Soft charge form factor of the pion

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We identify the source of a discrepancy between $F_\pi^{\rm soft}|_{\rm long}$ and $F_\pi^{\rm soft}|_{\rm trans}$ found by Isgur and Llewellyn Smith. We find that the discrepancy disappears when virtual $q\bar{q}$ pair production for longitudinal-momentum transfers is included. Our result rectifies the discussion by Isgur and Llewellyn Smith of soft nonperturbative effects at presently available values of Q^2.

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While perturbative QCD (PQCD) [1–5] is expected to be an adequate tool to study the asymptotic $Q \to \infty$ behavior of hard exclusive reactions, doubts have been raised [6, 7] about the applicability of PQCD at presently available Q^2. Currently a strong disagreement continues about a scale of momentum transfers characteristic for the transition from the nonperturbative to perturbative regime. While some argue [8–14] that the transition may take place for Q^2 as low as 4–6 $(\text{GeV}/c)^2$, or at some 15 $(\text{GeV}/c)^2$ [15], others maintain [16, 17] that the transition should take place for much higher values of Q^2, perhaps as high as 100 $(\text{GeV}/c)^2$.

Recently Isgur and Llewellyn Smith [16] (ILS) reexamined their original arguments [7] against the applications of PQCD in exclusive processes at presently available Q^2. The conclusion of ILS is that the applicability of PQCD is ruled out, but the processes in question could be explained by soft, nonperturbative effects. The latter point has been illustrated by the model study of the electromagnetic form factors of the pion and nucleon. Starting with an ansatz for the nonperturbative light-front wave function, ILS have derived formulas for calculations of form factors for both transverse- and longitudinal-momentum transfers. The form factors computed from these formulas should be identical, as the labeling of the axes is arbitrary. ILS concluded that to obtain such an identity within their scheme *"a special (and not obvious) relation"* between the x and p_T dependence of the model wave function is required. The actual function used by ILS yielded nevertheless a large discrepancy between $F^{\rm soft}|_{\rm trans}$ and $F^{\rm soft}|_{\rm long}$. Further, ILS observed that exotic wave functions, such as those proposed by Chernyak and Zhitnitsky [9], result in an even larger discrepancy between $F^{\rm soft}|_{\rm trans}$ and $F^{\rm soft}|_{\rm long}$.

In this Brief Report we remove this discrepancy, which we feel has been plaguing the examples given by ILS and in turn obscuring the main point of ILS about illegal end-point contributions. The discrepancy is traced back to the approximate formula used by ILS to calculate $F^{\rm soft}|_{\rm long}$. In this Brief Report we will derive a rigorous formula, which yields the same results for $F^{\rm soft}|_{\rm long}$ and $F^{\rm soft}|_{\rm trans}$. In view of this derivation we also find that the aforementioned relation between the x and p_T dependence of the light-front wave function is in fact provided by the covariance of the underlying Bethe-Salpeter wave function, and this issue is not relevant for the discrepancy reported by ILS. We hope that our paper will in this way help to expose the main point of ILS.

Our argument follows the results of our earlier work on the relation between the covariant and light-front approaches [18–22]. To make our point clear we consider the following model of a covariant Bethe-Salpeter wave function of a pion, treated as a $q\bar{q}$ system:

$$\Psi_P(k) = \frac{1}{k^2 - m^2 + i\epsilon}\,\frac{1}{(P-k)^2 - m^2 + i\epsilon}, \tag{1}$$

where k^μ is the four-momentum of the first constituent and P^μ is the total four-momentum of the pion. The same point could be made using a more sophisticated Bethe-Salpeter wave function at the cost of a much more complicated algebra, but the purpose of our paper is best served with the simple form given above.

Consider a process when the pion with an initial momentum P^μ absorbs a virtual photon with momentum q^μ and remains intact with a final momentum $P'^\mu = (P+q)^\mu$. We have then $P^2 = P'^2 = M^2$. In what follows we refrain from assuming that the pion was initially at rest; our results will hold for any initial momentum of the pion. The corresponding covariant bound-state current has the form [20]

$$J^\mu(0) = \int d^4k \frac{1}{(k+q)^2 - m^2 + i\epsilon}\,(2k^\mu + q^\mu) \times \frac{1}{k^2 - m^2 + i\epsilon}\,\frac{1}{(P-k)^2 - m^2 + i\epsilon}. \tag{2}$$

The invariant elastic form factor of the pion is then extracted:

$$J^\mu(0) = (2P^\mu + q^\mu)\,F(Q^2), \tag{3}$$

where $Q^2 = -q^2 \geq 0$. To expose the equivalent light-

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front structure of Eq. (2), we introduce the light-front variables $k^{\pm} = \frac{1}{\sqrt{2}}(k^0 \pm k^z), \mathbf{k}_\perp = (k^x, k^y)$, so that $k^2 = 2k^-k^+ - (\mathbf{k}_\perp)^2$, and $d^4k = dk^-dk^+d^2k_\perp$. We then perform the integration over the variable k^- in Eq. (2) analytically, arriving at the following result for the $\mu = +, x, y$ component:

$$J^\mu(0) = -2\pi i\,(I_a + I_b)^\mu, \tag{4}$$

where I_a^μ is given by

$$I_a^\mu = \int_0^{P^+} dk^+\, d^2k_\perp \frac{2k^\mu + q^\mu}{2k^+\, 2(k^+ + q^+)\, 2(P^+ - k^+)} \frac{1}{(P^- + q^-) - \left(q^- + \frac{m^2 + (\mathbf{P}_\perp - \mathbf{k}_\perp)^2}{2(P^+ - k^+)} + \frac{m^2 + \mathbf{k}_\perp^2}{2k^+}\right)}$$
$$\times \frac{1}{(P^- + q^-) - \left(\frac{m^2 + (\mathbf{P}_\perp - \mathbf{k}_\perp)^2}{2(P^+ - k^+)} + \frac{m^2 + (\mathbf{k}_\perp + \mathbf{q}_\perp)^2}{2(k^+ + q^+)}\right)}, \tag{5}$$

and I_b is

$$I_b^\mu = \int_0^{q^+} dk^+\, d^2k_\perp \frac{q^\mu - 2k^\mu}{2k^+\, 2(q^+ - k^+)\, 2(P^+ + k^+)} \frac{1}{(P^- + q^-) - \left(P^- + \frac{m^2 + (\mathbf{q}_\perp - \mathbf{k}_\perp)^2}{2(q^+ - k^+)} + \frac{m^2 + \mathbf{k}_\perp^2}{2k^+}\right)}$$
$$\times \frac{1}{(P^- + q^-) - \left(\frac{m^2 + (\mathbf{P}_\perp + \mathbf{k}_\perp)^2}{2(P^+ + k^+)} + \frac{m^2 + (\mathbf{q}_\perp - \mathbf{k}_\perp)^2}{2(q^+ - k^+)}\right)}. \tag{6}$$

One easily recognizes expressions (5) and (6) as the contributions from the light-front diagrams (a) and (b) of Fig. 1, respectively. All details of the calculation are given in Refs. [20, 21]. The key observation is that the second contribution, Eq. (6), corresponds to a conversion of a photon into a virtual $q\bar{q}$ pair *prior* (in the light front "time" x^+) to its interaction with the bound state. Because of the integration limits, this diagram is relevant only for a kick with a nonvanishing longitudinal-momentum transfer, $q^+ \neq 0$, and does not contribute for a purely transversal kick.

Consider first the case of a purely transversal-momentum transfer: i.e., $q^+ = 0, \mathbf{q}_\perp^2 = Q^2$. The contribution (6) vanishes. We introduce the light-front variables $x_1 = k^+/P^+, x_2 = 1 - x_1$, and remove the manifest $\mathbf{P}_\perp$ dependence from our result upon parametrizing the transversal momenta of quarks in the target in a standard way: $\mathbf{p}_{1\perp} = \mathbf{k}_\perp - x_1\mathbf{P}_\perp$, $\mathbf{p}_{2\perp} = \mathbf{P} - \mathbf{k}_\perp - x_2\mathbf{P}_\perp$. This yields $\mathbf{p}_{1\perp} + \mathbf{p}_{2\perp} = 0$ for any $\mathbf{P}_\perp$, and brings Eq. (5) exactly into the form (for $\mu = +$)

$$I_a^+ = P^+ \int_0^1 dx_1 \int d^2p_{1\perp}\, \psi_\pi(x_i, \tilde{\mathbf{p}}_{i\perp})\, \psi_\pi(x_i, \mathbf{p}_{i\perp}), \tag{7}$$

where we have introduced the $\mathbf{P}$-independent light-front wave function of the pion,

$$\psi_\pi(x_i, \mathbf{p}_{i\perp}) = \frac{1}{\sqrt{x_1 x_2}} \frac{1}{M^2 - \left(\frac{m^2 + \mathbf{p}_{1\perp}^2}{x_1} + \frac{m^2 + \mathbf{p}_{2\perp}^2}{x_2}\right)}, \tag{8}$$

and introduced final-state momenta $\tilde{\mathbf{p}}_{i\perp} = \mathbf{p}_{i\perp} - x_i\mathbf{q}_\perp + \mathbf{q}_\perp$ for the struck quark ($i = 1$), and $\tilde{\mathbf{p}}_{i\perp} = \mathbf{p}_{i\perp} - x_i\mathbf{q}_\perp$ for a spectator quark ($i = 2$), respectively. In this way Eq. (7) coincides fully with the classical Drell-Yan result [23] for an elastic form factor. Finally, a straightforward calculation verifies that the wave function of Eq. (8) appears as the light-front projection of the covariant wave function of Eq. (1), i.e.,

$$\sqrt{x_1 x_2}\, \psi_\pi(x_i, \mathbf{p}_{i\perp}) = \int dk^-\, \Psi_P(k), \tag{9}$$

with the relation between $\mathbf{p}_{i\perp}$ and $\mathbf{k}_\perp$ given above. In this way the Drell-Yan result [23], originally obtained as an approximate expression [up to the order $O(1/P^z)$ within the infinite-momentum frame], is now demonstrated to originate in an underlying covariant structure of the bound-state current, and holds rigorously for any value of the initial momentum of the target, including a target at rest.

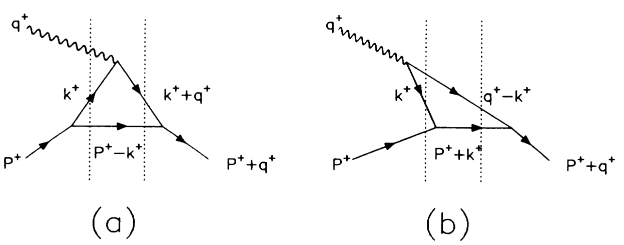

FIG. 1. Light-front representation of the covariant pion current of Eq. (2). (a) valence sector contribution, cf. Eq. (7) and Eq. (11) for transversal- and longitudinal-momentum transfers q^μ, respectively. (b) $q\bar{q}$ pair contribution, cf. Eq. (12) for longitudinal-momentum transfers.

We now turn to the case of purely longitudinal-momentum transfers: $q^+ \neq 0$, $\mathbf{q}_\perp = 0$. In this case we have $q^- = -Q^2/(2\alpha P^+)$, where

$$Q^2 = \frac{\alpha^2}{1+\alpha}\,(M^2 + \mathbf{P}_\perp^2), \tag{10}$$

and $\alpha = q^+/P^+$. The contribution (5) of the diagram (a) takes now the form

$$I_a^+ = P^+ \int_0^1 dx_1 \int d^2 p_{1\perp} \frac{x_1 + \alpha/2}{\sqrt{x_1(x_1+\alpha)}} \times \psi_\pi(y_i, \mathbf{p}_{i\perp})\, \psi_\pi(x_i, \mathbf{p}_{i\perp}), \quad (11)$$

where $y_1 = (x_1+\alpha)/(1+\alpha)$, $y_2 = x_2/(1+\alpha)$ is the fraction of the final pion longitudinal momentum carried by the struck and spectator quarks, respectively. The ordering of vertices in the pair production diagram (b) of Fig. 1 suggests that it should be possible to cast the contribution (6) into a form involving the light-front wave function of the outgoing (but not the incoming) pion. Indeed, after some manipulations we obtain

$$I_b^+ = P^+ \int_{-\alpha}^0 dx_1 \int d^2 p_{1\perp} \frac{x_1 + \alpha/2}{x_1\sqrt{(x_1+\alpha)x_2}}\, \psi_\pi(y_i, \mathbf{p}_{i\perp}) \frac{1}{Q^2/\alpha + \frac{m^2 + (\mathbf{p}_{1\perp} + x_1 \mathbf{P}_\perp)^2}{\alpha + x_1} - \frac{m^2 + (\mathbf{p}_{1\perp} + x_1\mathbf{P}_\perp)^2}{x_1}} \,. \quad (12)$$

The ILS approximation for $F_\pi^{\rm soft}|_{\rm long}$, cf. Eq. (23) of Ref. [16], includes the contribution (11), but neglects the (asymptotically dominant) contribution (12). This is the source of the observed discrepancy between $F_\pi^{\rm soft}|_{\rm long}$ and $F_\pi^{\rm soft}|_{\rm trans}$. We point out that Eq. (7) for transversal-momentum transfer and Eqs. (11) and (12) for longitudinal-momentum transfer are merely different representations of the covariant bound-state current (2). One could alternatively carry out the integration over the k^0 variable in Eq. (2), arriving at six different diagrams constituting the equal-time representation of the bound-state current [21, 24, 25]. In any case the form factor extracted, cf. Eq. (3), is an invariant function of Q^2 and does not depend on particular variables used to carry out the integration, nor the direction of the momentum transfer.

We thus conclude that if all contributions are accounted for, longitudinal- and transversal-momentum transfers yield identical invariant form factors which depend only on Q^2. Our proof was given for the case of the covariant wave function of Eq. (1), for which not only did we verify the validity of the Drell-Yan result, cf. Eq. (7), but we also explicitly evaluated the virtual pair production diagram. For the case of the light-front wave function used by ILS an underlying Bethe-Salpeter function is not known; thus, one is unable to give a corresponding manifest result for the pair production diagram; one is thus limited to the use of the Drell-Yan formula for transversal-momentum transfers. In view of this observation the ILS results [16] for longitudinal-momentum transfers should be disregarded, while the results of their numerical calculations for transversal-momentum transfers are in fact valid for both transversal- and longitudinal-momentum transfers. The same conclusion holds for the nucleon electromagnetic form factors obtained by ILS.

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