B_{l4} and D_{l4} decay

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We consider the predictions of chiral perturbation theory and heavy-quark symmetry for the decays $D\to K\pi\bar{l}\nu_l$, $D\to\pi\pi\bar{l}\nu_l$, $B\to\pi\pi l\bar{\nu}_l$, and $B\to D\pi l\bar{\nu}_l$ ($l=e$ or μ).

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I. INTRODUCTION

Heavy-quark symmetry and chiral symmetry put constraints on B_{l4} and D_{l4} semileptonic weak decay amplitudes [1–3]. In this paper we explicitly display the implications of these symmetries for $D\to K\pi\bar{l}\nu_l$, $D\to\pi\pi\bar{l}\nu_l$, $B\to\pi\pi l\bar{\nu}_l$, and $B\to D\pi l\bar{\nu}_l$ decays.

The strong interactions of the lowest-lying mesons containing a heavy quark Q with the pseudo Goldstone bosons π, K, η are determined by the chiral Lagrangian density

$$\begin{aligned}\mathcal{L}=&\frac{f^2}{8}\mathrm{Tr}(\partial^\mu\Sigma\partial_\mu\Sigma^\dagger)+\lambda_0\mathrm{Tr}[(m_q\Sigma+m_q\Sigma^\dagger)-i\,\mathrm{Tr}\,\bar{H}_a v_\mu\partial^\mu H_a+\frac{i}{2}\mathrm{Tr}\,\bar{H}_a H_b v^\mu(\xi^\dagger\partial_\mu\xi+\xi\partial_\mu\xi^\dagger)_{ba}\\&+\frac{ig}{2}\mathrm{Tr}\,\bar{H}_a H_b\gamma^\mu\gamma_5(\xi^\dagger\partial_\mu\xi-\xi\partial_\mu\xi^\dagger)_{ba}+\lambda_1\mathrm{Tr}\,\bar{H}_a H_b(\xi m_q\xi+\xi^\dagger m_q\xi^\dagger)_{ba}+\lambda_1'\mathrm{Tr}\,\bar{H}_a H_a(m_q\Sigma+m_q\Sigma^\dagger)_{bb}\\&+\frac{\lambda_2}{m_Q}\mathrm{Tr}\,\bar{H}_a\sigma_{\mu\nu}H_a\sigma^{\mu\nu}+\cdots\,,\end{aligned}\tag{1}$$

where the ellipsis denotes terms with additional derivatives, factors of the light-quark mass matrix

$$m_q=\begin{bmatrix}m_u&0&0\\0&m_d&0\\0&0&m_s\end{bmatrix}\tag{2}$$

associated with explicit violation of $SU(3)_L\times SU(3)_R$ chiral symmetry or factors of $1/m_Q$ associated with violation of heavy-quark spin-flavor symmetry. The Lagrangian is written in terms of a 4×4 matrix H_a that contains the pseudoscalar and vector-meson fields P_a and $P^*_{a\mu}$. (Note that $v^\mu P^*_{a\mu}=0$.) Explicitly [4,5]

$$H_a=\frac{1+\not{v}}{2}(P^*_{a\mu}\gamma^\mu-P_a\gamma_5)\,,\tag{3a}$$

$$\bar{H}_a=\gamma^0H_a^\dagger\gamma^0\,.\tag{3b}$$

This is a "shorthand notation." In cases where the type of heavy quark Q and its four-velocity v are important the 4×4 matrix is denoted by $H_a^{(Q)}(v)$.

In the Lagrangian density (1) the light-quark flavor indices a,b go over 1,2,3 and repeated indices are summed. For $Q=c$,

$$(P_1,P_2,P_3)=(D^0,D^+,D_s)$$

and

$$(P_1^*,P_2^*,P_3^*)=(D^{0*},D^{+*},D_s^*)$$

while for $Q=b$,

$$(P_1,P_2,P_3)=(B^-,\bar{B}^0,\bar{B}_s)$$

and

$$(P_1^*,P_2^*,P_3^*)=(B^{-*},\bar{B}^{0*},\bar{B}_s^*)\,.$$

The factors of $\sqrt{m_P}$ and $\sqrt{m_{P^*}}$ have been absorbed into the P and P^* fields. Consequently they have dimension 3/2.

The field H_a is a doublet under heavy-quark spin symmetry $SU(2)_v$ and a $\bar{3}$ under the unbroken $SU(3)_V$ light-quark flavor symmetry. Under $SU(2)_v$ and $SU(3)_L\times SU(3)_R$ it transforms as

$$H_a\to S(HU^\dagger)_a\,,\tag{4}$$

where $S\in SU(2)_v$ and U is the usual space-time dependent 3×3 unitary matrix that is introduced to transform matter fields in a chiral Lagrangian.

The pseudo Goldstone bosons appear in the Lagrangian density through

$$\xi=\exp(iM/f)\tag{5a}$$

and

$$\xi^2=\Sigma=\exp(2iM/f)\,.\tag{5b}$$

In Eqs. (5) M is the matrix of fields

$$M=\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{2/3}\eta \end{pmatrix}, \tag{6}$$

and f is the pion decay constant ($f\simeq 132$ MeV).

Under $SU(3)_L\times SU(3)_R$ chiral symmetry

$$\Sigma\to L\Sigma R^\dagger \tag{7a}$$

and

$$\xi\to L\xi U^\dagger=U\xi R^\dagger , \tag{7b}$$

where $L\in SU(3)_L$, $R\in SU(3)_R$, and U is a function of L,R and the meson fields. Typically U is space-time dependent. However, for $SU(3)_V$ transformations, $V=L=R$, U is equal to V.

Heavy-quark flavor symmetry implies that, to leading order in Λ_{QCD}/m_Q, g is independent of heavy-quark flavor. For $Q=c$ the $D^*\to D\pi$ decay width is determined by g:

$$\Gamma(D^{+*}\to D^0\pi^+)=\left[\frac{1}{6\pi}\right]\frac{g^2}{f^2}|\mathbf{p}_\pi|^3 . \tag{8}$$

The present experimental limit [6] on this width $[\Gamma(D^{+*}\to D^0\pi^+)<72$ keV$]$ implies that $g^2<0.4$. Applying the Noether procedure, the Lagrangian density (1) gives the following expression for the axial-vector current:

$$\bar{q}_a T_{ab}\gamma_\nu\gamma_5 q_b=-g\,\mathrm{Tr}\,\bar{H}_a H_b\gamma_\nu\gamma_5 T_{ba}+\cdots . \tag{9}$$

In Eq. (9) the ellipsis represents terms containing the pseudo-Goldstone-boson fields and T is a flavor-SU(3) generator. Treating the quark fields in Eq. (9) as constituent quarks and using the nonrelativistic quark model [i.e., static SU(6)] to estimate the D^* matrix element of the left-hand side (LHS) of Eq. (9) gives [3] $g=1$. (A similar estimate for the pion-nucleon coupling gives $g_A=\frac{5}{3}$.) In the chiral quark model [7] there is a constituent-quark pion coupling. Using the measured pion-nucleon coupling to determine the constituent-quark pion coupling gives that $g\simeq 0.75$. The decay $B^*\to B\pi$ is kinematically forbidden and so it will not be possible to use it to test the heavy-quark flavor independence of g. The amplitude for the semileptonic decay $B\to D\pi l\bar{\nu}_l$, in the kinematic region where the pion has low momentum (and the $D\pi$ mass is greater than that of the D^*), can be predicted using chiral perturbation theory. In principle, experimental study of this decay can give information on the flavor dependence of g.

In the next section we discuss the kinematics of weak semileptonic D_{l4} and B_{l4} decay. The fully differential decay rates are expressed in terms of form factors. The results of Sec. II are a slight modification of the kinematics of K_{l4} decay to the situation where the two hadrons in the final state have different masses. The generalization of K_{l4} decay kinematics to $D\to K\pi\bar{l}\nu_l$ decay was previously discussed by Kane and co-workers [8]. We have included a short review of the kinematics for completeness. Section III gives the predictions of chiral perturbation theory for $D\to K\pi\bar{l}\nu_l$, $D\to\pi\pi\bar{l}\nu_l$, and $B\to\pi\pi l\bar{\nu}_l$ decay form factors. In Sec. IV the predictions of chiral perturbation theory for $B\to D\pi l\bar{\nu}_l$ are given. Section V contains a brief discussion of the expected kinematic range where chiral perturbation theory for $B\to D\pi l\bar{\nu}_l$ is applicable. Concluding remarks are made in Sec. VI.

For B_{l4} and D_{l4} decay the kinematic region where chiral perturbation theory is applicable is small. In the kinematic region where chiral perturbation theory is applicable

$$B(B\to D\pi l\bar{\nu}_l)\sim(1/16\pi^2)B(B\to Dl\bar{\nu}_l)\sim 10^{-4} .$$

The situation is worse for the modes with two pseudo Goldstone bosons in the final state. For example, we expect that

$$B(D\to\pi\pi\bar{l}\nu_l)\sim(1/16\pi^2)\sin^2\theta_C(f_D/m_D)^2B(D\to X_s\bar{l}\nu_l) ,$$

where f_D is the decay constant for the D meson. For $f_D\sim 200$ MeV this crude order of magnitude estimate gives $B(D\to\pi\pi\bar{l}\nu_l)\sim 10^{-6}$. The factor of $\sin^2\theta_C$ is absent for the Cabibbo-allowed decay $D\to K\pi\bar{l}\nu_l$, but the fact that the kaon mass is not very small makes the validity of lowest-order chiral perturbation theory dubious. It will be very difficult, in the kinematic region where chiral perturbation theory applies, to observe B_{l4} and D_{l4} decay to two pseudo Goldstone bosons. However, the results of this paper may still prove useful for these decays. Phenomenological models that predict the form factors over the whole phase space should be constrained to agree with chiral perturbation theory in the kinematic region where it applies.

While this work was in progress we received Refs. [2] and [3]. Some of the work in these papers overlaps with that presented here. The form factors for $B^-\to\pi^+\pi^-l\bar{\nu}_l$ were considered in Ref. [2] and those for $B\to D\pi l\bar{\nu}_l$ in Ref. [3]. However, this paper gives the first detailed calculation of the rate for $B\to D\pi l\bar{\nu}_l$ in the kinematic region where chiral perturbation theory is expected to apply.

II. REVIEW OF THE KINEMATICS

Consider for definiteness the decay $D\to K\pi\bar{l}\nu_l$. At the end of this section we show how to modify the formulas so they apply to the other decays we are considering. It is convenient, following the analysis of K_{l4} decay by Pais and Treiman [9], to form the following combinations of four-momenta:

$$P=p_K+p_\pi , \quad Q=p_K-p_\pi ,$$
$$L=p_l+p_{\nu_l} , \quad N=p_l-p_{\nu_l} . \tag{10}$$

Like K_{l4} decay, D_{l4} decay is kinematically parametrized by five variables. For two of these we take the $K\pi$ and $\bar{l}\nu_l$ squared masses:

$$s_{K\pi}=P^2 , \quad s_{l\nu}=L^2 . \tag{11}$$

For the remaining three variables we choose θ_K, the angle formed by the kaon three-momentum in the $K\pi$ rest frame and the line of flight of the $K\pi$ in the D rest frame, θ_l, the angle formed by the $\bar{l}$ three-momentum in the $\bar{l}\nu_l$ rest frame and the line of flight of the $\bar{l}\nu_l$ in the D rest frame, and ϕ, the angle between the normals to the planes defined in the D rest frame by the $K\pi$ pair and the $\bar{l}\nu_l$ pair.

Over most of the available phase space (including the kinematic regime where chiral perturbation theory can be applied) the mass of the lepton can be neglected (i.e., $m_l^2/s_{l\nu} \ll 1$) and we find that, with $m_l=0$,

$$P\cdot L=\frac{m_D^2-s_{K\pi}-s_{l\nu}}{2}\ , \tag{12a}$$

$$L\cdot N=0\ ,\quad P\cdot Q=m_K^2-m_\pi^2\ , \tag{12b}$$

$$Q^2=2(m_K^2+m_\pi^2)-s_{K\pi}\ ,N^2=-s_{l\nu}\ , \tag{12c}$$

$$L\cdot Q=\left[\frac{m_K^2-m_\pi^2}{s_{K\pi}}\right]P\cdot L+\beta X\cos\theta_K\ , \tag{12d}$$

$$P\cdot N=X\cos\theta_l\ , \tag{12e}$$

$$Q\cdot N=\left[\frac{m_K^2-m_\pi^2}{s_{K\pi}}\right]X\cos\theta_l+\beta P\cdot L\cos\theta_K\cos\theta_l - \beta(s_{l\nu}s_{K\pi})^{1/2}\sin\theta_K\sin\theta_l\cos\phi\ , \tag{12f}$$

$$\epsilon_{\mu\nu\rho\sigma}Q^\mu P^\nu N^\rho L^\sigma=-\beta X(s_{l\nu}s_{K\pi})^{1/2}\sin\theta_K\sin\theta_l\sin\phi\ . \tag{12g}$$

In Eqs. (12),

$$X=[(P\cdot L)^2-s_{K\pi}s_{l\nu}]^{1/2}\ , \tag{13}$$

and β is $(2/\sqrt{s_{K\pi}})$ times the magnitude of the kaon three-momentum in the $K\pi$ rest frame,

$$\beta=(s_{K\pi}^2+m_\pi^4+m_K^4-2m_K^2m_\pi^2 -2s_{K\pi}m_K^2-2s_{K\pi}m_\pi^2)^{1/2}/s_{K\pi}\ . \tag{14}$$

Taking the limit, $m_K=m_\pi$, Eqs. (12) agree with the results of Pais and Treiman for K_{l4} decay.

The invariant matrix element for $D\to K\pi\bar{l}\nu_l$ semileptonic decay is

$$M_{fi}=\frac{G_F}{\sqrt{2}}V_{cs}\langle\pi(p_\pi)K(p_K)|\bar{s}\gamma_\mu(1-\gamma_5)c|D(p_D)\rangle \times\bar{u}(p_\nu)\gamma^\mu(1-\gamma_5)v(p_{\bar{l}})\ , \tag{15}$$

where V_{cs} is the $c\to s$ element of the Cabibbo-Kobayashi-Mashawa matrix and G_F is the Fermi constant. The hadronic matrix element can be written in terms of four form factors $w_\pm$, r, and h that are defined by

$$\langle\pi(p_\pi)K(p_K)|\bar{s}\gamma_\mu(1-\gamma_5)c|D(p_D)\rangle =[iw_+P_\mu+iw_-Q_\mu+ir(p_D-P)_\mu+h\epsilon_{\mu\alpha\beta\gamma}p_D^\alpha P^\beta Q^\gamma]\ . \tag{16}$$

The form factors $w_\pm$, r, and h are functions of $s_{l\nu}$, $s_{K\pi}$, and $\cos\theta_K$. Summing over the lepton polarizations the absolute value of the square of the matrix element is

$$\sum_{\text{spins}}|M_{fi}|^2=4G_F^2|V_{cs}|^2H_{\mu\nu}L^{\mu\nu}\ , \tag{17}$$

where

$$H_{\mu\nu}=\langle\pi(p_\pi)K(p_K)|\bar{s}\gamma_\mu(1-\gamma_5)c|D(p_D)\rangle \times\langle\pi(p_\pi)K(p_K)|\bar{s}\gamma_\nu(1-\gamma_5)c|D(p_D)\rangle^*\tag{18a}$$

$$L^{\mu\nu}=\tfrac{1}{2}(L^\mu L^\nu-N^\mu N^\nu-s_{l\nu}g^{\mu\nu}-i\epsilon^{\alpha\mu\gamma\nu}L_\alpha N_\gamma)\ . \tag{18b}$$

The differential decay rate takes the form

$$d^5\Gamma=\frac{G_F^2|V_{cs}|^2}{(4\pi)^6m_D^3}X\beta I(s_{K\pi},s_{l\nu},\theta_K,\theta_l,\phi) \times ds_{l\nu}ds_{K\pi}d\cos\theta_K d\cos\theta_l d\phi\ . \tag{19}$$

The dependence of I on θ_l and ϕ is given by

$$I=I_1+I_2\cos2\theta_l+I_3\sin^2\theta_l\cos2\phi+I_4\sin2\theta_l\cos\phi +I_5\sin\theta_l\cos\phi+I_6\cos\theta_l+I_7\sin\theta_l\sin\phi +I_8\sin2\theta_l\sin\phi+I_9\sin^2\theta_l\sin2\phi\ , \tag{20}$$

where $I_1,\ldots,I_9$ depend on $s_{K\pi}$, $s_{l\nu}$, and θ_K.

To display $I_1,\ldots,I_9$ in as compact a form as possible it is convenient to introduce the following combinations of kinematic factors and form factors:

$$F_1=Xw_++\left[\beta P\cdot L\cos\theta_K+\left[\frac{m_K^2-m_\pi^2}{s_{K\pi}}\right]X\right]w_-\ , \tag{21a}$$

$$F_2=\beta(s_{l\nu}s_{K\pi})^{1/2}w_-\ , \tag{21b}$$

$$F_3=\beta X(s_{l\nu}s_{K\pi})^{1/2}h\ . \tag{21c}$$

In terms of these combinations of form factors,

$$I_1=\tfrac{1}{4}[|F_1|^2+\tfrac{3}{2}\sin^2\theta_K(|F_2|^2+|F_3|^2)]\ , \tag{22a}$$

$$I_2=-\tfrac{1}{4}[|F_1|^2-\tfrac{1}{2}\sin^2\theta_K(|F_2|^2+|F_3|^2)]\ , \tag{22b}$$

$$I_3=-\tfrac{1}{4}(|F_2|^2-|F_3|^2)\sin^2\theta_K\ , \tag{22c}$$

$$I_4=\tfrac{1}{2}\mathrm{Re}(F_1^*F_2)\sin\theta_K\ , \tag{22d}$$

$$I_5=\mathrm{Re}(F_1^*F_3)\sin\theta_K\ , \tag{22e}$$

$$I_6=\mathrm{Re}(F_2^*F_3)\sin^2\theta_K\ , \tag{22f}$$

$$I_7=\mathrm{Im}(F_1F_2^*)\sin\theta_K\ , \tag{22g}$$

$$I_8=\tfrac{1}{2}\mathrm{Im}(F_1F_3^*)\sin\theta_K\ , \tag{22h}$$

$$I_9=-\tfrac{1}{2}\mathrm{Im}(F_2F_3^*)\sin^2\theta_K\ . \tag{22i}$$

Equations (20) and (22) are the same as Eqs. (11) of Pais and Treiman. However, the definitions of F_1, F_2, and F_3 are slightly different because $m_K\neq m_\pi$.

It is evident from Eqs. (22) that the partial-wave expansions for the form factors F_1, F_2, and F_3 are

$$F_1(s_{K\pi},s_{l\nu},\cos\theta_K)=\sum_{l=0}^{\infty}\tilde{F}_{1,l}(s_{K\pi},s_{l\nu})P_l(\cos\theta_K)\ , \tag{23a}$$

$$F_2(s_{K\pi},s_{l\nu},\cos\theta_K)$$
$$=\sum_{l=1}^{\infty}\frac{1}{[l(l+1)]^{1/2}}\tilde{F}_{2,l}(s_{K\pi},s_{l\nu})\frac{d}{d\cos\theta_K}P_l(\cos\theta_K)\ , \tag{23b}$$

$$F_3(s_{K\pi},s_{l\nu},\cos\theta_K)$$
$$=\sum_{l=1}^{\infty}\frac{1}{[l(l+1)]^{1/2}}\tilde{F}_{3,l}(s_{K\pi},s_{l\nu})\frac{d}{d\cos\theta_K}P_l(\cos\theta_K) \tag{23c}$$

Integrating over the angles gives

$$d^2\Gamma=\frac{G_F^2|V_{cs}|^2}{3(4\pi)^5m_D^3}X\beta \times\sum_l\frac{2}{2l+1}(|\tilde{F}_{1,l}|^2+|\tilde{F}_{2,l}|^2+|\tilde{F}_{3,l}|^2)ds_{l\nu}ds_{K\pi}\ , \tag{24}$$

and the total decay rate is

$$\Gamma=\int_{(m_K+m_\pi)^2}^{m_D^2}ds_{K\pi}\int_0^{(m_D-s_{K\pi}^{1/2})^2}ds_{l\nu}\left[\frac{d^2\Gamma}{ds_{l\nu}ds_{K\pi}}\right]\ . \tag{25}$$

One advantage of the variables θ_K, θ_l, ϕ, $s_{l\nu}$, and $s_{K\pi}$ is that in terms of these variables the region of phase-space integration is quite simple. The angles are unrestricted and Eq. (25) gives the region for $s_{K\pi}$ and $s_{l\nu}$.

Although we have focused on $D\to K\pi\bar{l}\nu_l$ decay the results presented above can be straightforwardly altered to apply to the other decays we discuss in this paper. For $D\to\pi\pi\bar{l}\nu_l$ decay one simply changes $V_{cs}\to V_{cd}$ and $m_K\to m_\pi$. For $B\to\pi\pi l\bar{\nu}_l$ decay one changes $V_{cs}\to V_{ub}^*$, $m_D\to m_B$, and $m_K\to m_\pi$. Also, in Eq. (15) $p_{\bar{l}}$ and p_ν are switched. Consequently the term proportional to the alternating tensor in Eq. (18b) and the expressions for I_5, I_6, and I_7 in Eqs. (22e), (22f), and (22g) change sign. Finally, for $B\to D\pi l\bar{\nu}_l$ decay the changes $V_{cs}\to V_{cb}^*$, $m_D\to m_B$, $m_K\to m_D$, and the same sign changes as for $B\to\pi\pi l\bar{\nu}_l$ decay are made.

III. DECAYS TO TWO PSEUDO GOLDSTONE BOSONS

The semileptonic decays $D\to K\pi\bar{l}\nu_l D\to\pi\pi\bar{l}\nu_l$ and $B\to\pi\pi l\bar{\nu}_l$ are determined by matrix elements of the left-handed current

$$L_{\nu a}=\bar{q}^a\gamma_\nu(1-\gamma_5)Q\ . \tag{26}$$

This operator transforms under chiral $SU(3)_L\times SU(3)_R$ as $(\bar{3}_L,1_R)$. In chiral perturbation theory its matrix elements are given by those of

$$L_{\nu a}=\left[\frac{i\alpha}{2}\right]\mathrm{Tr}\gamma_\nu(1-\gamma_5)H_b\xi_{ba}^\dagger+\cdots\ , \tag{27}$$

where the ellipsis denotes terms with derivatives, factors of the light-quark mass matrix m_q or factors of $1/m_Q$. The constant α is related to the decay constant of the heavy meson:

$$\langle 0|\bar{q}^a\gamma^\nu\gamma_5Q|P_a^{(Q)}(v)\rangle=if_{P_a^{(Q)}}m_{P_a^{(Q)}}v^\nu\ . \tag{28}$$

Taking the $P_a^{(Q)}$ to the vacuum matrix element of Eq. (27) (for this matrix element $\xi^\dagger$ can be replaced by unity) gives

$$\alpha=f_{P_a^{(Q)}}\sqrt{m_{P_a^{(Q)}}}\ . \tag{29}$$

The parameter α has a calculable logarithmic dependence on the heavy-quark [10,11] mass from perturbative QCD.

For D_{l4} and B_{l4} decay to two pseudo Goldstone bosons the Feynman diagrams in Fig. 1 determine the required matrix element. In Fig. 1 a solid line represents a heavy meson and a dashed line represents a pseudo Goldstone boson. The shaded square denotes an insertion of the left-handed current. The form factors $w_\pm$, r, and h that follow from calculation of these Feynman diagrams are given below.

(i) $D\to K\pi\bar{l}\nu_l$

$D\to K\pi\bar{l}\nu_l$ decays are determined by $Q=c$ matrix elements of $L_{\nu 3}$. For the decay $D^+\to K^-\pi^+\bar{l}\nu_l$ computation of the Feynman diagrams in Fig. 1 gives

$$w_-=-\left[\frac{f_Dm_Dg}{2f^2}\right]\frac{1}{v\cdot p_\pi+\Delta_c}\ w_+=-w_-+r\ , \tag{30a}$$

$$r=\left[\frac{f_D}{f^2}\right]\left[\frac{1}{2}-\frac{1}{2}\frac{v\cdot p_K-v\cdot p_\pi}{v\cdot(p_K+p_\pi)+\mu}-\frac{g(v\cdot p_\pi)}{v\cdot p_\pi+\Delta_c}-g^2\frac{p_\pi\cdot p_K-v\cdot p_Kv\cdot p_\pi}{[v\cdot(p_K+p_\pi)+\mu][v\cdot p_\pi+\Delta_c]}\right]\ , \tag{30b}$$

$$h=\left[\frac{f_Dg^2}{2f^2}\right]\frac{1}{v\cdot(p_\pi+p_K)+\Delta_c+\mu}\frac{1}{v\cdot p_\pi+\Delta_c}\ . \tag{30c}$$

In Eqs. (30),

$$\Delta_c=m_{D^*}-m_D\ , \tag{31a}$$

$$\mu=m_{D_s}-m_D\ , \tag{31b}$$

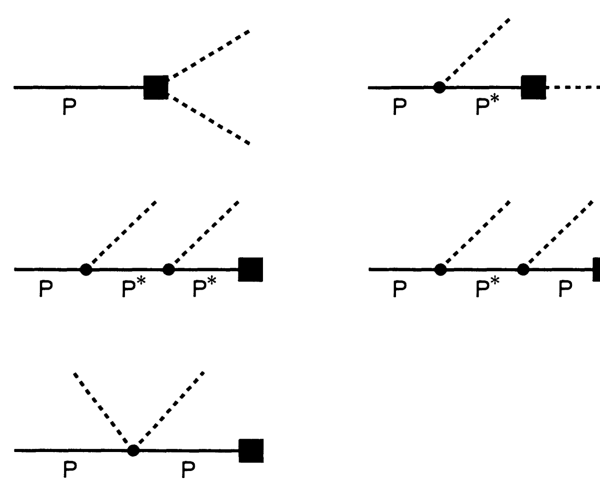

FIG. 1. Feynman diagrams for $D\to K\pi$, $D\to\pi\pi$, and $B\to\pi\pi$ matrix elements of the current $L_{\nu a}$. The shaded square denotes an insertion of the current in Eq. (27). Dashed lines denote pseudo Goldstone bosons.

and v^μ is the four-velocity of the D meson, i.e., $p_D^\mu = m_D v^\mu$. Isospin symmetry implies that the form factors for $D^0 \to K^- \pi^0 \bar{l} \nu_l$ are $1/\sqrt{2}$ times those above, the form factors for $D^+ \to \bar{K}^0 \pi^0 \bar{l} \nu_l$ are $-1/\sqrt{2}$ times those above, and the form factors for $D^0 \to \bar{K}^0 \pi^- \bar{l} \nu_l$ are equal to those above. It is straightforward using Eqs. (11) and (12) to express these form factors in terms of θ_K, $s_{K\pi}$, and $s_{l\nu}$.

(ii) $D^+ \to \pi^+ \pi^- \bar{l} \nu_l$

For this decay a $Q=c$ matrix element of $L_{\nu 2}$ is needed. It is straightforward to see that the form factors in this case are given by those in Eqs. (30) if the changes $p_K \to p_{\pi^-}$ and $p_\pi \to p_{\pi^+}$ are made and μ is set to zero. Again using Eqs. (11) and (12) these form factors can be expressed in terms of θ_{π^-}, $s_{\pi\pi}$, and $s_{l\nu}$.

(iii) $B^- \to \pi^+ \pi^- l \bar{\nu}_l$

In this case a $Q=b$ matrix element of $L_{\nu 1}$ is required. The form factors are given by those in Eqs. (30) if the changes $f_D \to f_B$, $m_D \to m_B$, $\Delta_c \to \Delta_b$, $p_K \to p_{\pi^+}$, and $p_\pi \to p_{\pi^-}$ are made and μ is set to zero. Using Eqs. (11) and (12) these form factors can be expressed in terms of θ_{π^+}, $s_{\pi\pi}$, and $s_{l\nu}$.

(iv) $D^0 \to \pi^- \pi^0 \bar{l} \nu_l$

In this case the $Q=c$ matrix element of $L_{\nu 2}$ is required. Computation of the Feynman diagrams in Fig. 1 gives that the form factors are

$$w_- = \left[\frac{g f_D m_D}{2\sqrt{2} f^2}\right] \left[\frac{1}{v\cdot p_{\pi^-} + \Delta_c} + \frac{1}{v\cdot p_{\pi^0} + \Delta_c}\right], \tag{32a}$$

$$w_+ = \left[\frac{g f_D m_D}{2\sqrt{2} f^2}\right] \left[\frac{1}{v\cdot p_{\pi^-} + \Delta_c} - \frac{1}{v\cdot p_{\pi^0} + \Delta_c}\right] + r, \tag{32b}$$

$$r = \left[\frac{f_D}{\sqrt{2} f^2}\right] \left[\frac{v\cdot(p_{\pi^-} - p_{\pi^0})}{v\cdot(p_{\pi^-} + p_{\pi^0})} + g\frac{v\cdot p_{\pi^0}}{v\cdot p_{\pi^0} + \Delta_c} - g\frac{v\cdot p_{\pi^-}}{v\cdot p_{\pi^-} + \Delta_c} - g^2 \frac{p_{\pi^-}\cdot p_{\pi^0} - (v\cdot p_{\pi^-})(v\cdot p_{\pi^0})}{v\cdot(p_{\pi^-} + p_{\pi^0})} \times \left[\frac{1}{v\cdot p_{\pi^-} + \Delta_c} - \frac{1}{v\cdot p_{\pi^0} + \Delta_c}\right]\right], \tag{32c}$$

$$h = -\left[\frac{f_D g^2}{2\sqrt{2} f^2}\right] \frac{1}{v\cdot(p_{\pi^-} + p_{\pi^0}) + \Delta_c} \left[\frac{1}{v\cdot p_{\pi^-} + \Delta_c} + \frac{1}{v\cdot p_{\pi^0} + \Delta_c}\right]. \tag{32d}$$

It is straightforward using Eqs. (11) and (12) to express these form factors in terms of θ_{π^-}, $s_{\pi\pi}$, and $s_{l\nu}$. (Here the difference of four-momenta $Q^\mu = p_{\pi^-}^\mu - p_{\pi^0}^\mu$.)

(v) $\bar{B}^0 \to \pi^+ \pi^0 l \bar{\nu}_l$

In this case the $Q=b$ matrix element of $L_{\nu 1}$ is needed. The form factors are given by those in Eqs. (32) if the following changes are made: $f_D \to f_B$, $m_D \to m_B$, $\Delta_c \to \Delta_b$, and $p_{\pi^-} \to p_{\pi^+}$. Using Eqs. (11) and (12) the form factors can be expressed in terms of θ_{π^+}, $s_{\pi\pi}$, and $s_{l\nu}$.

IV. $B \to D \pi l \bar{\nu}_l$

In this case matrix elements of the operator $\bar{c}\gamma_\mu(1-\gamma_5)b$ are needed. This operator is a singlet under chiral $SU(3)_L \times SU(3)_R$ and in chiral perturbation theory its matrix elements are equal to those of

$$\bar{c}\gamma_\mu(1-\gamma_5)b = -\eta(v\cdot v')\mathrm{Tr}\,\bar{H}_a^{(c)}(v') \times \gamma_\mu(1-\gamma_5) H_a^{(b)}(v) + \cdots . \tag{33}$$

The ellipsis in Eq. (33) denotes terms with derivatives, insertions of the light-quark mass matrix or factors of $1/m_Q$. The $B \to D$ and $B \to D^*$ matrix elements of this current are [10]

$$\langle D(v')|\bar{c}\gamma_\mu(1-\gamma_5)b|B(v)\rangle = \sqrt{m_B m_D}\,\eta(v\cdot v')(v+v')_\mu, \tag{34a}$$

$$\langle D^*(v',\epsilon)|\bar{c}\gamma_\mu(1-\gamma_5)b|B(v)\rangle = \sqrt{m_B m_{D^*}}\,\eta(v\cdot v')[-\epsilon_\mu^*(1+v\cdot v') + (\epsilon^*\cdot v)v'_\mu + i\epsilon_{\alpha\lambda\mu\sigma}\epsilon^{*\alpha} v'^\lambda v^\sigma]. \tag{34b}$$

The normalization of η at zero recoil, i.e., $v\cdot v'=1$, is determined by heavy-quark flavor symmetry and by high-momentum strong interaction effects that are computable using perturbative QCD methods, [10–14]

$$\eta(1) \simeq \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25}.$$

Since the operator in Eq. (33) does not involve the pseudo-Goldstone-boson fields, in the leading order of chiral perturbation theory $B \to D\pi$ matrix elements of the current are determined by the pole-type Feynman diagrams in Fig. 2. They give, for a charged pion,

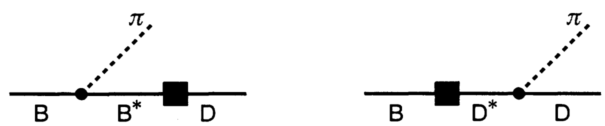

FIG. 2. Feynman diagrams for $B \to D\pi$ matrix element of $\bar{c}\gamma_\nu(1-\gamma_5)b$. The shaded square denotes an insertion of the current in Eq. (33).

$$w_+ - w_- = \frac{g}{f}\sqrt{m_B m_D}\,\eta(v\cdot v'+1)\left[\frac{1}{v'\cdot p_\pi - \Delta_c} - \frac{1}{v\cdot p_\pi + \Delta_b}\right] + r\ , \tag{35a}$$

$$w_+ + w_- = \frac{-g}{f}\left[\frac{m_B}{m_D}\right]^{1/2}\eta\left[\frac{p_\pi\cdot(v+v')}{v'\cdot p_\pi - \Delta_c}\right] + r\ , \tag{35b}$$

$$r = \frac{g}{f}\left[\frac{m_D}{m_B}\right]^{1/2}\eta\left[\frac{p_\pi\cdot(v\cdot v')}{v\cdot p_\pi + \Delta_b}\right]\ , \tag{35c}$$

$$h = \frac{g}{2f}\frac{\eta}{\sqrt{m_B m_D}}\left[\frac{1}{v'\cdot p_\pi - \Delta_c} - \frac{1}{v\cdot p_\pi + \Delta_b}\right]\ . \tag{35d}$$

In Eqs. (35),

$$\Delta_c = m_{D^*} - m_D \simeq 140\ \text{MeV}\ , \tag{36a}$$

$$\Delta_b = m_{B^*} - m_B \simeq 50\ \text{MeV}\ . \tag{36b}$$

The form factors for a neutral pion are obtained from the above by multiplying by $\pm 1/\sqrt{2}$.

We have assumed in writing Eqs. (35) that the kinematic region is chosen so that $v'\cdot p_\pi$ is not too close to Δ_c. For the use of the effective theory propagator to be appropriate it is necessary that

$$v'\cdot p_\pi - \Delta_c \gg m_\pi(m_\pi/2m_D) \simeq 5\ \text{MeV}\ . \tag{37}$$

This also ensures that the D^* width can be neglected in the propagator (it is expected to be only about a hundred keV).

It is convenient to reexpress some of the formulas of Sec. II in a way that makes the dependence of the heavy meson masses explicit and neglects terms suppressed by m_π/m_D or m_π/m_B. Introducing the pion's four-velocity $v_\pi^\mu = p_\pi^\mu/m_\pi$ we change integration variables from $s_{D\pi}$ and $s_{l\nu}$ to $v'\cdot v_\pi$ and $v\cdot v'$ using

$$ds_{D\pi}ds_{l\nu} \simeq 4 m_B m_\pi m_D^2\, d(v'\cdot v_\pi)d(v\cdot v')\ . \tag{38}$$

The form factors F_j are conveniently written in terms of dimensionless quantities $\hat{F}_j$:

$$F_j = \frac{m_B^{3/2} m_D^{1/2}}{f} g\eta(v\cdot v')\hat{F}_j\ . \tag{39}$$

Using

$$\beta \simeq (2m_\pi/m_D)[(v'\cdot v_\pi)^2 - 1]^{1/2}$$

and

$$X \simeq m_B m_D[(v\cdot v')^2 - 1]^{1/2}$$

the differential rate (after integrating over θ_l and ϕ) becomes

$$d^3\Gamma = \frac{8G_F^2 m_B^2 m_D^3 |V_{cb}|^2}{3(4\pi)^5}\left[\frac{m_\pi}{f}\right]^2 g^2\eta^2[(v'\cdot v_\pi)^2-1]^{1/2} \times[(v'\cdot v)^2-1]^{1/2}[|\hat{F}_1|^2 + \sin^2\theta_D(|\hat{F}_2|^2 + |\hat{F}_3|^2)]d(v'\cdot v_\pi)d(v'\cdot v)d\cos\theta_D\ . \tag{40}$$

Combining Eqs. (39), (35), and (21) the dimensionless form factors $\hat{F}_j$ are found to be

$$\begin{aligned}\hat{F}_1 = {} & [(v\cdot v')^2-1]^{1/2}(v+v')\cdot v_\pi\left[\left[\frac{m_D}{m_B}\right]\frac{1}{v\cdot v_\pi + \hat{\Delta}_b} - \frac{1}{v'\cdot v_\pi - \hat{\Delta}_c}\right] \\ & - v'\cdot v_\pi(v\cdot v'+1)[(v\cdot v')^2-1]^{1/2}\left[\frac{1}{v\cdot v_\pi + \hat{\Delta}_b} - \frac{1}{v'\cdot v_\pi - \hat{\Delta}_c}\right] \\ & + \cos\theta_D[(v'\cdot v_\pi)^2-1]^{1/2}(v\cdot v'+1)(v\cdot v' - m_D/m_B)\left[\frac{1}{v\cdot v_\pi + \hat{\Delta}_b} - \frac{1}{v'\cdot v_\pi - \hat{\Delta}_c}\right]\ ,\end{aligned} \tag{41}$$

$$\hat{F}_2 = [(v'\cdot v_\pi)^2-1]^{1/2}(v\cdot v'+1)[1+(m_D/m_B)^2 - 2(m_D/m_B)v\cdot v']^{1/2}\left[\frac{1}{v\cdot v_\pi + \hat{\Delta}_b} - \frac{1}{v'\cdot v_\pi - \hat{\Delta}_c}\right]\ , \tag{42}$$

$$\hat{F}_3 = -[(v'\cdot v_\pi)^2-1]^{1/2}[(v\cdot v')^2-1]^{1/2}[1+(m_D/m_B)^2 - 2(m_D/m_B)v\cdot v']^{1/2}\left[\frac{1}{v\cdot v_\pi + \hat{\Delta}_b} - \frac{1}{v'\cdot v_\pi - \hat{\Delta}_c}\right]\ . \tag{43}$$

TABLE I. $d^2\hat{\Gamma}/d(v\cdot v')d(v'\cdot v_\pi)$ for various values of $(v\cdot v')$ and $(v'\cdot v_\pi)$.

$d^2\hat{\Gamma}/d(v\cdot v')d(v'\cdot v_\pi)$	$(v\cdot v')$	$(v'\cdot v_\pi)$
0.030	1.2	1.2
0.042	1.4	1.2
0.024	1.2	1.3
0.034	1.4	1.3
0.021	1.2	1.4
0.030	1.4	1.4
0.018	1.2	1.5
0.027	1.4	1.5

In Eqs. (41)–(43)

$$v\cdot v_\pi=(v'\cdot v_\pi)(v\cdot v') - [(v'\cdot v_\pi)^2-1]^{1/2}[(v\cdot v')^2-1]^{1/2}\cos\theta_D \tag{44}$$

and

$$\hat{\Delta}_c=(m_{D^*}-m_D)/m_\pi\ ,\quad \hat{\Delta}_b=(m_{B^*}-m_B)/m_\pi\ . \tag{45}$$

Chiral perturbation theory should be valid for $v\cdot v_\pi$ and $v'\cdot v_\pi$ not too much greater than unity. From Eq. (44) it is clear that the kinematic region where $\cos\theta_D$ is positive yields (for given $v'\cdot v_\pi$ and $v\cdot v'$) a smaller value for $v\cdot v_\pi$. Note that because m_π and f are comparable, the rate for $B\to D\pi l\bar{\nu}_l$ is not suppressed by factors of m_π/m_D or m_π/m_B. In fact, the above formulas indicate that there is a significant rate for $B\to D\pi l\bar{\nu}_l$ in the kinematic region where chiral perturbation theory is expected to be applicable (and the $D\pi$ mass is large enough to neglect the width in the virtual D^* propagator). To illustrate this we write

$$d^3\Gamma=\frac{G_F^2 m_B^5}{192\pi^3}|V_{cb}|^2 g^2\eta^2 d^3\hat{\Gamma}\ . \tag{46}$$

In Table I we give $d^2\hat{\Gamma}/d(v\cdot v')d(v'\cdot v_\pi)$ for various values of $(v\cdot v')$ and $(v'\cdot v_\pi)$. Provided η does not fall off very rapidly as $v'\cdot v$ increases, the rate for $\beta\to D\pi l\bar{\nu}_l$, in the region where chiral perturbation theory is expected to be applicable (i.e., $v\cdot v_\pi$ and $v'\cdot v_\pi$ around unity) is comparable with what was estimated in the Introduction. In Table I we used $\hat{\Delta}_c=1$. The rate in the kinematic region where $v_\pi\cdot v'$ is near one is quite sensitive to the value of $\hat{\Delta}_c$. For $B^+\to D^+\pi^- l\bar{\nu}_l$ decay $\hat{\Delta}_c=1$ is consistent with the measured masses, but for $B^0\to D^0\pi^+ l\bar{\nu}_l$ decay $\hat{\Delta}_c=1$ is slightly less than the experimental value.

V. VALIDITY OF CHIRAL PERTURBATION THEORY

Chiral perturbation theory is an expansion in momenta so our results are expected to be valid for only a limited kinematic range. For $B\to D\pi l\bar{\nu}_l$ naive dimensional analysis suggests that the expansion parameters are $(v\cdot p_\pi)/\Lambda$ and $(v'\cdot p_\pi)/\Lambda$, where Λ is a nonperturbative strong-interaction scale around 1 GeV. However, it is far from clear precisely how small these quantities must be for the $B\to D\pi l\bar{\nu}_l$ differential decay rate given in Eqs. (40)–(45) to be a good approximation. We do have some experience from comparisons of the predictions of chiral perturbation theory for $\pi\pi$ scattering, weak kaon decays, etc., with experiment. As we shall see shortly, the situation in $B\to D\pi l\bar{\nu}_l$ decay is somewhat different.

For $B\to D\pi l\bar{\nu}_l$ the leading contribution is of order unity. One factor of p_π from the $D^*D\pi$ (or $B^*B\pi$) vertex is canceled by a factor of $1/p_\pi$ from the D^* (or B^*) propagator. At the next order of chiral perturbation theory, corrections come from two sources: (i) operators in the chiral Lagrangian for strong D^* and D (or B^* and B) interactions with pions containing two derivatives or one factor of the light-quark mass matrix; (ii) operators representing the weak current $\bar{c}\gamma_\mu(1-\gamma_5)b$ that contain one derivative.

For example, one term in the ellipsis of Eq. (33) is

$$\frac{i\tilde{\eta}(v\cdot v')}{\Lambda}\mathrm{Tr}[\bar{H}_a^{(c)}(v')\gamma_\mu(1-\gamma_5)H_d^{(b)}(v)\gamma^\lambda\gamma_5] \times(\xi^\dagger\partial_\lambda\xi-\xi\partial_\lambda\xi^\dagger)_{da}\ , \tag{47}$$

where $\tilde{\eta}(v\cdot v')$ is a new universal function of $v\cdot v'$. This "higher-order" contribution to the current $\bar{c}\gamma_\mu(1-\gamma_5)b$ gives rise to the following changes in the form factors $w_\pm, r$ and h:

$$\delta(w_+-w_-)=-\frac{2}{\Lambda f}\sqrt{m_B m_D}\,\tilde{\eta}(v\cdot v'+1)+\delta r\ , \tag{48a}$$

$$\delta(w_++w_-)=\frac{2}{\Lambda f}\left[\frac{m_B}{m_D}\right]^{1/2}\tilde{\eta}(p_\pi\cdot v)+\delta r\ , \tag{48b}$$

$$\delta r=\frac{2}{\Lambda f}\left[\frac{m_D}{m_B}\right]^{1/2}\tilde{\eta}(p_\pi\cdot v')\ , \tag{48c}$$

$$\delta h=\frac{-1}{\Lambda f}\frac{\tilde{\eta}}{\sqrt{m_B m_D}}\ . \tag{48d}$$

For the $\pi\pi$ phase shifts, the first corrections to the leading predictions of chiral perturbation theory are suppressed by s/Λ^2 and come from operators in the chiral Lagrangian with four derivatives and from one-loop diagrams. However, for $B\to D\pi l\bar{\nu}_l$ loops do not contribute to the leading correction which is only suppressed by $v\cdot p_\pi/\Lambda$ or $v'\cdot p_\pi/\Lambda$.

There are too many higher dimension operators with unknown coefficients to make any predictions for the next order contribution to the form factors for $B\to D\pi l\bar{\nu}_l$. However, it is certainly possible that our leading prediction for the $B\to D\pi l\bar{\nu}_l$ differential decay rate is valid at the 30% level over the kinematic range displayed in Table I. Eventually the range of validity of lowest-order chiral perturbation theory for $B\to D\pi l\bar{\nu}_l$ may be determined by experiment.

VI. CONCLUDING REMARKS

In this paper the semileptonic B and D meson decays, $D\to K\pi\bar{l}\nu_l$, $D\to\pi\pi\bar{l}\nu_l$, $B\to\pi\pi l\bar{\nu}_l$, and $B\to D\pi l\bar{\nu}_l$ were considered. Chiral symmetry and heavy-quark symmetry were combined to deduce the decay amplitudes in the kinematic region where the pseudo Goldstone bosons are soft. There was earlier work on these decays that considered the implications of chiral symmetry but it did not implement heavy-quark symmetry in a model-independent fashion [15].

For $B \to D\pi l\bar{\nu}_l$ decay the rate is large enough that detailed experimental study of the decay (in the kinematic regime where chiral perturbation theory is expected to be applicable) may be possible at a B factory. Table I gives $d^2\hat{\Gamma}/d(v\cdot v')d(v'\cdot v_\pi)$ for various values of $v\cdot v'$ and $v'\cdot v_\pi$ [see Eq. (46)]. These indicate that the branching ratio for semileptonic B_{l4} decay to nonresonant $D\pi$ (in the kinematic regime where the pion is soft, i.e., $v\cdot v_\pi$ and $v'\cdot v_\pi$ around unity) is about 10^{-4}.

The results of this paper rely on heavy-quark spin and flavor symmetry. There is experimental evidence from semileptonic B decay [16] and from the decays of excited charm mesons [17] that (at least in some cases) the charm quark is heavy enough for heavy-quark symmetry to be applicable. However, several theoretical analyses suggest that there are large $\Lambda_{\rm QCD}/m_c$ corrections to the prediction of heavy-quark symmetry for the relation between B and D meson decay constants [18–20]. If this is an isolated case, where the $\Lambda_{\rm QCD}/m_c$ corrections that break the flavor symmetry are anomalously large, then the results of this paper can still be used (with f_B and f_D in Sec. III treated as independent constants).

Semileptonic $B \to Dl\bar{\nu}_l$ and $B \to D^* l\bar{\nu}_l$ decay can be utilized to check that there are not large $\Lambda_{\rm QCD}/m_c$ corrections to the expression for the $b \to c$ transition current in Eq. (33). However, our predictions for $B \to D\pi l\bar{\nu}_l$ decay still depend on the validity of heavy-quark spin-flavor symmetry for the chiral Lagrangian in Eq. (1). The dependence on the flavor symmetry arises from the equality of the $B^*B\pi$ and $D^*D\pi$ couplings. If heavy-quark flavor symmetry is *not* used then the form factors for $B \to D\pi l\bar{\nu}_l$ decay given in Eq. (35) of Sec. IV become

$$w_+ - w_- = \frac{\sqrt{m_B m_D}}{f}(v\cdot v'+1) \times \eta\left[\frac{g_c}{v'\cdot p_\pi - \Delta_c} - \frac{g_b}{v\cdot p_\pi + \Delta_b}\right] + r\ , \tag{49a}$$

$$w_+ + w_- = -\frac{g_c}{f}\left[\frac{m_B}{m_D}\right]^{1/2}\eta\left[\frac{p_\pi\cdot(v+v')}{v'\cdot p_\pi - \Delta_c}\right] + r\ , \tag{49b}$$

$$r = \frac{g_b}{f}\left[\frac{m_D}{m_B}\right]^{1/2}\eta\left[\frac{p_\pi\cdot(v+v')}{v\cdot p_\pi + \Delta_b}\right]\ , \tag{49c}$$

$$h = \frac{1}{2f}\frac{\eta}{\sqrt{m_B m_D}}\left[\frac{g_c}{v'\cdot p_\pi - \Delta_c} - \frac{g_b}{v\cdot p_\pi + \Delta_b}\right]\ . \tag{49d}$$

It would be interesting to use $B \to D\pi l\bar{\nu}_l$ decay to test the heavy-quark flavor symmetry prediction, $g_b = g_c$.

It is not known precisely for what range of $v\cdot p_\pi$ and $v'\cdot p_\pi$ chiral perturbation theory will be valid. Our experience with light hadrons suggests that the relevant expansion parameters are roughly $v\cdot p_\pi/1$ GeV and $v'\cdot p_\pi/1$ GeV. It may be possible in $B \to D\pi l\bar{\nu}_l$ to study the range of validity of chiral perturbation theory for heavy-meson pion interactions.

A number of extensions and improvements on our work are possible. The decay $B \to D^*\pi l\bar{\nu}_l$ can be considered [3]. It is interesting to explore to what extent it can also be used to fix g and to test the heavy-quark flavor symmetry prediction $g = g_b = g_c$. There are computable $\alpha_s(m_b)$ and $\alpha_s(m_c)$ corrections to the form factors for the decays discussed in this paper [4,21,22] and it is worth examining their influence on the rates for B_{l4} and D_{l4} decays.

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