New formulation of pion-nucleon scattering and soft-pion theorems in the Skyrme model

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We present a new formulation of pion-nucleon scattering and the soft-pion theorems in the Skyrme model, where the reduction formula of the S matrix and the currents are described in terms of the original canonical fields, which are not yet transformed into the Skyrmion and fluctuation fields in the one-Skyrmion sector. Not referring to any constraints and gauge-fixing conditions imposed on fluctuation fields and collective coordinates, we can easily obtain the Born terms with both of the rotational and translational modes and the soft-pion theorems.

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I. INTRODUCTION

Since the Skyrme model was recognized as a realistic soliton model of the low-energy pion-nucleon system based on QCD [1], it has been expected that low-energy pion-nucleon scattering, as well as static properties of baryons, is well described in the Skyrme model. The static properties of a nucleon have been shown to be reproduced within about 30% error in the model [2]. It has also been shown that higher partial waves of pion-nucleon scattering are well reproduced by background scattering amplitudes in the model [3,4].

However, the background scattering amplitudes failed to describe P- and S-wave scattering [3,4]. The nucleon and the Δ isobar constructed as rotational levels of the Skyrmion cannot explicitly contribute to the P-wave Born amplitudes, because the Yukawa coupling to the Skyrmion vanishes owing to the stability condition of the classical Skyrmion configuration [5–7]. When the Skyrme Lagrangian is expanded in powers of $1/N_c$, however, the nonleading Yukawa coupling survives because of the mismatching of the rotating Skyrmion with the equation of motion. The surviving Yukawa coupling in the Lagrangian is of $O(N_c^{-3/2})$ in the standard gauge-fixing condition that the fluctuation fields are orthogonal to zero-mode wave functions [8]. Such an N_c scaling behavior of the surviving Yukawa coupling is a serious shortcoming and is called the Yukawa coupling problem.

Recently, the present authors have solved the Yukawa coupling problem in the $(1+1)$-dimensional soliton model [9–11] and in the Skyrme model [12,13] within the standard collective-coordinate quantization method. The Born terms are obtained through tedious calculation up to $O(N_c^{-3})$ within the tree approximation of the constrained field theory of fluctuation fields and collective coordinates. The Yukawa couplings defined as the residues of the Born terms are of $O(N_c^{1/2})$, and expressed in terms of the classical Skyrmion fields.

We have proved in a previous paper [11] that direct contributions from the soliton fields to the reduction formula of the S matrix vanish in the tree approximation, because there occurs a complete cancellation among them. As indicated in Ref. [11], this means that we can write the reduction formula in terms of the original canonical fields which are not yet decomposed into the soliton and fluctuation field.

In this paper we present a new formulation of pion-nucleon scattering and the soft-pion theorems in the Skyrme model, where these are successfully described in terms of the original canonical fields which appear in the original Skyrme Lagrangian. In this formulation we can easily obtain the correct Born terms having both of the rotational and translational modes without reference to any constraints and gauge-fixing conditions imposed on the fluctuation fields and the collective coordinates. This formulation is based on the consideration that the original field can be regarded as an interpolating field for the pions, which asymptotically plays the same role as the fluctuation field in the one-Skyrmion sector, as shown in the $(1+1)$-dimensional model by Steinmann [14]. A matrix element of a product of the original fields sandwiched between two single-baryon states is, then, reduced to that of the same product of the soliton fields at the tree level. Our formulation is also crucial to the study of the soft pion theorems in the Skyrme model.

Since the Skyrme model is described in terms of the spontaneously broken chiral-invariant Lagrangian, the soft pion theorems are naturally expected to hold within the model. Adkins *et al.* [2] showed that the Goldberger-Treiman (GT) relation [15] holds for the classical Skyrmion fields, while the axial-vector currents made of the classical Skyrmion fields do not satisfy the current algebra [16]. Furthermore, it is not clear whether or not the GT relation should be modified if we take into account the fluctuation fields. The Adler-Weisberger (AW) relation [17] has been expected to give the fact that $g_A > 1$ owing to the Δ contribution, while $g_A < 1$ in the Skyrme model [2] when the masses of the nucleon and Δ are fitted to the empirical values. The Tomozawa-Weinberg (TW) relation in the Skyrme model was given merely as a plane-wave approximation to the fluctuation fields ignoring any constraints [18]. Thus, the study of the soft pion theorems in the Skyrme model seems insufficient.

Noting that the vector and axial-vector currents written in terms of the original fields satisfy the current algebra, and that the equation to describe the partially conserving or the conserving axial-vector current is equivalent to the Lagrange equation to the original fields, we show that the GT relation holds as the relation between the classical Skyrmion fields even in the situation where the fluctuating pions are included. The AW relation also holds in the Skyrme model, but it is not a sum rule to the axial-vector coupling constant g_A, but a sum rule to the forward scattering amplitude from which the nucleon and Δ poles are subtracted. This shows that the Skyrme model with $g_A < 1$ is compatible with the AW relation even if there is the Δ contribution. It is also shown that the TW relation is obtained from the AW relation and the structure of the forward scattering amplitude.

In the next section, using the (1+1)-dimensional soliton model, we demonstrate that the elastic scattering amplitude is easily obtained by the reduction formula written in terms of the original field. The Skyrme model with both of the rotational and translational modes is discussed in Sec. III, and the dispersion relation of the forward scattering amplitude is given in Sec. IV. Axial-vector and vector currents and their hadronic matrix elements are given in Sec. V and Sec. VI is devoted to the formulation of the soft pion theorems in the Skyrme model. The conclusions and discussion are given in the last section. Some technical points are given in Appendixes.

II. MESON-SOLITON SCATTERING IN THE (1+1)-DIMENSIONAL MODEL

The starting Lagrangian is given as

$$L=\int dx\{\tfrac{1}{2}\dot{\Phi}(x,t)^2-\tfrac{1}{2}\Phi'(x,t)^2 -\tfrac{1}{2}m^2\Phi(x,t)^2-U[\Phi]\}, \tag{2.1}$$

where $\Phi'=\partial\Phi/\partial x$ and $\dot{\Phi}=\partial\Phi/\partial t$. The original canonical field $\Phi(x,t)$ was decomposed into the soliton and fluctuation fields in the constrained system of the fluctuation fields and the collective coordinates in previous papers:

$$\Phi(x,t)=\phi_s(x-R(t))+\chi(x,t), \tag{2.2}$$

where $R(t)$ is the center of the soliton and $\chi(x,t)$ is the fluctuation field in the laboratory frame. We note here that the original field $\Phi(x,t)$ behaves asymptotically the same as $\chi(x,t)$, and it is an interpolating field for the mesons in the one-soliton sector [14]. The one-soliton subspace of the Hilbert space on which the original field acts, is then taken to be the same Fock space spanned by the in and out states composed of the mesons of the fluctuation field and the one soliton. When an operator written in terms of the original fields is sandwiched between two single-soliton states, the original fields are reduced to the soliton fields, $\phi_s(x-R)$, in the tree approximation:

$$\langle p'|\Phi(x,t)|p\rangle=\langle p'|\phi_s(x-R(t))|p\rangle =\int\frac{dy}{2\pi}e^{i(p-p')y-i(E_p-E_{p'})t}\phi_s(x-y), \tag{2.3}$$

where an eigenvector of $R(T)$, $|y,t\rangle$, and $\langle y,t|p\rangle=(1/\sqrt{2\pi})\exp(ipy-iE_pt)$ are used, and $E_p=M_s+p^2/2M_s$ with M_s being the classical soliton mass. This is the same as the ansatz used in 1970s [19–21].

The momentum field $\Pi(x,t)=\partial L/\partial\dot{\Phi}(x,t)$ conjugate to Φ was also decomposed into the pure soliton part, the fluctuation momentum $\pi(x,t)$ conjugate to $\chi(x,t)$, and the mixed part of the collective coordinates and π's:

$$\Pi(x,t)=-\frac{1}{2M_s}\{\phi_s'(x-R(t)),P(t)\} +\text{mixed terms}+\pi(x,t), \tag{2.4}$$

where $P(t)$ is the soliton momentum operator conjugate to $R(t)$. Note here that the pure soliton part, the first term of Eq. (2.4), is gauge invariant, but the remaining terms are gauge dependent in the same way as the relation between ϕ_s and χ. In this paper we do not use such decompositions at all, but treat the original fields as a whole.

The original fields $\Phi(x,t)$ and $\Pi(y,t)$ satisfy the free canonical commutation relation in the tree approximation without any renormalization effects:

$$[\Phi(x,t),\Pi(y,t)]=i\delta(x-y). \tag{2.5}$$

The canonical commutation relation is also derived from the commutation relations $[\chi,\pi]$, $[\chi,P]$, and $[R,P]$ in the constrained system and Eqs. (2.2) and (2.4). The Hamiltonian written in terms of the original fields is given as

$$H=\int dx\left\{\frac{1}{2}\left[\Pi^2(x,t)+\frac{1}{2}\left[\frac{\partial\Phi(x,t)}{\partial x}\right]^2 +\frac{1}{2}m^2\Phi^2(x,t)\right]+U[\Phi]\right\}, \tag{2.6}$$

from which the Hamiltonian $H[\chi,\pi;R,P]$ in the constrained system is derived by using the fluctuation fields and the collective coordinates [11,21].

Now, we calculate the source term of $\Phi(x,t)$ to obtain the S matrix. It is given as

$$\mathcal{J}(x,t)=i^2[H,[H,\Phi(x,t)]]+(-\partial^2/\partial x^2+m^2)\Phi(x,t) =-U'[\Phi(x,t)] \tag{2.7}$$

through the canonical commutation relation (2.5), where $U'[\Phi]=\partial U/\partial\Phi$. We can write the S matrix for elastic scattering as

$$S_{fi}=\delta_{fi}+i^2\int dx\,dt\int dx'dt' f^*_{k'}(x',t')f_k(x,t)\{\langle p'|T[\mathcal{J}(x',t')\mathcal{J}(x,t)]+\delta(t-t')[\dot{\Phi}(x',t'),\mathcal{J}(x,t)]|p\rangle\}\ . \tag{2.8}$$

If U' is expanded in powers of χ, $\mathcal{J}=-U'[\Phi(x,t)]$ does not contain momentum fields π or P, in contrast with the source of χ, which is composed of complicated terms with the collective coordinates and momentum fields except for the $O(M_s^0)$ term, $-U''[\phi_s]\chi(x,t)$. (M_s plays the same role as N_c as an expansion parameter.)

The equal-time commutator is given as

$$[\dot{\Phi}(x',t),\mathcal{J}(x,t)]=i\delta(x'-x)U''[\Phi(x,t)]\ , \tag{2.9}$$

since $\dot{\Phi}(x',t)=\Pi(x',t)$. This potential is sandwiched between two single-soliton states $\langle p'|$ and $|p\rangle$. The contribution from the equal-time commutator is the same as the potential term of background scattering of $O(M_s^0)$ after the integration over x and x', since

$$\langle p'|U''[\Phi]|p\rangle=\int\frac{dy}{2\pi}e^{i(p-p')y-i(E_p-E_{p'})t}U''[\phi_s(x-y)]\ . \tag{2.10}$$

Next, we calculate the contribution from the single-soliton intermediate states in the time-ordered product term: We can write the matrix element of $\mathcal{J}$ as

$$\begin{aligned}\langle p''|\mathcal{J}(x,t)|p\rangle&=\langle p''|-U'[\phi_s(x-R)]|p\rangle\\&=\int\frac{dy}{2\pi}\exp\{-i(p''-p)y+i(E_{p''}-E_p)t\}\{-U'[\phi_s(x-y)]\}\\&=\int\frac{dy}{2\pi}\exp\{-i(p''-p)y+i(E_{p''}-E_p)t\}J_s(x-y)\ ,\end{aligned} \tag{2.11}$$

where we used the equation to $\phi_s(x)$, $-U'[\phi_s]=(-d^2/dx^2+m^2)\phi_s=J_s(x)$, the right-hand side of which is just the definition of the classical source term. Here note that $\ddot{\Phi}$ and $\dot{\Phi}$ are not equal to $\ddot{\phi}_s$ and $\dot{\phi}_s$, respectively, because the time derivatives of Φ, determined by the commutators between Φ and $H[\Phi]$, are not equal to the commutators between ϕ_s and $H[\chi,\pi;R,P]$. Thus, the whole contribution from the single-soliton intermediate states is written as

$$\frac{\tilde{J}_s^*(k')\tilde{J}_s(k)}{E_{p+k}-E_p-\omega_k}+\frac{\tilde{J}_s(k)\tilde{J}_s^*(k')}{E_{p-k'}-E_p+\omega_{k'}}\ , \tag{2.12}$$

with

$$\tilde{J}_s(k)=\int dx\,e^{ikx}J_s(x)\ , \tag{2.13}$$

which is just the same Born terms as obtained by tedious calculation in Ref. [11]. Contributions from other intermediate states form the unitarity cut of the amplitude so as to satisfy the unitarity.

Thus, starting with the original field and its source term, we have reached the same S matrix for elastic scattering as given in terms of the fluctuation fields alone in the one-soliton sector. We have not explicitly used any gauge fixing conditions on χ and π to get the S matrix, but we have used only the gauge-invariant soliton solution and its source function which appear as the matrix elements of Φ and $\mathcal{J}$ sandwiched between the two single-soliton states. This means that the scattering amplitude obtained on the basis of the standard gauge-fixing conditions is gauge invariant. We also note that if we start with the reduction formula written in terms of the original canonical fields, we must not decompose the original fields into the soliton and the fluctuation fields before we reach matrix elements sandwiched between two single-soliton states: If we do so, we encounter the cancellation of the soliton part, and the remaining terms are reduced to the results calculated in terms of the fluctuation fields and the collective coordinates, as in Ref. [11].

III. PION-NUCLEON SCATTERING IN THE SKYRME MODEL

In this paper we define the original canonical pion field $\Phi_a(x)$ through the SU(2) field $U(x)$ as

$$U(x)=\frac{1}{f_\pi}[\Phi_0(x)+i\tau_a\Phi_a(x)]\ , \tag{3.1}$$

where $x=(x^0,\mathbf{x})$, and $\Phi_0(x)$ is related to $\Phi_a(x)$ as

$$\Phi_0^2(x)=f_\pi^2-\sum\Phi_a^2(x)\ . \tag{3.2}$$

The Skyrme Lagrangian is written in terms of the original pion fields as

$$L=\frac{1}{2}\int d^3x\,\dot{\Phi}_aK_{ab}\dot{\Phi}_b-\mathcal{V}[\Phi,\nabla\Phi]\ , \tag{3.3}$$

with

$$\mathcal{V}[\Phi,\nabla\Phi]=\frac{1}{2}\int d^3x\{\partial_i\Phi_aG_{ab}\partial_i\Phi_b-m_\pi^2f_\pi^2\mathcal{M}[\Phi]\}\ , \tag{3.4}$$

where K_{ab} and G_{ab} are the metric kernels:

$$K_{ab}[\Phi]=X_{ab}+\frac{1}{\kappa^2f_\pi^2}[X_{ab}\partial_j\Phi_cX_{cd}\partial_j\Phi_d-X_{ac}\partial_j\Phi_cX_{bd}\partial_j\Phi_d]\ , \tag{3.5}$$

$$G_{ab}[\Phi]=X_{ab}+\frac{1}{2\kappa^2f_\pi^2}[X_{ab}\partial_j\Phi_cX_{cd}\partial_j\Phi_d-X_{ac}\partial_j\Phi_cX_{bd}\partial_j\Phi_d] \tag{3.6}$$

with $\kappa=ef_\pi$ and

$$X_{ab}=\delta_{ab}+\frac{\Phi_a\Phi_b}{\Phi_0^2}\ , \tag{3.7}$$

and the pion mass term is related to Φ_0 as

$$\mathcal{M}[\Phi]=2\left[1-\frac{\Phi_0}{f_\pi}\right]\ . \tag{3.8}$$

The metric kernels K_{ab} and G_{ab} are reduced to δ_{ab} for the weak field limit; that is, a "curved space" becomes flat asymptotically.

In previous papers, Φ_a was decomposed as

$$\Phi_a(x)=\phi_S^a(\mathbf{x}-\mathbf{X}(x^0))+\chi_a(x)\ , \tag{3.9}$$

where $\mathbf{X}$ is the center of the Skyrmion, ϕ_S^a the Skyrmion configuration of $O(N_c^{1/2})$ centered at $\mathbf{X}$, χ_a the laboratory fluctuation field of $O(N_c^0)$ around the Skyrmion. The Skyrmion solution is given as the static solution

$$\begin{aligned}\mathcal{E}^a[\phi_S,\nabla\phi_S]&\equiv\frac{\delta}{\delta\phi_S^a}\mathcal{V}[\phi_S,\nabla\phi_S]=-\partial_i\left[\frac{\delta\mathcal{V}}{\delta(\partial_i\phi_S^a)}\right]+\frac{\delta\mathcal{V}}{\delta\phi_S^a}\\&=(-\nabla^2+m_\pi^2)\phi_S^a(\mathbf{x})+U^a[\phi_S,\nabla\phi_S]=0\ ,\end{aligned} \tag{3.10}$$

where the form of the potential term U^a is not explicitly given here. The solution in the usual ansatz is written as

$$\phi_A^a(\mathbf{x})=f_\pi\sin F(r)R_{ai}\hat{x}_i \quad\text{and}\quad \phi_S^0(\mathbf{x})=f_\pi\cos F(r)\ , \tag{3.11}$$

where $F(r)$ is the profile function and $\hat{x}_i=x_i/r$, and R_{ai} is the orthogonal rotation matrix.

From the Skyrme Lagrangian, the momentum fields conjugate to Φ_a are given as

$$\Pi_a(x)=\frac{\partial L}{\partial\dot{\Phi}_a(x)}=K_{ab}\dot{\Phi}_b(x)\ . \tag{3.12}$$

The momentum field Π_a can also be transformed into the components transverse and parallel to the zero-mode wave functions in the constrained field theory, but the decomposition depends on gauge fixing conditions:

$$\begin{aligned}\Pi_a(x)&=\Pi_S^a(\mathbf{x}-\mathbf{X};\mathbf{I},\mathbf{P})+\pi_a(x)\ ,\\&\quad+\text{mixed terms of }\chi\text{ and collective coordinates}\ ,\end{aligned} \tag{3.13}$$

where the mixed terms and the transverse component π_a are gauge dependent, and $I_k(P_k)$ is the intrinsic isospin (linear momentum) operator of the Skyrmion, which is conjugate to the rotational angle around the kth axis (the kth component of the center of the skyrmion X_k). The pure Skyrmion part Π_S^a, which is of $O(N_c^{-1/2})$, is written as

$$\Pi_S^a(\mathbf{x}-\mathbf{X};\mathbf{I},\mathbf{P})=\frac{1}{2\Lambda_S}\sum_k\{K_{S_{ab}}\psi_{rk}^b,I_k\}+\frac{1}{2M_s}\sum_k\{K_{S_{ab}}\psi_{tk}^b,P_k\} \tag{3.14}$$

with the zero-mode wave function ψ_{rk}^b for the rotational mode and ψ_{tk}^b for the translational mode, which are given as

$$\psi_{rk}^a=\varepsilon_{kab}\phi_S^b(\mathbf{x}-\mathbf{X})\ , \tag{3.15}$$

$$\psi_{tk}^a=-\frac{\partial}{\partial x_k}\phi_S^a(\mathbf{x}-\mathbf{X})\ . \tag{3.16}$$

In the above $K_{S_{ab}}=K_{ab}[\phi_S,\nabla\phi_s]$, Λ_S and M_S are the moment of inertia and the mass of the Skyrmion, respectively. The commutation relations between the collective coordinates in the constrained system are:

$$\begin{aligned}&[X_i,P_j]=i\delta_{ij}\ ,\\&[P_i,P_j]=\text{terms of }O(\pi\chi)\ ,\\&[R_{ai},I_j]=i\varepsilon_{ajb}R_{bi}\ ,\\&[I_i,I_j]=-i\varepsilon_{ijk}I_k+\text{terms of }O(\pi\chi)\ ,\end{aligned} \tag{3.17}$$

where the $O(\pi\chi)$ terms come from the constraints imposed on the fluctuation fields and the collective coordinates. (See Ref. [12] for the expressions of them in the rotational mode.) If these commutators are sandwiched between two single-baryon states, the $O(\pi\chi)$ terms vanish at the tree level. In this sense we may call $\mathbf{P}$ and $\mathbf{I}$ the momentum and isospin operators of the baryon, respectively.

The commutation relation between Φ_a and Π_b is the unrenormalized canonical one at the tree level:

$$[\Phi_a(x),\Pi_b(y)]\delta(x^0-y^0)=i\delta_{ab}\delta^{(4)}(x-y)\ . \tag{3.18}$$

This commutation relation is also derived from the commutation relations among χ_a, π_b and the collective coordinates in the constrained system. The Hamiltonian is simply written as

$$H=\frac{1}{2}\int d^3x\{\Pi_aK_{ab}^{-1}\Pi_b\}+\mathcal{V}[\Phi,\nabla\Phi]\ , \tag{3.19}$$

which reduces to the Hamiltonian of the constrained system by using (3.9) and (3.13). In the above, we ignored the ordering problem of Π_a and K_{ab}^{-1} in the kinetic term, which produces a quantum correction of $O(\hbar^2)$.

The original field $\Phi_a(x)$ is regarded as the interpolating field for the pions as in the $(1+1)$-dimensional model, because asymptotically, $\phi_S^a(\mathbf{x}-\mathbf{X}(t))$ fades out, and $\Phi_a(x)$ reduces to $\chi_a(x)$. The one-baryon subspace of the Hilbert space, on which the original field operators act, is taken as the same Fock space spanned by the in and out states for the fluctuation fields in the constrained system. The matrix element of a product of the original fields sandwiched between two single-baryon states is, then, reduced to that of the same product of the Skyrmion fields in the tree approximation as in the $(1+1)$-dimensional soliton model.

Thus we write the S matrix for pion-nucleon elastic scattering by using the standard reduction formula in terms of the source terms $\mathcal{J}$, $\dot{\Phi}$, and Φ:

$$S_{fi}=\delta_{fi}+i^2\int d^3x'dt'\int d^3xdtf^*_{\mathbf{k}'}(x')f_{\mathbf{k}}(x)\{\langle N'(\mathbf{p}')|T(\mathcal{J}^b(x')\mathcal{J}^a(x))+\delta(t'-t)[\dot{\Phi}_b(x'),\mathcal{J}^a(x)]$$
$$-i\omega_{\mathbf{k}'}\delta(t'-t)[\Phi_b(x'),\mathcal{J}^a(x)]|N(\mathbf{p})\rangle\}\ , \qquad (3.20)$$

where $f_{\mathbf{k}}(x)=[(2\pi)^3 2\omega_{\mathbf{k}}]^{-1/2}\exp(-ikx)$ with $kx=\omega_{\mathbf{k}}t-\mathbf{k}\cdot\mathbf{x}$ is the asymptotic wave function of the pion. The source term of Φ_a is defined as

$$\mathcal{J}^a(x)=i^2[H,[H,\Phi_a(x)]]+(-\nabla^2+m_\pi^2)\Phi_a(x)\ . \qquad (3.21)$$

Calculating the commutator, we have

$$\mathcal{J}^a(x)=-K_{ab}^{-1}\mathcal{E}^b[\Phi,\nabla\Phi]+(-\nabla^2+m_\pi^2)\Phi_a(x)+\Pi_c[K_{bc}^{-1}\mathcal{K}_{ad,b}+\mathcal{K}_{ca,b}K_{bd}^{-1}-K_{ab}^{-1}\mathcal{K}_{cd,b}]\Pi_d\ , \qquad (3.22)$$

where $\mathcal{K}_{ad,b}=\delta K_{ad}^{-1}/\delta\Phi_b$, $\mathcal{E}^b$ is the variation of $\mathcal{V}$ by Φ_b, but terms with higher powers of $\hbar$ coming from the ordering problem are ignored.

In order to get the Born terms of pion-nucleon elastic scattering, we need the matrix element of $\mathcal{J}$ sandwiched between two single-baryon states. We can ignore the second line $\Pi[\cdots]\Pi$ in Eq. (3.22), because its matrix element is of $O(N_c^{-3/2})$ at the tree level, which is higher than that of the first line by $O(N_c^{-2})$. The matrix element of $\mathcal{J}^a$ is then written as

$$\langle B(\mathbf{p}'')|\mathcal{J}^a(x)|N(\mathbf{p})\rangle=\langle B(\mathbf{p}'')|-K_{ab}^{-1}\mathcal{E}^b[\Phi,\nabla\Phi]+(-\nabla^2+m_\pi^2)\Phi_a|N(\mathbf{p})\rangle$$
$$=e^{i(\mathbf{p}-\mathbf{p}'')\cdot\mathbf{x}-i(E_p^N-E_{p''}^B)t}\int\frac{d^3y}{(2\pi)^3}\langle B|(-\nabla^2+m_\pi^2)\phi_S^a(\mathbf{x}-\mathbf{y})|N\rangle e^{i(\mathbf{p}''-\mathbf{p})\cdot(\mathbf{x}-\mathbf{y})}$$
$$=e^{i(\mathbf{p}-\mathbf{p}'')\cdot\mathbf{x}-i(E_p^N-E_{p''}^B)t}\langle B|\tilde{J}_S^a(\mathbf{p}''-\mathbf{p})|N\rangle\ , \qquad (3.23)$$

since $\mathcal{E}^a[\phi_S,\ \nabla\phi_S]=0$, and $|B\rangle$ denotes the nucleon or the Δ state, and we used the eigenstate $|\mathbf{y},t\rangle$ of $\mathbf{X}(t)$. The last factor is the Fourier transform of the classical source term $J_S^a(\mathbf{x})=(-\nabla^2+m_\pi^2)\phi_S^a(\mathbf{x})$. In the above we define $E_{\mathbf{p}}^B$ as

$$E_{\mathbf{p}}^B=M_S+\frac{I_B(I_B+1)}{2M_S}+\frac{\mathbf{p}^2}{2M_S}$$
$$=M_B+\frac{\mathbf{p}^2}{2M_B}+O(N_c^{-2})\ , \qquad (3.24)$$

which is an eigenvalue of the Skyrmion part of the Hamiltonian $H[\chi,\pi;\mathbf{X},\mathbf{I},\mathbf{P}]$ in the constrained system, where I_B is the magnitude of the isospin of the baryon B. The term of $O(N_c^{-2})$ is discarded.

Thus, we have the Born terms which contain both of the rotational and the translational modes:

$$\sum_B\left\{\frac{\langle B|\tilde{J}_S^b(\mathbf{k}')|N\rangle^\dagger\langle B|\tilde{J}_S^a(\mathbf{k})|N\rangle}{E_{\mathbf{p}+\mathbf{k}}^B-E_{\mathbf{p}}^N-\omega_{\mathbf{k}}}+\frac{\langle B|\tilde{J}_S^a(-\mathbf{k})|N\rangle^\dagger\langle B|\tilde{J}_S^b(-k')|N\rangle}{E_{\mathbf{p}'-\mathbf{k}}^B-E_{\mathbf{p}'}^N+\omega_{\mathbf{k}}}\right\}. \qquad (3.25)$$

Note that the Yukawa coupling $\langle B|\tilde{J}_S^a(\mathbf{k})|N\rangle$ defined as the residue of the Born term is of $O(N_c^{1/2})$, as seen from Eq. (3.23) with ϕ_S^a being of $O(N_c^{1/2})$.

The equal-time commutator between $\mathcal{J}$ and $\dot{\Phi}$ in the reduction formula is written as

$$[\dot{\Phi}_b(y),\mathcal{J}^a(x)]\delta(x^0-y^0)$$
$$=-iK_{bd}^{-1}\frac{\delta\mathcal{J}^a}{\delta\Phi_d}(x)\delta^4(x-y)$$
$$+[K_{bd}^{-1}(y),\mathcal{J}^a(x)]\Pi_d(x)\delta(x^0-y^0)\ . \qquad (3.26)$$

In contrast with the $(1+1)$-dimensional model, another equal-time commutator between Φ and $\mathcal{J}$ appears, because $\mathcal{J}$ involves the momentum field: It is given as

$$[\Phi_b(y),\mathcal{J}^a(x)]\delta(x^0-y^0)=i\frac{\delta\mathcal{J}^a(x)}{\delta\Pi_b(y)}\delta^{(4)}(x-y)\ . \qquad (3.27)$$

These equal-time commutators are sandwiched between two single-baryon states in the S matrix. Both of the $\Pi[\cdots]\Pi$ terms in the first term and the second term in Eq. (3.26) are higher order than the leading term of $O(N_c^0)$ by $O(N_c^{-2})$, and the commutator of Eq. (3.27) is higher order by $O(N_c^{-1})$, when they are sandwiched between two single-baryon states. We, therefore, discard them in this paper.

The source term $\mathcal{J}^a$ is finally rewritten as

$$\mathcal{J}^a(x)=-U^a[\Phi,\nabla\Phi]+K_{ab}^{-1}(K_{bc}-\delta_{bc})\mathcal{E}^c[\Phi,\nabla\Phi]\ , \qquad (3.28)$$

where we used $\mathcal{E}^a=(-\nabla^2+m_\pi^2)\Phi_a+U^a[\Phi,\nabla\Phi]$. By defining

$$\mathcal{U}_{,d}^a\equiv-\frac{\delta\mathcal{J}^a}{\delta\Phi_d}$$
$$=\frac{\delta U^a}{\delta\Phi_d}-K_{ab}^{-1}(K_{bc}-\delta_{bc})\frac{\delta\mathcal{E}^c}{\delta\Phi_d}$$
$$-\text{terms proportional to }\mathcal{E}^c\ , \qquad (3.29)$$

we can write the contribution from the equal-time commutators to the scattering amplitude as

$$-\int d^3y\ e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{y}}\langle N'|K_{S\,bd}^{-1}\mathcal{U}_{S,d}^a(\mathbf{y})|N\rangle\ , \qquad (3.30)$$

which we denote as $-\langle N'|\tilde{U}_{ba}|N\rangle$, where $K_{S\,bd}^{-1}=K_{bd}^{-1}[\phi_S,\nabla\phi_S]$ and $\mathcal{U}_{S,d}^a=U_{,d}^a[\phi_S,\nabla\Phi_S]$ with

$\mathcal{E}^c[\phi_s]=0$.

It is easy to see that $\mathcal{U}^a_{S,b}$ is the same potential term as the one appearing in the equation to the normal-mode amplitudes:

$$\ddot{\chi}_a+(-\nabla^2+m_\pi^2)\chi_a=-\mathcal{U}^a_{S,d}\chi_d , \tag{3.31}$$

which is obtained by expanding both sides of the equation $\ddot{\Phi}_a+(-\nabla^2+m_\pi^2)\Phi_a=\mathcal{J}^a$ in powers of χ_a and by taking linear terms. The left-hand side is the source of χ_a of $O(N_c^0)$, which we write as $J_a^{(0)}$, and we calculate the equal-time commutator

$$[\dot{\chi}_b(\mathbf{y},t),J_a^{(0)}(\mathbf{x},t)]=K_{S\,bc}^{-1}[\pi_c(\mathbf{y},t),\chi_d(\mathbf{x},t)](-\mathcal{U}^a_{S,d})$$
$$=iK_{S\,bc}^{-1}\mathcal{U}^a_{S,c}\delta(\mathbf{x}-\mathbf{y})-\frac{i}{\Lambda_S}\psi^b_{rk}(\mathbf{y})\mathcal{U}^a_{S,d}\psi^d_{rk}(\mathbf{x})-\frac{i}{M_S}\psi^b_{tk}(\mathbf{y})\mathcal{U}^a_{S,d}\psi^d_{tk}(\mathbf{x}) , \tag{3.32}$$

where only $O(N_c^0)$ terms are taken, and the commutation relation $[\pi_c,\chi_d]$ is calculated in the conventional gauge-fixing condition. The second and third terms contribute to the Born terms as well as higher-order terms, and the first is the potential term in the constrained field theory [21].

Thus, we reach the following expression of the scattering amplitude:

$$T^{ba}_{\pi N}=-\langle N'|\tilde{U}_{ba}|N\rangle+\sum_B\left\{\frac{\langle B|\tilde{J}^b_S(\mathbf{k}')|N'\rangle^\dagger\langle B|\tilde{J}^a_S(\mathbf{k})|N\rangle}{E^B_{\mathbf{p}+\mathbf{k}}-E^N_{\mathbf{p}}-\omega_{\mathbf{k}}}+\frac{\langle B|\tilde{J}^a_S(-\mathbf{k})|N'\rangle^\dagger\langle B|\tilde{J}^b_S(-\mathbf{k}')|N\rangle}{E^B_{\mathbf{p}'-k}-E^N_{\mathbf{p}'}+\omega_{\mathbf{k}}}\right\}$$
$$+\text{unitarity}-\text{cut terms} . \tag{3.33}$$

IV. FORWARD SCATTERING AMPLITUDE

For the purpose of the following sections, we write explicitly the forward scattering amplitude in the laboratory system, where momenta are given as

$$\mathbf{p}'=\mathbf{p}=0 , \quad p^0=p'^0=M_N , \quad \text{and } k'_\mu=k_\mu . \tag{4.1}$$

In order to have a dispersion relation for the forward scattering amplitude, we redefine the scattering amplitude as

$$F^{ba}(\omega_l)=i\int d^4y\, e^{iky}K_y\langle \mathbf{p}|\theta(y)[\Phi_b(y),\mathcal{J}^a(0)]|\mathbf{p}\rangle , \tag{4.2}$$

where $K_y=\partial^2/\partial y_\mu\partial y^\mu+m_\pi^2$ is the Klein-Gordon operator, and ω_l is the laboratory energy of a pion. This definition is essentially the same as T^{ba} in the preceding section, except for the analytic structure for $\omega_l<0$ [22].

Since we are restricted to the nonrelativistic kinematics for the baryon, we have redundant poles which are absent in the relativistic kinematics: The denominator of the direct nucleon-pole term is written as

$$D^N_d=\frac{1}{2M_N}(\omega_l^2-m_\pi^2-2M_N\omega_l) \tag{4.3}$$

for forward scattering in the laboratory system, which develops zeros at $\omega_l=M_N\pm\sqrt{M_N^2+m_\pi^2}$. In order to avoid the fictitious zero at $M_N+\sqrt{M_N^2+m_\pi^2}$ we modify the denominator as

$$D^N_d=-\frac{m_\pi^2}{2M_N}-\omega_l . \tag{4.4}$$

Similarly, we modify the denominator of the crossed nucleon pole as

$$D^N_c=-\frac{m_\pi^2}{2M_N}+\omega_l . \tag{4.5}$$

These denominators are the same as the relativistic kinematics, and we put $\omega_N=-m_\pi^2/(2M_N)$, hereafter. Defining $\omega_\Delta=M_\Delta-M_N-m_\pi^2/(2M_\Delta)$, we also modify the denominators of the direct and crossed Δ poles as

$$D^\Delta_{d,c}=\omega_\Delta\mp\omega_l . \tag{4.6}$$

The forward scattering amplitude in the Skyrme model is then written as

$$F^{ba}(\omega_l)=-\langle N'|\tilde{U}_{ba}|N\rangle+\left[\frac{G_{NN\pi}}{2M_N}\right]^2\left[\frac{k^2\tau^b\tau^a}{\omega_N-\omega_l}+\frac{k^2\tau^a\tau^b}{\omega_N+\omega_l}\right]$$
$$+\frac{2}{9}\left[\frac{G_{\Delta N\pi}}{2M_N}\right]^2\left[\frac{k^2(2\delta_{ba}-\frac{1}{2}[\tau^b,\tau^a])}{\omega_\Delta-\omega_l}+\frac{k^2(2\delta_{ba}+\frac{1}{2}[\tau^b,\tau^a])}{\omega_\Delta+\omega_l}\right]+\frac{1}{\pi}\int_{m_\pi}^\infty d\omega'\left\{\frac{\text{Im}F^{ba}(\omega')}{\omega'-\omega_l}+\frac{\text{Im}F^{ab}(\omega')}{\omega'+\omega_l}\right\} , \tag{4.7}$$

where we have used a relation $T^bT^a=\frac{1}{3}(2\delta_{ba}-\frac{1}{2}[\tau^b,\tau^a])$ and a similar one for S_iS_j. We have tentatively assumed that the unsubtracted dispersion integrals converge in Eq. (4.7). Note that the Δ state is extracted from the dispersion integral, and that its width is ignored. The imaginary part is given as

$$\begin{aligned}\mathrm{Im}F^{ba}(\omega_l)&=\frac{1}{2}\int d^4y\, e^{iky}\langle N'|[\mathcal{A}^b(y),\mathcal{A}^a(0)]|N\rangle\\&=\frac{1}{2}\sum_{n\neq\Delta}\{(2\pi)^4\delta(\mathbf{p}_n-\mathbf{k})\delta(\omega'-\omega_l)\langle N'|\mathcal{A}^b(0)|n\rangle\langle n|\mathcal{A}^a(0)|N\rangle\\&\qquad+(2\pi)^4\delta(\mathbf{p}_n+\mathbf{k})\delta(\omega'+\omega_l)\langle N'|\mathcal{A}^a(0)|n\rangle\langle n|\mathcal{A}^b(0)|N\rangle\}\ ,\end{aligned} \tag{4.8}$$

where $\omega'=p_n^0-M_N$, and the Δ is not included, because it is extracted as a pole.

The pseudoscalar-pseudoscalar coupling constant is explicitly given as

$$G_{BN\pi}(\mathbf{k}^2)=-\Lambda_{BN}2M_Nf_\pi\frac{\omega_{\mathbf{k}}^2}{|\mathbf{k}|}\int d^3y\, j_1(ky)\sin F(y)\ , \tag{4.9}$$

where $\Lambda_{\Delta N}=1/\sqrt{2}$, $\Lambda_{NN}=-1/3$, $j_1(z)$ is the spherical Bessel function of order 1, and $F(y)$ is the Skyrmion profile function. This is the same as given by many authors [23], but a little different from the definition in Ref. [12], because the definition of the fields are different from each other, but the two definitions give the same values at $\omega_{\mathbf{k}}=0$. Here note that $G_{BN\pi}$ is of $O(N_c^{3/2})$, but it always appears in the form of $G_{BN\pi}/2M_N$ in the π-N scattering amplitude.

Now we decompose the forward scattering amplitude F^{ba} into the isospin even and odd amplitudes:

$$F^{ba}(\omega_l)=\delta_{ba}F^+(\omega_l)+\tfrac{1}{2}[\tau^b,\tau^a]F^-(\omega_l)\ . \tag{4.10}$$

Each amplitude is written as

$$F^+(\omega_l)=-\langle N'|\tfrac{1}{2}\{\tilde{U}_{ba}+\tilde{U}_{ab}\}|N\rangle+\left[\frac{G_{NN\pi}}{2M_N}\right]^2\frac{2\omega_Nk^2}{\omega_N^2-\omega_l^2}+\frac{4}{9}\left[\frac{G_{\Delta N\pi}}{2M_N}\right]^2\frac{2\omega_\Delta k^2}{\omega_\Delta^2-\omega_l^2}+\frac{1}{\pi}\int 2\omega'd\omega'\frac{\mathrm{Im}F^+(\omega')}{\omega'^2-\omega_l^2}\ , \tag{4.11}$$

$$F^-(\omega_l)=\left[\frac{G_{NN\pi}}{2M_N}\right]^2\frac{2\omega_lk^2}{\omega_N^2-\omega_l^2}-\frac{2}{9}\left[\frac{G_{\Delta N\pi}}{2M_N}\right]^2\frac{2\omega_lk^2}{\omega_\Delta^2-\omega_l^2}+\frac{2\omega_l}{\pi}\int d\omega'\frac{\mathrm{Im}F^-(\omega')}{\omega'^2-\omega_l^2}\ . \tag{4.12}$$

The potential term $\tilde{U}_{ba}$ is symmetric in interchanging a and b: The equal-time commutator $[\dot{\Phi}_b,\mathcal{A}^a]$ should be equivalent to $[\dot{\Phi}_a,\mathcal{A}^b]$, because the reduction formula does not depend on the order of operation of the Klein-Gordon operators. The odd amplitude does not contain the potential term, therefore. Using the equality

$$9G_{NN\pi}^2=2G_{\Delta N\pi}^2 \tag{4.13}$$

in the Skyrme model, which comes from the values of Λ_{BN} in Eq. (4.9), we can rewrite Eq. (4.12) as

$$\begin{aligned}\frac{1}{\omega_l}F^-(\omega_l)=&\,2\left[\frac{G_{NN\pi}}{2M_N}\right]^2\frac{m_\pi^2-\omega_N^2}{\omega_l^2-\omega_N^2}\\&-\frac{4}{9}\left[\frac{G_{\Delta N\pi}}{2M_N}\right]^2\frac{m_\pi^2-\omega_\Delta^2}{\omega_l^2-\omega_\Delta^2}\\&+\frac{2}{\pi}\int d\omega'\frac{\mathrm{Im}F^-(\omega')}{\omega'^2-\omega_l^2}\ .\end{aligned} \tag{4.14}$$

In the chiral limit the first nucleon pole term vanishes, because the numerator is proportional to m_π^2.

We should notice that the Born terms in the odd amplitude is of $O(N_c^{-1})$, because both of ω_N and ω_Δ are of $O(N_c^{-1})$ and the leading terms are canceled out, and the contribution from forward background scattering of $O(N_c^0)$ to $\mathrm{Im}F^-$ vanishes as proved in Ref. [24]. Thus the odd amplitude F^- is of $O(N_c^{-1})$ in the physical region. On the other hand, the even amplitude is of $O(N_c^0)$ because of the potential term of $O(N_c^0)$. The dispersion integral in F^- is convergent, while the one in F^+ needs a subtraction.

V. AXIAL-VECTOR AND VECTOR CURRENTS

The axial-vector and vector currents written in terms of the original canonical fields are given as

$$V_a^\mu(x)=\varepsilon_{abc}\Phi_bK_{cd}\partial^\mu\Phi_d+(\dot{\Phi}^2\text{ terms})\ , \tag{5.1}$$

$$A_a^\mu(x)=-\Phi_0K_{ab}\partial^\mu\Phi_b+(\dot{\Phi}^2\text{ terms})\ , \tag{5.2}$$

where $\dot{\Phi}^2$ terms exist only in the spatial components, which we ignore throughout this section. Since the time components are particularly important, we write them explicitly:

$$V_a^0(x)=\varepsilon_{abc}\Phi_b(x)\Pi_c(x)\ , \tag{5.3}$$

$$A_a^0(x)=-\tfrac{1}{2}\{\Phi_0(x),\Pi_a(x)\}\ , \tag{5.4}$$

where the symmetrization is done for the axial-vector current.

From the symmetries of the Skyrme Lagrangian, the following conservation laws hold:

$$\partial_\mu V_a^\mu(x)=0\ , \tag{5.5}$$

$$\partial_\mu A_a^\mu(x)=m_\pi^2f_\pi\Phi_a(x)\ , \tag{5.6}$$

We call the case that $m_\pi=0$ the chiral limit hereafter. The partial conservation or the conservation law of the axial vector current is equivalent to the Lagrange equation to Φ_a. The linearity of the left-hand side of Eq. (5.6) in Φ is due to the definition of the original pion field Φ, Eq. (3.1). The currents defined above contain the baryon (Skyrmion) current, pion currents and their mixed currents.

It is easy to see that the time-components of the currents satisfy the SU(2)×SU(2) current algebra in the form of the current-current commutators

$$[V_a^0(x), V_b^0(y)]\delta(x^0-y^0)=i\varepsilon_{abc}V_c^0(x)\delta^{(4)}(x-y)\ ,$$
$$[V_a^0(x), A_b^0(y)]\delta(x^0-y^0)=i\varepsilon_{abc}A_c^0(x)\delta^{(4)}(x-y)\ , \quad (5.7)$$
$$[A_a^0(x), A_b^0(y)]\delta(x^0-y^0)=i\varepsilon_{abc}V_c^0(x)\delta^{(4)}(x-y)\ .$$

Also we can show that the time and one of the space components of the axial vector current satisfy the commutation relation

$$[A_a^0(x), A_b^i(y)]\delta(x^0-y^0)=i\varepsilon_{abc}V_c^i(x)\delta^{(4)}(x-y)\ , \quad (5.8)$$

except for the Schwinger term. (See the Appendixes.)

We note that the above current algebra holds only for the currents written in terms of the original fields. It is known that the currents made of the skyrmion fields alone do not satisfy the current algebra, even if the translational motion is allowed [16].

Now, we consider how to evaluate matrix elements of the axial-vector currents sandwiched between two hadronic states. The axial vector current $A_a^\mu(x)$ is a weak current for the pionic decay such as $\pi^a\to\mu+\bar{\nu}$. This part of the matrix element of the axial vector current in the meson sector is given as

$$\langle 0|A_a^\mu(x)|\pi^b(\mathbf{q})\rangle=if_\pi q_\mu\delta_{ab}f_{\mathbf{q}}(x)\ , \quad (5.9)$$

where $f_{\mathbf{q}}(x)$ is the asymptotic pion wave function as a solution of the Klein-Gordon equation.

If we take a matrix element of the axial vector current sandwiched between two hadronic states, $\langle\beta$ out$|$ and $|\alpha$ in$\rangle$, we have to take into account the fact that the axial-vector current can be converted into a pion propagation. According to the chiral field theory [25–27] we can extract the pion-pole term from the hadronic matrix element of the axial-vector current:

$$\langle\beta\ \text{out}|A_a^\mu(0)|\alpha\ \text{in}\rangle=\frac{-iq^\mu}{m_\pi^2-q^2}f_\pi\langle\beta\ \text{out}|\mathcal{J}^a(0)|\alpha\ \text{in}\rangle + \langle\beta\ \text{out}|A_a^\mu(0)|\alpha\ \text{in}\rangle_{\text{direct}} \quad (5.10)$$

with $q^\mu=p_\beta^\mu-p_\alpha^\mu$. The first term is the pion-pole term and the second one is the direct coupling term which gives the coupling constant of the axial-vector current to the $(\alpha\beta)$ state. The above equation is regarded as an equation to determine the direct coupling term. When it is combined with PCAC (partial conservation of axial-vector current), Eq. (5.6), the Adler consistency condition is obtained [25]:

$$f_\pi\langle\beta\ \text{out}|\mathcal{J}^a(0)|\alpha\ \text{in}\rangle=iq_\mu\langle\beta\ \text{out}|A_a^\mu(0)|\alpha\ \text{in}\rangle_{\text{direct}}\ , \quad (5.11)$$

The same relation holds for the chiral limit, too. In the chiral limit it is crucial to extract the pion pole term to get Eq. (5.11): The left-hand side of Eq. (5.10) multiplied by iq_μ vanishes by CAC (conservation of axial-vector current), while the direct coupling term should not. If the axial-vector current is sandwiched between two single-baryon states, $A_a^\mu[\Phi]$ in the left-hand side of Eq. (5.10) is reduced to the skyrmion current $A_a^\mu[\phi_s]$ in the tree approximation.

There are no vector particles in our model. The matrix element of the vector current is only the direct coupling term:

$$\langle\beta\ \text{out}|V_a^\mu(x)|\alpha\ \text{in}\rangle=\langle\beta\ \text{out}|V_a^\mu(x)|\alpha\ \text{in}\rangle_{\text{direct}}\ . \quad (5.12)$$

When both states are single-baryon states, V_a^μ is reduced to the current written in terms of the skyrmion fields.

VI. SOFT PION THEOREMS

At first, we put $\langle\beta$ out$|=\langle B(\mathbf{p}')|$ and $|\alpha$ in$\rangle=|N(\mathbf{p})\rangle$ in Eqs. (5.10) and (5.11) for the case of the massive pion. Taking the limit that $q_\mu\to 0$ in Eq. (5.10), the pion pole term vanishes, and then we have

$$\lim_{q_\mu\to 0}\langle B(\mathbf{p}')|A_a^\mu|N(\mathbf{p})\rangle=\lim_{q_\mu\to 0}\langle B(\mathbf{p}')|A_a^\mu|N(\mathbf{p})\rangle_{\text{direct}}\ , \quad (6.1)$$

where A_a^μ in the left-hand side reduces to $A_a^\mu[\phi_s]$. Using this relation, we can calculate the axial-vector coupling constant $g_A(0)$ from $\langle B|A_a^\mu[\phi_s]|N\rangle$ at the tree level:

$$\lim_{q_\mu\to 0} iq_\mu\langle B(\mathbf{p}')|A_a^\mu(0)|N(\mathbf{p})\rangle_{\text{direct}} = \lim_{q_\mu\to 0}\left[\langle B|g_A(0)i\mathbf{S}\cdot\mathbf{q}\frac{T^a}{2}|N\rangle+O(N_c^{-1})\right]\ . \quad (6.2)$$

The $O(N_c^{-1})$ term comes from the time component $q^0A_a^0[\phi_s]$ in Eq. (5.10), which has terms proportional to $q^0\mathbf{q}$ and to $q^0(\mathbf{p}+\mathbf{p}')$, when we take account of the translational mode. The contribution from the time component is given in the Appendixes. We discard the time component of the axial-vector current hereafter. The left-hand side of Eq. (5.11) is reduced to the $BN\pi$ coupling constant $G_{BN\pi}$ at the tree level:

$$f_\pi\langle B(\mathbf{p}')|\mathcal{J}^a(0)|N(\mathbf{p})\rangle=f_\pi\langle B(\mathbf{p})|J_s^a[\phi_s(-\mathbf{X})]N(\mathbf{p})\rangle = f_\pi\frac{G_{BN\pi}(q)}{2M_N}\langle B|i\mathbf{S}\cdot\mathbf{q}T^a|N\rangle\ . \quad (6.3)$$

Thus, we obtain the Goldberger-Treiman (GT) relation [15]

$$f_\pi\frac{G_{BN\pi}(0)}{2M_N}=\frac{1}{2}g_A(0)\ . \quad (6.4)$$

The same relation also holds in the chiral limit, provided that we note that the direct coupling term is not equal to $\langle B|A_a^i[\phi_s]|N\rangle$ in the chiral limit as seen from Eq. (5.10). (See the Appendixes.)

The GT relation written in terms of the classical skyrmion fields is valid at the tree level if the fluctuation around the Skyrmion is taken into account.

Now, we proceed to the Ward identity of the axial vector currents: According to the standard prescription the Ward identity is written as [27]

$$q_\mu q'_\nu F^{\nu\mu}_{ba} = i\int d^4y\, e^{iq'y} q_\mu q'_\nu \langle \mathbf{p}'|\theta(y^0)[A^\nu_b(y), A^\mu_a(0)]|\mathbf{p}\rangle$$
$$= i\int d^4y\, e^{iq'y}\{\langle \mathbf{p}'|\theta(y^0)[D_b(y)D_a(0)]|\mathbf{p}\rangle + i\delta(y^0)q_\mu\langle \mathbf{p}'|[A^0_b(y), A^\mu_a(0)]|\mathbf{p}\rangle - \delta(y^0)\langle \mathbf{p}'|[D_b(y), A^0_a(0)]|\mathbf{p}\rangle\} , \quad (6.5)$$

where $D_b(y) = m_\pi^2 f_\pi \Phi_b(y)$, which is zero in the chiral limit, of course. Extracting the pion pole terms from $F^{ba}_{\nu\mu}$ and combining them with the first term in the right-hand side of Eq. (6.5), we have

$$-f_\pi^2 F^{ba}_{\pi N}(q',q) + [q_\mu q'_\nu F^{\nu\mu}_{ba}]_{\rm direct} = -i\varepsilon_{bac} q_\mu \langle \mathbf{p}'|V^\mu_c(0)|\mathbf{p}\rangle - \delta_{ba} m_\pi^2 f_\pi \langle \mathbf{p}'|\Phi_0(0)|\mathbf{p}\rangle , \quad (6.6)$$

where the commutation relations (5.8) and (3.18) are used. The same equation holds in the chiral limit, except for the last term of the right-hand side.

The spatial component of the vector current vanishes, and the time component is the isospin matrix in forward scattering:

$$\langle \mathbf{p}|V^i_c(0)|\mathbf{p}\rangle = 0 ,$$
$$\langle \mathbf{p}|V^0_c(0)|\mathbf{p}\rangle = \int d^3y \langle N'|V^0_c[\phi_s(y)]|N\rangle = \tfrac{1}{2}\tau^c . \quad (6.7)$$

The first term in the right-hand side of Eq. (6.6) is then given as

$$-\frac{1}{2}[\tau^b,\tau^a]\frac{q^0}{2} . \quad (6.8)$$

The second term is written as

$$-\delta_{ba} m_\pi^2 f_\pi \langle \mathbf{p}'|\Phi_0[\phi_s(-\mathbf{X})]|\mathbf{p}\rangle = -\delta_{ba} m_\pi^2 f_\pi^2 \int d^3y\, e^{i(\mathbf{p}'-\mathbf{p})\mathbf{y}} \cos F(y) . \quad (6.9)$$

This is divergent for $\mathbf{p}'=\mathbf{p}$. But the same divergence appears in the meson sector, where $F(y)=0$. Subtracting the divergence at $F(y)=0$ we have a finite result:

$$\sigma = m_\pi^2 f_\pi^2 \int d^3y [1-\cos F(y)] , \quad (6.10)$$

which is the same sigma term discussed in literature [2,18]. Thus we have

$$f_\pi^2 F^{ba}_{\pi N}(q',q) = [q_\mu q_\nu F^{\nu\mu}_{ba}]_{\rm direct} + \frac{1}{2}[\tau^b,\tau^a]\frac{q^0}{2} - \sigma\delta_{ba} . \quad (6.11)$$

Here we take a limit $\mathbf{q}\to 0$ under the condition $q_\mu^2=0$, and pick up terms linear in q^0 from $[F^{\nu\mu}_{ba}]_{\rm direct}$. Such terms come from the nucleon pole diagrams [27]: Calculating the nucleon pole terms in the Skyrme model we have

$$[q_\mu q_\nu F^{\nu\mu}_{ba}]_{\rm direct} = -\frac{1}{2}[\tau^b,\tau^a]\frac{\mathbf{q}^2}{2q^0} g_A^2(0) + O(\mathbf{q}^2) . \quad (6.12)$$

Thus, we reach the soft pion limit

$$\lim_{q^0\to 0}\left\{\frac{1}{q^0}F^-(q,q)\right\} = \frac{1}{2f_\pi^2}(1-g_A^2) \quad (6.13)$$

for the isospin odd forward scattering amplitude. The left-hand side is rewritten through Eq. (4.14) under the condition that $m_\pi=0$ and $\omega_l=q^0$ as follows:

$$\lim_{\omega_l\to 0}\left\{\frac{1}{\omega_l}F^-(\omega_l)\right\} = -\frac{4}{9}\left[\frac{G_{\Delta N\pi}}{2M_N}\right]^2 + \frac{2}{\pi}\int d\omega' \frac{{\rm Im}F^-(\omega')}{\omega'^2} . \quad (6.14)$$

Thus, we finally obtain the Adler-Weisberger (AW) relation [17] as

$$\frac{1}{2f_\pi^2}(1-g_A^2) = -\frac{4}{9}\left[\frac{G_{\Delta N\pi}}{2M_N}\right]^2 + \frac{2}{\pi}\int d\omega' \frac{{\rm Im}F^-(\omega')}{\omega'^2} . \quad (6.15)$$

However, if we use the GT relation and the algebraic relation between $G_{NN\pi}$ and $G_{\Delta N\pi}$, Eq. (4.13), we observe that the g_A^2 term is completely canceled by the $\Delta N\pi$ coupling term, and the AW relation in the Skyrme model reduces to the sum rule

$$\frac{1}{2f_\pi^2} = \frac{2}{\pi}\int d\omega' \frac{{\rm Im}F^-(\omega')}{\omega'^2} . \quad (6.16)$$

Here remember that ${\rm Im}F^-$ is of $O(N_c^{-1})$ as stated below Eq. (4.14).

The ratio of $G_{NN\pi}$ to $G_{\Delta N\pi}$, Eq. (4.13), in the Skyrme model is equal to the ratio in the case of the infinite-N_c limit [28]. We have already shown that the g_A^2 term should be canceled out from the AW relation in the infinite-N_c limit in order that it is consistent with the large-N_c behavior [24,29,30]. Thus, the AW relation in the Skyrme model is not a sum rule to g_A, but a sum rule to the dispersion integral even if the Δ state is taken into account, in contrast with the standard understanding of the AW relation.

From Eq. (4.14) we observe that

$$F^-(m_\pi) = \frac{2m_\pi}{\pi}\int d\omega' \frac{{\rm Im}F^-(\omega')}{\omega'^2 - m_\pi^2} \quad (6.17)$$

at the threshold, $\omega_l = m_\pi$. The right-hand side is approximated as

$$F^-(m_\pi) = m_\pi\left[\frac{2}{\pi}\int d\omega' \frac{{\rm Im}F^-(\omega')}{\omega'^2} + O(m_\pi^2)\right] , \quad (6.18)$$

which reduces to

$$F^-(m_\pi) = 4\pi a_- = \frac{m_\pi}{2f_\pi^2}\{1+O(m_\pi^2)\} \quad (6.19)$$

by using the AW relation (6.16), where a_- is the isospin-odd S-wave scattering length. This is just the Tomozawa-Weinberg (TW) relation in the Skyrme model.

It should be noticed that in order to obtain the TW relation we have not referred to any special interaction Hamiltonian such as $\varepsilon_{abc}\chi_b\dot{\chi}_c V_c^0[\phi_s]/f_\pi^2$, but have used the sum rule to the dispersion integral of the forward scattering amplitude.

VII. CONCLUSIONS AND DISCUSSION

We have shown in this paper that the pion-nucleon elastic scattering amplitude in the Skyrme model is successfully formulated by the reduction formula described in terms of the original canonical fields in the Skyrme lagrangian. In this formulation it is not explicitly required to refer to any constraints and gauge-fixing conditions imposed on the fluctuation fields and the collective coordinates. A matrix element of a product of the original fields sandwiched between two single-soliton (single-baryon) states is reduced to the matrix element of the soliton (Skyrmion) fields with appropriate collective coordinates in the tree approximation. The original field is regarded as the interpolating field playing the same role as the fluctuation field in the one-soliton sector, and then the one-soliton subspace of the Hilbert space on which the original fields act is taken to be the same one spanned by the in and out states for the fluctuation fields. Our prescription to elastic scattering amplitude is applicable to other processes such as $\pi N \to \pi\Delta$, $\pi\Delta \to \pi\Delta$, and photoproduction of a pion.

While our formulation does not refer to any gauge-fixing conditions, previous works to obtain the Born terms based on the constrained field theories of the collective coordinates and the fluctuation fields are gauge dependent. In the theory of the conventional gauge-fixing conditions [12,13], where both of the fluctuation fields χ_a and π_a are to be orthogonal to the rotational and translational zero modes, the Yukawa interaction Hamiltonian surviving from the stability condition of the Skyrmion is of $O(N_c^{-3/2})$, and one needs to calculate higher-order seagull terms up to $O(N_c^{-2})$ in addition to the zero-mode pole term of $O(N_c^0)$ in order to get the proper Born terms in the tree approximation. In the theory of the nonrigid gauge-fixing condition [31] the surviving Yukawa interaction Hamiltonian is of $O(N_c^{-1/2})$, since the orthogonality of χ_a to the zero modes is not required. Combining with the zero-mode pole term one gets the gauge-invariant Born terms, but one has to calculate the loop corrections at the same time. Thus, in the formulation with the use of the collective coordinates and the fluctuation fields we have to calculate complicated higher-order terms or loop corrections in order to get the gauge-invariant Born terms from the gauge-dependent interaction Hamiltonian. There is another approach [32]: In this approach one calculates the matrix element of the surviving interaction lagrangian linear in $\dot{\chi}_a$ in the plane-wave approximation and adds its Born terms with the intermediate Δ states to the background scattering amplitude. In this way one obtains the correct P-wave Born terms, but all the constraints imposed on the fluctuating fields and the collective coordinates are simply ignored. Contrasting with this, our approach is free from the constraints, because the fields we treat are the original fields satisfying canonical commutation relations.

We have also shown that the soft pion theorems are satisfactorily formulated in the Skyrme model in the situation where the fluctuating pions are included. The Adler-Weisberger relation in the Skyrme model is, however, not the sum rule to the axial-vector coupling constant, but the sum rule to the scattering amplitude of $O(N_c^{-1})$, in contrast with the standard understanding of the AW relation. The fact that g_A may be less than unity in the Skyrme model is, therefore, compatible with the AW relation.

As to the even scattering length the threshold value of the amplitude is written as

$$F^+(m_\pi) = -\frac{1}{f_\pi^2}\sigma + \lim_{\substack{q^2=m_\pi^2 \\ \mathbf{q}\to 0}} \frac{1}{f_\pi^2}[q_\mu q_\nu F_+^{\nu\mu}]_{\text{direct}}\ , \quad (7.1)$$

where the nucleon pole terms in the direct coupling term vanish at the threshold, but the isoeven direct coupling term as a whole remains finite at the physical threshold [33]. The small value of the even scattering length a_+ would be due to the cancellation between the two terms, but we did not attempt to evaluate a_+ in this paper.

We have often noted that our formulas are valid at the tree level. The Yukawa coupling defined as the residue of the Born term is of $O(N_c^{1/2})$, but this does not mean that effective one-pion interactions are always of $O(N_c^{1/2})$. For example, consider a one-loop self-energy of a baryon: If the one-pion interaction is of $O(N_c^{1/2})$, the one-loop self-energy is of $O(N_c)$ and may have a different value depending on a spin and isospin value. If this is the case, the mass difference between Δ and N is of $O(N_c)$. This destroys the N_c^{-1} expansion scheme in the soliton model.

The one-loop correction may be written as

$$\Delta M_{1\ \text{loop}} = \langle B|H'|B\rangle - \sum_{\mathbf{p},\mathbf{k}} \frac{\langle B|H'|\mathbf{p},\mathbf{k}\rangle\langle \mathbf{p},\mathbf{k}|H'|B\rangle}{E_{\mathbf{p}}^{B'}+\omega_{\mathbf{k}}-M_B}\ , \quad (7.2)$$

where H' is the interaction Hamiltonian. The first term denotes possible one-loop corrections in the first-order perturbation theory, and H' in the second-order perturbation is sandwiched between two states with different pion numbers, where the pion states should not be the asymptotic ones. Therefore, if H' is written in terms of Φ and Π, its matrix elements cannot be written in terms of ϕ_s and Π_s, but should be recalculated within the constrained system of the fluctuation fields and collective coordinates. The lowest-order one-loop correction comes from a seagull term of $O(N_c^{-2})$. $\Delta M_{1\ \text{loop}}$ is of $O(N_c^{-2})\times O(\hbar)$ at most, while the ordering correction in the kinetic term is of $O(N_c^{-1})\times O(\hbar^2)$. Thus, the mass difference between Δ and N remains in $O(N_c^{-1})$.

We have given the Born terms with the translational mode as well as the rotational mode of the collective coordinates. The P wave nature of the Born terms is, however, not altered even if we take into account of the translational mode. In the case of the Dirac particle the Born terms contain diagrams with pair creation and annihilation, which give the S wave nature in addition to the P wave nature. Thus, the soliton model is essentially non-relativistic for the soliton, similar to quark models of the nucleon.

APPENDIX A: COMMUTATOR BETWEEN THE TIME AND SPATIAL AXIAL-VECTOR CURRENTS

$$\begin{aligned}[A_b^0(\mathbf{y}), A_a^i(\mathbf{x})] &= \tfrac{1}{2}\{\Phi_0(\mathbf{y}), [\Pi_b(\mathbf{y}), \Phi_0 K_{ac}\partial^i\Phi_c(\mathbf{x})]\} \\ &= \tfrac{1}{2}\{\Phi_0(\mathbf{y}), [\Pi_b(\mathbf{y}), \Phi_0(\mathbf{x})]K_{ac}\partial^i\Phi_c(\mathbf{x})\} \quad (1) \\ &\quad + \tfrac{1}{2}\{\Phi_0(\mathbf{y}), \Phi_0(\mathbf{x})[\Pi_b(\mathbf{y}), K_{ac}(\mathbf{x})]\partial^i\Phi_c(\mathbf{x})\} \quad (2) \\ &\quad + \tfrac{1}{2}\{\Phi_0(\mathbf{y}), \Phi_0(\mathbf{x})K_{ac}(\mathbf{x})[\Pi_b(\mathbf{y}), \partial^i\Phi_c(\mathbf{x})]\} , \quad (3) \end{aligned} \tag{A1}$$

where we omit t in the argument, and each line is numbered:

$$(1) = i\Phi_b K_{ac}\partial^i\Phi_c(\mathbf{x})\delta(\mathbf{y}-\mathbf{x}) . \tag{A2}$$

$$(2) = -i\frac{1}{2}\left\{\Phi_0(\mathbf{y}), \Phi_0(\mathbf{x})\left[\frac{\partial K_{ac}(\mathbf{x})}{\partial\Phi_b(\mathbf{x})}\delta(\mathbf{y}-\mathbf{x}) + \frac{\partial K_{ac}(\mathbf{x})}{\partial(\partial_j\Phi_b(\mathbf{x}))}\partial_j\delta(\mathbf{y}-\mathbf{x})\right]\partial^i\Phi_c(\mathbf{x})\right\} , \tag{A3}$$

each term of which is reduced to

$$(2)_1 = -i[\Phi_a K_{bc}\partial^i\Phi_c(\mathbf{x}) - K_{ab}\Phi^i(\mathbf{x}) + \tilde{K}_{ab}^{ij}\Phi_j(\mathbf{x})]\delta(\mathbf{y}-\mathbf{x}) , \tag{A4}$$

where $\Phi^i = -\phi_c\partial^i\Phi_c = \Phi_0\partial^i\Phi_0$ and

$$\tilde{K}_{ab}^{ij} = \frac{1}{\kappa^2}\{X_{ab}\partial^i\Phi_c X_{cd}\partial^j\Phi_d + X_{ac}\partial^j\Phi_c X_{bd}\partial^i\Phi_d - 2X_{ac}\partial^i\Phi_c X_{bd}\partial^j\Phi_d\} \tag{A5}$$

with $X_{ab} = \delta_{ab} + (\Phi_a\Phi_b)/\Phi_0^2$,

$$\begin{aligned}(2)_2 &= i\Phi_0(\mathbf{y})\Phi_0(\mathbf{x})\tilde{K}_{ab}^{ij}\frac{\partial\delta(\mathbf{y}-\mathbf{x})}{\partial x^j} \\ &= i\tilde{K}_{ab}^{ij}\Phi_j(\mathbf{x})\delta(\mathbf{y}-\mathbf{x}) - i\frac{\partial}{\partial y_j}[\Phi_0^2(\mathbf{x})\mathbf{K}_{ab}^{ij}\delta(\mathbf{y}-\mathbf{x})] ,\end{aligned} \tag{A6}$$

where we used $\partial\delta(\mathbf{y}-\mathbf{x})/\partial x_j = -\partial\delta(\mathbf{y}-\mathbf{x})/\partial y_j$. This cancels the third term in $(2)_1$:

$$(3) = -i\Phi_0(\mathbf{y})\Phi_0(\mathbf{x})K_{ab}\frac{\partial\delta(\mathbf{y}-\mathbf{x})}{\partial x_j} , \tag{A7}$$

which cancels the second term in $(2)_1$. Finally we get

$$\begin{aligned}&[A_b^0(\mathbf{y},t), A_a^i(\mathbf{x},t)] \\ &\quad = i\{\Phi_b K_{ac}\partial^i\Phi_c(\mathbf{x},t) - \Phi_a K_{bc}\partial^i\Phi_c(\mathbf{x},t)\}\delta(\mathbf{y}-\mathbf{x}) ,\end{aligned} \tag{A8}$$

except for the Schwinger term

$$i(\partial/\partial y_j)[\delta(\mathbf{y}-\mathbf{x})\Phi_0^2(\mathbf{x})(K_{ab}\delta_{ij} - \tilde{K}_{ab}^{ij})] .$$

APPENDIX B: AXIAL-VECTOR COUPLING CONSTANTS

Putting $r_i = x_i - X_i$, we have

$$K_{S_{ab}} = R_{ai}R_{bj}[g_L(r)\hat{r}_i\hat{r}_j + g_T(r)\delta_{ij}^T] , \tag{B1}$$

where

$$g_L = \frac{1}{c^2}\left[1 + \frac{2}{\kappa^2}\frac{s^2}{r^2}\right] , \quad g_T = 1 + \frac{1}{\kappa^2}\left[\frac{s^2}{r^2} + F'^2\right] \tag{B2}$$

with $c = \cos F(r)$, $s = \sin F(r)$, and $F' = dF(r)/dr$. The axial-vector current is written as

$$A_i^a[\phi_s] = -R_{aj}[\delta_{ij}^T f_\pi^2\frac{sc}{r}g_T(r) + \hat{r}_i\hat{r}_j f_\pi^2 c^2 F' g_L(r)] . \tag{B3}$$

Since the pion-pole term vanishes in the limit $q^i \to 0$ for the massive-pion case, $g_A(0)$ is written as

$$g_A S^i\frac{T^a}{2} = \lim_{\mathbf{q}\to 0}\int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}\langle B|A_a^i[\phi_s(\mathbf{r})]|N\rangle , \tag{B4}$$

which gives

$$g_A(0) = -8\pi\Lambda_{BN}f_\pi^2\int dr\, r^2\left[c^2F'g_L - \frac{1}{3}\left[c^2F'g_L - \frac{sc}{r}g_T\right]\right] . \tag{B5}$$

By using the equation of motion to $F(r)$, we can rewrite the above definition as

$$g_A(0) = -\frac{8\pi}{3}f_\pi^2\Lambda_{BN}m_\pi^2\int dr\, r^3 s , \tag{B6}$$

while the coupling constant $G_{BN\pi}$ in Eq. (4.9) is given as

$$\frac{G_{BN\pi}(0)}{2M_N} = -\frac{4\pi f_\pi}{3}m_\pi^2\int dr\, r^3 s . \tag{B7}$$

Thus, the GT relation results.

In the chiral limit the $BN\pi$ coupling constant is written as

$$\frac{G_{BN\pi}(0)}{2M_N} = -4\pi f_\pi\Lambda_{BN}B , \tag{B8}$$

where $F(r) \to B/r^2$ as $r \to \infty$. If we use Eq. (B6) under $m_\pi = 0$ naively, we get a value smaller than the above constant by a factor 2/3. The pion-pole contribution to the direct coupling term is

$$\lim_{\mathbf{q}\to 0}\frac{iq^i}{\mathbf{q}^2}f_\pi\langle B|J_s^a[\phi_s]|N\rangle = -4\pi f_\pi^2\Lambda_{BN}\frac{B}{3}S^iT^a , \tag{B9}$$

where q_0^2 is neglected as it is of $O(N_c^{-2})$. This gives the required factor. The result $g_A = -8\pi f_\pi^2 \Lambda_{BN} B$ is the same as given by Adkins *et al.* [2].

The time-component of $O(N_c^0)$ is written as

$$A_0^a[\phi_s] = -\frac{f_\pi}{2\Lambda_s}\{cK_{S_{ab}}\psi_{rj}^b, I_j\} - \frac{f_\pi}{2M_s}\{cK_{S_{ab}}\psi_{tj}^b, P_j\} , \tag{B10}$$

which is rewritten as

$$A_0^a[\phi_s] = -f_\pi^2 csg_T[I^2/2\Lambda_s, R_{ai}]i\hat{q}_i - \frac{1}{2M_s}\{A_j^a[\phi_s], P_j\} . \tag{B11}$$

Thus,

$$\langle B(\mathbf{p}')|iq^0 A_0^a[\phi_s]|N(\mathbf{p})\rangle = i\mathbf{S}\cdot\mathbf{q}\frac{T^a}{2}\hat{f}_A(q) + i\mathbf{S}\cdot\left[\frac{\mathbf{p}+\mathbf{p}'}{2}\right]\frac{T^a}{2}f_A(q) + i\mathbf{S}\cdot\mathbf{q}\frac{T^a}{2}(\mathbf{p}'^2-\mathbf{p}^2)h_A(q) , \tag{B12}$$

where

$$\hat{f}_A(q) = f_\pi^2(M_B - M_N)q^0\int d^3r\frac{j_1(qr)}{q^r}\frac{scg_T}{r} ,$$

$$f_A(q) = \frac{1}{M_s}q^0[g_A(q)] ,$$

$$h_A(q) = -\frac{q^0}{M_s}\frac{f_\pi}{m_\pi^2+\mathbf{q}^2}\frac{G_{BN\pi}(q)}{2M_N} . \tag{B13}$$

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