Self-dual Chern-Simons solitons in (2+1)-dimensional Einstein gravity

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We consider a generalization of the Abelian Higgs model in curved space, by adding a Chern-Simons term. The static equations are self-dual provided we choose a suitable potential. The solutions give a self-dual Maxwell-Chern-Simons soliton that possesses a mass and a spin.

Einstein gravity in $2+1$ dimensions has attracted much attention recently, not only as a theoretical laboratory for studying effects of quantized gravity but also due to its direct physical relevance in cosmic-string dynamics [1]. The $(2+1)$-dimensional Einstein gravity is trivial in the absence of matter, while introducing point [2,3] or line [4] sources alters the global geometry of space-time in the following way. The metric describing N point particles located at $\mathbf{r}_i$ ($i = 1, \ldots, N$), with mass M_i and spins J_i, is known to have the form [5]

$$ds^2 = - \left[dt + G \sum_{i=1}^{N} J_i \frac{\mathbf{r} - \mathbf{r}_i}{|\mathbf{r} - \mathbf{r}_i|^2} \times d\mathbf{r} \right]^2 + \left[\prod_{i=1}^{N} |\mathbf{r} - \mathbf{r}_i|^{-2GM_i} \right] (d\mathbf{r})^2 . \tag{1}$$

The spatial geometry is thus multiconical, and with nonzero J_i one finds a time helical structure [2].

As has been known for some time, the nonsingular counterparts for spinless multiparticle systems are provided by static multivortex solutions of the curved-space Abelian Higgs model in the self-dual limit [6]. One expects a regular configuration of nonzero spin as a result of introducing a Chern-Simons term. Linet [7] introduced a non-self-dual model including this term. With some assumptions he shows that the asymptotic geometry is analogous to that of a spinning particle. Recently, Valtancoli [8] considered a curved-space self-dual model taking the Chern-Simons term as the entire gauge field action. We shall present here a model including both the Maxwell and the Chern-Simons terms and show how it leads to self-dual equations. The Higgs model and the pure Chern-Simons model correspond to special limiting cases of our treatment.

In flat space it is known [9] that, with both Maxwell and Chern-Simons terms, the simplest self-dual system is described by the action

$$I_{\text{flat}} = \int d^3x \left[-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{4}\kappa\epsilon^{\mu\nu\lambda} F_{\mu\nu} A_\lambda - |D_\mu \phi|^2 \right.$$
$$- \tfrac{1}{2}(\partial_\mu S)^2 - \tfrac{1}{2}(e|\phi|^2 + \kappa S - ev^2)^2$$
$$\left. - e^2 S^2 |\phi|^2 \right] , \tag{2}$$

where $D_\mu = \partial_\mu - ieA_\mu$; ϕ is a complex scalar field and S a real one. For a related curved-space self-dual system, it is then natural to consider the action

$$I = \frac{1}{4\pi G} \int d^3x \sqrt{-g}\, R + I_M , \tag{3a}$$

$$I_M = \int d^3x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right.$$
$$+ \frac{\kappa}{4} \frac{1}{\sqrt{-g}} \epsilon^{\mu\nu\rho} F_{\mu\nu} A_\rho$$
$$- g^{\mu\nu}(D_\mu\phi)^* D_\nu\phi$$
$$\left. - \frac{1}{2} g^{\mu\nu} \partial_\mu S\, \partial_\nu S - V(\phi, S) \right] , \tag{3b}$$

where the precise form of the scalar potential $V(\phi, S)$ is still to be determined. Our interest is in time-independent solitonlike configurations satisfying corresponding matter field equations and Einstein's equations ($R^{\mu\nu} - \tfrac{1}{2} g^{\mu\nu} R = 2\pi G T^{\mu\nu}$), and we here want specifically a model for which the governing equations for the solitons can be reduced to first-order self-dual (or Bogomol'nyi-type) equations.

We may assume the general stationary metric

$$ds^2 = -N^2(dt + K_i dx^i)^2 + \gamma_{ij} dx^i dx^j \quad (i,j = 1,2) , \tag{4}$$

viz.,

$$g_{00} = -N^2 , \quad g_{0i} = -N^2 K_i , \quad g_{ij} = \gamma_{ij} - N^2 K_i K_j ,$$
$$g^{00} = -1/N^2 + \gamma^{ij} K_i K_j , \quad g^{0i} = -\gamma^{ij} K_j , \quad g^{ij} = \gamma^{ij} . \tag{5}$$

Here, $N \geq 0$; K_i and γ_{ij} are functions of $\mathbf{r} = (x^1, x^2)$ only. The spatial metric γ_{ij} will be used to move indices.

Instead of A_i we find it convenient to use the fields

$$\overline{A}_i = A_i - K_i A_0 , \tag{6}$$

so that $\overline{A}^i \equiv \gamma^{ij} \overline{A}_j = g^{i\mu} A_\mu$. Then, denoting $(1/\sqrt{\gamma})\epsilon^{ij}\partial_i \overline{A}_j = \overline{B}$, $(1/\sqrt{\gamma})\epsilon^{ij}\partial_i K_j = H$, and $\overline{D}_i = \partial_i - ie\overline{A}_i$, we obtain the following *static* action from the action (3) with time-independent fields:

$$I = \int d^3x \sqrt{\gamma} N \left[\frac{1}{4\pi G} \bar{R} + \frac{1}{2} \left(-A_0^2 + \frac{N^2}{4\pi G} \right) H^2 - A_0 \bar{B} H + \frac{1}{2} \frac{1}{N^2} \gamma^{ij} \partial_i A_0 \partial_j A_0 - \frac{1}{2} \bar{B}^2 \right.$$
$$\left. + \kappa \frac{1}{N} A_0 \bar{B} + \frac{\kappa}{2} \frac{1}{N} A_0^2 H + e^2 \frac{1}{N^2} A_0^2 |\phi|^2 - \gamma^{ij} (\bar{D}_i \phi)^* \bar{D}_j \phi - \frac{1}{2} \gamma^{ij} \partial_i S \, \partial_j S - V(\phi,S) \right] . \tag{7}$$

Here $\bar{R}$ is the Ricci scalar associated with the metric γ_{ij}. The equation of motion coming from an N variation gives

$$\frac{1}{4\pi G} \sqrt{\gamma} \bar{R} = \sqrt{\gamma} \left[\left[\frac{1}{2} A_0^2 - \frac{3N^2}{8\pi G} \right] H^2 + A_0 \bar{B} H \right.$$
$$+ \frac{1}{2} \frac{1}{N^2} \gamma^{ij} \partial_i A_0 \partial_j A_0 + \frac{1}{2} \bar{B}^2$$
$$+ \frac{1}{N^2} e^2 A_0^2 |\phi|^2 + \gamma^{ij} (\bar{D}_i \phi)^* \bar{D}_j \phi$$
$$\left. + \frac{1}{2} \gamma^{ij} \partial_i S \, \partial_j S + V(\phi,S) \right] . \tag{8}$$

This is one of Einstein's equations. In what follows we will take as an ansatz that

$$N(\mathbf{r}) = 1 . \tag{9}$$

This means putting $N = 1$ in the action (7) and keeping Eq. (8) as an extra constraint that our solutions must satisfy.

The static field equation related to the K_i variation of the action (7) can readily be integrated to yield

$$\left[-A_0^2 + \frac{1}{4\pi G} \right] H - A_0 \bar{B} + \frac{\kappa}{2} A_0^2 = -\frac{\kappa}{2} C , \tag{10}$$

C being an integration constant. Then, making use of the identity

$$\sqrt{\gamma} \gamma^{ij} (\bar{D}_i \phi)^* \bar{D}_j \phi$$
$$= \frac{1}{2} \sqrt{\gamma} \gamma^{ij} [(\bar{D}_i \phi)^* \mp i \sqrt{\gamma} \epsilon_{il} \gamma^{lm} (\bar{D}_m \phi)^*]$$
$$\times (\bar{D}_j \phi \pm i \sqrt{\gamma} \epsilon_{jk} \gamma^{kn} \bar{D}_n \phi)$$
$$\pm e \bar{B} |\phi|^2 \mp \frac{i}{2} \frac{1}{\sqrt{\gamma}} e^{ij} \partial_i [\phi^* \bar{D}_j \phi - (\bar{D}_j \phi)^* \phi] \tag{11}$$

(note that $\epsilon_{ij} = \epsilon^{ij}$ in our convention), we observe that the action (7), up to surface terms, can be rearranged into the form

$$I = \int d^3x \sqrt{\gamma} \left[\frac{1}{4\pi G} \bar{R} + \frac{1}{2} \left(-A_0^2 + \frac{1}{4\pi G} \right) \left[H + \frac{-A_0 \bar{B} + \frac{1}{2}\kappa(A_0^2 + C)}{-A_0^2 + 1/4\pi G} \right]^2 \right.$$
$$- \frac{1}{2} \frac{1}{1 - 4\pi G A_0^2} \{ \bar{B} - 2\pi G \kappa A_0 (A_0^2 + C) - (1 - 4\pi G A_0^2)[\kappa A_0 \mp e(|\phi|^2 - v^2)] \}^2 - \frac{\pi G \kappa^2}{2(1 - 4\pi G A_0^2)} (A_0^2 + C)^2$$
$$+ \frac{1}{2} (1 - 4\pi G A_0^2) \left[\frac{2\pi G \kappa A_0 (A_0^2 + C)}{1 - 4\pi G A_0^2} + \kappa A_0 \mp e(|\phi|^2 - v^2) \right]^2 + e^2 A_0^2 |\phi|^2 - V(\phi,S) + \frac{1}{2} \gamma^{ij} \partial_i A_0 \partial_j A_0$$
$$\left. - \frac{1}{2} \gamma^{ij} \partial_i S \, \partial_j S - \frac{1}{2} \gamma^{ij} [(\bar{D}_i \phi)^* \mp i \sqrt{\gamma} \epsilon_{il} \gamma^{lm} (\bar{D}_m \phi)^*](\bar{D}_j \phi \pm i \sqrt{\gamma} \epsilon_{jk} \gamma^{kn} \bar{D}_n \phi) \right] . \tag{12}$$

Equation (10) is manifestly incorporated in this form, and see below for the role for an arbitrary constant v^2. [Note that all v^2-dependent terms on the right-hand side of Eq. (12) cancel, except for the surface term $\mp ev^2 \int d^3x \sqrt{\gamma} \bar{B}$.] Now, as a natural extension of the flat-space self-duality equations [9], let us suppose that the following equations hold:

$$\bar{D}_i \phi \pm i \sqrt{\gamma} \epsilon_{ij} \gamma^{jk} \bar{D}_k \phi = 0 , \tag{13a}$$

$$A_0 = \mp S . \tag{13b}$$

Then the field equation related to the A_i variation will effectively take the form

$$\bar{B} = 2\pi G \kappa A_0 (A_0^2 + C)$$
$$+ (1 - 4\pi G A_0^2)[\kappa A_0 \mp e(|\phi|^2 - v^2)] , \tag{14}$$

with v^2 interpreted as the associated integration constant. We have already incorporated this equation in the action (12) in a manifest way.

We still have to consider the field equations related to the γ_{ij}, ϕ, S, and A_0 variations of the action (12), and for these the first, second, and last terms in the right-hand side of the action can be ignored thanks to Eqs. (10), (13), and (14). The equations from the γ_{ij} variation then forces the remaining terms in the action (12) to vanish locally when $A_0 = \mp S$; this forces the scalar potential for a consistent self-dual system to take the sixth-order form

$$V(\phi,S) = \frac{1}{2} (e|\phi|^2 + kS - ev^2)^2 + e^2 S^2 |\phi|^2$$
$$- 2\pi G [\frac{1}{2} \kappa(S^2 - C) + (e|\phi|^2 - ev^2)S]^2 . \tag{15}$$

On the other hand, it follows that the only nontrivial equation obtained from the ϕ, S, and A_0 variations is (here $\bar{\nabla}$ is the two-dimensional covariant derivative)

$$\bar{\nabla}^i \bar{\nabla}_i S - \frac{\partial V(\phi,S)}{\partial S} = 0 , \tag{16}$$

which, as an equation for A_0, describes Gauss' law. We can now state that if the scalar potential of a given sys-

tem is the one given in Eq. (15), any static configuration satisfying the conditions (8)–(10), (13), (14), and (16), the desired curved-space generalization of the flat-space self-duality equations, provides a solution to the full coupled field equations. Also notice that Eqs. (10) and (14) are equivalent to the relations

$$H = 4\pi G[-\tfrac{1}{2}\kappa(S^2+C)+S(e|\phi|^2+\kappa S-ev^2)] ,$$
$$\bar{B} = \mp[(e|\phi|^2+\kappa S-ev^2)-SH] ,$$
$$(17)$$

while Eq. (8) can be simplified (thanks to other equations in the set) as

$$\frac{1}{4\pi G}\bar{R} = \pm ev^2\bar{B}+\kappa CH+\frac{1}{2}\bar{\nabla}^i\bar{\nabla}_i|\phi|^2+\frac{1}{2}\bar{\nabla}^i\bar{\nabla}_iS^2 . \quad (18)$$

Actually, the integration constant C above is subject to a physical constraint: at spatial infinity, the fields (S,ϕ) should approach some constant values (S_∞,ϕ_∞) and H and $\bar{B}$ should tend to zero (since their integrated values correspond to physical observables to be discussed below). Then we have, from Eq. (17),

$$e|\phi_\infty|^2+\kappa S_\infty-ev^2=0 , \quad S_\infty^2+C=0 , \quad (19)$$

while Eq. (16) requires, in addition,

$$S_\infty|\phi_\infty|^2=0 . \quad (20)$$

Thus the allowed values for C are

$$C = \begin{cases} 0, & \text{with } |\phi_\infty|=v \text{ and } S_\infty=0 , \\ -e^2v^4/\kappa^2, & \text{with } |\phi_\infty|=0 \text{ and } S_\infty=ev^2/\kappa . \end{cases} \quad (21)$$

With $C=0$ we have the broken vacuum and topological soliton solutions, while with $C=-e^2v^4/\kappa^2$ we have the unbroken vacuum and nontopological solitons only. [But, if we turned off gravity (i.e., set $G=0$), both would, of course, lead to a theory in which broken and unbroken vacua are degenerate [9].] Regardless of whether $C=0$ or $C=-e^2v^4/\kappa^2$, the total energy [2,10] of the given soliton is

$$E = \frac{1}{4\pi G}\int d^2\mathbf{r}\sqrt{\gamma}\bar{R} = \pm ev^2\Phi \quad \left[\Phi \equiv \int d^2\mathbf{r}\,\epsilon^{ij}\partial_iA_j\right] , \quad (22)$$

where we have used Eq. (18) and the relation $A_0=\mp S$. As in the flat-space case, the magnetic flux of a topological soliton must be quantized, i.e., $\Phi=\pm(2\pi/e)n$ (n is a positive integer), while the Φ value of a nontopological soliton is not. These solitons also have a nonzero angular momentum, as determined by the formula [2,10]

$$J = \frac{1}{2\pi G}\int d^2\mathbf{r}\sqrt{\gamma}H$$
$$= \int d^2\mathbf{r}\sqrt{\gamma}[-\kappa(S^2+C)+2S(|\phi|^2+\kappa S-ev^2)] . \quad (23)$$

Note that this definition is consistent with the asymptotic form $K_i(\mathbf{r})\sim -GJ\epsilon_{ij}x^j/|\mathbf{r}|^2$ [cf. Eqs. (1) and (4)], and for $G=0$ it reduces to the usual flat-space angular momentum appropriate to the topological ($C=0$) or nontopological ($C=-e^2v^4/\kappa^2$) soliton case.

We here mention some limiting cases. When the Chern-Simons coupling κ is equal to zero, it is consistent

to set $A_0=S=K_i=0$ and our system trivially reduces to the model considered in Ref. [6]. The scalar potential becomes simply $V(\phi)=\tfrac{1}{2}e^2(|\phi|^2-v^2)^2$, and we here have the self-duality equations

$$D_i\phi\pm i\sqrt{\gamma}\epsilon_{ij}\gamma^{jk}D_k\phi=0 ,$$
$$\frac{1}{\sqrt{\gamma}}\epsilon^{ij}\partial_iA_j = \mp e(|\phi|^2-v^2) . \quad (24)$$

Only topological solitons are possible with $\kappa=0$. More interesting will be the limit $\kappa\to\infty$ for fixed κ/e^2. In this limit, the kinetic term for S and the Maxwell term become negligible, and up to order-$(1/\kappa)$ corrections one can identify

$$A_0 = \mp S = \pm\frac{e}{\kappa}(|\phi|^2-v^2) . \quad (25)$$

Then one finds that the appropriate matter action and self-duality equations read

$$I_M = \int d^3x \left\{\frac{\kappa}{4}\epsilon^{\mu\nu\lambda}F_{\mu\nu}A_\lambda-\sqrt{-g}\,g^{\mu\nu}(D_\mu\phi)^*D_\nu\phi \right.$$
$$-\sqrt{-g}\left[\frac{e^4}{\kappa^2}|\phi|^2(|\phi|^2-v^2)^2\right.$$
$$-\frac{1}{2}\pi G\frac{e^4}{\kappa^2}\left[(|\phi|^2-v^2)^2\right.$$
$$\left.\left.\left.+\frac{\kappa^2}{e^2}C\right]^2\right]\right\} , \quad (26)$$

$$\bar{D}_i\phi\pm i\sqrt{\gamma}\epsilon_{ij}\gamma^{jk}\bar{D}_k\phi=0 , \quad (27a)$$

$$H = -2\pi G\frac{e^2}{\kappa}\left[(|\phi|^2-v^2)^2+\frac{\kappa^2}{e^2}C\right] , \quad (27b)$$

$$\bar{B} = \mp\frac{2e^3}{\kappa^2}|\phi|^2(|\phi|^2-v^2)$$
$$\pm2\pi G\frac{e^3}{\kappa^2}(|\phi|^2-v^2)\left[(|\phi|^2-v^2)^2+\frac{\kappa^2}{e^2}C\right] , \quad (27c)$$

where $C=0$ for the broken-phase case (topological soliton solutions only) and $C=-e^2v^4/\kappa^2$ for the unbroken-phase case (nontopological soliton solutions only). Note that we have now an eight-order potential. The total energy is still given by Eq. (22), while the angular momentum formula simplifies as

$$J = -\frac{e^2}{\kappa}\int d^2\mathbf{r}\sqrt{\gamma}\left[(|\phi|^2-v^2)^2+\frac{\kappa^2}{e^2}C\right] . \quad (28)$$

The action (26), with C set to zero, was first obtained by Valtancoli [8]. (But this paper contains a few sign mistakes.)

To analyze the curved-space self-duality equations, particularly convenient is the conformal coordinate system in which $\gamma_{ij}=\rho(\mathbf{r})\delta_{ij}$ and so

$$\sqrt{\gamma}\bar{R} = -\Delta\ln\rho , \quad \sqrt{\gamma}\bar{\nabla}^i\bar{\nabla}_i|\phi|^2=\Delta|\phi|^2 , \quad (29)$$

where Δ is the flat-space Laplacian. Moreover, to re-

move the arbitrariness in K_i associated with the time reparametrization $t \to t' = t + \Lambda(\mathbf{r})$, we adopt the gauge condition

$$\overline{\nabla}_i K^i = 0 . \tag{30}$$

In the conformal coordinates, this condition is equivalent to $\partial_i K_i = 0$, and therefore we may write

$$K_i = \epsilon_{ij} \partial_j U(\mathbf{r}) , \quad \sqrt{\gamma} H = -\Delta U(\mathbf{r}) . \tag{31}$$

Similarly, we may express the vector potential $\overline{A}_i$ as

$$\overline{A}_i = -\frac{1}{2}\epsilon_{ij}\partial_j \mathscr{A}(\mathbf{r}) , \quad \sqrt{\gamma} \overline{B} = \frac{1}{e}\Delta \mathscr{A}(\mathbf{r}) . \tag{32}$$

Using these in Eqs. (13a) and (18), and choosing in particular the value $C = 0$, we can solve the equations for $\phi(\mathbf{r})$ and $\rho(\mathbf{r})$ to obtain

$$\phi(\mathbf{r}) = \exp[\mp \mathscr{A}(\mathbf{r})] f(z) \quad (z \equiv x \pm iy) , \tag{33a}$$

$$\rho(\mathbf{r}) = \left[\frac{|\phi(\mathbf{r})|^2}{|f(z)|^2} \right]^{2\pi G v^2} \exp\{-2\pi G[|\phi(\mathbf{r})|^2 + S(\mathbf{r})^2]\} , \tag{33b}$$

where $f(z)$ can be any finite polynomial in z. The zeros of the function $f(z)$, located arbitrarily in space, can be identified with the positions of the vortices [comparable to the "particle" position $\mathbf{r}_i$ in the metric (1)]. Here note that Eqs. (32) and (33a) allow us to write

$$\rho(\mathbf{r}) \overline{B}(\mathbf{r}) = \mp \frac{1}{2e} \Delta \left[\ln \frac{|\phi(\mathbf{r})|^2}{|f(z)|^2} \right] . \tag{34}$$

Now, if we use Eqs. (33b) and (34) in Eq. (14), we are left with the equation

$$\Delta \ln|\phi|^2 = 2e \left[\frac{|\phi|^2}{|f(z)|^2} \right]^{2\pi G v^2} \exp[-2\pi G(|\phi|^2 + S^2)][(1 - 4\pi G S^2)(e|\phi|^2 + \kappa S - e v^2) + 2\pi G \kappa S^3] , \tag{35}$$

which is valid away from the zeros of $|\phi|^2$. In this way we can reduce the whole problem to the analysis of the two coupled equations involving $|\phi|^2$ and S, i.e., Eqs. (35) and (16). Once $|\phi|^2$ and S are determined, K_i and A_i follow from Eq. (17). In the $\kappa \to \infty$ limit mentioned earlier [i.e., for the system described by the action (26)], they become just one nontrivial equation:

$$\Delta \ln|\phi|^2 = \left[\frac{|\phi|^2}{|f(z)|^2} \right]^{2\pi G v^2} \exp(-2\pi G|\phi|^2) \left[\frac{4e^4}{\kappa^2} |\phi|^2(|\phi|^2 - v^2) - 4\pi G \frac{e^4}{\kappa^2}(|\phi|^2 - v^2)^3 \right] . \tag{36}$$

But, even for the latter case, some numerical analysis appears to be necessary for more detailed information. For discussions on the asymptotic behaviors of the solutions [which are consistent with the point-particle metric (1)], see Ref. [8].

Finally, we would like to add some comments on the stability of our system. A specific concern here is that, since our scalar potential (15) is unbounded from below (for $G > 0$), the vacua assumed in Eq. (21) will be at most local minima classically. But it must be noted that, in the presence of gravity, the definition of energy depends on the asymptotic behaviors of the metric, and so there is no simple way to compare the energies of two different vacua. Furthermore, self-dual systems are generally believed to be the bosonic sector of some extended supersymmetric theories [11,12], and we naturally expect our present system to be related to a certain extended supergravity theory. In the latter framework, the vacuum stability is likely to follow automatically [10].

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