Chiral Schwinger model based on the Batalin-Fradkin-Vilkovisky formalism

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We quantize the bosonized chiral Schwinger model by using the systematic Batalin-Fradkin-Vilkovisky formalism. We derive a Becchi-Rouet-Stora-Tyutin gauge-fixed covariant action showing that the auxiliary fields introduced in the formalism turn into the Wess-Zumino scalar.

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I. INTRODUCTION

There has been great progress in understanding the physical meaning of anomalies in quantum field theory through the study of the chiral Schwinger model (CSM) [1–4]. Jackiw and Rajaraman [1] showed that a consistent and unitary quantum field theory is possible even in the gauge-noninvariant formulation. Alternatively, a gauge-invariant version [2–4] can be obtained by adding a Wess-Zumino action to the Lagrangian, as was proposed by Faddeev and Shatashvili [5].

On the other hand, Batalin and Fradkin (BF) [6] have proposed a new kind of quantization procedure for second-class constraint systems. Furthermore, when combined with the Batalin, Fradkin, and Vilkovisky (BFV) [7] formalism for the first-class constraint systems, the BF formalism is particularly powerful for deriving a covariantly gauge-fixed action in the configuration space. Recently, Fujiwara, Igarashi, and Kubo (FIK) [8] have proposed a systematic treatment of anomalous gauge theories based on the BF formalism making a second-class constraint system into a first-class one with BF fields. They have applied this formalism to an anomalous chiral massive U(1) gauge theory in four dimensions [9] and to subcritical bosonic string theories.

On the other hand, several authors [10] had applied the BFV formalism to the CSM. Unfortunately, they did not identify the Wess-Zumino scalar [11] with the BF fields as pointed out by FIK.

In this paper, we apply FIK's improved method based on the BFV formalism to the bosonized CSM. As a pedagogical illustration of this formalism, we quantize the bosonized CSM through the generalized Hamiltonian formalism and derive a Becchi-Rouet-Stora-Tyutin (BRST)-invariant action showing that the auxiliary BF fields turn into the Wess-Zumino scalar through the proper gauge choice. In Sec. II, we briefly review the essence of the systematic BFV formalism. We consider the bosonized CSM for the regularization ambiguity $a=1$ in Sec. III. Through this analysis we expect that we may get insight into the algebraic structure of constraints and symmetry properties in an anomalous chiral U(1) gauge theory in four dimensions because this theory also has four constraints that are fully second class [9]. In Sec. IV, we consider the bosonized CSM with $a>1$, which has two constraints, as an example of different constraint structure. Section V is devoted to a conclusion.

II. THE ESSENCE OF THE BFV FORMALISM

In this section, we summarize the essence of the systematic BFV formalism [7,8], which is applicable for the general theories with first-class constraints. We recapitulate this formalism in terms of a finite number of phase-space variables. This makes the discussion simpler and conclusions more apparent.

First of all, consider a phase space of canonical variables q^i, p_i $(i=1,2,\ldots,n)$ in terms of which the canonical Hamiltonian $H_c(q^i,p_i)$ and the constraints $\Omega_a(q^i,p_i)\approx 0$ $(a=1,2,\ldots,m)$ are given. We assume that the constraints satisfy the constraint algebra [12]

$$[\Omega_a,\Omega_b]=i\Omega_c U^c_{ab}, \quad [H_c,\Omega_a]=i\Omega_b V^b_a , \tag{1}$$

where the structure coefficients U^c_{ab} and V^b_a are generally functions of the canonical variables. We also assume that the constraints are irreducible, which means that an invertible change of variables locally exists such that Ω_a can be identified with the m unphysical momenta.

In order to single out the physical variables, we introduce the additional conditions $\Phi^a(q^i,p_i)\approx 0$ with $|\det[\Phi^a,\Omega_b]|\neq 0$ at least in the vicinity of the constraint surface with $\Phi^a\approx 0$ and $\Omega_a\approx 0$. Then the Φ^a play the role of gauge-fixing functions. That is to say, from the condition of the time stability of the constraints, there exists a family of phase-space trajectories. By selecting one of these trajectories through the conditions $\Phi^a\approx 0$, we can get the $2(n-m)$-dimensional physical phase space described by the canonical variables q^*, p^*. Then, $\Phi^a(q^i,p_i)$ can be identified with the m unphysical coordinates.

The quantum theory of the described dynamical system only depends on q^*, p^* of the physical phase space. Therefore, the partition function is given by

$$\begin{aligned} Z&=\int [dq^i dp_i]\delta(\Omega_a)\delta(\Phi^b)|\det[\Phi^b,\Omega_a]| \\ &\quad\times \exp\left[i\int dx(p\dot{q}-H_c)\right] \\ &=\int [dq^*dp^*]\exp\left\{i\int dx[p^*\dot{q}^*-H_{\text{phys}}(q^*,p^*)]\right\} . \end{aligned} \tag{2}$$

And the constraints $\Omega_a\approx 0$ and $\Phi^a\approx 0$ together with the Hamiltonian equations may be obtained from an action

$$S=\int dt(p_i\dot{q}^i-H_c-\lambda^a\Omega_a+\pi_a\Phi^a) , \tag{3}$$

where λ^a and π_a are the Lagrange multiplier fields canonically conjugate to each other, obeying the commutation relations

$$[\lambda^a, \pi_a] = i\delta_b^a \ . \tag{4}$$

Note that the gauge-fixing conditions generally contain λ^a in the form

$$\Phi^a = \dot{\lambda}^a + \chi^a(q^i, p_i, \lambda^a) \ , \tag{5}$$

where χ^a are the arbitrary functions. And we can see that the Lagrange multipliers λ^a become dynamically active, and π_a serve as their conjugate momenta. This consideration naturally leads to the canonical formalism in an extended phase space.

In order to make the equivalence to the initial theory with constraints in the reduced phase space, we may introduce two sets of canonically conjugate, anticommuting ghost coordinates and momenta $\mathcal{C}^a, \overline{\mathcal{P}}_a$ and $\mathcal{P}^a, \overline{\mathcal{C}}_a$ such that

$$[\mathcal{C}^a, \overline{\mathcal{P}}_b] = [\mathcal{P}^a, \overline{\mathcal{C}}_b] = i\delta_b^a \ . \tag{6}$$

The quantum theory is defined by the extended phase-space functional integral

$$Z_\Psi = \int [dq^i dp_i][d\lambda^a d\pi_a][d\mathcal{C}^a d\overline{\mathcal{P}}_a][d\mathcal{P}^a d\overline{\mathcal{C}}_a] e^{iS_\Psi} \ , \tag{7}$$

where the action is now

$$S_\Psi = \int dt \{ p_i \dot{q}^i + \pi_a \dot{\lambda}^a + \overline{\mathcal{P}}_a \dot{\mathcal{C}}^a + \overline{\mathcal{C}}_a \dot{\mathcal{P}}^a - H_m + i[Q, \Psi] \} \ . \tag{8}$$

Here, the BRST charge Q and the fermionic gauge-fixing function Ψ are defined by

$$\begin{aligned} Q &= \mathcal{C}^a \Omega_a - \tfrac{1}{2} \mathcal{C}^b \mathcal{C}^c U_{cb}^a \overline{\mathcal{P}}_a + \mathcal{P}^a \pi_a \ , \\ \Psi &= \overline{\mathcal{C}}_a \chi^a + \overline{\mathcal{P}}_a \lambda^a \ , \end{aligned} \tag{9}$$

respectively. H_m is the BRST-invariant Hamiltonian, which one calls the minimal Hamiltonian,

$$H_m = H_c + \overline{\mathcal{P}}_a V_b^a \mathcal{C}^b \ . \tag{10}$$

The measure in Z_Ψ is the Liouville measure on the covariant phase space. Furthermore, if we choose the fermionic gauge-fixing function Ψ properly [4,13], we can obtain a manifestly covariant expression. And the equivalence of the dimensionality $2n+6m$ in the extended phase space, including the canonical ghost variables, to the original dimensionality $2n-2m$ in the reduced phase space can be seen by identifying the ghost variables with the negative-dimensional canonical degrees of freedom, which is suggested by the original work of Parisi and Sourlas concerned with the super-rotation Osp(1,1|2) in the extended phase space [14].

Although we have only discussed first-class-constraint systems up to now, we may face the problem that a certain theory has not a first-class but rather a second-class constraint system. For such a theory, however, we can make the system first class by introducing BF fields.

The first-class Hamiltonian with BF fields is achieved by solving some kinds of the coupled differential equations, which can be obtained by the requirement of the time stability of new constraints, as long as the solution exists. And then, with the first-class constraint system, the procedure is straightforward. In the next section, we will show the concrete analysis through the CSM with $a=1$.

III. CSM IN THE CASE $a=1$

First consider the CSM in the case of $a=1$, which gives the algorithm for the application of the BFV formalism with BF fields. These new fields, which recover a local symmetry, were originally proposed by Stueckelberg [15] in the theory of massive vector fields. Furthermore, the CSM with BF fields shows how the BFV formalism is used to find the effective covariant action. In particular, it is very interesting to formulate the CSM with $a=1$ in terms of the BFV formalism because the four constraint structures resemble those of four-dimensional anomalous theory.

We start with the following Lagrangian density of the bosonized CSM [2–4] with $a=1$:

$$\begin{aligned} \mathcal{L} = &-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} e^2 A_\mu A^\mu + \tfrac{1}{2} \partial_\mu \phi \partial^\mu \phi \\ &+ e A_\nu (\eta^{\mu\nu} - \epsilon^{\mu\nu}) \partial_\mu \phi \ , \end{aligned} \tag{11}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $\epsilon^{01} = -\epsilon_{01} = 1$, and $\eta_{\mu\nu} = \mathrm{diag}(1,-1)$. The canonical momenta corresponding to A_0, A_1, and ϕ are

$$\begin{aligned} \Pi^0 &= \frac{\delta \mathcal{L}}{\delta \dot{A}_0} \approx 0 \ , \\ \Pi^1 &= \frac{\delta \mathcal{L}}{\delta \dot{A}_1} = \partial^1 A^0 - \dot{A}^1 \ , \\ \Pi_\phi &= \frac{\delta \mathcal{L}}{\delta \dot{\phi}} = \dot{\phi} + e(A^0 + A^1) \ , \end{aligned} \tag{12}$$

where the overdot represents a time derivative and ∂^1 a partial spatial derivative. Performing the Legendre transformation, we obtain the primary Hamiltonian

$$\begin{aligned} H_p &= \int dx (\Pi_1 \dot{A}^1 + \Pi_\phi \dot{\phi} - \mathcal{L}) \ , \\ &= \int dx [\tfrac{1}{2}(\Pi^1)^2 + \tfrac{1}{2}(\Pi_\phi)^2 + \tfrac{1}{2}(\partial_1 \phi)^2 \\ &\qquad - e(\Pi_\phi + \partial_1 \phi)(A^0 + A^1) - A^0 \partial_1 \Pi^1 + e^2 A^0 A^1 \\ &\qquad + e^2 (A^1)^2] \ , \end{aligned} \tag{13}$$

and obtain the primary constraint $\Omega_1 \equiv \Pi^0 \approx 0$. Thus the canonical Hamiltonian is given by

$$H_c = H_p + \int dx \, u \Omega_1 \ , \tag{14}$$

where u is an undetermined multiplier field. For the stability of the primary constraint with respect to the time evolution, we require the secondary constraints, and we get

$$\begin{aligned} \Omega_2 &\equiv \partial_1 \Pi^1 + e(\Pi_\phi + \partial_1 \phi) - e^2 A^1 \ , \\ \Omega_3 &\equiv -\Pi^1 \ , \\ \Omega_4 &\equiv e(\Pi_\phi + \partial_1 \phi) - e^2 A_0 - 2e^2 A^1 \ . \end{aligned} \tag{15}$$

On the other hand, we could fix the multiplier field u from $\dot{\Omega}_4=0$ as follows:

$$u=\frac{1}{e}(-\partial_1\partial^1\phi+\partial_1\Pi_\phi)-2\partial_1 A^1+2\Pi^1 . \qquad (16)$$

These four constraints are fully second class, and the nonvanishing equal-time commutators are given by

$$\begin{aligned}
&[\Omega_1(x),\Omega_4(y)]=ie^2\delta(x-y) ,\\
&[\Omega_2(x),\Omega_3(y)]=-ie^2\delta(x-y) ,\\
&[\Omega_3(x),\Omega_4(y)]=2ie^2\delta(x-y) ,\\
&[\Omega_4(x),\Omega_4(y)]=2ie^2\partial_1\delta(x-y) .
\end{aligned} \qquad (17)$$

In order to make these constraints first class in the phase space [2,5], we introduce four auxiliary fields θ_1, θ_2, Π_{θ_1}, and Π_{θ_2}, which are called the BF fields, such that

$$\begin{aligned}
&[\theta_1(x),\Pi_{\theta_1}(y)]=i\delta(x-y) ,\\
&[\theta_2(x),\Pi_{\theta_2}(y)]=i\delta(x-y) ,
\end{aligned} \qquad (18)$$

and we then have the extended phase space. Mixing the constraints with θ_1, θ_2, Π_{θ_1}, and Π_{θ_2}, we could easily find the following first-class constraints in the extended phase space,

$$\begin{aligned}
&\tilde{\Omega}_1=\Omega_1-e\theta_1 ,\\
&\tilde{\Omega}_2=\Omega_2+e\theta_2 ,\\
&\tilde{\Omega}_3=\Omega_3-e\theta_1+e\Pi_{\theta_2} ,\\
&\tilde{\Omega}_4=\Omega_4+e(\Pi_{\theta_1}-\partial_1\theta_1)+e\theta_2 .
\end{aligned} \qquad (19)$$

On the other hand, using Eq. (14) with Eq. (15), we find that the involutional relations between the canonical Hamiltonian and the constraints are

$$\begin{aligned}
&[\Omega_1,H_c]=i\Omega_2 ,\\
&[\Omega_2,H_c]=2ie^2\Omega_1-ie^2\Omega_3 ,\\
&[\Omega_3,H_c]=-2i\partial_1\Omega_1+i\Omega_4 ,\\
&[\Omega_4,H_c]=i(2\partial_1\partial^1+4e^2)\Omega_1 .
\end{aligned} \qquad (20)$$

In order to make the new constraints in Eq. (19) consistent with the time evolution of the system, we require Eq. (20) to be preserved in the extended phase space, i.e.,

$$\begin{aligned}
&[\tilde{\Omega}_1,\tilde{H}_c]=i\tilde{\Omega}_2 ,\\
&[\tilde{\Omega}_2,\tilde{H}_c]=2ie^2\tilde{\Omega}_1-ie^2\tilde{\Omega}_3 ,\\
&[\Omega_3,\tilde{H}_c]=-2i\partial_1\tilde{\Omega}_1+i\tilde{\Omega}_4 ,\\
&[\tilde{\Omega}_4,\tilde{H}_c]=i(2\partial_1\partial^1+4e^2)\tilde{\Omega}_1 .
\end{aligned} \qquad (21)$$

Furthermore, the change of the constraint structure requires the modification of the Hamiltonian. Thus, a new Hamiltonian in the extended phase space should be constructed by adding some polynomials $H_{\rm BF}(\theta_1,\theta_2,\Pi_{\theta_1},\Pi_{\theta_2})$ of the BF fields to the Hamiltonian (4). That is to say, we should solve the following relations:

$$[\tilde{\Omega}_i,\tilde{H}_c]=[\tilde{\Omega}_i,H_c+H_{\rm BF}]=f(\tilde{\Omega}_i), \quad i=1,2,3,4 . \qquad (22)$$

For example, in the case of $i=1$,

$$\begin{aligned}
[\tilde{\Omega}_1,\tilde{H}_c]&=[\Omega_1,H_c]+[-e\theta_1,H_{\rm BF}]\\
&=i\Omega_2+ie\theta_2=i\tilde{\Omega}_2 .
\end{aligned} \qquad (23)$$

Thus we get

$$-i\frac{\delta H_{\rm BF}}{\delta\Pi_{\theta_1}}=i\theta_2 . \qquad (24)$$

Similarly, for the other cases, we also obtain some kind of coupled differential equation. After some tedious calculations, the solutions give the desired Hamiltonian, which contains θ_1, θ_2, Π_{θ_1}, and Π_{θ_2} variables in the extended phase space,

$$\begin{aligned}
\tilde{H}_c=H_c-\int dx\,[&\theta_2\Pi_{\theta_1}+\tfrac{1}{2}e^2(\Pi_{\theta_1})^2+e^2\theta_1\Pi_{\theta_2}+\theta_2\partial_1\theta_1\\
&-(\partial_1\theta_1)^2-\tfrac{3}{2}e^2(\theta_1)^2] .
\end{aligned} \qquad (25)$$

Then, the modified system described by Eqs. (19) and (25) is first class, and we can apply the BFV algorithm to this modified system in the extended phase space.

We introduce four canonical sets of ghost and antighost fields along with auxiliary fields as follows:

$$(\mathcal{C}^i,\bar{\mathcal{P}}_i),\quad (\mathcal{P}^i,\bar{\mathcal{C}}_i),\quad (N^i,B_i) , \qquad (26)$$

with

$$\begin{aligned}
&[N^i(x),B_j(y)]=i\delta^i_j\delta(x-y) ,\\
&[\mathcal{C}^i(x),\bar{\mathcal{P}}_j(y)]=i\delta^i_j\delta(x-y) ,\\
&[\mathcal{P}^i(x),\bar{\mathcal{C}}_j(y)]=i\delta^i_j\delta(x-y) ,
\end{aligned} \qquad (27)$$

where $i,j=1,2,3,4$. From the systematic BFV formalism, the nilpotent BRST charge Q and the fermionic gauge-fixing function Ψ are given by

$$\begin{aligned}
Q=\int dx\,[&B_1\mathcal{P}^1+B_2\mathcal{P}^2+B_3\mathcal{P}^3+B_4\mathcal{P}^4+\mathcal{C}^1\tilde{\Omega}_1+\mathcal{C}^2\tilde{\Omega}_2\\
&+\mathcal{C}^3\tilde{\Omega}_3+\mathcal{C}^4\tilde{\Omega}_4] ,
\end{aligned} \qquad (28)$$

$$\begin{aligned}
\Psi=\int dx\,[&\bar{\mathcal{C}}_1\chi^1+\bar{\mathcal{C}}_2\chi^2+\bar{\mathcal{C}}_3\chi^3+\bar{\mathcal{C}}_4\chi^4+\bar{\mathcal{P}}_1N^1+\bar{\mathcal{P}}_2N^2\\
&+\bar{\mathcal{P}}_3N^3+\bar{\mathcal{P}}_4N^4] ,
\end{aligned} \qquad (29)$$

where we choose gauge-fixing conditions as follows:

$$\chi^1=A_0,\quad \chi^2=\partial_1A^1+\frac{\alpha}{2}B_2,\quad \chi^3=\frac{1}{\beta}\theta_2,\quad \chi^4=\frac{1}{\gamma}\Pi_{\theta_2} . \qquad (30)$$

Here α, β, and γ are arbitrary parameters. It will be proved later that these are proper gauge conditions, which is crucial for the identification of the BF fields with the Wess-Zumino scalar. The BRST-invariant Hamiltonian takes the form

$$\begin{aligned}
H_m=\tilde{H}_c+\int dx\,[&2e^2\bar{\mathcal{P}}_1\mathcal{C}^2+2\bar{\mathcal{P}}_1\partial_1\mathcal{C}^3+\mathcal{P}_1(2\partial_1\partial^1+4e^2)\mathcal{C}^4\\
&+\bar{\mathcal{P}}_2\mathcal{C}^1-e^2\bar{\mathcal{P}}_3\mathcal{C}^2-\bar{\mathcal{P}}_4\mathcal{C}^3] .
\end{aligned} \qquad (31)$$

The BRST charge Q, the fermionic gauge-fixing function Ψ, and the minimal Hamiltonian H_m satisfy the following relations:

$$i[Q,H_m]=0\ ,$$
$$Q^2=[Q,Q]=0\ , \qquad (32)$$
$$[[\Psi,Q],Q]=0\ ,$$

where they are the conditions of the physical subspace being imposed.

We are now ready to derive the covariant effective action. The action is given by

$$S_{\rm eff}=\int d^2x[\Pi_0\dot{A}^0+\Pi_1\dot{A}^1+\Pi_\phi\dot{\phi}+\Pi_{\theta_1}\dot{\theta}_1+\Pi_{\theta_2}\dot{\theta}_2+B_2\dot{N}^2+B_3\dot{N}^3+B_4\dot{N}^4+\overline{\mathcal{P}}_1\dot{\mathcal{C}}^1+\overline{\mathcal{P}}_2\dot{\mathcal{C}}^2+\overline{\mathcal{P}}_3\dot{\mathcal{C}}^3$$
$$+\overline{\mathcal{P}}_4\dot{\mathcal{C}}^4+\overline{\mathcal{C}}_2\dot{\mathcal{P}}^2+\overline{\mathcal{C}}_3\dot{\mathcal{P}}^3+\overline{\mathcal{C}}_4\dot{\mathcal{P}}^4]-H_{\rm total}\ , \qquad (33)$$

where $H_{\rm total}=H_m-i[Q,\Psi]$. Note that we could suppress the term $\int d^2x(B_1\dot{N}^1+\overline{\mathcal{C}}_1\dot{\mathcal{P}}^1)=-i[Q,\int d^2x\ \overline{\mathcal{C}}_1\dot{N}^1]$ in the Legendre transformation by replacing χ^1 with $\chi^1+\dot{N}^1$. The generating functional is

$$Z=\int[d\mu]\exp(iS_{\rm eff})\ , \qquad (34)$$

where $[d\mu]$ is the Liouville measure of the extended phase space

$$[d\mu]=[d\phi][d\Pi_\phi]\prod_{i=0}^{1}[dA^i][d\Pi_i]\prod_{j=1}^{2}[d\theta_j][d\Pi_{\theta_j}]\prod_{k=1}^{4}[dN^k][dB_k][d\mathcal{C}^k][d\overline{\mathcal{P}}_k][d\mathcal{P}^k][d\overline{\mathcal{C}}_k]\ , \qquad (35)$$

and a normalization factor in Eq. (34) is understood.

In order to derive the covariant action, we first need to eliminate N^1, B_1, B_3, B_4, $\mathcal{P}^1$, $\overline{\mathcal{C}}_1$, $\mathcal{P}^3$, $\overline{\mathcal{C}}_3$, $\mathcal{P}^4$, $\overline{\mathcal{C}}_4$, $\mathcal{C}^1$, $\overline{\mathcal{P}}_1$, $\mathcal{C}^3$, $\overline{\mathcal{P}}_3$, $\mathcal{C}^4$, $\overline{\mathcal{P}}_4$, θ_2, Π_{θ_2}, A_0, and Π_0 by Gaussian integration. After integration, we take the limits $\beta,\gamma\to 0$. Then, the covariant effective action is

$$S_{\rm eff}=\int d^2x\left[\Pi_1\dot{A}^1+\Pi_\phi\dot{\phi}+\Pi_\theta\dot{\theta}+B\dot{N}^2+\overline{\mathcal{P}}\dot{\mathcal{C}}+\overline{\mathcal{C}}\dot{\mathcal{P}}-\tfrac{1}{2}(\Pi^1)^2-\tfrac{1}{2}(\Pi_\phi)^2-\tfrac{1}{2}(\partial_1\phi)^2+e(\Pi_\phi+\partial_1\phi)A^1\right.$$
$$-e^2(A^1)^2-(\partial_1\theta)^2-\theta(\partial_1\Pi_\phi-\partial_1\partial^1\phi)+2e\theta\partial_1A^1-2e\theta\Pi^1-\tfrac{1}{2}e^2(\Phi_\theta)^2-\tfrac{3}{2}e^2\theta^2$$
$$-N^2(\partial_1\Pi^1+e\Pi_\phi+e\partial_1\phi-e^2A^1)+N^3(e\theta+\Pi^1)-N^4(e\Pi_\phi+e\partial_1\phi-2e^2A^1+e\Pi_\theta-e\partial_1\theta)$$
$$\left.-B\left[\partial_1A^1+\frac{\alpha}{2}B\right]-\partial_1\overline{\mathcal{C}}\partial^1\mathcal{C}-\overline{\mathcal{P}}\mathcal{P}\right]\ , \qquad (36)$$

with $\theta_1\equiv\theta$, $\Pi_{\theta_1}\equiv\Pi_\theta$, $B_2\equiv B$, $\overline{\mathcal{C}}_2\equiv\overline{\mathcal{C}}$, $\mathcal{C}^2\equiv\mathcal{C}$, $\mathcal{P}^2\equiv\mathcal{P}$, and $\overline{\mathcal{P}}_2\equiv\overline{\mathcal{P}}$.

Second, the variations with respect to Π^1, Π_θ, Π_ϕ, $\mathcal{P}$, and $\overline{\mathcal{P}}$ in Eq. (35) yield

$$\Pi^1=\partial_0A_1+\partial_1N^2-2e\theta+N^3\ ,$$
$$\Pi_\phi=\partial_0\phi+e(A^1-N^2)+\partial_1\theta-eN^4\ ,$$
$$N_4=\frac{1}{e}\partial_0\theta\ , \qquad (37)$$
$$\mathcal{P}=\dot{\mathcal{C}},\quad \overline{\mathcal{P}}=-\dot{\overline{\mathcal{C}}}\ .$$

Finally, if we substitute Eq. (37) into the action (36), and identify N^2 with $-A_0$, which should not be confused with the original A_0, we obtain the covariant effective action

$$S_{\rm eff}=\int d^2x[-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}e^2A_\mu A^\mu+\tfrac{1}{2}\partial_\mu\Phi\partial^\mu\Phi$$
$$+eA_\nu(\eta^{\mu\nu}-\epsilon^{\mu\nu})\partial_\mu\Phi+e\epsilon^{\mu\nu}(\partial_\mu\theta)A_\nu$$
$$+A^\mu\partial_\mu B-\tfrac{1}{2}\alpha(B)^2-\partial_\mu\overline{\mathcal{C}}\partial^\mu\mathcal{C}]\ , \qquad (38)$$

where we have redefined the fields ϕ,θ into $\Phi\equiv\phi-\theta$, and used the relation $N^3=e\theta$. This action is invariant under the BRST transformation

$$\delta_BA_\mu=-\frac{1}{e}\lambda\partial_\mu\mathcal{C},\quad \delta_B\theta=-\lambda\mathcal{C},\quad \delta_B\Phi=-\lambda\mathcal{C}\ ,$$
$$\delta_B\mathcal{C}=0,\quad \delta_B\overline{\mathcal{C}}=-\lambda B,\quad \delta_BB=0\ , \qquad (39)$$

where λ is a constant Grassmann parameter.

By applying the systematic BFV formulation, we have shown that the Wess-Zumino term for the case $a=1$ in the CSM naturally appears in the effective action.

IV. CSM IN THE CASE $a>1$

In this section, we consider the CSM Lagrangian density in the case of $a>1$ as follows:

$$\mathcal{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}ae^2A_\mu A^\mu+\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi$$
$$+eA_\nu(\eta^{\mu\nu}-\epsilon^{\mu\nu})\partial_\mu\phi\ . \qquad (40)$$

The canonical momenta corresponding to A_0, A_1, and ϕ are

$$\Pi^0\approx 0\ ,$$
$$\Pi^1=F^{10}=\partial^1A^0-\dot{A}^1\ , \qquad (41)$$
$$\Pi_\phi=\dot{\phi}+e(A^0+A^1)\ ,$$

and the primary Hamiltonian is

$$H_p=\int dx(\Pi_1\dot{A}^1+\Pi_\phi\dot{\phi}-\mathcal{L}) ,$$
$$=\int dx\{\tfrac{1}{2}(\Pi^1)^2+\tfrac{1}{2}(\Pi_\phi)^2+\tfrac{1}{2}(\partial_1\phi)^2 -e(\Pi_\phi+\partial_1\phi)(A^0+A^1)-A^0\partial_1\Pi^1 -\tfrac{1}{2}ae^2[(A^0)^2-(A^1)^2]+\tfrac{1}{2}e^2(A^0+A^1)^2\} . \quad (42)$$

The primary constraint is $\Omega_1\equiv\Pi^0\approx 0$, and its time stability gives the only secondary constraint

$$\Omega_2=\partial_1\Pi^1+e(\Pi_\phi+\partial_1\phi)-e^2A^1+(a-1)e^2A^0 . \quad (43)$$

The constraints Ω_1,Ω_2 are second class, and the multiplier field v is fixed as follows:

$$v=-\partial_1A^1-\frac{1}{a-1}\Pi^1 . \quad (44)$$

Then, the canonical Hamiltonian is given by

$$H_c=H_p-\int dx\left[\partial_1A^1+\frac{1}{a-1}\Pi^1\right]\Pi^0 , \quad (45)$$

and the relations between the canonical Hamiltonian and the constraints are

$$[\Omega_1,H_c]=i\Omega_2 ,$$
$$[\Omega_2,H_c]=-i\left[\partial_1\partial^1+\frac{e^2}{a-1}\right]\Omega_1 . \quad (46)$$

As before, we introduce two BF fields θ,Π_θ, and then the modified constraint algebra forms a first-class system with the following constraints:

$$\tilde{\Omega}_1=\Omega_1+e(a-1)\theta ,$$
$$\tilde{\Omega}_2=\Omega_2+e\Pi_\theta . \quad (47)$$

And also, requiring the constraint equations in the extended phase space to be maintained, we can construct the first-class Hamiltonian so as to have the involutional relations with $\tilde{\Omega}_1$ and $\tilde{\Omega}_2$. For the constraint $\tilde{\Omega}_1$ one should add $\int dx[1/2(a-1)](\Pi_\theta)^2$ to H_c. Similarly, the constraint $\tilde{\Omega}_2$ can be made consistent by adding another term

$$\int dx[\tfrac{1}{2}e^2\theta^2+\tfrac{1}{2}(a-1)(\partial_1\theta)^2] .$$

As a result, we obtain the following first-class Hamiltonian:

$$\tilde{H}_c=H_c+\int dx\left[\frac{1}{2(a-1)}(\Pi_\theta)^2+\tfrac{1}{2}e^2\theta^2 +\tfrac{1}{2}(a-1)(\partial_1\theta)^2\right] . \quad (48)$$

According to the BFV formalism in the extended phase space, let us introduce the ghosts and antighosts along with auxiliary fields as follows:

$$(\mathcal{C}^i,\bar{\mathcal{P}}_i),\ (\mathcal{P}^i,\bar{\mathcal{C}}_i),\ (N^i,B_i) , \quad (49)$$

where $i=1,2$. The nilpotent BRST charge Q, the fermionic gauge-fixing function Ψ, and the minimal Hamiltonian H_m are

$$Q=\int dx[\mathcal{C}^1\tilde{\Omega}_1+\mathcal{C}^2\tilde{\Omega}_2+\mathcal{P}^1B_1+\mathcal{P}^2B_2] ,$$
$$\Psi=\int dx[\bar{\mathcal{C}}_1\chi^1+\bar{\mathcal{C}}_2\chi^2+\bar{\mathcal{P}}_1N^1+\bar{\mathcal{P}}_2N^2] , \quad (50)$$
$$H_m=\tilde{H}-\int dx\left[\bar{\mathcal{P}}_2\mathcal{C}^2+\bar{\mathcal{P}}_1\partial_1\partial^1\mathcal{C}^2-\frac{e^2}{a-1}\bar{\mathcal{P}}_1\mathcal{C}^2\right] ,$$

where $\chi^1=A^0$, $\chi^2=\partial_1A^1+(\alpha/2)B_2$, and α is an arbitrary parameter. Then, the effective action is given by

$$S_{\text{eff}}=\int d^2x[\Pi_0\dot{A}^0+\Pi_1\dot{A}^1+\Pi_\phi\dot{\phi}+\Pi_\theta\dot{\theta}+B_2\dot{N}^2 +\bar{\mathcal{P}}_1\dot{\mathcal{C}}^1+\bar{\mathcal{P}}_2\dot{\mathcal{C}}^2+\bar{\mathcal{C}}_2\dot{\mathcal{P}}^2]-H_{\text{total}} , \quad (51)$$

where $H_{\text{total}}=H_m-i[Q,\Psi]$, and also $B_1\dot{N}+\bar{\mathcal{C}}_1\dot{\mathcal{P}}^1$ terms could be suppressed as in Sec. III. The fields B_1, N^1, $\bar{\mathcal{C}}_1$, $\mathcal{P}^1$, $\bar{\mathcal{P}}_1$, $\mathcal{C}^1$, A^0 are eliminated, and integration of Π_0 gives the δ functional through Gaussian integration. Then we obtain

$$S_{\text{eff}}=\int d^2x\Big[\Pi_1\dot{A}^1+\Pi_\phi\dot{\phi}+\Pi_\theta\dot{\theta}+B\dot{N}+\bar{\mathcal{P}}\dot{\mathcal{C}}+\bar{\mathcal{C}}\dot{\mathcal{P}}-\tfrac{1}{2}(\Pi^1)^2-\tfrac{1}{2}(\Pi_\phi)^2-\tfrac{1}{2}(\partial_1\phi)^2+e(\Pi_\phi+\partial_1\phi)A^1 -\tfrac{1}{2}ae^2(A^1)^2-\tfrac{1}{2}e^2(A^1)^2-e(a-1)\theta\partial_1A^1-e\theta\Pi^1-\frac{1}{2(a-1)}(\Pi_\theta)^2-\tfrac{1}{2}e^2\theta^2 -\tfrac{1}{2}(a-1)(\partial_1\theta)^2-N\partial_1\Pi^1-eN(\Pi_\phi+\partial_1\phi)+e^2NA^1-eN\Pi_\theta-B(\partial_1A^1+\tfrac{1}{2}\alpha B)-\partial_1\bar{\mathcal{C}}\partial^1\mathcal{C}-\bar{\mathcal{P}}\mathcal{P}\Big] , \quad (52)$$

with $N^2\equiv N$, $B_2\equiv B$, $\bar{\mathcal{C}}_2\equiv\bar{\mathcal{C}}$, $\mathcal{C}^2\equiv\mathcal{C}$, $\bar{\mathcal{P}}_2\equiv\bar{\mathcal{P}}$, and $\mathcal{P}^2\equiv\mathcal{P}$. Using the variations of Π^1, Π_θ, Π_ϕ, $\mathcal{P}$, and $\bar{\mathcal{P}}$, we obtain the following relations:

$$\Pi^1=\dot{A}_1-e\theta+\partial_1N ,$$
$$\Pi_\theta=(a-1)(\dot{\theta}-eN) ,$$
$$\Pi_\phi=\dot{\phi}+eA^1-eN , \quad (53)$$
$$\bar{\mathcal{P}}=-\dot{\bar{\mathcal{C}}},\ \mathcal{P}=\dot{\mathcal{C}} ,$$

and identifying $N=-A^0$, we finally get the covariant effective action

$$S_{\rm eff}=\int d^2x\{-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}e^2aA_\mu A^\mu+\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi+eA_\nu(\eta^{\mu\nu}-\epsilon^{\mu\nu})\partial_\mu\phi+\tfrac{1}{2}(a-1)\partial_\mu\theta\partial^\mu\theta$$
$$+eA_\nu[\epsilon^{\mu\nu}+(a-1)\eta^{\mu\nu}]\partial_\mu\theta+A^\mu\partial_\mu B-\tfrac{1}{2}\alpha(B)^2-\partial_\mu\bar{\mathcal{C}}\partial^\mu\mathcal{C}\}\ , \tag{54}$$

which is invariant under the BRST transformation

$$\delta_B A_\mu=-\frac{1}{e}\lambda\partial_\mu\mathcal{C},\quad \delta_B\phi=\lambda\mathcal{C},\quad \delta_B\theta=-\mathcal{C}\ ,$$
$$\delta_B\mathcal{C}=0,\quad \delta_B\bar{\mathcal{C}}=-B,\quad \delta_B B=0\ . \tag{55}$$

In Eq. (54) we see that the auxiliary BF field θ is exactly the well-known Wess-Zumino scalar [2–4].

V. CONCLUSION

The generalized Hamiltonian formalism of Batalin and Fradkin for quantization of systems with second-class constraints is based on the idea that any second-class constraint can be made effectively a first class one in an extended phase space. On the other hand, in the Lagrangian formalism it has been realized that a broken gauge symmetry can be restored by extending the original configuration space. With this point of view, the derived covariant effective actions (38) and (54) of the bosonized CSM with $a=1$ and $a>1$ are the BRST gauge-fixed versions in the extended phase space, showing that BF fields turn into the Wess-Zumino scalar.

Following the BFV formalism, we obtained the BRST-invariant action, which is equivalent to the original anomalous action. Although the BRST-invariant action is superficially different from the original one, the physical contents are the same. That is a merit of the BFV formalism. It exhibits the freedom of gauge fixing and we may analyze the anomalous gauge theory in terms of a BRST-invariant theory. This is related to the renormalization program, although we did not discuss this issue. For $a=1$ in our analysis, the constraint structures resemble those of four-dimensional chiral gauge theory. Thus further study in this direction may be interesting.

In order to successfully carry out this program, we have pointed out the importance of the gauge choice, and have stressed the use of the minimal number of BF fields. We may add that even if one takes BF fields different from ours, one may obtain effective covariant actions like Eqs. (38) and (54).

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[1] R. Jackiw and R. Rajaraman, Phys. Rev. Lett. **54**, 1219 (1985); **54**, 2060(E) (1985).

[2] R. Rajaraman, Phys. Lett. **162B**, 148 (1985); H. O. Girotti, H. J. Rothe, and K. D. Rothe, Phys. Rev. D **33**, 514 (1986); **34**, 592 (1986); N. K. Falck and G. Kramer, Ann. Phys. (N.Y.) **176**, 330 (1987).

[3] I. Halliday, E. Rabinovici, A. Schwimmer, and M. Chanowitz, Nucl. Phys. **B268**, 413 (1986).

[4] O. Babelon, F. Schaposnik, and C. Viallet, Phys. Lett. B **177**, 385 (1986); K. Harada and I. Tsutsui, *ibid.* **183**, 311 (1987); S. Miyake and K. Shizuya, Phys. Rev. D **36**, 3781 (1987).

[5] L. D. Faddeev and S. S. Shatashvili, Phys. Lett. **167B**, 225 (1986).

[6] I. A. Batalin and E. S. Fradkin, Nucl. Phys. **B279**, 514 (1987); Phys. Lett. B **180**, 157 (1986).

[7] E. S. Fradkin and G. A. Vilkovisky, Phys. Lett. **55B**, 224 (1975); I. A. Batalin and G. A. Vilkovisky, *ibid.* **69B**, 309 (1977).

[8] T. Fujiwara, Y. Igarashi, and J. Kubo, Nucl. Phys. **B341**, 695 (1990).

[9] R. Rajaraman, Phys. Lett. B **184**, 369 (1987); S. Miyake and K. Shizuya, Mod. Phys. Lett. A **4**, 2675 (1989); F. S. Otto, H. J. Rothe, and A. Recknagel, Phys. Rev. D **42**, 1203 (1990); F. S. Otto, *ibid.* **43**, 548 (1991); J.-G. Zhou, C.-Y Xiao, and Y.-Y. Liu, *ibid.* **45**, 705 (1992).

[10] M. Moshe and Y. Oz, Phys. Lett. B **224**, 145 (1989); P. P. Srivastava, *ibid.* **235**, 287 (1990).

[11] J. Wess and B. Zumino, Phys. Lett. **37B**, 95 (1971); E. Witten, Nucl. Phys. **B233**, 422 (1983).

[12] To define the equal-time supercommutation relations, we simply replace the super-Poisson brackets $\{A,B\}$ by the supercommutator $-i[A,B]$ defined as $[A,B]=AB-BA(-1)^{\epsilon(A)\epsilon(B)}$, where $\epsilon(A)$ is the Grassmann parity of A.

[13] M. Henneaux, Phys. Rep. **126**, 1 (1985).

[14] G. Parisi and N. Sourlas, Phys. Rev. Lett. **43**, 774 (1979); H. Aratyn, R. Ingermanson, and A. J. Niemi, Nucl. Phys. **B307**, 157 (1988); A. J. Niemi, Phys. Rev. D **36**, 3731 (1987).

[15] E. G. Stueckelberg, Helv. Phys. Acta. **30**, 209 (1957).