Two-photon contribution to polarization in $K^+ \to \pi^+ \mu^+ \mu^-$

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Short-distance physics involving virtual top and charm quarks contributes to μ^+ (and μ^-) polarization in the decay $K^+ \to \pi^+ \mu^+ \mu^-$. Measurement of the parity-violating asymmetry $(\Gamma_R - \Gamma_L)/(\Gamma_R + \Gamma_L)$, where Γ_R and Γ_L are the rates to produce right- and left-handed μ^+, may provide valuable information on the unitarity triangle. The parity-violating asymmetry also gets a contribution from Feynman diagrams with two-photon intermediate states. We estimate this two-photon contribution to the asymmetry and discuss briefly the two-photon contribution to time-reversal-odd asymmetries that involve both the μ^+ and μ^- polarizations.

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I. INTRODUCTION

In the minimal standard model the coupling of the quarks to the W bosons has the form

$$\mathcal{L}_{\rm int} = -\frac{g_2}{\sqrt{2}} \bar{u}_L^j \gamma_\mu V^{jk} d_L^k W^\mu + \text{H.c.} \tag{1}$$

Here the repeated generation indices j and k are summed over 1,2,3 and g_2 is the weak SU(2) gauge coupling. V is a 3×3 unitary matrix that arises from diagonalization of the quark mass matrices. By redefining the phases of the quark fields it is possible to write V in terms of the four angles θ_1, θ_2, θ_3 and δ [For N_g generations there are $(N_g-1)^2$ angles.] The θ_j are analogous to the Euler angles and δ is a phase that gives rise to CP violation. Explicitly [1],

$$V = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} , \tag{2}$$

where $c_i \equiv \cos\theta_i$ and $s_i \equiv \sin\theta_i$. It is possible to choose the θ_j to lie in the first quadrant by redefining the quark fields. Then the quadrant of δ has physical significance and cannot be chosen by convention. A value of δ not equal to zero or π gives rise to CP violation.

Experimental information on nuclear β decay and weak decays of kaons, hyperons, and B mesons shows that the angles θ_j are small (but different from zero). The angle θ_1 is essentially the Cabibbo angle. It is by far the best known of the angles [2]:

$$\sin\theta_1 = 0.22 \tag{3}$$

(with an error at the percent level).

Unitarity of the Cabibbo-Kobayashi-Maskawa matrix V gives

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 . \tag{4}$$

We can think of each of the three complex numbers ($V_{ud}V_{ub}^*$, etc.) on the left-hand-side (LHS) of Eq. (4) as vectors in the complex plane. These vectors add to zero and so by translating them they form the sides of a triangle that is often called the unitarity triangle. With the parametrization of the Cabibbo-Kobayashi-Maskawa matrix in Eq. (2) we have

$$V_{ud}V_{ub}^* \simeq -s_1 s_3 , \tag{5a}$$

$$V_{td}V_{tb}^* \simeq -s_1 s_2 e^{-i\delta} , \tag{5b}$$

$$V_{cd}V_{cb}^* \simeq s_1(s_3 + s_2 e^{-i\delta}) . \tag{5c}$$

The unitarity triangle specifies the angles θ_2, θ_3, and δ. From Eqs. (5) it is clear that the length of two sides gives θ_2 and θ_3 while the angle between two of the sides is $\pi-\delta$.

The orientation of the unitarity triangle in the complex plane depends on the phase convention in the Cabibbo-Kobayashi-Maskawa matrix. The length of the sides and the angles at each vertex α, β, and γ are independent of the phase convention. When there is no CP violation the unitarity triangle collapses to a line. One common orientation for the triangle has $V_{cd}V_{cb}^*$ lying along the real axis. It is conventional to rescale the side on the real axis to unit length and locate one vertex at the origin of the complex plane. This is shown in Fig. 1. With this convention the unitarity triangle is specified by the coordinates in the complex plane, $\rho + i\eta$, of the vertex associated with the angle α.

It is important to determine the unitary triangle by measuring quantities that do not violate CP. The resulting values of the weak mixing angles can then be used to predict the expected values of CP-violating quantities. In this way the standard six-quark model for CP violation can be tested. At the present time it is not known if the CP violation observed in kaon decays is due to the phase in the Cabibbo-Kobayashi-Maskawa matrix or from new physics, beyond that in the minimal standard model, or both.

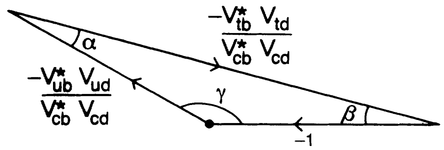

FIG. 1. The unitary triangle.

B-meson decays give valuable information on the unitarity triangle. However, rare kaon decays where a virtual top quark plays an important role can also be useful. For example, an accurate measurement of the branching ratio for $K^+\rightarrow\pi^+\nu\bar{\nu}$ would restrict the α corner of the unitarity triangle to lie on a circle.

In Ref. [3] it was pointed out that the measurement of polarization in $K^+\rightarrow\pi^+\mu^+\mu^-$ decay can also lead to valuable information on the weak mixing angles. The dominant contribution to the $K^+\rightarrow\pi^+\mu^+\mu^-$ decay amplitude comes from Feynman diagrams where a single photon produces the $\mu^+\mu^-$ pair. Even though the weak interactions violate parity maximally the one photon part of the decay amplitude is necessarily parity conserving and does not contribute to the parity-violating asymmetry

$$\Delta_{LR}=(\Gamma_R-\Gamma_L)/(\Gamma_R+\Gamma_L)\ ,$$

where Γ_R and Γ_L are the rates to produce right- and left-handed μ^+, respectively.[1] This parity-violating asymmetry arises predominantly from two sources: (i) the interference of W-box and Z-penguin Feynman diagrams (see Fig. 2) with the one-photon piece; (ii) the interference of Feynman diagrams where two photons create the $\mu^+\mu^-$ pair with the one-photon piece. Is the short-distance W-box and Z-penguin part dominates the asymmetry then its measurement can lead to important information on the unitarity triangle. The main purpose of this paper is to examine the long-distance two-photon contribution to the $K^+\rightarrow\pi^+\mu^+\mu^-$ decay amplitude and in particular its influence on the parity-violating asymmetry Δ_{LR}. Reference [3] also noted that there are T-odd asymmetries which involve both the μ^+ and μ^- polarizations and can arise from the interference of the Z-penguin and W-box Feynman diagrams with the one-photon piece. Detailed predictions for the short-distance contribution to these T-odd asymmetries were made in Ref. [4]. Here we stress that the T-odd asymmetries also receive a contribution from the interference of the absorptive part of the parity-violating two-photon contribution with the one-photon piece.

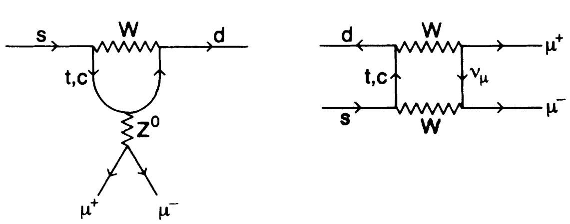

FIG. 2. Some of the Z-penguin and W-box Feynman diagrams that contribute to the $K^+\rightarrow\pi^+\mu^+\mu^-$ decay amplitude.

[1]By right- (left-)handed we mean that the spin is directed along (opposite) the direction of motion, i.e., helicity $+\frac{1}{2}$ (or $-\frac{1}{2}$).

II. KINEMATICS

The dominant part of the $K^+\rightarrow\pi^+\mu^+\mu^-$ decay amplitude comes from Feynman diagrams where a single photon produces the $\mu^+\mu^-$ pair. The one-photon contribution to the invariant matrix element has the form

$$\mathcal{M}^{\rm PC}=\frac{s_1G_F}{\sqrt{2}}\alpha f(s)(p_K+p_\pi)^\mu\bar{u}(p_-,s_-)\gamma_\mu v(p_+,s_+)\ , \tag{6}$$

where p_K and p_π are the four-momentum of the kaon and pion and $p_\pm$ are the four-momenta of the $\mu^\pm$. In Eqs. (6) $s_\pm$ are the spin vectors for the $\mu^\pm$ while $\sqrt{s}$ is the invariant mass of the $\mu^+\mu^-$ pair

$$s=(p_++p_-)^2\ . \tag{7}$$

We shall parametrize the differential decay rate in terms of s and θ the angle between the three-momentum of the kaon and the three-momentum of the μ^- in the $\mu^+\mu^-$ pair rest frame. In terms of these variables the inner products of four-momenta are

$$p_-\cdot p_+=s/2-m_\mu^2\ , \tag{8a}$$

$$(p_K+p_\pi)^2=2(m_K^2+m_\pi^2)-s\ , \tag{8b}$$

$$\begin{aligned}2p_+\cdot(p_K+p_\pi)&\\ =(m_K^2-m_\pi^2)&+(1-4m_\mu^2/s)^{1/2}[(s+m_K^2-m_\pi^2)^2\\ &-4sm_K^2]^{1/2}\cos\theta\ .\end{aligned} \tag{8c}$$

For a right- or left-handed μ^+ the dot products of the polarization four-vector s_+^μ with the μ^- and kaon four-momenta are

$$s_+^{(R)}\cdot p_-=-s_+^{(L)}\cdot p_-=\frac{s}{2m_\mu}\left[1-\frac{4m_\mu^2}{s}\right]^{1/2}\ , \tag{9a}$$

$$\begin{aligned}s_+^{(R)}\cdot p_K&=-s_+^{(L)}\cdot p_K\\ &=\frac{1}{4m_\mu}\left\{(1-4m_\mu^2/s)^{1/2}(s+m_K^2-m_\pi^2)\right.\\ &\quad\left.+[(s+m_K^2-m_\pi^2)^2-4sm_K^2]^{1/2}\cos\theta\right\}\ .\end{aligned} \tag{9b}$$

The total differential decay rate is dominated by the one-photon piece and the invariant amplitude in Eq. (6) gives

$$
\begin{aligned}
d(\Gamma_R+\Gamma_L)/d\cos\theta\, ds
&= \frac{s_1^2 G_F^2 \alpha^2 |f(s)|^2}{2^9 m_K^3 \pi^3}(1-4m_\mu^2/s)^{1/2} \\
&\quad \times[(m_K^2-m_\pi^2+s)^2-4sm_K^2]^{3/2} \\
&\quad \times[1-(1-4m_\mu^2/s)\cos^2\theta]\ . \qquad (10)
\end{aligned}
$$

The parity-violating part of the decay amplitude has the form

$$
\begin{aligned}
\mathcal{M}^{\mathrm{PV}} &= \frac{s_1 G_F \alpha}{\sqrt{2}}[B(p_K+p_\pi)^\mu + C(p_K-p_\pi)^\mu] \\
&\quad \times \bar{u}(p_-,s_-)\gamma_\mu\gamma_5 v(p_+,s_+)\ . \qquad (11)
\end{aligned}
$$

The parameters B and C in Eq. (11) get contributions from the Z-penguin and W-box Feynman diagrams as well as from Feynman diagrams with two photons.

The difference in decay amplitudes for right- and left-handed μ^+ arises from the interference of the parity-conserving part of the decay amplitude in Eq. (6) with the parity-violating part of the decay amplitude in Eq. (11). This gives

$$
\begin{aligned}
d(\Gamma_R-\Gamma_L)/d\cos\theta\, ds &= \frac{-s_1^2 G_F^2 \alpha^2}{2^8 m_K^3 \pi^3}(1-4m_\mu^2/s)^{1/2}[(s+m_K^2-m_\pi^2)^2-4sm_K^2] \\
&\quad \times \Bigg\{ \mathrm{Re}[f^*(s)B](1-4m_\mu^2/s)^{1/2}[(s+m_K^2-m_\pi^2)-4sm_K^2]^{1/2}\sin^2\theta \\
&\quad + 4\left[\mathrm{Re}[f^*(s)B]\left[\frac{m_K^2-m_\pi^2}{s}\right] + \mathrm{Re}[f^*(s)C]\right] m_\mu^2 \cos\theta \Bigg\}\ . \qquad (12)
\end{aligned}
$$

Note that in Eq. (12) the contribution of C vanishes when the difference of decay rates is integrated over θ.

III. THE PARITY-CONSERVING AMPLITUDE

The parity-conserving amplitude arises predominantly from Feynman diagrams where a single photon produces the $\mu^+\mu^-$ pair. It is characterized by the function $f(s)$ introduced in Eq. (6) of Sec. II. The absolute value of $f(s)$ has been determined by experimental data on $K^+\to\pi^+e^+e^-$. A good fit to the differential decay rate is obtained from [5]

$$
|f(s)| = |f(0)|(1+\lambda s/m_\pi^2)\ , \qquad (13)
$$

with $\lambda=0.11$ and $|f(0)|=0.31$.

Using chiral perturbation theory, the imaginary part of $f(s)$ arises from the Feynman diagrams in Fig. 3, with the pions in the loop on their mass shell. The strong interactions of the pseudo Goldstone bosons π, K, and η are described by the effective chiral Lagrangian

$$
\mathcal{L} = (f^2/8)\,\mathrm{Tr}\,\partial_\mu\Sigma\partial^\mu\Sigma^\dagger + v\,\mathrm{Tr}(m_q\Sigma+\Sigma^\dagger m_q) + \cdots\ . \qquad (14)
$$

In Eq. (14) f is the pion decay constant, $f \simeq 132$ MeV and m_q is the quark mass matrix,

$$
m_q = \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{bmatrix}\ . \qquad (15)
$$

The pseudo-Goldstone-boson field occur in the 3×3 special unitary matrix Σ. Explicitly

$$
\Sigma = \exp(2iM/f)\ , \qquad (16)
$$

where

$$
M = \begin{bmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{bmatrix}\ . \qquad (17)
$$

In Fig. 3 a shaded circle denotes an interaction vertex arising from the strong interaction effective Lagrangian density in Eq. (14). The effective Lagrangian for $\Delta s=1$ weak nonleptonic decays transforms under chiral

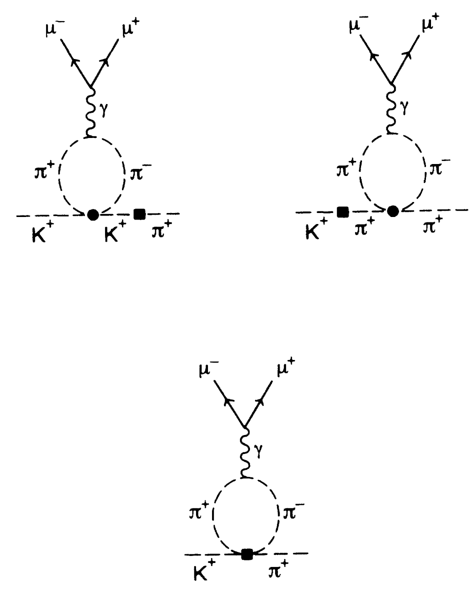

FIG. 3. Feynman diagrams that give the leading contribution to Im $f(s)$ in chiral perturbation theory.

$SU(3)_L \times SU(3)_R$ as $(8_L, 1_R)+(27_L, 1_R)$. In terms of Σ the $(8_L, 1_R)$ part of the effective Lagrangian density[2] for weak nonleptonic kaon decays is

$$\mathcal{L}=g_8(G_F/4\sqrt{2})s_1 f^4 \operatorname{Tr} O_W \partial^\mu \Sigma \partial_\mu \Sigma^\dagger + \cdots , \tag{18}$$

where

$$O_W = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} . \tag{19}$$

The measured $K_S \to \pi^+\pi^-$ decay rate implies that [6] $|g_8| \simeq 5.1$. In Fig. 3 a shaded square denotes an interaction vertex from the effective Lagrangian in Eq. (18). The Feynman diagrams in Fig. 3 give [6]

$$\operatorname{Im} f(s) = -(g_8/24)(1-4m_\pi^2/s)^{3/2}\theta(s-4m_\pi^2) . \tag{20}$$

The imaginary part of $f(s)$ is largest at the maximum value of s, $s_{\max}=(m_K - m_\pi)^2$. Equation (20) implies that (up to a sign) $\operatorname{Im} f(s_{\max}) \simeq 0.05$ and so the imaginary part of $f(s)$ is expected to be much smaller than its real part.

Chiral perturbation theory also predicts $\operatorname{Re} f(s)$ up to an s-independent constant that is determined by the total decay rate [6]. The measured s dependence given in Eq. (13) is somewhat greater than what chiral perturbation theory gives but the experimental error is still quite large, i.e., $\lambda=0.105\pm0.035\pm0.015$.

IV. SHORT-DISTANCE CONTRIBUTION TO THE PARITY-VIOLATING AMPLITUDE

The Z-penguin and W-box Feynman diagrams contribute to both B and C of the parity-violating amplitude in Eq. (11). Explicitly,

$$B=f_+(s)\xi, \quad C=f_-(s)\xi , \tag{21}$$

where $f_+(s)$ and $f_-(s)$ are the form factors for K_{l3} semileptonic decay. Conventionally their s dependence is parametrized by

$$f_\pm(s)=f_\pm(0)(1+\lambda_\pm s/m_\pi^2) .$$

We use [2] $f_+(0)=1.02$, $\lambda_+=0.03$, $f_-(0)=-0.17$ and $\lambda_-=0$. ξ is a quantity that, apart from mixing angles, is essentially the same as occurs in $B \to X_s e^+e^-$. As noted in Ref. [3] it is given by

$$\xi \simeq -\tilde{\xi}_c + \left[\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} \right] \tilde{\xi}_t , \tag{22}$$

where

$$\tilde{\xi}_q = \tilde{\xi}_q^{(Z)} + \tilde{\xi}_q^{(W)} \tag{23}$$

is the sum of the contributions of the Z-penguin (superscript Z) and W-box (superscript W) diagrams. In Eq. (23),

$$\tilde{\xi}_t^{(Z)} = \frac{x}{\sin^2\theta_W} \frac{1}{16\pi} \left[\frac{(x-6)(x-1)+(3x+2)\ln x}{(x-1)^2} \right] , \tag{24a}$$

$$\tilde{\xi}_t^{(W)} = \frac{x}{\sin^2\theta_W} \frac{1}{8\pi} \left[\frac{x-1-\ln x}{(x-1)^2} \right] , \tag{24b}$$

with $x=m_t^2/M_W^2$ and

$$\tilde{\xi}_c^{(Z)} \simeq \frac{\eta^{(Z)}}{\sin^2\theta_W} \frac{1}{8\pi} \left[\frac{m_c^2}{M_W^2} \right] \ln(m_c^2/M_W^2) , \tag{25a}$$

$$\tilde{\xi}_c^{(W)} \simeq -\frac{\eta^{(W)}}{\sin^2\theta_W} \frac{1}{8\pi} \left[\frac{m_c^2}{M_W^2} \right] \ln(m_c^2/M_W^2) . \tag{25b}$$

The QCD correction factors $\eta^{(Z)}$ and $\eta^{(W)}$ have been computed in the leading-logarithmic approximation [7] and they have the values $\eta^{(Z)} \simeq 0.3$ and $\eta^{(W)} \simeq 0.6$. Using $m_c=1.5$ GeV and $M_W=81$ GeV and $\sin^2\theta_W=0.23$, Eqs. (25a) and (25b) imply that $\tilde{\xi}_c=1.4\times10^{-4}$. The value of $\tilde{\xi}_t$ depends sensitively on the top-quark mass. For $m_t=140$ GeV, $\tilde{\xi}_t \simeq 0.51$ and for $m_t=200$ GeV, $\tilde{\xi}_t \simeq 0.89$.

The coefficient of $\tilde{\xi}_t$ depends on the weak mixing angles. It is convenient to reexpress this combination of elements of the Cabibbo-Kobayashi-Maskawa matrix in terms of $|V_{cb}|$ and the complex coordinates $\rho+i\eta$ of the α vertex of the unitarity triangle:

$$V_{ts}^* V_{td}/V_{us}^* V_{ud} = (\rho-1+i\eta)|V_{cb}|^2 . \tag{26}$$

The value of $|V_{cb}|$ can be obtained from the semileptonic decays $B \to D^* e\bar{\nu}_e$ and $B \to De\bar{\nu}_e$. Using heavy-quark spin-flavor symmetry the hadronic form factors for these decays can be expressed in terms of a single universal function of "velocity transfer [8]." Furthermore, the normalization of this universal function is fixed at zero recoil where both the B and D^* or D are at rest [8–10]. A comparison of the predictions of heavy-quark symmetry with experimental data on these decays gives [11] $|V_{cb}|=0.043\pm0.007$. At zero recoil there are no $\Lambda_{\rm QCD}/m_c$ to corrections to the $m_b, m_c \to \infty$ predictions of heavy-quark symmetry for the $B \to D^*$ and $B \to D$ matrix elements of the weak current [12]. Consequently this method for determining $|V_{cb}|$ is on a very sound theoretical footing. Eventually, with improved data on semileptonic B decay, the error on $|V_{cb}|$ should be substantially reduced.

Experimental information on the end point of the electron spectrum in semileptonic B-meson decay and $B^0-\bar{B}^0$ mixing constrains the values for ρ and η. However, in these cases there are large theoretical uncertainties that arise from the influence of nonperturbative strong interaction physics on the relevant hadronic matrix elements.

At the present time the value of $|V_{ub}|$ is determined by comparing data on the end point of the electron spectrum in B-meson semileptonic decay with phenomenological models. This gives [2] $|V_{ub}/V_{cb}|=0.10\pm0.03$, leading to the constraint $\sqrt{\rho^2+\eta^2}=0.5\pm0.2$. Because of the model dependence in extracting $|V_{ub}|$ the error quoted

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[2] It dominates over the $(27_L, 1_R)$ part of the Lagrangian.

above should be interpreted as providing a rough measure of the uncertainty. In Fig. 4 we have plotted semicircles corresponding to $\sqrt{\rho^2+\eta^2}=0.7$ and 0.3. In the future exclusive decays may also provide valuable information on $|V_{ub}|$. Heavy-quark symmetry plus isospin symmetry relates the hadronic form factors for $D\to\rho\bar{e}\nu_e$ and $B\to\rho e\bar{\nu}_e$ decay [8]. Since the weak mixing angles are known in the D decay case the form factors needed for the B decay can be determined from experimental data on semileptonic D decay. If light-quark SU(3) is used instead of isospin then the needed decay is $D\to K^*\bar{e}\nu_e$. There is already experimental information on form factors for this decay. Unfortunately there is no theorem that protects the $m_b, m_c\to\infty$ heavy-quark symmetry prediction for the relationship between form factors in $B\to\rho e\bar{\nu}_e$ decay and $D\to\rho\bar{e}\nu_e$ decay from $\Lambda_{\rm QCD}/m_c$ corrections. Lattice Monte Carlo calculations can also provide valuable information on the needed hadronic form factors [13].

The measurement $\Delta M/\Gamma=0.75$ for $B^0-\bar{B}^0$ mixing provides information on the magnitude of V_{td} (for a given top-quark mass). This constraint depends on the hadronic matrix element of a local four-quark operator, which is usually written as $B_B f_B^2$. Heavy-quark symmetry [14] and the constituent quark model suggest a value for f_B around 120 MeV, while recent lattice QCD results [15] and QCD sum-rule calculations [16] suggest a large value[3] for f_B around 250 MeV. Constraining $|V_{td}|$ from B^0-$\bar{B}^0$ mixing corresponds to a constraint on $\sqrt{(1-\rho)^2+\eta^2}$, which would appear as a semicircle in the ρ-η plane centered around $\rho=1$, $\eta=0$. Because the range of f_B's mentioned above gives rise to a very large uncertainty in $|V_{td}|$ we have not plotted the implications of the measured value for B^0-$\bar{B}^0$ mixing in Fig. 4. Qualitatively the smaller value $\sqrt{B_B}f_B\simeq 120$ MeV implies, for top-quark masses less than 200 GeV, that ρ is negative while the larger value $\sqrt{B_B}f_B\simeq 250$ MeV implies that ρ is positive.

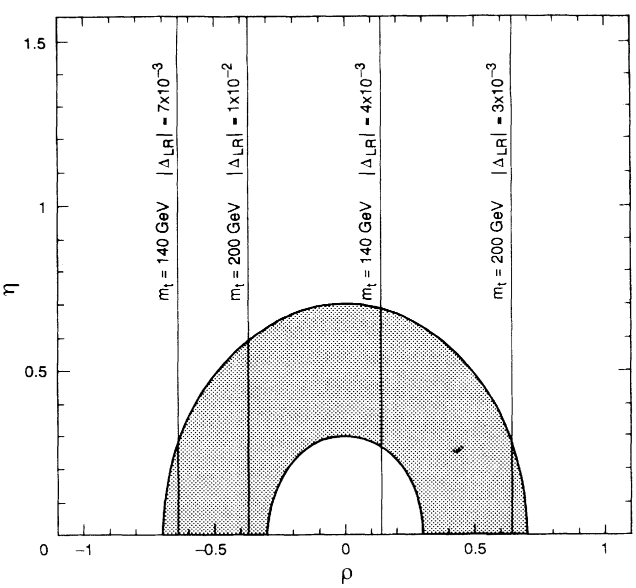

FIG. 4. Implications of measurement of the asymmetry Δ_{LR} for the location of the α vertex of the unitarity triangle.

Since Im $f(s)$ is small (provided the two-photon contribution to the parity violating amplitude is negligible) measurement of the polarization asymmetry Δ_{LR} determines the value of ρ restricting the α vertex of the unitary triangle to lie on a vertical line in the ρ-η plane. Integrating over the whole available phase space we find that the interference of the short-distance contribution to the parity-violating amplitude with the parity-conserving part implies that[4]

$$|\Delta_{LR}|=|2.3\,\mathrm{Re}\,\xi| \ . \tag{27}$$

For $m_t=140$ GeV and $\rho=-0.51$ this gives $|\Delta_{LR}|=3.7\times10^{-3}$ while for $m_t=200$ GeV and $\rho=-0.12$ this gives $|\Delta_{LR}|=4.7\times10^{-3}$.

The magnitude of the asymmetry Δ_{LR} is larger for $\cos\theta$ positive than for $\cos\theta$ negative as Eq. (12) of Sec. II indicates. Hence, the asymmetry can be increased by a cut on $\cos\theta$. If $\cos\theta$ is restricted to lie in the region

$$-0.5<\cos\theta<1.0 \ , \tag{28}$$

the asymmetry arising from the interference of the short-distance parity-violating amplitude with the parity-conserving part is

$$|\Delta_{LR}|=|4.1\,\mathrm{Re}\,\xi| \ . \tag{29}$$

For $m_t=140$ GeV, and $\rho=-0.51$ this gives $|\Delta_{LR}|=6.5\times10^{-3}$ while for $m_t=200$ GeV and $\rho=-0.12$, Eq. (29), implies that $|\Delta_{LR}|=8.3\times10^{-3}$. This cut increases the magnitude of the asymmetry by almost a factor of 2 and reduces the number events by only a factor of 0.77. In Fig. 4 we show the constraint on ρ extracted from a Δ_{LR} measurement [with the cut in Eq. (28)] for some values of the top-quark mass and asymmetry. The values of the asymmetry and top-quark mass are chosen to be compatible with the measured value for $B^0-\bar{B}^0$ mixing when $\sqrt{B_B}f_B$ lies between 120 and 250 MeV. ξ is dominated by the top-quark loop for the values of the asymmetry shown in Fig. 4.

V. TWO-PHOTON CONTRIBUTION TO THE PARITY-VIOLATING AMPLITUDE

In this section we use chiral perturbation theory to examine the two-photon contribution to the parity-violating form factors B and C. There are local operators that can contribute to the parity-violating $K^+\to\pi^+\mu^+\mu^-$ amplitude. At the leading order of chiral perturbation theory there are included in the effective Lagrange density

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[3]Calculations using two-dimensional (2D) QCD in the large-N_c limit [17] suggest there are large $\Lambda_{\rm QCD}/m_c$ corrections to the prediction of heavy-quark symmetry for the relationship between f_D and f_B.

[4]This differs slightly from the result of Ref. [3] because in this paper the measured s dependence of $f(s)$ has been used.

$$\mathcal{L}=\frac{iG_F\alpha s_1}{\sqrt{2}}\bar\mu\gamma_\mu\gamma_5\mu[\gamma_1\,\mathrm{Tr}(O_W Q^2\Sigma\partial^\mu\Sigma^\dagger)+\gamma_2\,\mathrm{Tr}(O_W\partial^\mu\Sigma Q^2\Sigma^\dagger-O_W\Sigma Q^2\partial^\mu\Sigma^\dagger)+\gamma_3\,\mathrm{Tr}(O_W\partial^\mu\Sigma Q\Sigma^\dagger Q-O_W\Sigma Q\partial^\mu\Sigma^\dagger Q)]\ . \tag{30}$$

In Eq. (30) Q is the electromagnetic charge matrix:

$$Q=\begin{bmatrix}\frac{2}{3} & 0 & 0\\ 0 & -\frac{1}{3} & 0\\ 0 & 0 & -\frac{1}{3}\end{bmatrix}\ . \tag{31}$$

Each term contains two factors of Q because the Lagrange density in Eq. (30) arises from Feynman diagrams with two photons. When the photons (and other virtual particles) are off shell by an amount that is large compared with the pseudo-Goldstone-boson masses their effects are reproduced by those of the local operators in Eq. (30). *CPS* symmetry [18] has been used to reduce the effective Lagrangian to the form in Eq. (30). Under a *CPS* transformation

$$\Sigma(\mathbf{x},t)\to S\Sigma^*(-\mathbf{x},t)S\ , \tag{32}$$

where S is the matrix that switches strange and down quarks:

$$S=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}\ . \tag{33}$$

It is *CPS* symmetry that forces the two terms in the last two traces of Eq. (30) to occur with a relative minus sign (the linear combination with a relative plus sign is not invariant under *CPS*). Expanding out the Σ matrices in terms of the pseudo-Goldstone-boson fields it is easy to see that the effective Lagrange density in Eq. (30) gives a contribution to B proportional to $\gamma_1-8\gamma_2-4\gamma_3$, but gives *no* contribution to C. We shall not be able to predict B using chiral perturbation theory as γ_1, γ_2, and γ_3 are not known.

CPS symmetry forces the contribution to C from local operators (without factors of m_q) to vanish. This symmetry is broken by the difference between strange- and down-quark masses. In the pole-type graphs of Fig. 5 the quark masses cannot be neglected and it is these diagrams that (in chiral perturbation theory) give the dominant contribution to C. In Fig. 5 the shaded square is an interaction vertex from the weak $\Delta s=1$ Lagrangian in Eq. (18), the shaded circle is a $\eta\gamma\gamma$ or $\pi^0\gamma\gamma$ vertex from the Wess-Zumino term [19]

$$\mathcal{L}_{\mathrm{WZ}}=\frac{\alpha}{4\pi f}\epsilon_{\mu\nu\lambda\sigma}F^{\mu\nu}F^{\lambda\sigma}(\pi^0/\sqrt{2}+\eta/\sqrt{6})+\cdots\ . \tag{34}$$

The cross denotes a $\eta\mu^+\mu^-$ or $\pi^0\mu^+\mu^-$ vertex that arises from the local terms in the effective Lagrange density for strong and electromagnetic interactions,

$$\mathcal{L}=\frac{i\alpha^2}{4\pi^2}\bar\mu\gamma^\mu\gamma_5\mu[\chi_1\,\mathrm{Tr}(Q^2\Sigma^\dagger\partial_\mu\Sigma-Q^2\partial_\mu\Sigma^\dagger\Sigma)+\chi_2\,\mathrm{Tr}(Q\Sigma^\dagger Q\partial_\mu\Sigma-Q\partial_\mu\Sigma^\dagger Q\Sigma)]\ , \tag{35}$$

that couple a π^0 or η to a $\mu^+\mu^-$ pair.

In the Feynman diagrams of Fig. 5 the "infinite part" of the loop integrals is canceled by the terms from Eq. (35) yielding the following prediction for C:

$$C=\frac{g_8}{12}\left[\frac{3m_\eta^2-m_\pi^2-2m_{K^+}^2}{s-m_\eta^2}\right]\mathcal{A}(s)\ , \tag{36}$$

where

$$\mathrm{Re}\mathcal{A}(s)=\frac{\alpha}{4\pi^2}\Bigg[w+\tfrac{1}{2}(s/m_\mu^2)-\tfrac{1}{4}(s/m_\mu^2)^2+(s/m_\mu^2)\ln(s/m_\mu^2)+\tfrac{1}{2}(s/m_\mu^2)^2\ln(s/m_\mu^2)$$
$$-\int_0^1 dx\left[3+\frac{2[(s/4m_\mu^2)-1]\sqrt{x}}{\sqrt{x+(4m_\mu^2/s)(1-x)}}\right]\lambda_+^2\ln|\lambda_+/2|$$
$$-\int_0^1 dx\left[3-\frac{2[(s/4m_\mu^2)-1]\sqrt{x}}{\sqrt{x+(4m_\mu^2/s)(1-x)}}\right]\lambda_-^2\ln|\lambda_-/2|\Bigg]\ , \tag{37a}$$

and[5]

$$\mathrm{Im}\mathcal{A}(s)=\frac{\alpha}{\pi}\frac{1}{\sqrt{1-(4m_\mu^2/s)}}\ln\left[\frac{1+\sqrt{1-(4m_\mu^2/s)}}{2m_\mu/\sqrt{s}}\right]\ . \tag{37b}$$

The Feynman diagrams in Fig. 5 give no contribution to B. In Eq. (37a) w is a constant independent of s and

$$\lambda_\pm=\sqrt{x(s/m_\mu^2)}\pm\sqrt{x(s/m_\mu^2)+4(1-x)}\ . \tag{38}$$

The constant w gets contributions both from the one-

[5]The imaginary part is related to the unitarity limit for $\eta\to\mu^+\mu^-$. This was computed in Ref. [20]. The real part of the $\eta\to\mu^+\mu^-$ amplitude was also computed in Ref. [20] using a phenomenological model for the form factor associated with the $\eta\to\gamma\gamma$ vertex.

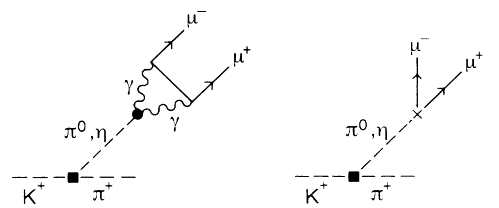

FIG. 5. Feynman diagrams that give the dominant two-photon contribution to C in chiral perturbation theory.

loop diagrams and from the tree diagrams in Fig. 5. It can be determined from the relative strength of the decays $\eta\to\gamma\gamma$ and $\eta\to\mu^+\mu^-$. At the leading order of chiral perturbation theory,

$$\Gamma(\eta\to\gamma\gamma)=\alpha^2 m_\eta^3/96\pi^3 f^2 , \tag{39}$$

and

$$\Gamma(\eta\to\mu^+\mu^-)=\frac{|\alpha\mathcal{A}(m_\eta^2)|^2}{48\pi}\left[\frac{m_\mu}{f}\right]^2\sqrt{m_\eta^2-4m_\mu^2} . \tag{40}$$

The recent measurement [21] of the branching ratio for $\eta\to\mu^+\mu^-$, $B(\eta\to\mu^+\mu^-)=(5\pm1)\times10^{-6}$, is within 1σ of the unitarity limit which is 4.3×10^{-6} (arising from an on-shell two-photon intermediate state.) The measured branching ratio for $\eta\to\mu^+\mu^-$ implies that $|\mathrm{Re}\,A(m_\eta^2)|<2.5\times10^{-3}$ which gives $-2<w<25$. Using the cut on $\cos\theta$, given in Eq. (28), we find that the two-photon contribution of the parity-violating form factor C to the asymmetry satisfies $|\Delta_{LR}|<1.2\times10^{-3}$. Improving the measurement of the branching ratio for $\eta\to\mu^+\mu^-$ would reduce the uncertainty in w and consequently improve our knowledge of the two-photon contribution to C.

If the short-distance contribution to the asymmetry Δ_{LR} [with the cut on $\cos\theta$ given in Eq. (28)] is at the $\frac{1}{2}\%$ level then it is likely that the two photon contribution to C can be neglected. (Of course, if the full range of $\cos\theta$ is used then the contribution of C to the asymmetry vanishes.) We have not been able to predict using chiral perturbation theory, the two-photon contribution to the parity-violating form factor B. However, we do not expect its influence on Δ_{LR} [with the cut on $\cos\theta$ given in Eq. (28)] to be larger than that of C. [Our naive expectation is that it gives $|\Delta_{LR}|=O(\alpha/\pi)\sim2\times10^{-3}$.] It would be interesting to try to estimate the two photon contribution to B using phenomenological models. Experimental information on the decay $K^+\to\pi^+\gamma\gamma$ may also prove useful.

There are T-odd asymmetries that involve both the μ^+ and μ^- polarizations. They will be much more difficult to measure than the parity-violating asymmetry we have been discussing. The T-odd asymmetries also violate parity and are determined by $\mathrm{Im}\,Bf^*(s)$ and $\mathrm{Im}\,Cf^*(s)$. They get a contribution from the interference of the two-photon contribution to the imaginary part of C, given in Eqs. (36) and (37), with the real part of the parity-conserving amplitude (as well as from short-distance physics).

VI. CONCLUDING REMARKS

We have calculated the two-photon contribution to the parity-violating $K^+\to\pi^+\mu^+\mu^-$ decay amplitude arising from the diagrams in Fig. 5. They give rise to an invariant matrix element with Lorentz structure $(p_K-p_\pi)^\mu\bar{u}\gamma_\mu\gamma_5 v$ and do not contribute to the other possible form for the parity-violating amplitude $(p_K+p_\pi)^\mu\bar{u}\gamma_\mu\gamma_5 v$. CPS symmetry of the chiral Lagrangian forces the contact terms (that arise from Feynman diagrams where the virtual particles have large momentum) to have the structure $(p_K+p_\pi)^\mu\bar{u}\gamma_\mu\gamma_5 v$. Therefore, the diagrams in Fig. 5 give the leading value for the coefficient of $(p_K-p_\pi)^\mu\bar{u}\gamma_\mu\gamma_5 v$ in chiral perturbation theory. The prediction of chiral perturbation theory contains an s-independent constant that is fixed by the measured $\eta\to\mu^+\mu^-$ decay rate. Improving the experimental value for the $\eta\to\mu^+\mu^-$ branching ratio would reduce the uncertainty in this constant and hence improve our prediction for the coefficient of $(p_K-p_\pi)^\mu\bar{u}\gamma_\mu\gamma_5 v$. Unfortunately, we cannot compute the coefficient of $(p_K+p_\pi)^\mu\bar{u}\gamma_\mu\gamma_5 v$ using chiral perturbation theory since there are several local contact terms that contribute to it which we cannot fix experimentally. These contact terms also contribute to the $K_L\to\mu^+\mu^-$ decay amplitude, but for this amplitude they enter in a different linear combination than for the $K^+\to\pi^+\mu^+\mu^-$ matrix element and furthermore the measured $K_L\to\mu^+\mu^-$ branching ratio is not accurate enough to provide a significant constraint.

If all the available phase space is integrated over then the $(p_K-p_\pi)^\mu\bar{u}\gamma_\mu\gamma_5 v$ piece of the parity-violating decay amplitude does not contribute to the parity-violating asymmetry

$$\Delta_{LR}\equiv(\Gamma_R-\Gamma_L)/(\Gamma_R+\Gamma_L) .$$

However, it is advantageous to make the cut, $-0.5<\cos\theta<1$, since it increases the short-distance contribution to the asymmetry by almost a factor of 2 and diminishes the number of events by only a factor of 0.77. With this cut the measured $\eta\to\mu^+\mu^-$ branching ratio implies that the two-photon contribution to Δ_{LR} from the diagrams in Fig. 5 satisfies $|\Delta_{LR}|<1.2\times10^{-3}$. This asymmetry is much less than the asymmetry arising from short-distance physics involving virtual top and charm quarks, provided that ρ is negative. For ρ positive, the Feynman diagrams in Fig. 5 may contribute a non-negligible portion of the asymmetry. It seems likely to us that the asymmetry coming from the two-photon contribution to the part of the $K^+\to\pi^+\mu^+\mu^-$ decay amplitude of the form $(p_K+p_\pi)^\mu\bar{u}\gamma_\mu\gamma_5 v$ is not much larger than that arising from the diagrams in Fig. 5. Our naive expectation is that it gives rise to an asymmetry $|\Delta_{LR}|=O(\alpha/\pi)\sim2\times10^{-3}$. It would be interesting to estimate this part of the parity-violating $K^+\to\pi^+\mu^+\mu^-$ decay amplitude using phenomenological models. (Such calculations may reveal a further physical suppression of this amplitude.) Experimental information on the decay $K^+\to\pi^+\gamma\gamma$ could also be valuable.

The asymmetry Δ_{LR} can provide information on the unitarity triangle. Even an experimental limit at the percent level would provide interesting information on ρ.

This may be within the reach of a dedicated experiment at existing facilities [22].

Short-distances physics contributes to T-odd (and P-odd) correlations involving both the μ^+ and μ^- polarizations. We have found that the imaginary part of the Feynman diagrams in Fig. 5 (that arises from on-shell photons) also contributes. A crude measure of the importance of this long-distance physics contribution is the ratio, $r(s)=\mathrm{Im}\,C(s)/|f(s)|$. Using Eqs. (13), (36), and (37b) we find, for example, that $r(4m_\pi^2)\simeq -5\times 10^{-3}$. The effect of the long-distance contribution to $\mathrm{Im}\,C(s)$ should be included in analysis of the implications of measuring these T-odd correlations.

ACKNOWLEDGMENTS

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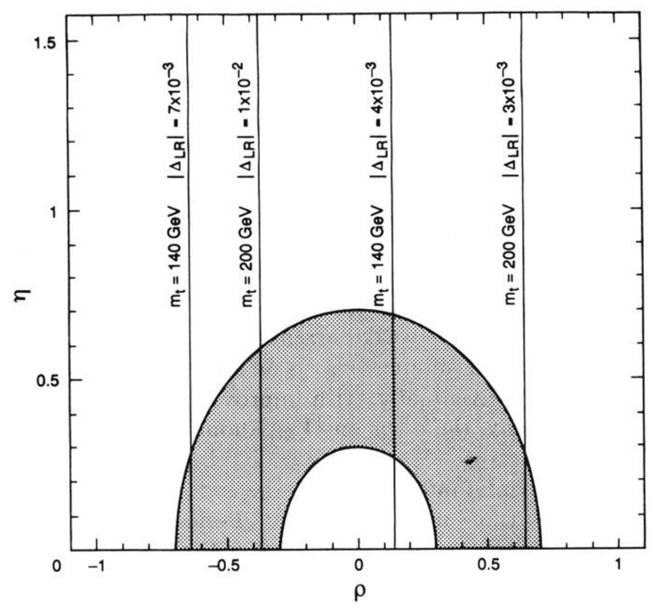

FIG. 4. Implications of measurement of the asymmetry Δ_{LR} for the location of the α vertex of the unitarity triangle.