \mathrm{GeV}^{3/2}\ , \tag{3.19}$$

which yields

$$|R(0)|=0.64\pm0.15\ \mathrm{GeV}^{3/2}\ . \tag{3.20}$$

Finally, from

$$\Gamma(\eta_c\to p\bar{p})=\frac{1}{32\pi^4M_\eta}(M_\eta^2-4m_p^2)^{1/2}|A_{++}(\eta_c\to p\bar{p})|^2\ , \tag{3.21}$$

we can compute the decay width for the process $\eta_c\to p\bar{p}$. In Table IV we show the values obtained with the different distribution amplitudes (DA's) described in Eqs. (3.12).

Let us now comment on the results summarized in Tables III and IV.

As usual, the asymptotic and nonrelativistic wave functions give much smaller results than the other distribution amplitudes.

The QCD sum rule wave functions [Eqs. (3.12b)–(3.12d)] yield sizable values of $\Gamma(\chi_{c0}\to p\bar{p})$, in agreement with the only available experimental information [5], a huge upper bound. These results are only a factor $\sim2-10$ smaller than the values measured for $\Gamma(\chi_{c1,c2}\to p\bar{p})$ [6,7,9], decays which are not forbidden by the helicity-conservation rule and have been successfully computed in pure perturbative QCD schemes with massless quarks [14]. Actually, on phase-space considerations

TABLE IV. Values of $\Gamma(\eta_c \to p\bar{p})$, for the different distribution amplitudes considered in the text. All results are in eV. Experimentally, $\Gamma(\eta_c \to p\bar{p}) = 12.1 \pm 7.9$ keV.

DA	NR	as	CZ	COZ	KS	GS
Γ	0	$(4\pm2)\times10^{-4}$	1.8 ± 1.0	1.0 ± 0.5	0.4 ± 0.2	14.0 ± 7.0

alone, one expects the $p\bar{p}$ decay rate for χ_{c0} to be indeed smaller than for $\chi_{c1,c2}$. Also, the Gari-Stefanis wave function [Eq. (3.12e)] gives a large result.

The values of $\Gamma(\chi_{c0} \to p\bar{p})$, obtained here with massive quarks, are somewhat smaller than the values obtained within the quark-diquark model of the nucleon [21], where both constituent quarks *and* diquarks are present. A definite measurement of $\Gamma(\chi_{c0} \to p\bar{p})$ would help in understanding whether or not a genuine diquark contribution is present; some common features of the quark and the quark-diquark models are expected from the observation, made in Ref. [35], that the QCD sum-rule wave functions, strongly asymmetric in the sharing of the proton momentum by the three quarks, correspond to a quark-scalar diquark configuration.

The results obtained for $\Gamma(\eta_c \to p\bar{p})$ are not zero, but much smaller than the experimental data [8]: Once more [27], η_c decays seem to defeat any attempt of explanation. The values obtained here (Table IV) are comparable to those obtained in the quark-diquark scheme [21,22]. The Gari-Stefanis wave function [Eq. (3.12e)] gives the relatively best result; it is not clear, however, if it satisfies the QCD sum rules, and before concluding that it indeed gives a better description of the quark momentum distribution, more detailed phenomenological analyses should be done. We shall comment again on the η_c in Sec. IV.

IV. η_c DECAYS AND CONCLUSIONS

We have shown that mass corrections to charmonium decays into $H\bar{H}$ are, as expected, large and proportional to m_H/m_c; in cases where massless perturbative QCD gives a fair description of the process, mass corrections can be, depending on m_H, as large as 40–60 % of the zero-mass result. Such is the case of $\chi_{c0,c2}$ decays into longitudinally polarized ρ, K^*, and ϕ. Encouraged by these results, we have computed, with massive quarks, the decay widths $\Gamma(\chi_{c0} \to p\bar{p})$ and $\Gamma(\eta_c \to p\bar{p})$: The latter processes are forbidden in massless perturbative QCD by the helicity-conservation rule, which can be broken by terms proportional to m_H/m_c.

We have found that, using the proton wave functions suggested by QCD sum rules, it is easy to obtain sizable values of $\Gamma(\chi_{c0} \to p\bar{p})$, values comparable, taking into account the smaller mass of the χ_{c0}, with those measured for the analogous decays $\Gamma(\chi_{c1,c2} \to p\bar{p})$; although there is no precise experimental value for $\Gamma(\chi_{c0} \to p\bar{p})$, but only a very large upper bound, we still consider it significant to have obtained such results. Of course, a definite measurement of the process would help in clarifying the situation.

The decay $\eta_c \to p\bar{p}$ is different: Even with massive quarks, the results obtained, although nonzero, are still consistently smaller than the data. Moreover, one should remember that mass corrections to other forbidden η_c decays, previously computed, such as $\eta_c \to$vector-meson pairs, are strictly zero [27]. Other nonperturbative corrections to the massless QCD scheme, like those modeled in the quark-diquark scheme, also fail with $\eta_c \to p\bar{p}$ decay [21,22]. While it remains to be seen if further corrections, such as higher-order Fock states or intrinsic transverse momentum of the quarks, might help with the η_c decays, one has to admit, at this stage, that there is no clear way of computing them. It might be that a more drastic modification is needed, such as assuming that the η_c is not a pure $c\bar{c}$ state, but has a large gluonic component [20].

In the energy region of the charmonium decays, $Q^2 \lesssim 10$ GeV^2, one expects several nonperturbative or higher-order corrections to play a non-negligible role; we have here explored mass corrections alone, leaving aside other possible effects, such as the Fermi motion of the quarks or higher-order components of the hadron wave functions. The importance of two quark correlations, diquarks, has already been studied and found to be relevant [21], in the case of decays into $p\bar{p}$, with the usual exception of the η_c. As, in the quark-diquark model, the nucleons also contain massive constituent quarks, it is difficult to evaluate separately the pure mass corrections and pure diquark ones.

Charmonium decays are clearly in a Q^2 transition region where both perturbative and nonperturbative effects are present; moreover, the latter are of different natures and this makes it difficult to disentangle the various contributions. Nevertheless, a detailed study of these decays, with better and more refined experimental information, should yield a valuable information about the structure of hadrons in this transition region of few GeV^2, a region of great importance for most of the high-energy experiments. Of course, it would be most interesting also to have data on the decays of heavier mesons, such as $b\bar{b}$ bound states, to see if indeed the pure perturbative regime eventually takes over.

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