Axial-vector coupling constants and chiral-symmetry restoration

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The isovector axial-vector coupling constant g_A is determined by using the method of QCD sum rules. A sum rule for (g_A-1) is obtained, and it is shown that, with standard values of the quark condensates, $g_A=1.26\pm0.08$. It is also shown that the isovector axial-vector coupling $(g_A-1)=0$ in the limit in which chiral symmetry is restored, and the quark condensate vanishes. A sum rule is also obtained for the "isoscalar" axial-vector coupling constant g_A^S, which is found to be 0.13 if the isovector values of susceptibilities are used. On the other hand, $g_A^S=-0.68$ if the quark condensate is set to zero while $g_A^S=-1.00$ if both the quark and gluon condensates vanish in the event of chiral-symmetry restoration. The values of g_A and g_A^S allow us to deduce Δu and Δd in the proton.

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I. INTRODUCTION

The magnitude of the isovector axial-vector coupling constant $g_A^{(1)}$, abbreviated simply as g_A, has long been of considerable interest. The prediction of the Goldberger-Treiman relation [1] that, for a free nucleon, $g_A=1.25$, consistent with its experimental value, is a remarkable achievement of the theory of hadronic strong interactions. At a momentum transfer $q^2=0$, this relation is $M_N g_A = f_\pi g_{\pi NN}$, so that the axial-vector coupling constant is determined by the pion decay constant f_π, the pion-nucleon coupling constant $g_{\pi NN}$, and the nucleon mass M_N.

It has also been of considerable interest to determine g_A from the theories of electroweak interactions and QCD. Although it is not possible to use the QCD Lagrangian directly for this purpose, since this would require the solution to the problem of determining the structure of a nucleon and the low-energy interaction of a nucleon with a gauge boson, it has been possible to use the methods of QCD sum rules [2] to determine the value of g_A. Following the method of introducing an external electromagnetic field [3] for the evaluation of nucleon magnetic moments, studies of a nucleon in the presence of an external axial-vector field have been carried out within the framework [4,5] of QCD sum rules, with results consistent with the experimental value of $g_A=1.26$. However, this result was obtained with a smaller value of the quark condensate than that used to determine the mass, the magnetic moments, and other static properties of nucleons. Since one of the features of the method is to be able to use the same condensate values for all physical processes within a given system, this result might be taken to suggest that the method of QCD sum rules may not be totally adequate for determining the coupling of the axial-vector current to nucleons. In particular, one should use the same value of the quark condensate in deriving the response of a nucleon to an external axial-vector field to calculate g_A as in deriving the response to an external vector field to calculate the magnetic dipole moments. In the present work, we find that the solution to this problem lies in the addition of terms consistently up to dimension 8 ($d=8$); we obtain a value of g_A consistent with experiment by means of standard QCD sum-rule parameters.

Recently, the polarized structure function of the proton $g_1^p(x)$ has been measured by the European Muon Collaboration (EMC) [6] for the Bjorken variable x down to $\cong 0.01$, making possible extraction of the first moment $\int_0^1 dx\, g_1^p(x)$, which turns out to be considerably smaller than the Ellis-Jaffe sum rule [7]. When combined with the known values for g_A and the F/D ratio, the EMC data [6] yield a value for the "isoscalar" axial-vector coupling g_A^S ($g_A^S=\Delta u+\Delta d$ in the language of the EMC data analysis [6]) with a value $g_A^S=0.28\pm0.08$, where the most recent values of F and D have been used [8] in extracting Δu and Δd from the EMC data [6]. In the context of the QCD sum-rule approach, Belyaev, Ioffe, and Kogan [9] were able, a few years before the EMC data came about, to predict $g_A^S\approx 0.5$, a value already significantly below the naive nonrelativistic quark-model value $g_A^S\approx 1$. A slightly smaller value $g_A^S\approx 0.35$ was obtained somewhat later by Gupta, Murthy, and Pasupathy [10], who also used the QCD sum-rule approach. With the addition of terms consistently up to dimension 8, we reinvestigate the problem of g_A^S and obtain $g_A^S=0.13\pm0.08$ significantly lower than the previously reported theoretical values [9,10] and in approximate agreement with, though somewhat smaller than, that obtained from the EMC data.

It is the goal of the present authors to apply the methods of QCD sum rules for the studies of weak interactions of free hadrons and for those in nuclei. Of particular current interest are the values of g_A, g_A^S, and g_P, the induced pseudoscalar coupling constant, for the free nucleon and for the nucleon embedded in a nuclear medium. This has motivated us to carry out a study of g_A with emphasis on the role of chiral-symmetry violation, since it is expected that chiral symmetry will be restored at high densities and that at normal nuclear densities chiral symmetry may be partially restored. Recently, several authors using the method of QCD sum rules [11,12] in nuclear matter and in the study of the Nolan-Schiffer effect [13] have suggested that the quark condensate, which vanishes in the chiral limit, is reduced in nuclear matter. A QCD sum-rule analysis of g_A in nuclear matter [14] has also been carried out. Recently, the question of the value of g_A in nuclear matter has become even more urgent with the recognition [15] that the method of using the correlation between magnetic dipole moments and Gamow-Teller β decay lifetimes to obtain the often-accepted value (of 1.0) for the value of g_A in nuclear matter may not be reliable.

In the present work, we carry out studies similar to those in Refs. [4] and [5] in which we trace out the role of the various condensates which appear. In addition to the well-known quark and gluon condensates of Shifman, Vainshtein, and Zakharov [2] (SVZ), new condensates are induced by the external axial-vector field Z_μ. At first glance one worries that, with the introduction of new parameters, one will not be able to determine additional properties probed by the external field. Here, as in Ref. [4], we show that the sum rule for g_A can be combined with that for the mass to obtain a sum rule for g_A-1, which yields predictions relatively stable against reasonable variations in the Borel mass M_B. Because the quark condensate plays the dominant role in the latter sum rule, it follows that $(g_A-1)=0$ if chiral symmetry is restored. In addition, we obtain a value of $g_A=1.26\pm0.08$ with standard values of the condensates. In this study we assume that the Z field is static, and so no relations can be obtained for g_P. We intend to investigate g_P in the near future.

II. QCD SUM RULE FOR g_A-1 AND $g_A+g_A^S$

Although the method of QCD sum rules as originally developed [1] was applied to the study of hadronic properties in the region of about 1 GeV, Ioffe and Smilga [3] developed techniques for embedding hadrons in an external field in order to derive static properties in terms of the condensates, including induced condensates which introduce new parameters. In Refs. [4] and [5] the method was applied to an external static axial-vector field. We briefly review the method here for an external axial-vector field.

The starting point is the polarization function in an external axial-vector field, which we call Z_μ. The correlation operator $\Pi(p)$ is defined as [3–5]

$$\Pi(p)\equiv i\int d^4x\, e^{ip\cdot x}\langle 0|T[\eta(x)\bar{\eta}(0)]|0\rangle\ , \tag{1}$$

where for the nucleon current we use a standard (but not unique) form [16]:

$$\eta(x)=\epsilon^{abc}\{u^a(x)^T C\gamma_\mu u^b(x)\}\gamma^\mu\gamma^5 d^c(x)\ , \tag{2a}$$

$$\langle 0|\eta(0)|N(p)\rangle\equiv\lambda_N v_N(p)\ , \tag{2b}$$

with C the charge-conjugation operator, a,b,c color indices, and $v_N(p)$ the nucleon spinor normalized such that $\bar{v}v=2M_N$. Embedding the system in an external Z_μ field and introducing intermediate states, we can express the polarization operator in the limit of a constant external field, $Z_\mu(x)=Z_\mu$, as [5]

$$\Pi(p)=-|\lambda_N|^2\frac{1}{\hat{p}-M_N}g_A\hat{Z}\gamma_5\frac{1}{\hat{p}-M_N}+\cdots\ , \tag{3}$$

if we adopt the on-shell definition of the nucleon axial-vector form factor

$$\langle N(p',\lambda')|J_\mu^5(0)|N(p,\lambda)\rangle =\bar{u}_{\lambda'}(p')\{g_A(q^2)\gamma_\mu\gamma_5+g_P(q^2)q_\mu\gamma_5\}u_\lambda(p)\ , \tag{4}$$

with $q_\mu\equiv p'_\mu-p_\mu$ and $\hat{a}\equiv\gamma_\mu a^\mu$. The term shown in Eq. (3) corresponds to nucleon intermediate states; continuum contributions to Π are shown simply by the ellipsis in that equation. The axial-vector coupling constant g_A in Eq. (3) is defined at $q^2=0$. Equation (3) is the expression for the phenomenological form, in which $\Pi(p)$ is evaluated at the baryon level. When evaluating the polarization operator $\Pi(p)$ at the quark level and comparing it with Eq. (3), one is led to three sum rules involving g_A, which [5] may not be consistent among themselves, although there is indeed one sum rule which seems most appropriate for g_A.

We note that Eq. (4) gives the on-shell matrix element of the axial-vector current. In treating the correlator $\Pi(p)$ in the presence of an external axial field $Z_\mu(x)$, one may consider the slightly off-shell nucleon matrix elements, where additional off-shell form factors can occur. For instance,

$$\langle N(p',\lambda')|J_\mu^5(0)|N(p,\lambda)\rangle =\bar{u}_{\lambda'}(p')\{G_1\gamma_\mu\gamma_5+G_2q_\mu\gamma_5+G_3i\sigma_{\mu\nu}P^\nu\gamma_5\}u_\lambda(p)\ , \tag{5}$$

with $P_\mu\equiv p'_\mu+p_\mu$. In the on-shell limit, this reduces to Eq. (4) with $g_A=G_1+2M_NG_3$ and $g_P=G_2-G_3$. Among the three sum rules which one obtains by comparing coefficients of $p\cdot Z\hat{p}\gamma_5$, $\hat{Z}\gamma_5$, and $i\sigma_{\mu\nu}Z^\mu p^\nu\gamma_5$, there is one sum rule in which only g_A enters.

We now find the polarization function at the quark level by evaluating $\Pi(p)$ via the quark propagators in the presence of gluonic and Z fields. The starting point is the quark propagator

$$S_{ij}^{ab}\equiv\langle 0|T[q_i^a(x)\bar{q}_j^b(0)]|0\rangle\ .$$

Following the method of Ref. [3] including terms up to the second order in the Taylor expansion, we find

$$S^{ab}=\frac{\delta^{ab}}{2\pi^2x^4}(i\hat{x}-gx\cdot Z\hat{x}\gamma_5)+\frac{i}{32\pi^2x^2}g_c\frac{\lambda^n_{ab}}{2}G^n_{\mu\nu}(\hat{x}\sigma^{\mu\nu}+\sigma^{\mu\nu}\hat{x})$$
$$+\delta^{ab}\langle\bar{q}q\rangle\{-\tfrac{1}{12}(1+\tfrac{1}{16}x^2m_0^2)+\tfrac{1}{12}g\chi\hat{Z}\gamma_5+\tfrac{1}{36}gx^\alpha Z^\beta\sigma_{\alpha\beta}\gamma_5+\tfrac{1}{216}g\kappa(\tfrac{5}{2}x^2\hat{Z}-x\cdot Z\hat{x})\gamma_5\}+\cdots \quad (6)$$

The first three terms in Eq. (6) are the perturbative free-quark propagator and the quark propagator with a Z and a gluon, depicted in Figs. 1(a)–1(c). The next five nonperturbative terms, proportional to $\langle\bar{q}q\rangle$, are the quark condensate and this same condensate in the presence of gluonic and external Z fields, depicted in the five diagrams of Figs. 1(d)–1(h). The other quantities appearing in Eq. (6) are the Z-quark coupling constant ($g=g_u=-g_d$ for the isovector axial-vector coupling g_A or $g=g_u=g_d$ for the isoscalar axial-vector coupling g_A^S) and the condensate parameters defined by

$$\langle 0|\bar{q}g_c\sigma\cdot Gq|0\rangle=-m_0^2\langle\bar{q}q\rangle\ ,$$
$$\langle 0|\bar{q}g_c\tilde{G}_{\mu\nu}\gamma^\nu q|0\rangle=g\kappa Z_\mu\langle\bar{q}q\rangle\ , \quad (7)$$
$$\langle 0|\bar{q}\gamma_\mu\gamma_5 q|0\rangle=g\chi Z_\mu\langle\bar{q}q\rangle\ .$$

Our definition of κ differs in sign from that of Ref. [5]. Although the last term in the quark propagator (6) differs in sign and by a factor of 3 with that of Ref. [5], the sign is due to the difference in definition and the factor of 3 is absorbed in the definition of κ in Ref. [5]. In addition to the quark and gluon condensates, one has the parameter m_0^2 and the two susceptibilities κ and χ. We shall show that for the sum rule for (g_A-1) there is a very weak dependence on κ and χ. The parameter m_0^2 does enter the mass sum rule and also that for g_A-1.

As in Refs. [4] and [5], we find it most useful to use the sum rule which is derived with the Borel transformation [1] of the coefficient of the covariant $p\cdot Z\hat{p}\gamma_5$ after the Fourier transform of both the phenomenological and quark-level forms of the polarization function $\Pi(p)$. As pointed out in detail in Ref. [5], this sum rule depends less on the treatment of excited states than the other ones. The processes which enter the calculation are shown in Figs. 2(a)–2(h). These diagrams can readily be evaluated by using the relationship

$$\langle 0|T(\eta(x)\bar{\eta}(0))|0\rangle$$
$$=-2\epsilon^{abc}\epsilon^{a'b'c'}\mathrm{Tr}\{S(x)_u^{bb'}\gamma_\nu CS(x)_u^{aa'T}C\gamma_\mu\}$$
$$\times\gamma_5\gamma^\mu S(x)_d^{cc'}\gamma^\nu\gamma_5\ . \quad (8)$$

Note that Figs. 2(b) and 2(h) are evaluated with the aid of the identity for the gluon condensate:

$$\langle g_c^2G^n_{\rho\sigma}G^m_{\alpha\beta}\rangle=\frac{\delta^{nm}}{96}(g_{\rho\alpha}g_{\sigma\beta}-g_{\rho\beta}g_{\sigma\alpha})\langle g_c^2G^2\rangle\ . \quad (9)$$

On the other hand, Figs. 2(f) are evaluated using the relation

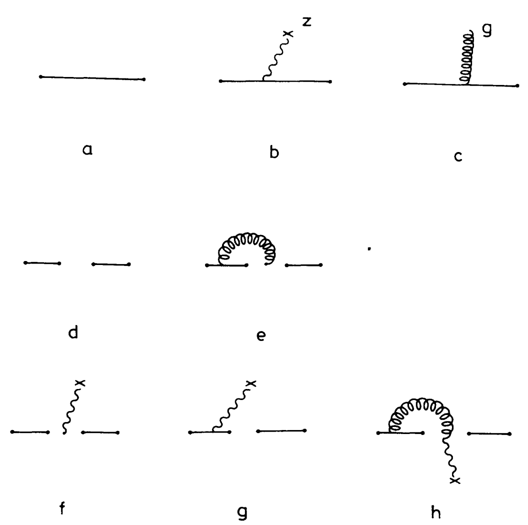

FIG. 1. Diagrams included in the quark propagator of Eq. (6).

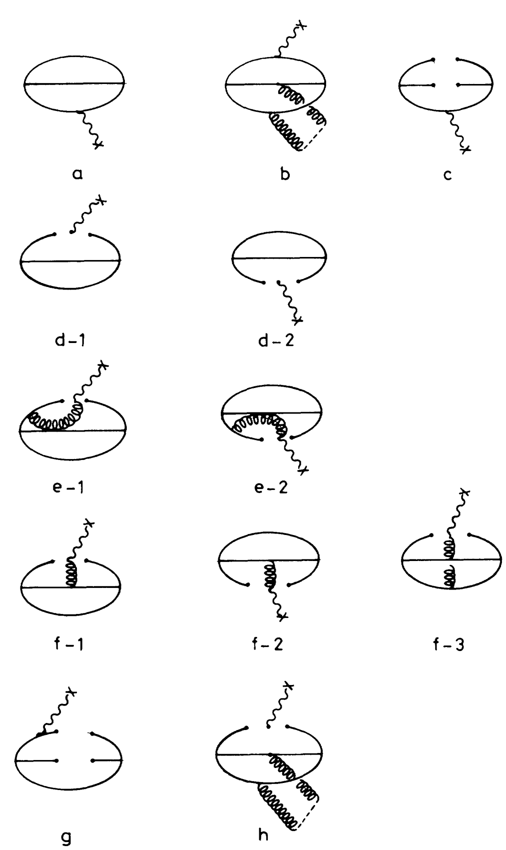

FIG. 2. Processes included in the polarization function leading to the sum rules when the coefficients of $p\cdot Z\hat{p}\gamma_5$ and $\hat{Z}\gamma_5$ are compared.

$$\langle q_i^a G_{\mu\nu}^m \bar{q}_j^b \rangle_Z = \tfrac{1}{96}(\gamma_\alpha \sigma_{\mu\nu} + \sigma_{\mu\nu}\gamma_\alpha)\gamma_5 \frac{\lambda_{ab}^m}{2} \langle \bar{q}\tilde{G}^{\alpha\beta}\gamma_\beta q \rangle \ , \tag{10}$$

with $\tilde{G}^{\alpha\beta} \equiv \frac{1}{2}\epsilon^{\alpha\beta\rho\sigma} G_{\rho\sigma}^n \lambda^n/2$. Note that, in Eqs. (9) and (10), all the field operators are evaluated at $x=0$.

In addition to terms included in Refs. [4], [5], [9], and [10], we have added Fig. 2(h) and others so that contribution are included consistently up to dimension 8 ($d=8$). Note also that Figs. 2(a)–2(h) enter the sum rules when the coefficients of $p\cdot Z\hat{p}\gamma_5$ and $\hat{Z}\gamma_5$ are compared. On the other hand, the processes which enter the sum rule by comparing the coefficients of $i\sigma_{\mu\nu}Z^\mu p^\nu \gamma_5$ are depicted in Figs. 3(a)–3(d).

After carrying out a Borel transformation to improve convergence, we obtain a sum rule for g_A:

$$\frac{M_B^6 E_2}{8L^{4/9}} + \frac{M_B^2}{32L^{4/9}}\langle g_c^2 G^2\rangle E_0 - \frac{M_B^2}{18L^{68/81}}\kappa a E_0 + \frac{5}{18}a^2 L^{4/9} + \frac{1}{288L^{4/9}}\chi a \langle g_c^2 G^2 \rangle = \beta_N^2 (g_A + A M_B^2)\exp(-M_N^2/M_B^2) \ , \tag{11a}$$

where $a = -(2\pi)^2\langle \bar{q}q\rangle$ and $L = 0.621\ln(10M_B)$, corresponding to $\Lambda_{\rm QCD}=0.1$ GeV with the Borel mass M_B in GeV and $\beta_N^2 \equiv (2\pi)^4\lambda_N^2/4$. The most important terms on the left-hand side (LHS) are the first term and that proportional to a^2, corresponding to Figs. 2(a), 2(c), and 2(g), respectively. The factors $E_0 = 1-e^{-x}$, $E_1 = 1-(1+x)e^{-x}$, and $E_2 = 1-(1+x+\frac{1}{2}x^2)e^{-x}$, with $x \equiv W^2/M_B^2 \approx (2.3\ {\rm GeV}^2)/M_B^2$ (see Ref. [3]) are used to correct the sum rule to obtain consistent M_B^2 dependence for contributions from excited states through perturbative QCD techniques [9,16]. They also serve to restrict the range of the integration and increase the weight given to the nucleon. We have thus made the usual assumption in Eq. (11a). The constant A is introduced to represent the residual continuum contribution to the dispersion integral. Note that only the standard quark and gluon condensates and the susceptibilities κ and χ enter, and that the term involving the latter is numerically small.

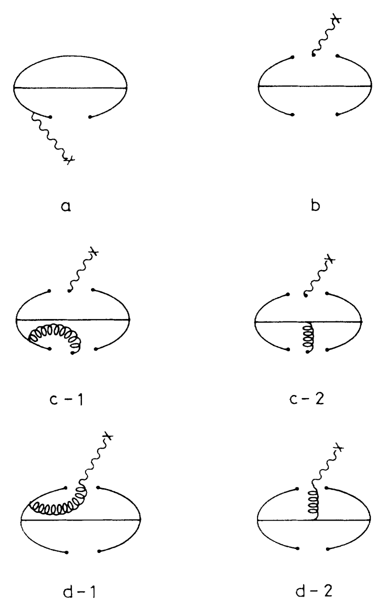

FIG. 3. Processes included in the polarization function leading to the sum rule when the coefficients of $i\sigma_{\mu\nu}Z^\mu p^\nu\gamma_5$ are compared.

On the same footing, we may obtain the sum rule for g_A^S (with $g_u = g_d = 1$):

$$-\frac{M_B^6 E_2}{8L^{4/9}} + \frac{M_B^2}{32L^{4/9}}\langle g_c^2 G^2\rangle E_0 + \frac{1}{6L^{4/9}}\chi a M_B^4 E_1 - \frac{M_B^2}{18L^{68/81}}\kappa a E_0 - \frac{1}{18}a^2 L^{4/9} + \frac{1}{288L^{4/9}}\chi a\langle g_c^2 G^2\rangle = \beta_N^2(g_A^S + A^S M_B^2)\exp(-M_N^2/M_B^2) \ . \tag{11b}$$

This is the sum rule for the "isoscalar" axial-vector coupling constant g_A^S; it agrees with that of Ref. [10], except that their κ should be $\kappa/3$. It is assumed that the susceptibilities and W^2 are identical to those for the isovector case. This assumption can be investigated [17], but we adopt it here for simplicity. The most important terms on the left-hand side are the first term and that proportional to $\chi a M_B^4$, corresponding to Figs. 2(a), 2(d-1), and 2(d-2), respectively. Note that the susceptibility χ is very important in the sum rule for g_A^S, but only makes a small correction to g_A.

In order to find a sum rule for $(g_A - 1)$, we make use of a Belyaev-Ioffe sum rule [16] for the determination of the nucleon mass:

$$\frac{M_B^6}{8L^{4/9}}E_2 + \frac{M_B^2}{32L^{4/9}}\langle g_c^2 G^2\rangle E_0 + \frac{1}{6}a^2 L^{4/9} - \frac{1}{24M_B^2}a^2 m_0^2 = \beta_N^2 \exp(-M_N^2/M_B^2) \ . \tag{12}$$

Note that the first two terms in the left-hand side of the two sum rules [Eqs. (11a) and (12)] are equal. By subtracting Eq. (12) from Eq. (11a), one obtains a sum rule for $(g_A - 1)$ involving the condensates a, m_0^2, and the susceptibilities κ and χ. These parameters have been estimated to be [2,3,16,18]

$$\begin{aligned} &a \approx 0.55\ {\rm GeV}^3, \quad \kappa a \approx 0.140\ {\rm GeV}^4 \ , \\ &\chi a \approx 0.70\ {\rm GeV}^2, \quad \langle g_c^2 G^2\rangle \approx 0.47\ {\rm GeV}^4 \ , \\ &m_0^2 \approx 0.8\ {\rm GeV}^2 \ . \end{aligned} \tag{13}$$

Because κ is less well known than the other constants, we also consider $\kappa a \approx -0.140\ {\rm GeV}^4$ in order to estimate (roughly) the error of the sum-rule method. The parameter β_N^2 has been determined [3] through the mass sum rule to be $\beta_N^2 \approx 0.26\ {\rm GeV}^6$. In Eq. (13) we use the standard value [3] of the quark condensate. Subtracting Eq. (12) from Eq. (11a), we obtain a sum rule very similar to one obtained [4] by Belyaev and Kogan:

$$\frac{1}{9}a^2L^{4/9}+\frac{1}{24}\frac{a^2m_0^2}{M_B^2}-\frac{1}{18}\frac{\kappa a M_B^2}{L^{68/81}}E_0 + \frac{1}{288L^{4/9}}\chi a\langle g_c^2G^2\rangle = \beta_N^2\{(g_A-1)+AM_B^2\}\exp(-M_N^2/M_B^2)\ . \quad (14)$$

This sum rule is only very weakly dependent on χa; its dominant contribution on the left-hand side is the first term. The second term is less important, and the other ones are small.

Analogously, we obtain, by adding together Eqs. (11a) and (11b),

$$\frac{M_B^2}{16L^{4/9}}\langle g_c^2G^2\rangle E_0+\frac{1}{6L^{4/9}}\chi a M_B^4E_1-\frac{M_B^2}{9L^{68/81}}\kappa a E_0 + \frac{2}{9}a^2L^{4/9}+\frac{1}{144L^{4/9}M_B^2}\chi a\langle g_c^2G^2\rangle = \beta_N^2(g_A+g_A^S+A'M_B^2)\exp(-M_N^2/M_B^2)\ . \quad (15)$$

Equations (14) and (15) are our main result. It is clear from Eqs. (13) and (14) that, for (g_A-1), the quark condensate (represented by a) dominates and that the induced condensates (proportional to the susceptibilities χ and κ) are not important. This is not so for the "isoscalar" (g_A^S) sum rule, and it causes greater uncertainty in our results for this quantity.

In our numerical analysis, after moving the factor $\exp(-M_N^2/M_B^2)$ to the LHS, we compare the LHS to a straight-line approximation $C+DM_B^2$. In practice, for a given Borel mass M_B, we may determine the straight line which goes through the points $M_B\pm\delta M_B$ (with, say, $\delta M_B=0.1$ GeV) and then compare the values of the LHS and RHS of the sum rule at M_B. When both sides agree with the desired accuracy, the sum rule is said to hold to that accuracy and it allows for extraction of the constants C and D.

We obtain solutions for the (g_A-1) sum rule [Eq. (14)] for values of the Borel mass $M_B\geq 1.8$ GeV. It is worrisome that such large values of M_B are present in our analysis, since it is expected that M_B is of the same magnitude as the mass of the baryon of interest. That is, one expects solutions for M_B in the range of 1 GeV. Larger values of M_B might appear to indicate that our g_A is distorted by coupling to baryon resonances and perhaps other states of higher energy. However, in our analysis these continuum contributions to the sum rule for (g_A-1) are very small. Continuum contributions show up at two places for (g_A-1): (1) the term proportional to A and (2) the deviation of E_0 from unity in Eq. (14). The former is very small, and we obtain almost identical solutions to Eq. (14) if we let $E_0=1$. Therefore we conclude that the continuum contributions are small and are handled reasonably well in our calculation, even through M_B is larger than expected. The large values of M_B are still puzzling, and may be a consequence of the fact that in the sum rule for (g_A-1) the continuum contributions of the g_A and mass sum rules almost cancel.

For the $g_A+g_A^S$ sum rule [Eq. (15)], we also obtain solutions for values of the Borel mass $M_B\geq 1.8$ GeV. The situation here, however, is quite different: The continuum corrections provided by E_0, E_1, and E_2 are quite important for our final results. For this reason we have examined the dependence of g_A^S on the value of M_B. We seek solutions to Eq. (15) for $g_A+g_A^S$ simultaneously with the mass sum rule [Eq. (12)] by fixing the value of the nucleon mass at its physical value and adjusting β_N^2 accordingly. We can obtain solutions of the g_A^S sum rule for $M_B\approx 1.2$ GeV, but with about a 35% increase in the value of the parameter β_N^2. The resulting g_A^S is not significantly changed. In other words, the value of g_A^S is quite insensitive to the value of M_B for $M_B\geq 1.2$ GeV. Numerically, we obtain (with $M_B\geq 1.8$ GeV)

$$g_A=1.26\pm0.08\ , \quad (16a)$$

$$g_A^S=0.13\pm0.08\ . \quad (16b)$$

There are a number of points to note in understanding the significance of the result shown in Eqs. (16a) and (16b). First, the errors shown are based, presumably, on variations in the parameter A (used to represent the residual continuum contribution) and uncertainties in the quark susceptibility κ. This method yields an approximate uncertainty of 30% in (g_A-1). We have introduced other parametrizations of the continuum, such as those discussed in Ref. [5], with no significant alteration in our result.

A most satisfactory aspect of our result is that we obtain a value of g_A consistent with experiment with a value of the quark condensate parameter a which gives rise to the correct magnetic moments of nucleons [3]. On the other hand, the value for g_A^S, which is very sensitive to the susceptibility χ, is not in agreement with the EMC data. The EMC data, together with an analysis of strange-baryon decays, yield [6,8]

$$g_A^S=\Delta u+\Delta d=0.28\pm0.08\ ,$$

a value slightly larger than ours. [Note that Δu and Δd extracted from the EMC data contain contributions from antiquarks, which have been neglected in our prediction (16b).] The sensitivity of our result to the induced condensate and susceptibilities can be judged by the fact that Gupta, Murthy, and Pasupathy [10] obtain $g_A^S\approx 0.35$ with a 20% smaller value of a.

If one takes $\kappa=\chi=0$, so that the last two terms in Eqs. (14) and (15) vanish, the resulting value of g_A is almost the same as before, but the value of g_A^S changes substantially:

$$g_A=1.30\pm0.08\ , \quad (17a)$$

$$g_A^S=-0.56\pm0.08\ . \quad (17b)$$

That is, the induced condensates are not very important for g_A, but the situation for g_A^S is completely different.

Indeed, the quark condensate a almost completely determines (g_A-1). Thus, if we take $a=0.0$, corresponding to vanishing quark condensate, we find

$$g_A=1.00\pm0.02\ , \quad (18a)$$

$$g_A^S = -0.68 \ . \tag{18b}$$

In the perturbative limit of free quarks, in which the gluon condensate is also zero, we find

$$g_A^S = -1.00 \ . \tag{18c}$$

This follows directly from the sum rule for $g_A - g_A^S - 2$, which vanishes in this limit; since $g_A = 1$, Eq. (18c) follows immediately. It is clear that g_A^S is very sensitive to the immersion of a nucleon into a nuclear medium. The value for g_A is that expected when chiral symmetry is restored at high densities or high temperature. The values $g_A^S = -1$ and $g_A = 1$ imply $\Delta u = 0$ and $\Delta d = -1$, a very strange result, but the free-quark perturbative limit is not physical. The error given in Eq. (18a) indicates the expected size of higher-dimensional terms which would appear in the chiral-symmetric limit. Note that the κ and χ condensates each contribute terms of the order of 0.01, but that they tend to cancel [see Eq. (14)].

It is interesting to speculate from this result on the value of g_A^*, the value of the axial-vector coupling constant in finite nuclei. If one uses the result of Cohen, Furnstahl, and Griegel [11], that the quark condensate is reduced by about 50% at nuclear matter density, one would naively expect that $g_A^* \approx 1.15$ in finite nuclei. As it is in a many-body environment, however, other covariants may appear [11,12], which may change the result for g_A^* and might even result in a g_A^* being near the free-nucleon value. On the other hand, the value for g_A^{S*}, which is likely to differ significantly from the free-space value of approximately zero, may have very important consequences for the measurements on the polarized structure function of a proton bound in a nucleus.

As a footnote, it is interesting to compare our result to the Adler-Weisberger sum rule [19], which can be written as

$$g_A^2 - 1 = \frac{1}{\pi} f_\pi^2 \int_{\nu_t}^{\infty} \frac{k \, d\nu}{\nu^2} \{\sigma(\pi^+ p) - \sigma(\pi^- p)\} \ , \tag{19}$$

with $\nu \equiv p \cdot q / M$, k the magnitude of the pion three-momentum in the laboratory frame, $\nu_t = m_\pi(1 + m_\pi / 2M_N)$, and $\sigma(\pi^{\pm} p)$ the total $\pi^+ p$ and $\pi^- p$ cross sections. From the PCAC (partial conservation of axial-vector current) relation, we have

$$f_\pi^2 m_\pi^2 \approx -2 m_q \langle \bar{q} q \rangle \ . \tag{20}$$

In the σ model, f_π vanishes with $\langle \bar{q} q \rangle$ [20], so that Eq. (19) gives $g_A \to 1$ when $\langle \bar{q} q \rangle \to 0$, in agreement with our result. Note, however, from Eq. (14) and the discussion following that equation that $g_A - 1 \propto \langle \bar{q} q \rangle^2$, while Eqs. (19) and (20) seem to imply $g_A - 1 \propto \langle \bar{q} q \rangle$ if m_π and $\sigma(\pi^{\pm} p)$ are independent of $\langle \bar{q} q \rangle$. Furthermore, the method of QCD sum rules refers to the physics at the scale characterized by M_B^2, while Eq. (20) is a low-energy theorem. What is remarkable is that both methods yield similar numerical results for g_A.

III. CONCLUSION

In this work we have reexamined the problem of extracting both the isovector and isoscalar axial-vector coupling constant g_A and g_A^S via the method of QCD sum rules. Our major contribution lies in recalculating g_A and g_A^S including terms consistently up to dimension 8 ($d = 8$), resulting in values of g_A and g_A^S consistent with experimental findings by means of standard QCD sum-rule parameters, and in studying the limits of chiral-symmetry restoration. By using sum rules for $g_A - 1$ and $g_A + g_A^S$, we have shown that almost all of the departure of g_A from 1.0 arises from the quark condensate. We find $g_A = 1.26 \pm 0.08$ and $g_A^S = 0.13 \pm 0.08$ with standard values of the condensates. We find $g_A \approx 1.00 \pm 0.02$ in the chiral-symmetry limit of $\langle \bar{q} q \rangle = 0.0$, certainly an interesting result. In the chiral limit, we also obtain $g_A^S = -0.68$ (if only the quark condensate vanishes) or $g_A^S = -1.00$ (in the free-field limit), also an interesting and somewhat puzzling result.

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