ARTICLES

Testing local Lorentz invariance of gravity with binary-pulsar data

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As gravity is a long-range force, one might *a priori* expect the Universe's global matter distribution to select a preferred rest frame for local gravitational physics. Two parameters α_1 and α_2 suffice to describe the phenomenology of preferred-frame effects in post-Newtonian gravity. One of them has already been very tightly constrained ($|\alpha_2| < 2.4 \times 10^{-7}$). We show here that binary-pulsar data provide a bound on the other one ($|\alpha_1| < 5.0 \times 10^{-4}$, 90% C.L.) which is quantitatively comparable to previous solar-system limits, but qualitatively more powerful because it is derived for systems comprising strong-gravitational-field regions. Our results correct a previous claim that α_1 could be very tightly constrained *via* a purported semisecular effect in the orbital period of binary pulsars.

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Local Lorentz invariance, i.e., the absence of preferred frames in local experiments, is an essential ingredient of our present understanding of the constitution and interactions of matter and is verified every day in high-energy experiments. If gravity is mediated only by a second-rank symmetric tensor field (as assumed in general relativity), or, more generally, by one symmetric tensor field and an arbitrary number of scalar fields, the gravitational physics of localized systems will also be boost-invariant (at least within a good approximation). On the other hand, it has been pointed out some time ago by Will and Nordtvedt [1] that if gravity is mediated in part by a long-range vector field (or by a second tensor field) one expects the Universe's global matter distribution to select a preferred rest frame for the gravitational interaction. In the post-Newtonian limit all the gravitational effects associated with the possible existence of such a preferred cosmic frame are phenomenologically describable by two parameters α_1 and α_2 [1]. These parameters contribute additional, non-boost-invariant, velocity-dependent terms in the gravitational many-body post-Newtonian Lagrangian, beyond the usual boost-invariant terms obtained in general relativity and its minimal extensions described by the Eddington parameters γ and β (which correspond to adding one or several scalar fields [2]). More precisely, the N-body post-Newtonian Lagrangian reads

$$L^{N\ \mathrm{body}} = L_{\beta,\gamma} + L_{\alpha_1} + L_{\alpha_2}, \tag{1}$$

$$\begin{aligned} L_{\beta,\gamma} = \sum_A -m_A c^2[1-(\mathbf{v}_A^0)^2/c^2]^{1/2} + \frac{1}{2}\sum_{A\neq B}\frac{G m_A m_B}{r_{AB}}\Bigg[1 &+ \frac{1}{2c^2}[(\mathbf{v}_A^0)^2+(\mathbf{v}_B^0)^2] - \frac{3}{2c^2}(\mathbf{v}_A^0\cdot\mathbf{v}_B^0) \\ &- \frac{1}{2c^2}(\mathbf{n}_{AB}\cdot\mathbf{v}_A^0)(\mathbf{n}_{AB}\cdot\mathbf{v}_B^0) + \frac{\gamma}{c^2}(\mathbf{v}_A^0-\mathbf{v}_B^0)^2\Bigg] \\ -\frac{1}{2}\sum_{B\neq A\neq C}(2\beta-1)\frac{G^2 m_A m_B m_C}{c^2 r_{AB} r_{AC}}\,, \end{aligned} \tag{2}$$

$$L_{\alpha_1} = -\frac{\alpha_1}{4}\sum_{A\neq B}\frac{G m_A m_B}{c^2 r_{AB}}(\mathbf{v}_A^0\cdot\mathbf{v}_B^0)\,, \tag{3}$$

$$L_{\alpha_2} = \frac{\alpha_2}{4} \sum_{A \neq B} \frac{G m_A m_B}{c^2 r_{AB}} \left[(\mathbf{v}_A^0 \cdot \mathbf{v}_B^0) - (\mathbf{n}_{AB} \cdot \mathbf{v}_A^0)(\mathbf{n}_{AB} \cdot \mathbf{v}_B^0) \right] , \tag{4}$$

in which $\mathbf{v}_A^0$ denotes the velocity of the mass m_A with respect to the Universe's preferred rest frame.

Bounds on the magnitudes of α_1 and α_2 have been obtained by several authors [3–7], based upon various effects associated with Eqs. (3) and (4) in the weak-gravitational-field context of the solar system. When deriving such bounds, it is necessary to make a definite assumption about the preferred rest frame entering the Lagrangians (3) and (4). The standard assumption [4–7] that we shall take up in the present paper is to take the frame defined by the cosmic microwave background. (If local Lorentz violation is due to an extra vector or tensor interaction, this assumption means essentially that its range is infinite or, at least, of cosmological magnitude.) The final results are that the close alignment of the Sun's spin axis with the solar system's planetary angular momentum yields an extremely tight bound on α_2 [7],

$$|\alpha_2| < 2.4 \times 10^{-7} , \tag{5}$$

while combined orbital data on the planetary system [6] yield a much weaker bound on α_1:

$$\alpha_1 = (2.1 \pm 1.9) \times 10^{-4} . \tag{6}$$

The limit (6) is only a factor five better than the present limits on the (more conservative) Eddington post-Newtonian parameters β and γ [8, 9].

In view of this situation, it seems important to investigate whether or not binary-pulsar data, which have proven to be marvelous gravitational probes [10–14], cannot be used to set more stringent bounds on α_1. In fact, it has been claimed [15] that a very stringent bound ($|\alpha_1| \lesssim 10^{-7}$) could be deduced from the agreement between the observed orbital period change of the binary pulsar PSR 1913+16 and the general relativistic prediction. Actually, we found that this claim was incorrect (see below), and this motivated us to look in detail at other ways of using binary-pulsar data to constrain the Lorentz-invariance-violation parameter α_1.

The α_1-dependent terms, Eq. (3), have several different types of observable consequences in the dynamics of a binary pulsar. First, let us consider a pair of mass elements (m_A, m_B) within the pulsar, and let us decompose (with, sufficient, Newtonian accuracy) each "absolute" velocity $\mathbf{v}_A^0$ according to $\mathbf{v}_A^0 = \mathbf{w} + \mathbf{v}_1 + \mathbf{u}_A$. Here, $\mathbf{w}$ denotes the velocity of the center of mass of the binary system with respect to the preferred rest frame, $\mathbf{v}_1$ denotes the velocity of the pulsar with respect to the center of mass of the binary system, and $\mathbf{u}_A$ denotes the velocity of the considered mass element with respect to the center of mass of the pulsar. From Eqs. (2) and (3) one sees that the matter within the pulsar experiences an effective gravitational constant [16, 3],

$$G^{\text{eff}} = G \left[1 - \frac{1}{2c^2} \alpha_1 (\mathbf{w} + \mathbf{v}_1)^2 \right] , \tag{7}$$

which is modulated, because of $\mathbf{v}_1$, at the orbital frequency. This modulation causes a corresponding modulation in the spin angular velocity of the pulsar, $\Delta\omega_1/\omega_1 = \kappa \Delta G^{\text{eff}}/G^{\text{eff}}$, which, after a time integration, contributes additional terms in the timing formula, giving the arrival times of the pulses. However, the integral of the term $-\alpha_1 \mathbf{w} \cdot \mathbf{v}_1 / c^2$ can be completely reabsorbed in the main ("Römer") term of the timing model [17], while the integral of the term $-\alpha_1 \mathbf{v}_1^2 / 2c^2$ can be reabsorbed in the "Einstein" time delay. In terms of observables, these reabsorptions lead to fractional modifications of the timing parameters $x^{\text{timing}} = a_1 \sin i / c$ and γ^{timing} of order $\Delta x^{\text{timing}} / x^{\text{timing}} \sim \alpha_1 \kappa w / c$ and $\Delta \gamma^{\text{timing}} / \gamma^{\text{timing}} \sim \alpha_1 \kappa$. In view of the current and foreseeable precision of the tests obtainable by combining the measurements of several timing parameters [13], these "internal" effects of α_1 cannot compete with the existing solar-system limit (6) and we shall not consider them any further.

Let us now consider the effects of α_1 on the orbital motion of the pulsar. Decomposing the "absolute" velocities according to $\mathbf{v}_1^0 = \mathbf{w} + \mathbf{v}_1$, $\mathbf{v}_2^0 = \mathbf{w} + \mathbf{v}_2$, where $\mathbf{w}$ and $\mathbf{v}_1$ have the same meaning as above, and where $\mathbf{v}_2$ denotes the velocity of the pulsar's companion with respect to the center of mass of the binary system, one gets

$$L_{\alpha_1}^{\text{orbital}} = -\frac{\widehat{\alpha}_1}{2} \frac{\widehat{G} m_1 m_2}{c^2 r_{12}} \left[\mathbf{w}^2 + \mathbf{w} \cdot (\mathbf{v}_1 + \mathbf{v}_2) + \mathbf{v}_1 \cdot \mathbf{v}_2 \right] . \tag{8}$$

We have added a caret to α_1 in Eq. (8) to denote a possible modification of the weak-field value of α_1 by strong-field-gravity effects in the pulsar and its companion. Similarly $\widehat{G} \equiv G_{12}$ denotes the effective gravitational coupling constant between the pulsar and its companion, including self-gravity effects (see, e.g., [2]).

One can verify that Eq. (8) predicts no overall secular acceleration of a binary system. One then defines $\mathbf{w}$ as the absolute velocity of the frames with respect to which the binary system is, on the average, at rest. In such a frame the (usually defined) instantaneous relativistic center of mass of the binary oscillates around its fixed average position by an amount $\Delta \mathbf{x}_{\text{c.m.}}(t)$, obtained by integrating

$$\frac{d}{dt} \Delta \mathbf{x}_{\text{c.m.}} = \frac{1}{c^2} \widehat{\alpha}_1 \widehat{G} M X_1 X_2 \left[\mathbf{w} \left(\frac{1}{r_{12}} - \frac{1}{a} \right) - \frac{X_1 - X_2}{2} \left(\frac{\mathbf{v}_{12}}{r_{12}} - \left\langle \frac{\mathbf{v}_{12}}{r_{12}} \right\rangle \right) \right] . \tag{9}$$

In Eq. (9) $M \equiv m_1 + m_2$, $X_1 \equiv m_1/M$, $X_2 \equiv m_2/M$, a is the semimajor axis of the relative orbit, $\mathbf{v}_{12} \equiv \mathbf{v}_1 - \mathbf{v}_2$ the relative velocity, and the angular brackets denote the time average (e.g. $\langle r_{12}^{-1}\rangle = a^{-1}$). Given the solar-system limit (6), and $v_{12}/c \lesssim w/c \sim 10^{-3}$, the wobbling $\Delta\mathbf{x}_{\rm c.m.}(t)$ is easily seen to give a negligible contribution to the timing of a pulsar such as PSR 1913+16: $\Delta t \sim \Delta x_{\rm c.m.}/c \lesssim 10^{-9}$ s.

It remains to study the effects of α_1 on the relative motion of the pulsar around its companion. It is easy to see in advance that, given the solar-system limit (6), all the periodic effects (that do not build up beyond one orbital period) give negligible contributions to the timing of the pulsar (similar to that associated with $\Delta\mathbf{x}_{\rm c.m.}$). Finally, our only hope of getting new, tight constraints on $\widehat{\alpha}_1$ is to study the secular effects in the relative motion.

Adding the contributions from β and γ [Eq. (2)] to the α_1 contributions from Eq. (8), and averaging over one orbital period the time derivatives of the energy, the angular momentum, and the Lagrange-Laplace (-Runge-Lenz) vector, one finds the following equations for the secular evolution of the Keplerian elements of the relative orbit:

$$\left\langle \frac{da}{dt} \right\rangle = 0 \,, \tag{10}$$

$$\left\langle \frac{d\boldsymbol{l}}{dt} \right\rangle = \frac{e}{1+(1-e^2)^{1/2}}\, \mathbf{b}\times\mathbf{k} \,, \tag{11}$$

$$\left\langle \frac{d\mathbf{e}}{dt} \right\rangle = \omega_R\, \mathbf{c}\times\mathbf{e} + \frac{1}{1+(1-e^2)^{1/2}} \times \left[(1-e^2)^{1/2}(\mathbf{k}\cdot\mathbf{a})\mathbf{a} + (\mathbf{k}\cdot\mathbf{b})\mathbf{b} - \frac{e^2}{(1-e^2)^{1/2}}(\mathbf{k}\cdot\mathbf{c})\mathbf{c} \right]. \tag{12}$$

In Eqs. (10)–(12) $\boldsymbol{l} \equiv (1-e^2)^{1/2}\mathbf{c}$, $\mathbf{e} \equiv e\mathbf{a}$,

$$\mathbf{k} = \frac{1}{2c^2}\,\widehat{\alpha}_1\, \frac{\widehat{G}M}{a^2}\,(X_1 - X_2)\mathbf{w} \,, \tag{13}$$

$$\omega_R = (2\widehat{\gamma} - \widehat{\beta} + 2 + X_1X_2\widehat{\alpha}_1)\, \frac{\widehat{G}M}{c^2a(1-e^2)}\, n \,, \tag{14}$$

where e is the eccentricity, and $n \equiv 2\pi/P_b = (\widehat{G}M/a^3)^{1/2}$ is the orbital frequency, $(\mathbf{a},\mathbf{b},\mathbf{c})$ being an orthonormal triad with $\mathbf{a}$ in the direction of the periastron of the pulsar orbit, and $\mathbf{c} = \mathbf{a}\times\mathbf{b}$ along the orbital angular momentum. Moreover, $\widehat{\gamma}$ and $\widehat{\beta}$ in Eq. (14) denote the effective values of the Eddington parameters for the relative orbital dynamics of two compact objects, including possible strong-field effects. (See [2] for the exact definition of these quantities.)

The result (10) shows that α_1 has no secular effect on the orbital period, contrarily to the claim of Ref. [15]. On the other hand, Eqs. (11) and (12) show that both the eccentricity and the spatial orientation of the Keplerian binary ellipse undergo secular changes when $\widehat{\alpha}_1 \neq 0$. [Eqs. (12) and (14) exhibit also the influence of $\widehat{\alpha}_1$ on the secular advance of the periastron.] In the case of the binary pulsar PSR 1913+16 the secular variation of e is observationally constrained at the level $|\dot{e}| < 1.9\times10^{-14}\ \mathrm{s}^{-1}$ [10], while Eq. (12) yields $\dot{e} = 1.39\widehat{\alpha}_1(\mathbf{w}\cdot\mathbf{a})/c\times10^{-9}\ \mathrm{s}^{-1}$. Even if $\mathbf{a}$ is favorably oriented (so that $|\mathbf{w}\cdot\mathbf{a}|/c \sim 10^{-3}$), this limits $\widehat{\alpha}_1$ only at the $\sim 10^{-2}$ level, which pales in comparison with Eq. (6). [The observational constraints on the variation of the timing parameter $x = aX_2\sin i/c$ [10] puts a limit on the change of the orbital inclination which, after using Eq. (11), yields an even weaker bound on $\widehat{\alpha}_1$.]

Fortunately the class of low-companion-mass, small-eccentricity, long-orbital-period binary pulsars turns out to provide a better testing ground for a possible violation of the local Lorentz invariance of the gravitational interaction. Taking into account the very small eccentricity of these systems, we can simplify very much the secular evolution system (11), (12). Equation (11) shows that the orbital plane is fixed, $\langle d\mathbf{c}/dt\rangle = O(e|\mathbf{k}|) \simeq 0$, while Eq. (12) becomes

$$\left\langle \frac{d\mathbf{e}}{dt} \right\rangle = \omega_R\, \mathbf{c}\times\mathbf{e} + \frac{1}{2}\mathbf{k}_\perp \,, \tag{15}$$

where $\mathbf{k}_\perp \equiv (\mathbf{k}\cdot\mathbf{a})\mathbf{a} + (\mathbf{k}\cdot\mathbf{b})\mathbf{b}$ is the projection of $\mathbf{k}$ onto the orbital plane. Equation (15) shows that the main new effect of an α_1-type Lorentz-invariance violation is to add a constant forcing term in the time evolution of the eccentricity vector which tries to "polarize" the orbit in the direction of the projection of $\mathbf{w}$ onto the orbital plane, $\mathbf{w}_\perp$. The familiar relativistic periastron precession term ω_R in Eq. (15) cuts off the build up of the constantly polarizing term $\frac{1}{2}\mathbf{k}_\perp$ and deflects its action by 90° (in a gyroscopelike way). More precisely the general solution of the linear evolution equation (15) can be written as the vectorial superposition

$$\mathbf{e}(t) = \mathbf{e}_F + \mathbf{e}_R(t) \,, \tag{16}$$

$$\mathbf{e}_F = \frac{1}{2\omega_R}\,\mathbf{c}\times\mathbf{k}_\perp = \frac{\widehat{\alpha}_1(X_1 - X_2)}{4(2\widehat{\gamma} - \widehat{\beta} + 2 + X_1X_2\widehat{\alpha}_1)}\, \frac{\mathbf{c}\times\mathbf{w}}{na} \,. \tag{17}$$

In Eq. (16) $\mathbf{e}_R(t)$ is a vector of constant magnitude which rotates in the orbital plane with angular velocity ω_R (usual relativistic periastron advance). By contrast, $\mathbf{e}_F$ is a fixed vector which represents a constant, $\widehat{\alpha}_1$-induced, polarization of the orbit (or "forced eccentricity"). Note that, because of the small denominator ω_R in Eq. (17), the velocity of light has dropped from the final expression of $\mathbf{e}_F$ whose magnitude depends essentially on $\widehat{\alpha}_1$ and the ratio $w_\perp/na$, where $w_\perp$ is the projection on the orbital plane of the absolute velocity of the center of mass of the binary system, and na is the relative orbital velocity. Note also that, although the instantaneous form of the α_1-type perturbing forces [derived from Eq. (8)] is very different from that corresponding to a possible differential free-fall acceleration in the gravitational field of the Galaxy (equivalence principle violation), their secular effects in small-eccentricity binary systems have exactly the same structure [compare Eqs. (15)–(17) above to Eqs. (3) and (4) of Ref. [12]]. We can therefore take up the method of Ref. [12] for deriving an upper bound on $|\hat{\alpha}_1|$ from the observations of binary pulsars having a very small eccentricity and a long orbital period. Let us first remark that a recent work [14] has derived experimental constraints on the magnitude of the parameters (β' and β'') that drive (in a class of theories) the possible strong-field effects in $\hat{G}$, $\hat{\beta}$, and $\hat{\gamma}$. These constraints are sufficiently tight to ensure that, in a system such as PSR 1855+09 that we shall consider in the following, one will have $\hat{G} = G(1 \pm 0.06)$ and $2\hat{\gamma} - \hat{\beta} + 2 = 3(1 \pm 0.04)$ at the 90% confidence level. Because of these results (which will be probably tightened in the future), we shall simplify Eq. (17) to $|\mathbf{e}_F| = \hat{\alpha}_1 |X_1 - X_2| w_\perp / 12(GMn)^{1/3}$. Applying to this expression the probabilistic reasoning of Ref. [12], we can conclude that the observation of an (old) binary-pulsar system having a (small) observed eccentricity e allows one to set an upper bound on $|\hat{\alpha}_1|$ given by

$$|\hat{\alpha}_1| < (10/\pi)\, I_{i,\lambda}\, e/\hat{e} \quad (90\%\ \text{C.L.})\ , \tag{18a}$$

where

$$I_{i,\lambda} = (2\pi)^{-1} \int_0^{2\pi} d\Omega\, [1 - (\cos i \cos\lambda + \sin i \sin\lambda \sin\Omega)^2]^{-1/2}\ , \tag{18b}$$

$$\hat{e} = \tfrac{1}{12} |X_1 - X_2|\, w/(GMn)^{1/3}\ , \tag{18c}$$

in which the full magnitude of the absolute velocity appears, $w \equiv |\mathbf{w}|$. The complete elliptic integral (18b) arises from making a probabilistic argument about the *a priori* unknown values of two angles in the problem: the time-dependent angle θ between $\mathbf{e}_F$ and $\mathbf{e}_R(t)$, and the longitude of the node Ω of the binary orbit with respect to the line of sight (see [12]). The factor $(10/\pi) I_{i,\lambda}$ represents a quantitatively precise way of allowing for unfavorable configurations of the angles θ and Ω when trying to estimate $\hat{\alpha}_1$ from the observed e. All the quantities appearing in the final results (18) are (in favorable circumstances) measurable from Earth: i is as above the inclination of the orbital plane, while λ is the angle between $\mathbf{w}$ and the line of sight.

From Eqs. (18), one sees that $P_b^{1/3}/e$ defines a figure of merit for selecting the binary-pulsar systems giving the best limits on $\hat{\alpha}_1$. A survey of existing small-eccentricity long-orbital-period binary pulsars show that the two systems PSR 0655+64 and PSR 1855+09 have, by far, the highest figures of merit. We shall consider solely the latter system which is the only one for which all the needed quantities have been measured (and which is known to be old enough for our probabilistic argument to be applicable).

The experimental results of Ryba and Taylor [18] on PSR 1855+09 yield $e = 2.167 \times 10^{-5}$, $P_b = 1.0650676 \times 10^6$ s, $i = 88.28°$, $X_1 - X_2 = 0.690$, $M = 1.50\, M_\odot$. On the other hand, the experimental results of the Cosmic Background Explorer (COBE) on the velocity of the solar system with respect to the cosmic microwave background [19] give $w = 365\ \text{km}\,\text{s}^{-1}$, in the direction $(\alpha, \delta) = (11.2\ \text{h}, -7°)$, i.e., making an angle $\lambda = 117°$ with the line of sight towards PSR 1855+09. (We are using the fact that PSR 1855+09 is a nearby system with small observed apparent transverse motion as seen from the solar system to estimate that $\mathbf{w}_{\text{PSR}} \simeq \mathbf{w}_\odot$.) Inserting all numbers in Eqs. (18) we get $I_{i,\lambda} = 1.43$, and the result

$$|\hat{\alpha}_1| < 5.0 \times 10^{-4} \qquad (90\%\ \text{C.L., PSR 1855+09 data})\ . \tag{19}$$

Our final (90% C.L.) upper bound (19) on a possible gravitational violation of local Lorentz invariance is interesting in two respects. First, it is quite comparable with the best existing solar-system limit (6) which allows, at the 90% confidence level (1.64σ), an α_1 as big as $+5.2 \times 10^{-4}$. Second, it is the first limit obtained for a gravitationally bound system which comprises strong-field regions (namely the $1.27 M_\odot$ neutron star seen as a pulsar in PSR 1855+09). Recently, it has been shown [2] that there existed classes of boost-invariant theories which had the same Eddington parameters as general relativity in the weak-field limit but for which the effective values of $\hat{\beta}$ and $\hat{\gamma}$ for a system containing compact objects would generically be given by power series in the compactness parameters $c_A = -2\partial \ln m_A / \partial \ln G$ of the type $\hat{\beta} = 1 - \frac{1}{2}\beta'(X_1 c_2 + X_2 c_1) + \cdots$, $\hat{\gamma} = 1 - \beta'(c_1^2 + c_2^2) + \cdots$. If a similar result holds for non-boost-invariant theories, we see that our limit (19), taken in conjunction with the solar-system limit (6), provides already tight limits on the coefficients of any conceivable strong-field modification of α_1: $\hat{\alpha}_1 = \alpha_1 + \alpha_1'(c_1 + c_2) + \cdots$ ($c_1 \approx 0.27$ for a $1.27 M_\odot$ neutron star).

Let us also note that if one could extract from the PSR 1855+09 data a bound on the secular variation of the eccentricity vector at a level $|d\mathbf{e}/dt| < 2 \times 10^{-16}\ \text{s}^{-1}$ (i.e., about a factor 10 below the present limit on de/dt) one could both render more secure (by freeing it from the need to use probabilistic considerations) and tighten the limit (19). Indeed, we have from Eq. (16) $d\mathbf{e}/dt = \omega_R \mathbf{c} \times \mathbf{e}_R$ so that $|d\mathbf{e}/dt|$ gives access to the magnitude of $\mathbf{e}_R$. The level $2 \times 10^{-16}\ \text{s}^{-1}$ quoted above corresponds to $e_R/e < (10/\pi) I_{i,\lambda} = 4.55$, i.e., a level for e_R where the probability argument behind the derivation of the safety factor $(10/\pi) I_{i,\lambda}$ in Eq. (18a) is becoming too pessimistic.

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