Faddeev-Popov ghosts and (1+1)-dimensional black-hole evaporation

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Recently Callan, Giddings, Harvey, and the author derived a set of one-loop semiclassical equations describing black-hole formation and/or evaporation in two-dimensional dilaton gravity conformally coupled to N scalar fields. These equations were subsequently used to show that an incoming matter wave develops a black-hole-type singularity at a critical value ϕ_{cr} of the dilaton field. In this paper a modification to these equations arising from the Faddeev-Popov determinant is considered and shown to have dramatic effects for $N<24$, in which case ϕ_{cr} becomes complex. The $N<24$ equations are solved along the leading edge of an incoming matter shock wave and found to be nonsingular. The shock wave arrives at future null infinity in a zero-energy state, gravitationally cloaked by negative-energy Hawking radiation. Static black-hole solutions supported by a radiation bath are also studied. The interior of the event horizon is found to be nonsingular and asymptotic to de Sitter space for $N<24$, at least for sufficiently small mass. It is noted that the one-loop approximation is *not* justified by a small parameter for small N. However an alternate theory (with different matter content) is found for which the same equations arise to leading order in an adjustable small parameter.

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In recent work [1] it was proposed that two-dimensional dilaton gravity coupled to conformal matter is a useful and simple model in which progress might be made in unraveling the mysteries associated with black-hole evaporation [2]. It was shown that the process of black-hole formation and/or evaporation, in an approximation which includes one-loop matter effects and treats gravity semiclassically, can be described by a set of partial differential equations which incorporate the back reaction of Hawking radiation on the geometry. It was further pointed out that this approximation is formally exact in a limit as the number N of matter fields is taken to infinity.

It was subsequently shown [3,4] that in the large-N approximation a collapsing matter wave forms a black hole containing a singularity.[1] This singularity no longer occurs at the value $\phi=\infty$ of the dilation as in the classical theory, but rather moves up to the finite value $\phi_{cr}=-\frac{1}{2}\ln(N/12)$. The black hole then evaporates, presumably leaving a massless, singular "remnant" [5–7].

In this paper we will consider the equations describing dilaton gravity coupled to N conformal scalars in the one-loop approximation for finite N. These equations differ from those derived in [1] by the addition of terms arising from the gravity-ghost measures which are negligible for $N\to\infty$. We shall see that, for $N<24$, these terms remove the singularity found in [3,4].

Some evidence consistent with the absence of other types of singularities is presented, but the equations are sufficiently complex that the question is not settled here. We hope to analyze the problem numerically in the near future [8].

We begin with a discussion of the gauge fixing and quantization of pure dilaton gravity:

$$S_0=\frac{1}{2\pi}\int d^2\sigma\sqrt{-g}e^{-2\phi}[R+4(\nabla\phi)^2+4\lambda^2]\ , \qquad (1)$$

where g and ϕ are the metric and dilaton fields, respectively, and λ^2 is a cosmological constant.[2] This is a theory with no local degrees of freedom: the $3+1$ fields in g and ϕ may be eliminated by two gauge conditions and two constraints. There is, however, a one-parameter family of classical black-hole solutions labeled by the black-hole mass [10]. We wish to gauge fix (1) to conformal gauge

$$\begin{aligned} g_{+-}&=-\tfrac{1}{2}e^{2\rho}\ ,\\ g_{++}&=g_{--}=0\ , \end{aligned} \qquad (2)$$

where $\sigma^{\pm}=\tau\pm\sigma$. In so doing the action will be shifted by the usual logarithm of the Faddeev-Popov ghost determinant. This term may be expressed in a covariant notation as

$$S_{FP}=\frac{13}{48\pi}\int d^2\sigma\sqrt{-g}R\Box^{-1}R\ . \qquad (3)$$

However, an ambiguity in this procedure arises in the present context. There is a family of metrics g_α given by

$$g_\alpha=e^{-2\alpha\phi}g\ , \qquad (4)$$

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[1]Or more precisely, the fields become so large that the large-N approximation breaks down.

[2]A related discussion of this quantization with similar conclusions has been given in [9].

any of which might be used to construct S_{FP} in (3). Although we have chosen g_0 in order to write down (1), it is not especially preferred. Indeed, the difference between two choices of metric is given by

$$S_{FP}(g_\alpha)-S_{FP}(g)=\frac{13\alpha}{12\pi}\int d^2\sigma\sqrt{-g}(R+\alpha\Box\phi)\phi \ . \qquad (5)$$

This is a local expression which might have been added to the action (1) either in the first place or as a finite counterterm during one-loop renormalization. Thus, there is no right or wrong choice of metric in (3): different choices simply correspond to different theories. One must choose a theory which contains the physical phenomena one wishes to investigate.

In fact, g is not a good choice of metric to use for defining the ghost measures—it leads to a sick theory. Using the metric g in (3) means that the Faddeev-Popov b-c ghosts couple to the geometry in the same way as the conformal f fields described in [1]. It immediately follows that black holes will grow in mass by Hawking radiation of negative-energy ghosts.[3] This is clearly nonsense.

This problem is avoided by defining the ghost measure with the alternate metric

$$\hat{g}=e^{-2\phi}g \ . \qquad (6)$$

This metric turns out to be flat for all classical solutions of (1). Black holes will therefore not radiate ghosts to leading order.

As is familiar in Liouville gravity, there is an additional term of the form (3) arising from the dependence of the ρ,ϕ measures on the metric. Again there is an ambiguity in these measures. Since ρ,ϕ are not local, propagating degrees of freedom, it is natural to demand that there be no Hawking radiation in these modes. This is accomplished by using the metric $\hat{g}$ to define their measures as well, which changes the 13 in Eqs. (3) to a 12. This definition ensures that there is a stable black-hole solution of the quantum theory for each value of the mass M.

The gauge-fixed action, including all the measure terms, is then

$$S_0+S_M=\frac{1}{\pi}\int d^2\sigma[e^{-2\phi}(2\partial_+\partial_-\rho-4\partial_+\phi\partial_-\phi+\lambda^2e^{2\rho}) +2\partial_+(\rho-\phi)\partial_-(\rho-\phi)] \ , \qquad (7)$$

while the stress tensor is[4]

$$T^0_{++}+T^M_{++}=e^{-2\phi}(4\partial_+\phi\partial_+\rho-2\partial_+^2\phi)+2(\partial_+\rho-\partial_+\phi)^2 -2\partial_+^2\rho+2\partial_+^2\phi+t_+ \ , \qquad (8)$$

where, as explained in [1], $t_+(\sigma^+)$ is determined by boundary conditions.

In order to study the problem of black-hole evaporation, we must complicate the theory by adding matter fields with local degrees of freedom. Following [1] we add N conformally coupled scalar matter fields f_i. Again an ambiguity in defining the f measure arises. However, this time we do not wish to use metric $\hat{g}$. This leads to a presumably sensible theory which does not contain the phenomena we wish to study: black holes do not Hawking radiate. Indeed this theory in a sense does not even contain black holes, since the matter sees only the flat metric $\hat{g}$. As explained in [1], if we instead use g to define the f measure, Hawking radiation of f particles indeed occurs, and closely resembles the four-dimensional phenomena. One thereby arrives at the final action

$$S=\frac{1}{\pi}\int d^2\sigma\left[e^{-2\phi}(2\partial_+\partial_-\rho-4\partial_+\phi\partial_-\phi+\lambda^2e^{2\rho}) -\frac{N}{12}\partial_+\rho\partial_-\rho+\frac{1}{2}\sum_{i=1}^{N}\partial_+f_i\partial_-f_i +2\partial_+(\rho-\phi)\partial_-(\rho-\phi)\right] . \qquad (9)$$

The constraints will be discussed shortly.

The quantum theory is described by functional integration with the "naive" measure weighted by S. In the large-N limit, all terms are of order N (after shifting ϕ) except the last one, which is order one and may therefore be dropped. In [1] it was argued that these order-N terms may be treated as a quantum effective action which describes the process of black-hole formation and evaporation, with the modifications of the gravitational action accounting for the stress energy carried by Hawking radiation.

However, the large-N approximation is not necessarily a reliable barometer of finite-N physics, particularly for $N\leq 24$. One way to see this is from the behavior of the ρ-ϕ kinetic operator $\mathcal{K}$ at large positive and negative ϕ. At large negative ϕ, the theory is essentially classical and $\mathcal{K}$ has one positive and one negative eigenvalue. At large positive ϕ and finite N, the classical action may be treated as a perturbation about the free measure-induced terms, and $\mathcal{K}$ still has one positive and one negative eigenvalue.[5] In the large-N limit, however, there are two negative eigenvalues for large ϕ and consequently a zero eigenvalue at an intermediate value of ϕ. This zero eigenvalue leads to singular behavior in the large-N limit [3,4] which may

[3]Although we cannot build detectors to see the ghosts directly, we can still observe their effects on the geometry.

[4]Ignoring the cosmological constant term, (7) becomes a free theory in terms of the variables $v=e^{-2\phi}$ and $w=\rho-\phi$. It is then easy to check that the stress tensor has $c=26$, as required by coordinate invariance. It also easily follows that the cosmological constant operator e^w is dimension (1,1) (with no renormalization of the exponent). Thus, there are many similarities with Liouville theory, and the methods developed there may be useful in the present context.

[5]Although for $N>24$, there is a region whose size grows with N in which there are two negative eigenvalues.

not be present for small N. Thus, other methods should be found for analyzing the theory at small N.

In this paper the action (9) including the last term (and corresponding modification of the constraints) will be treated as a quantum effective action for finite N. This amounts to a one-loop semiclassical approximation. For small N, there is no obvious small parameter which justifies this approximation. The loop expansion may break down when $e^{2\phi}$ gets large, and we cannot be confident that our conclusions are qualitatively correct. Nevertheless, we shall find in the one-loop approximation that the behavior of the theory changes dramatically at $N=24$, and we hope that the one-loop semiclassical approximation is at least qualitatively correct in the $N<24$ regime.

While treating (9) semiclassically has not been justified as a systematic approximation to dilaton gravity coupled to N scalar fields for small N, it can be formally justified as a systematic approximation to dilaton gravity coupled to a different matter system: let there be NM scalar f fields, and include an additional $c=-24M$ conformal matter sector with measure defined with respect to $\hat{g}$. After a shift of ϕ, one recovers an action of the form (9) multiplied by M. One then expects a semiclassical treatment to be valid for large M. The following analysis may be taken to apply to this system.

We now proceed to analyze the dynamics following from (9). The ρ and ϕ equations may be cast in the useful form

$$8P\partial_+\partial_-\phi=-P'(4\partial_+\phi\partial_-\phi+\lambda^2e^{2\rho}) , \tag{10}$$

$$2P\partial_+\partial_-\rho=e^{-4\phi}(4\partial_+\phi\partial_-\phi+\lambda^2e^{2\rho}) , \tag{11}$$

where

$$P\equiv e^{-4\phi}-\frac{N}{12}e^{-2\phi}+\frac{N}{24} ,$$
$$P'\equiv\frac{\delta P}{\delta\phi}=4e^{-2\phi}\left[\frac{N}{24}-e^{-2\phi}\right] . \tag{12}$$

The $++$ constraint equation is

$$T_{++}=e^{-2\phi}(4\partial_+\phi\partial_+\rho-2\partial_+^2\phi)+\tfrac{1}{2}\sum_{i=1}^{N}\partial_+f_i\partial_+f_i$$
$$-\frac{N}{12}(\partial_+\rho\partial_+\rho-\partial_+^2\rho)$$
$$+2[\partial_+(\rho-\phi)\partial_+(\rho-\phi)-\partial_+^2(\rho-\phi)]+t_+=0 , \tag{13}$$

and a similar equation holds for T_{--}.

The effect of the ghost-induced terms in these equations is immediately evident from (10). The prefactor P in (10) has zeros at

$$e^{-2\phi}=\frac{N}{24}\left[1\pm\left[1-\frac{24}{N}\right]^{1/2}\right] . \tag{14}$$

As pointed out in [3,4], these zeros are very dangerous: because the right-hand side of (10) is generically nonzero, $\partial_+\partial_-\phi$ is forced to diverge whenever ϕ crosses a zero.

However, for $N<24$, there are no real solutions of (14), and P is a positive definite quantity with a minimum at $\phi=-\frac{1}{2}\ln(N/24)\equiv\phi_c$.[6]

Thus, the singularities described in [3,4] do not arise. While we do not know if other types of singularities arise in the $N<24$ equations, they are clearly far better behaved.[7]

These equations can be solved following [3,4] perturbatively about the leading edge of an f shock wave incident on the linear dilaton vacuum, as illustrated in Fig. 1. The linear dilaton vacuum is a solution of (10)–(13) given by

$$\phi=-\lambda\sigma ,$$
$$\rho=0 . \tag{15}$$

An f shock wave is defined by

$$\tfrac{1}{2}\sum_{i=1}^{N}\partial_+f_i\partial_+f_i\equiv T^f_{++}=M\delta(\sigma^+-\sigma_0^+) . \tag{16}$$

Classically, the f stress energy leads to a black hole of mass M above σ_0^+. The quantum equations, however, exhibit different behavior. Below σ_0^+ one still has (15), and the solution above σ_0^+ can be computed in a Taylor expansion in $(\sigma^+-\sigma_0^+)$. Defining $\Sigma(\sigma^-)=\partial_+\phi(\sigma_0^+,\sigma_-)$, Eq. (10) becomes a simple equation for Σ:

$$8P\partial_-\Sigma=-P'(4\partial_-\phi\Sigma+\lambda^2e^{2\rho}) , \tag{17}$$

where (15) should be substituted for the values of ϕ,ρ along (σ_0^+,σ^-). This is easily integrated to yield

$$\Sigma(\sigma^-)=\frac{1}{2}\left[\frac{M}{\sqrt{P(\sigma_0^+,\sigma^-)}}-\lambda\right] , \tag{18}$$

just above the shock wave. The integration constant here is fixed by requiring that asymptotically as $\sigma^-\to-\infty$ (on $\mathcal{I}_R^-$) Σ agree with the classical f shock wave solution.

For $N<24$, P has no zeros, and Σ is perfectly finite. (For $N\geq24$, Σ diverges at a finite value of σ^-.) As explained in [3], an apparent horizon occurs whenever Σ vanishes, and one may say that an "apparent black hole" has formed. Since the minimum value of P (at $\phi=\phi_c$) is $(N/24)(1-N/24)$ there is no apparent horizon for sufficiently weak shock waves, i.e., small M. For $M=\lambda\sqrt{(N/24)(1-N/24)}$, Σ has a double zero where

[6]The stability of the zeros (or lack thereof) of P—on which our results strongly depend—against higher loop quantum corrections is an important question to which we do not have a definitive answer. However, it can be easily seen, by considering perturbation theory in e^{ϕ} and $e^{-\phi}$, that, in the large-N limit (in which the last term in P is neglected), P must change sign between weak and strong coupling, and consequently must have at least one zero. For $N<24$, it does not change sign, and must therefore have an even number of zeros, though we are not sure if that even number is zero in the exact theory.

[7]The weak-coupling singularities of the quantum kink solutions found in [5,7], are presumably still present.

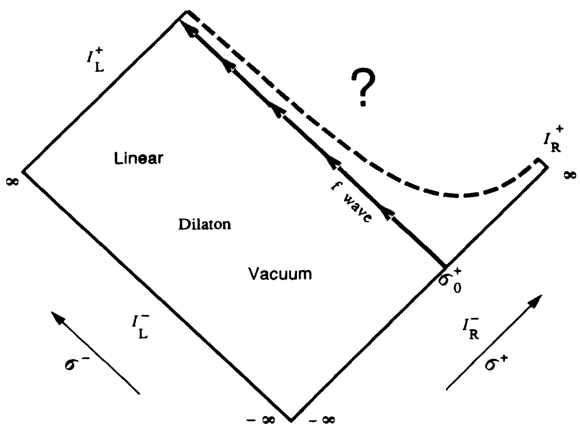

FIG. 1. An f shock wave incident on the linear dilaton vacuum. For $N<24$, no singularities are encountered in a Taylor expansion above the shock wave, but this expansion does not probe the region above the dashed line.

the shock wave crosses $\phi=\phi_c$, which splits into two apparent horizons (containing a region of trapped points[8]) as M increases. Since P approaches $N/24$ as $\sigma^-\to\infty$ (on $\mathcal{I}_L^+$), for $M>\lambda\sqrt{N/24}$, the region of trapped points extends all the way from the first apparent horizon up to $\mathcal{I}_L^+$ along the f shock wave.

It is of interest to determine the fate of the apparent black hole—or, equivalently, the region of trapped points—above the shock wave. This is a difficult problem in general, but some progress on it can be made as follows. At the apparent horizon, where $\partial_+\phi=0$, Eq. (10) reduces to

$$\partial_-(\partial_+\phi)=-\frac{P'\lambda^2e^{2\rho}}{8P}\ , \tag{19}$$

where P' is negative (positive) in the weak- (strong-) coupling region where $\phi<\phi_c$ ($\phi>\phi_c$). It follows that if one moves across an outer (inner) apparent horizon in the direction of increasing σ^-, one always enters (leaves) the interior of the apparent black hole where $\partial_+\phi>0$.

Thus, if the outer apparent horizon (in the weak-coupling region) is followed above the shock wave, σ^+ will monotonically increase until (or unless) it meets the line $\phi=\phi_c$ (which is spacelike inside the apparent horizon). If it does meet this line, the boundary (now the inner horizon) must subsequently continue along decreasing σ^+, back toward the shock wave. Thus, the apparent black hole ceases to exist for σ^+ greater than the value at which the apparent horizon meets ϕ_c.[9]

It is thus crucial to determine whether or not the apparent horizon actually reaches ϕ_c. Following [3], the outer horizon may be parametrized as the curve $\hat\sigma^-(\sigma^+)$. Russo, Susskind, and Thorlacius [3] derive several formulas for $\hat\sigma^-$ from the condition

$$\frac{d}{d\sigma^+}\partial_+\phi(\hat\sigma)=\partial_+^2\phi(\hat\sigma)+\frac{d\hat\sigma^-}{d\sigma^+}\partial_+\partial_-\phi(\hat\sigma)=0\ . \tag{20}$$

For the present system these become

$$\frac{d\hat\sigma^-}{d\sigma^+}=-\frac{\partial_+^2\phi}{\partial_+\partial_-\phi}=\frac{4e^{2\phi-2\rho}P}{\lambda^2P'}T^Q_{++}\ , \tag{21}$$

where

$$T^Q_{++}=\frac{N}{12}(\partial_+\rho\partial_+\rho-\partial_+^2\rho)+2[\partial_+(\rho-\phi)\partial_+(\rho-\phi)-\partial_+^2(\rho-\phi)]+t_+ \tag{22}$$

can be thought of as minus the energy in Hawking radiation leaving the apparent black hole. The second relation in (21) implies that if T^Q_{++} is always negative (i.e., the black hole is evaporating), the outer apparent horizon is timelike and will tend to meet the spacelike line $\phi=\phi_c$.[10] However, we have unfortunately been unable to prove that T^Q_{++} is indeed everywhere negative.

Further progress can be made by considering a very small apparent black hole, for which M is just over the threshold for production of an apparent horizon along the shock wave. In that case the apparent black hole is formed in a small neighborhood of ϕ_c, and its evolution can be determined by a Taylor expansion about the point

$$(\sigma_0^+,\sigma_c^-)\equiv\left[\sigma_0^+,\sigma_0^+-\frac{1}{\lambda}\ln\frac{N}{24}\right]\ ,$$

where the shock wave crosses the line $\phi=\phi_c$. Using the first equation in (21) and (19), one finds that the trajectory of the horizon is determined by

$$\frac{d\hat\sigma^-}{d\sigma^+}\approx\frac{2(24/N-1)\partial_+^2\phi(\sigma_0^+,\sigma_c^-)}{\lambda^3(\sigma^--\sigma_c^-)}\equiv-\frac{k}{\sigma^--\sigma_c^-}\ . \tag{23}$$

One then finds, for $\sigma^+>\sigma_0^+$,

$$\hat\sigma^-=\sqrt{2k(\sigma_0^+-\sigma^+)+c^2}+\sigma_c^-\ , \tag{24}$$

where $(\sigma_0^+,\sigma_c^-\pm c)$ are the initial coordinates of the inner and outer horizons along the shock wave. The behavior of the trajectory depends crucially on the sign of $\partial_+^2\phi(\sigma_0^+,\sigma_c^-)$ or k. According to (24), if k is positive the apparent black hole shrinks and disappears at $\sigma^+=\sigma_0^++c^2/2k$, soon after formation. On the other hand, if k is negative the apparent black hole will initially grow in size, and perturbation theory about the shock wave cannot be used to determine its ultimate fate. The

[8]A trapped point is one point for which ϕ increases along both outgoing null geodesics. This corresponds to the four-dimensional definition of a trapped surface when $e^{-2\phi}$ is interpreted as the size of the two spheres [3,4,11].

[9]Though it may (and will for $M>\lambda\sqrt{N/24}$) exist for large values of σ^-.

[10]If T^Q goes to zero sufficiently fast, the horizon could be asymptotically null and avoid ϕ_c.

sign of k is determined by continuing to one higher order the Taylor expansion about σ_0^+ used to find $\Sigma(\sigma^-)$. A tedious computation reveals that, rather curiously, k is positive for some values of N and negative for others. We do not understand the significance of this. Perhaps the theory depends qualitatively on N even within the range $0<N<24$.

It is of interest to study the behavior of the quantum stress tensor T^Q_{++} of (22) along the shock wave. At $\sigma^+=\sigma_0^+$, $\partial_+\phi$ (as well as $\partial_+\rho$) is discontinuous. T^Q_{++} will therefore have a δ function at σ_0^+. One easily finds

$$T^Q_{++}(\sigma_0^+,\sigma^-)=M\delta(\sigma^+-\sigma_0^+)\left[\frac{e^{\lambda(\sigma_0^+-\sigma^-)}}{\sqrt{P}}-1\right] . \quad (25)$$

As the shock wave enters from $\mathcal{I}_R^-(\sigma^-=-\infty)$, T^Q_{++} vanishes. As it moves in, P increases, and negative-energy quantum fluctuations begin to accumulate along the shock wave. This energy approaches a constant up on $\mathcal{I}_L^+$ which obeys

$$T^Q_{++}+T^f_{++}=0 \quad (26)$$

at (σ_0^+,∞). Thus, the f shock wave is gravitationally cloaked by a cloud of quantum fluctuations and arrives at $\mathcal{I}_L^+$ as a "zero-energy bound state."[11]

This is in line with the conjecture made in [1] that the incoming state from $\mathcal{I}_R^-$ evaporates before forming a black hole, and arrives as a zero-energy bound state on $\mathcal{I}_L^+$. It is tempting to speculate that this conjecture, though disproved for $N>24$ [3,4], might be applicable to $N<24$. However, it is not clear if this is consistent with the behavior of the apparent horizons.

We hope to answer these questions in the near future. For the moment some further insight can be gained by investigating static black-hole solutions of the type found for $N>24$ in [6,7]. Following [7], this is best accomplished in terms of the variable

$$s=\lambda^{-2}e^{\lambda(\sigma^+-\sigma^-)} , \quad (27)$$

which vanishes on the horizon, is spacelike and positive outside, and timelike and negative inside. The equations of motion (10) and (11) for fields depending only on s become

$$8P(s\ddot{\phi}+\dot{\phi})=P'(\lambda^2e^{2\rho}-4s\dot{\phi}^2) , \quad (28)$$

$$2P(s\ddot{\rho}+\dot{\rho})=-e^{-4\phi}(\lambda^2e^{2\rho}-4s\dot{\phi}^2) , \quad (29)$$

while the constraint is

$$e^{-2\phi}(4\dot{\phi}\dot{\rho}-2\ddot{\phi})=\frac{N}{12}(\dot{\rho}^2-\ddot{\rho})-2[(\dot{\rho}-\dot{\phi})^2-\ddot{\rho}+\ddot{\phi}]+\frac{\hat{t}}{s^2} , \quad (30)$$

where an overdot denotes differentiation with respect to s and $\hat{t}$ is a constant. The finiteness of $\ddot{\phi}$ and $\ddot{\rho}$ at the horizon gives constraints for initial data at $s=0$:

$$\dot{\rho}(0)=-\frac{\lambda^2e^{2\rho(0)-4\phi(0)}}{2P(0)} ,$$
$$\dot{\phi}(0)=\frac{\lambda^2e^{2\rho(0)}P'(0)}{8P(0)} , \quad (31)$$
$$\hat{t}=0 .$$

Since $\rho(0)$ can be set to zero by a global coordinate transformation, there is a one-parameter family of inequivalent solutions labeled by $\phi(0)$, or equivalently, the black-hole mass.

The behavior of these solutions depends on whether $\phi(0)$ is less than or greater than the critical value, $\phi_c=\frac{1}{2}\ln(24/N)$, where P' changes sign. For large negative $\phi(0)$ (corresponding to large black holes), the solutions will differ little outside the horizon from those found in [6,7]. Asymptotically the solution approaches the linear dilaton vacuum, but with a linearly divergent Arnowitt-Deser-Misner (ADM) mass corresponding to the infinite radiation bath required to stabilize the black hole against Hawking decay. As $\phi(0)$ approaches ϕ_c the solutions begin to differ. This can be seen from the fact that, at $\phi(0)=\phi_c$, the solution is exactly given by [in the gauge $\rho(0)=0$]

$$\phi=\frac{1}{2}\ln\frac{24}{N}\equiv\phi_c ,$$
$$\rho=-\ln\left[1+\frac{s}{\alpha}\right] , \quad (32)$$

where $\alpha=2\lambda^{-2}(24/N-1)$. This corresponds to de Sitter space filled with Hawking radiation. We presume that, as $\phi(0)$ approaches ϕ_c, there is a growing de Sitter–like region outside the black hole. At $\phi(0)=\phi_c$, this region engulfs the entire spacetime and the black-hole horizon becomes a de Sitter horizon.

One expects that a large slowly evaporating black hole is approximated within some region by these static solutions with a slowly increasing $\phi(0)$. However, one should not conclude from the above that the end point of an apparent black hole formed by a massive incoming shock wave is de Sitter space. In that situation, the spacetime is always asymptotic to the linear dilaton vacuum with a finite mass. One possibility is that it looks like de Sitter space within some region which then decays back to the linear dilaton vacuum.

Inside the horizon, the ghost modifications have a crucial effect even for $\phi(0)\ll\phi_c$. When $N>24$, it was shown that ϕ increases (now in a timelike direction) until a zero of P and a singularity is reached. The resulting spacetime has a causal structure identical to that of the static classical black-hole solution. However, for $N<24$ there are no zeros of P. In this case one finds from (28) that, for $\phi<\phi_c$, both ϕ and $s\dot{\phi}$ are initially increasing as before. ϕ will then inevitably cross ϕ_c, at which point $s\dot{\phi}$ must start to decrease, and ϕ "bounces" off of some maximum value rather than becoming singular. This behavior can be understood from a linearized analysis for small $\phi(0)$. To leading order the ρ equation is independent of ϕ and yields the de Sitter solution (32). The linearization of (10) is then

[11]This greatly strengthens the analogy made in [5,12] to the Schwinger model with a position-dependent mass.

$$s\ddot{\phi}+\dot{\phi}=\frac{2\alpha}{(s+\alpha)^2}\phi \ . \tag{33}$$

Define the new timelike variable

$$u=\ln\left[\frac{\sqrt{\alpha}+\sqrt{-s}}{\sqrt{\alpha}-\sqrt{-s}}\right], \tag{34}$$

which runs from zero at the horizon to plus infinity at future timelike infinity ($s=-\alpha$). The linearized ϕ equation then becomes

$$\partial_u^2\phi+\coth u\,\partial_u\phi+2\phi=0 \ , \tag{35}$$

while the boundary condition (31) implies $\partial_u\phi$ vanishes at the horizon. As the coefficient of the first derivative term is positive for $u>0$, this is a damped harmonic oscillator. Thus, excursions of ϕ are damped inside the horizon and the linearized approximation does not break down. We therefore conclude that, at least for small $\phi(0)$, the interior of the black hole is nonsingular and asymptotic to de Sitter space, as illustrated in Fig. 2.

Evidently this system is very resistant to singularity formation: even if a small black hole is forced into existence by continuously pumping in energy from infinity, there is no singularity in its interior.

For large $\phi(0)$ the equations are harder to analyze, but we expect similar behavior. Numerical work on this question is in progress [8].

Clearly this set of equations exhibits complex and unusual behavior that we do not yet fully understand, and which merits further investigation. Our preliminary investigations have failed to uncover any black-hole-type singularities, but their existence is certainly not ruled out. We also do not know if the equations give a qualitatively correct description of $N<24$ matter fields coupled to dilaton gravity because higher-loop corrections could be important. What has been established, however, is that the nature of the black-hole formation-evaporation process, including the singularity structure, depends qualitatively on the properties of the matter sector. It is an urgent problem to characterize the possible behaviors.

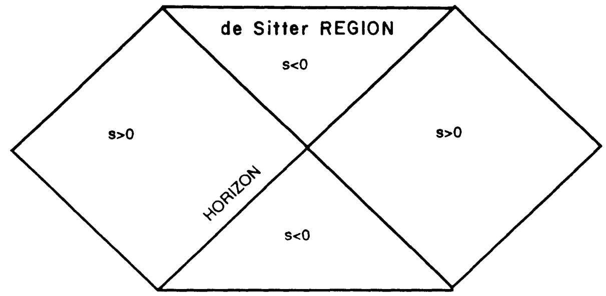

FIG. 2. Penrose diagram for an $N<24$ quantum black hole in equilibrium with a radiation bath. The singularity present in the classical Kruskal diagram is replaced by an asymptotically de Sitter region.

It is intriguing that the outcome of two-dimensional gravitational collapse depends qualitatively on the matter content of the Universe. Perhaps this will also turn out to be true in four dimensions, and lead to constraints on the spectrum of elementary particles.

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