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# SU(3)⊗U(1) model for electroweak interactions

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We consider a gauge model based on a SU(3)⊗U(1) symmetry in which the lepton number is violated explicitly by charged scalar and gauge bosons, including a vector field with double electric charge.

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## I. INTRODUCTION

Some years ago, it was pointed out that processes such as $e^-e^- \rightarrow W^-V^-$ in Fig. 1(a), if induced by right-handed currents coupled to the vector $V^-$, imply violation of unitarity at high energies. Then, if the right-handed currents are part of a gauge theory, it has been argued that at least some neutrinos must have a nonzero mass [1].

The argument to justify this follows exactly the same way as in the usual electroweak theory for the process $\nu\bar{\nu} \rightarrow W^+W^-$. The graph induced by an electron exchange has bad high-energy behavior; when the energy goes to infinity, the respective amplitude violates unitarity [2].

In Fig. 1(a) the lower vertex indicates a right-handed current which absorbs the right-handed antineutrino coming from the upper vertex, which represents the left-handed current of the electroweak standard model. The part of the amplitude, corresponding to Fig. 1(a), in which we are interested is

$$\sum_{\nu_m} U^L_{\mathrm{em}} \frac{\not{q}}{q^2 - M^2_{\nu_m}} U^R_{\mathrm{em}} \,, \tag{1}$$

where $U^L$ ($U^R$) is the mixing matrix in the left- (right)-handed current, and $q$ is the four-momentum transfer [1]. The space-time structure of Eq. (1) is the same as the charged-lepton exchange amplitude in the process $\nu\bar{\nu} \rightarrow W^+W^-$ [2]. Then, we must have the same bad high-energy behavior of the last process. One way to avoid this is to have a cancellation among the contributions from the various $\nu_m$ exchanges when we add them up; at high energy and large $q^2$, the latter dominates the denominator in Eq. (1), and if we require that

$$\sum_{\nu_m} U^L_{\mathrm{em}} U^R_{\mathrm{em}} = 0, \tag{2}$$

the amplitude in Eq. (1) vanishes even at low energies, unless at least one of the masses $M_{\nu_m}$ is nonzero. On the other hand, the diagram in Fig. 1(a) or its time-reversed one $W^-W^- \rightarrow e^-e^-$ appearing in Fig. 1(b), when both vertices are left handed, proceeds via Majorana massive neutrinos.

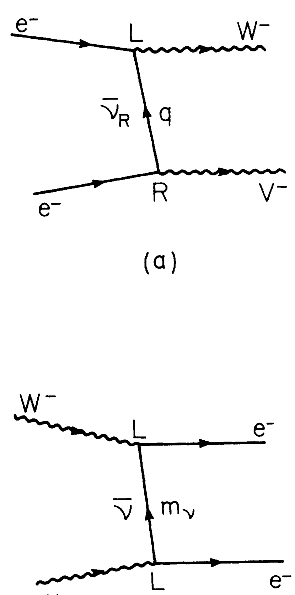



FIG. 1. (a) $e^-e^- \rightarrow W^-V^-$ process induced by right-handed currents; $L$ and $R$ denote the handedness of the current at the vertex, and $q$ is the momentum transfer. (b) Diagram for $W^-W^- \rightarrow e^-e^-$ with massive Majorana neutrinos; both vertices are left-handed.

Here we are concerned with a gauge model based on a $\mathrm{SU}_L(3)\otimes\mathrm{U}_N(1)$ symmetry. The original motivation leading to the study of this model stemmed from the observation that a gauge theory must be consistent, that is, unitary and renormalizable, independently of the values of some parameters, such as mixing angles. Then, from this point of view, instead of using the condition in Eq. (2) in order to solve the problem of the graph in Fig. 1(a), we prefer the introduction of a doubly charged gauge boson which, like the $Z^0$ in the standard model, will restore the good high-energy behavior.

Although there exist in the literature several models based on a SU(3)⊗U(1) gauge symmetry [3–7], our model has a different representation content and a quite different new physics at an, in principle, arbitrary mass scale. The main new features of our model occur in processes in which the initial electric charge is not zero. Even from the theoretical point of view, that sort of processes have not been well studied; for instance, general results exist only for zero initial charge [8].

The plan of this paper is as follows. Section II is devoted to present the model. Some phenomenological consequences are given in Sec. III. In this way we can estimate the allowed value for the mass scale characterizing the new physics. In Sec. IV we study briefly the scalar potential and show that there is no mixing between the lepton-number-conserving and lepton-number-violating scalar fields which could induce decays such as the neutrinoless double $\beta$ decay. The last section is devoted to our conclusions and some comments and in the Appendix we give more details about the definition of the charge-conjugation operation we have used in this work.

## II. THE MODEL

As we said before, the gauge model that we shall consider is one in which the gauge group is $\mathrm{SU}_L(3)\otimes\mathrm{U}_N(1)$. This is possibly the simplest way to enlarge the gauge group $\mathrm{SU}_L(2)\otimes\mathrm{U}_Y(1)$ in order to have doubly charged gauge bosons, without losing the natural features of the standard electroweak model. The price we must pay is the introduction of exotic quarks, with electric charges 5/3 and $-4/3$.

In this model we have the processes appearing in Figs. 2(a) and 2(b); the last diagram plays the same role as the similar diagram with $Z^0$ in the standard model and it restores the "safe" high-energy behavior of the model. Both vector bosons $V^-$ and $U^{--}$ in Figs. 2(a) and 2(b) are very massive, and their masses depend on the mass scale of the breaking of the $\mathrm{SU}_L(3)\otimes\mathrm{U}_N(1)$ symmetry into $\mathrm{SU}_L(2)\otimes\mathrm{U}_Y(1)$. Phenomenological bounds on this mass scale will be given in the next section.

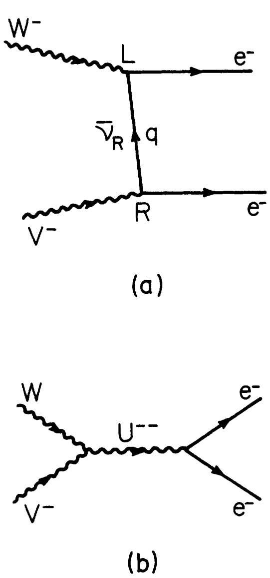



FIG. 2. Diagram for $W^-V^- \to e^-e^-$ due to the existence of right-handed current (a) and doubly charged gauge boson (b).

### A. Yukawa interactions

We start by choosing the following triplet representations for the left-handed fields of the first family:

$$E_L = \begin{pmatrix} \nu_e \\ e \\ e^c \end{pmatrix}_L (\mathbf{3},0), \quad Q_{1L} = \begin{pmatrix} u \\ d \\ J_1 \end{pmatrix}_L (\mathbf{3},+\tfrac{2}{3}), \tag{3}$$

and

$$u_R\,(\mathbf{1},+\tfrac{2}{3}), d_R\,(\mathbf{1},-\tfrac{1}{3}), J_{1R}\,(\mathbf{1},+\tfrac{5}{3}), \tag{4}$$

for the respective right-handed fields. Notice that we have not introduced right-handed neutrinos. The numbers $0, 2/3$ in Eq. (3), and $2/3, -1/3$, and $5/3$ in Eq. (4) are $\mathrm{U}_N(1)$ charges. The electric charge operator has been defined as

$$\frac{Q}{e} = \frac{1}{2}\left(\lambda_3 - \sqrt{3}\lambda_8\right) + N, \tag{5}$$

where $\lambda_3$ and $\lambda_8$ are the usual Gell-Mann matrices; $N$ is proportional to the unit matrix. Then, the exotic quark $J_1$ has an electric charge $+5/3$.

The other two lepton generations also belong to triplet representations:

$$M_L = \begin{pmatrix} \nu_\mu \\ \mu \\ \mu^c \end{pmatrix}_L (\mathbf{3},0), \quad T_L = \begin{pmatrix} \nu_\tau \\ \tau \\ \tau^c \end{pmatrix}_L (\mathbf{3},0). \tag{6}$$

The model is anomaly-free if we have equal number of triplets and antitriplets, counting the color of $\mathrm{SU}(3)_c$, and furthermore requiring the sum of all fermion charges to vanish. As in the model of Ref. [3], the anomaly cancellation occurs for the three generations together and not generation by generation.

Then, we must introduce the antitriplets

$$Q_{2L} = \begin{pmatrix} J_2 \\ c \\ s \end{pmatrix}_L (\mathbf{3}^*,-\tfrac{1}{3}), \quad Q_{3L} = \begin{pmatrix} J_3 \\ t \\ b \end{pmatrix}_L (\mathbf{3}^*,-\tfrac{1}{3}), \tag{7}$$

also with the respective right-handed fields in singlets. The quarks $J_2$ and $J_3$ have both charge $-4/3$.

In order to generate fermion masses, we introduce the following Higgs triplets, $\eta, \rho$, and $\chi$:

$$\begin{pmatrix} \eta^0 \\ \eta_1^- \\ \eta_2^+ \end{pmatrix} (\mathbf{3},0), \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} (\mathbf{3},1), \qquad \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} (\mathbf{3},-1), \tag{8}$$

These Higgs triplets will produce the following hierarchical symmetry breaking:

$$\mathrm{SU}_L(3)\otimes\mathrm{U}_N(1) \xrightarrow{\langle\chi\rangle} \mathrm{SU}_L(2)\otimes\mathrm{U}_Y(1) \xrightarrow{\langle\rho,\eta\rangle} \mathrm{U}_{\mathrm{em}}(1), \tag{9}$$

The Yukawa Lagrangian, without considering the mixed terms between quarks, is

$$-\mathcal{L}_Y = \frac{1}{2}\sum_l G_l \varepsilon^{ijk}\bar{\psi}^c_{li}\psi_{lj}\eta_k + \bar{Q}_{1L}(G_u u_R \eta + G_d d_R \rho + G_{J1} J_{1R}\chi) + (G_c \bar{Q}_{2L} c_R + G_t \bar{Q}_{3L} t_R)\,\rho^* + (G_s \bar{Q}_{2L} s_R + G_b \bar{Q}_{3L} b_R)\,\eta^* + (G_{J2}\bar{Q}_{2L} J_{2R} + G_{J3}\bar{Q}_{3L} J_{3R})\,\chi^* + \text{H.c.} \quad (10)$$

with $l = e, \mu, \tau$. Explicitly, we have, for the leptons,

$$2\mathcal{L}_{lY} = \sum_l G_l \left[ (\bar{l}^c_R l^c_L - \bar{l}_R l_L)\eta^0 - (\bar{\nu}^c_{lR} l^c_L - \bar{l}_R \nu_{lL})\eta_1^- + (\bar{\nu}^c_{lR} l_L - \bar{l}^c_R \nu_{lL})\eta_2^+ \right] + \text{H.c.}, \quad (11)$$

and, using the definition of charge conjugation $\psi^c = \gamma^5 C \bar{\psi}^T$ that we shall discuss in the Appendix, we can write Eq. (11) as

$$\mathcal{L}_{lY} = \sum_l G_l (-\bar{l}_R l_L \eta^0 + \bar{l}_R \nu_L \eta_1^- + \bar{\nu}^c_R l_L \eta_2^+ + \text{H.c.}). \quad (12)$$

In Eq. (12) there is lepton-number violation through the coupling with the $\eta_2^+$ Higgs scalar.

For the first and second quark generations we have the Yukawa interactions

$$\begin{aligned} -\mathcal{L}_{QY} = {} & G_u(\bar{u}_L u_R \eta^0 + \bar{d}_L u_R \eta_1^- + \bar{J}_{1L} u_R \eta_2^+) \\ & + G_d(\bar{u}_L d_R \rho^+ + \bar{d}_L d_R \rho^0 + \bar{J}_{1L} d_R \rho^{++}) \\ & + G_c(\bar{J}_{2L} c_R \rho^{--} + \bar{c}_L c_R \rho^{0*} + \bar{s}_L c_R \rho^-) \\ & + G_s(\bar{J}_{2L} s_R \eta_2^- + \bar{c}_L s_R \eta_1^+ + \bar{s}_L s_R \eta^{0*}) \\ & + G_{J_1}(\bar{u}_L J_{1R}\chi^- + \bar{d}_L J_{1R}\chi^{--} + \bar{J}_{1L} J_{1R}\chi^{0*}) \\ & + G_{J_2}(\bar{J}_{2L} J_{2R}\chi^0 + \bar{c}_L J_{2R}\chi^{++} + \bar{s}_L J_{2R}\chi^+) + \text{H.c.} \end{aligned} \quad (13)$$

The Yukawa interactions for the third quark generation are obtained from those of the second generation making $c \to t$, $s \to b$, and $J_2 \to J_3$. In Eq. (10), since the neutrinos are massless there is no mixing between leptons, so it is not necessary at all to consider terms such as $\frac{1}{2}\sum_{n,m} h_{lm}\bar{\psi}^c_{niL}\psi_{mjL}H^{[ij]} + \text{H.c.}$ where $n, m = e, \mu, \tau$, the coupling constants $h_{nm} = -h_{mn}$ and $H^{[ij]} = \varepsilon^{ijk}\eta^k$.

The neutral component of the Higgs fields develops the vacuum expectation value

$$\langle \eta^0 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_\eta \\ 0 \\ 0 \end{pmatrix}, \quad \langle \rho^0 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_\rho \\ 0 \end{pmatrix}, \quad \langle \chi^0 \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ v_\chi \end{pmatrix}. \quad (14)$$

So, the masses of the fermions are $m_l = G_l \frac{v_\eta}{\sqrt{2}}$ for the charged leptons and

$$\begin{aligned} m_u &= G_u \tfrac{v_\eta}{\sqrt{2}}, & m_c &= G_c \tfrac{v_\rho}{\sqrt{2}}, & m_t &= G_t \tfrac{v_\rho}{\sqrt{2}}, \\ m_d &= G_d \tfrac{v_\rho}{\sqrt{2}}, & m_s &= G_s \tfrac{v_\eta}{\sqrt{2}}, & m_b &= G_b \tfrac{v_\eta}{\sqrt{2}}, \\ m_{J_1} &= G_{J_1} \tfrac{v_\chi}{\sqrt{2}}, & m_{J_2} &= G_{J_2} \tfrac{v_\chi}{\sqrt{2}}, & m_{J_3} &= G_{J_3} \tfrac{v_\chi}{\sqrt{2}}, \end{aligned} \quad (15)$$

for the quarks. The exotic quarks obtain their masses from the $\chi$ triplet. Notice that, if we had introduced right-handed neutrinos, we would have massive Dirac neutrinos through their couplings with the $\eta$ Higgs triplet.

### B. The gauge bosons

The gauge bosons of this theory consist of an octet $W^a_\mu$ associated with $\mathrm{SU}_L(3)$ and a singlet $B_\mu$ associated with $\mathrm{U}_N(1)$. The covariant derivatives are

$$\mathcal{D}_\mu \varphi_i = \partial_\mu \varphi_i + ig(\mathbf{W}_\mu \cdot \boldsymbol{\lambda}/2)^j_i \varphi_j + ig' N_\varphi \varphi_i B_\mu, \quad (16)$$

where $N_\varphi$ denotes the $N$ charge for the $\varphi$ Higgs multiplet, $\varphi = \eta, \rho, \chi$. Using Eqs. (14) in Eq. (16) we obtain the symmetry-breaking pattern appearing in Eq. (9).

The gauge bosons $\sqrt{2}W^+ \equiv -(W^1 - iW^2)$, $\sqrt{2}V^- \equiv -(W^4 - iW^5)$, and $\sqrt{2}U^{--} \equiv -(W^6 - iW^7)$ have the masses

$$M_W^2 = \frac{1}{4}g^2\left(v_\eta^2 + v_\rho^2\right), \quad M_V^2 = \frac{1}{4}g^2\left(v_\eta^2 + v_\chi^2\right), \quad M_U^2 = \frac{1}{4}g^2\left(v_\rho^2 + v_\chi^2\right). \quad (17)$$

Notice that even if $v_\eta = v_\rho \approx v/\sqrt{2}$, $v$ being the usual vacuum expectation value of the Higgs boson in the standard model, the $v_\chi$ must be large enough to keep the new gauge bosons $V^+$ and $U^{++}$ sufficiently heavy in order to have consistency with low-energy phenomenology. On the other hand, the neutral gauge bosons have the following mass matrix in the $(W^3, W^8, B)$ basis:

$$M^2 = \frac{1}{4}g^2 \begin{pmatrix} v_\eta^2 + v_\rho^2 & \frac{1}{\sqrt{3}}(v_\eta^2 - v_\rho^2) & -2\frac{g'}{g}v_\rho^2 \\ \frac{1}{\sqrt{3}}(v_\eta^2 - v_\rho^2) & \frac{1}{3}(v_\eta^2 + v_\rho^2 + 4v_\chi^2) & \frac{2}{\sqrt{3}}\frac{g'}{g}(v_\rho^2 + 2v_\chi^2) \\ -2\frac{g'}{g}v_\rho^2 & \frac{2}{\sqrt{3}}\frac{g'}{g}(v_\rho^2 + 2v_\chi^2) & 4\frac{g'^2}{g^2}(v_\rho^2 + v_\chi^2) \end{pmatrix}, \tag{18}$$

and, since $\det M^2 = 0$, we must have a photon after the symmetry breaking. If we had introduced a $\mathbf{6}^*$, the matrix $M^2$ in Eq. (18) would be such that $\det M^2 \neq 0$. In fact, the eigenvalues of the matrix in Eq. (18) are

$$M_A^2 = 0, \qquad M_Z^2 \simeq \frac{g^2}{4}\frac{g^2+4g'^2}{g^2+3g'^2}(v_\eta^2+v_\rho^2),$$
$$M_{Z'}^2 \simeq \frac{1}{3}(g^2+3g'^2)v_\chi^2, \tag{19}$$

where we have used $v_\chi \gg v_{\rho,\eta}$ for the case of $M_Z$ and $M_{Z'}$. Notice that the $Z'^0$ boson has a mass proportional to $v_\chi$ and, like the charged bosons $V^+, U^{++}$, must be very massive. In the present model we have

$$\frac{M_Z^2}{M_W^2} = \frac{1+4t^2}{1+3t^2}, \tag{20}$$

where $t = g'/g \equiv \tan\theta$, and in order to obtain the usual relation $\cos^2\theta_W M_Z^2 = M_W^2$, with $\cos^2\theta_W \approx 0.78$, we must have $\theta \approx 54°$, i.e., $\tan^2\theta \approx 11/6$. Then, we can identify $Z^0$ as the neutral gauge boson of the standard model.

The neutral physical states are

$$A_\mu = \frac{1}{(1+4t^2)^{\frac{1}{2}}}\left[(W_\mu^3 - \sqrt{3}W_\mu^8)t + B_\mu\right],$$

$$Z_\mu^0 \simeq -\frac{1}{(1+4t^2)^{\frac{1}{2}}}\left[(1+3t^2)^{\frac{1}{2}}W_\mu^3 + \frac{\sqrt{3}t^2}{(1+3t^2)^{\frac{1}{2}}}W_\mu^8 - \frac{t}{(1+3t^2)^{\frac{1}{2}}}B_\mu\right], \tag{21}$$

$$Z_\mu'^0 \simeq \frac{1}{(1+3t^2)^{\frac{1}{2}}}\left(W_\mu^8 + \sqrt{3}tB_\mu\right).$$

Concerning the vector bosons, we have the trilinear interactions $W^+W^-N$, $V^+V^-N$, $U^{++}U^{--}N$, and $W^+V^+U^{--}$, where $N$ could be any of the neutral vector bosons $A, Z^0$, or $Z'^0$.

### C. Charged and neutral currents

The interactions among the gauge bosons and fermions are read off from

$$\mathcal{L}_F = \bar{R}i\gamma^\mu(\partial_\mu + ig'B_\mu N)R + \bar{L}i\gamma^\mu\left(\partial_\mu + ig'B_\mu N + \frac{ig}{2}\boldsymbol{\lambda}\cdot\mathbf{W}_\mu\right)L, \tag{22}$$

where $R$ represents any right-handed singlet and $L$ any left-handed triplet.

Let us consider first the leptons. For the charged leptons, we have the electromagnetic interaction by identifying the electron charge as (see the Appendix)

$$e = \frac{g\sin\theta}{(1+3\sin^2\theta)^{\frac{1}{2}}} = \frac{g'\cos\theta}{(1+3\sin^2\theta)^{\frac{1}{2}}}, \tag{23}$$

and the charged-current interactions are

$$\mathcal{L}_l^{CC} = -\frac{g}{\sqrt{2}}\sum_l\left(\bar{\nu}_{lL}\gamma^\mu l_L W_\mu^+ + \bar{l}_L^c\gamma^\mu\nu_{lL}V_\mu^+ + \bar{l}_L^c\gamma^\mu l_L U_\mu^{++} + \text{H.c.}\right). \tag{24}$$

Notice that, as we have not assigned to the gauge bosons a lepton number, we have explicit breakdown of this quantum number induced by the $V^+, U^{++}$ gauge bosons. A similar mechanism for lepton number violation was proposed in Ref. [9] but in that reference the lepton-number-violating currents are coupled to the standard gauge bosons and they are proportional to a small parameter appearing in this model.

For the first generation of quarks we have the charged-current interactions

$$\mathcal{L}_{Q_1W}^{CC} = -\frac{g}{\sqrt{2}}\left(\bar{u}_L\gamma^\mu d_{\theta L}W_\mu^+ + \bar{J}_{1L}\gamma^\mu u_L V_\mu^+ + \bar{d}_{\theta L}\gamma^\mu J_{1L}U^{--} + \text{H.c.}\right), \tag{25}$$

and, for the second generation of quarks we have

$$\mathcal{L}_{Q_2W}^{CC} = -\frac{g}{\sqrt{2}}\left(\bar{c}_L\gamma^\mu d_{\theta L}W_\mu^+ - \bar{s}_{\theta L}\gamma^\mu J_{2\phi L}V_\mu^+ + \bar{c}_L\gamma^\mu J_{2\phi L}U^{--} + \text{H.c.}\right). \tag{26}$$

The charge-changing interactions for the third generation of quarks are obtained from those of the second generation, making $c \to t$, $s \to b$, and $J_2 \to J_3$. We have mixing only in the $Q = -\frac{1}{3}$ and $Q = -\frac{4}{3}$ sectors, then in Eqs. (25) and (26) $d_\theta, s_\theta$, and $J_{2\phi}$ mean Cabibbo-Kobayashi-Maskawa states in the three- and two-dimensional flavor space $d, s, b$ and $J_2, J_3$ respectively.

Similarly, we have the neutral currents coupled to both $Z^0$ and $Z'^0$ massive vector bosons, according to the Lagrangian

$$\mathcal{L}_\nu^{NC} = -\frac{g}{2}\frac{M_Z}{M_W}\bar{\nu}_{lL}\gamma^\mu\nu_{lL}\left(Z_\mu - \frac{1}{\sqrt{3}}\frac{1}{\sqrt{h(t)}}Z'_\mu\right), \tag{27}$$

with $h(t) = 1 + 4t^2$, for neutrinos and

$$\mathcal{L}_l^{NC} = -\frac{g}{4}\frac{M_Z}{M_W}[\bar{l}\gamma^\mu(v_l + a_l\gamma^5)lZ_\mu + \bar{l}\gamma^\mu(v'_l + a'_l\gamma^5)lZ'_\mu], \tag{28}$$

for the charged leptons, where we have used $\bar{l}^c_L\gamma^\mu l^c_L = -\bar{l}_R\gamma^\mu l_R$ and defined

$$v_l = -1/h(t), \quad a_l = 1, \tag{29a}$$

$$v'_l = -\sqrt{3/h(t)}, \quad a'_l = v'_l/3. \tag{29b}$$

With $t^2 = 11/6$, $v_l$ and $a_l$ have the same values of the standard model.

As it was said before, the quark representations in Eqs. (3) and (7) are symmetry eigenstates; that is, they are related to the mass eigenstates by Cabibbo-like angles. As we have one triplet and two antitriplets, it should be expected flavor-changing neutral currents exist. Notwithstanding, as we shall show below, when we calculate the neutral currents explicitly, we find that *all* of them, for the same charge sector, have equal factors and the Glashow-Iliopoulos-Maiani (GIM) [2] cancellation is automatic in neutral currents coupled to $Z^0$. Remember that, in the standard electroweak model, the GIM mechanism is a consequence of having each charge sector the same coupling with $Z^0$; for example, for the charge +2/3 sector,

$$v^U_{\rm SM} = 1 - \frac{8}{3}\sin^2\theta_W, \quad a^U_{\rm SM} = -1. \tag{30}$$

The Lagrangian interaction among quarks and the $Z^0$ is

$$\mathcal{L}_{ZQ} = -\frac{g}{4}\frac{M_Z}{M_W}\sum_i[\bar\Psi_i\gamma^\mu(v^i + a^i\gamma^5)\Psi_i]Z_\mu, \tag{31}$$

where $i = u,c,t,d,s,b,J_1,J_2,J_3$; with

$$v^U = (3+4t^2)/3h(t), \quad a^U = -1, \tag{32a}$$

$$v^D = -(3+8t^2)/3h(t), \quad a^D = 1, \tag{32b}$$

$$v^{J1} = -20t^2/3h(t), \quad a^{J_1} = 0, \tag{32c}$$

$$v^{J_2} = v^{J_3} = 16t^2/3h(t), \quad a^{J_2} = a^{J_3} = 0; \tag{32d}$$

$U$ and $D$ mean the charge +2/3 and −1/3 respectively, the same for $J_{1,2,3}$. Notice that, as was said above, there is no flavor-changing neutral current coupled to the $Z^0$ field and the exotic quarks couple to $Z^0$ only through vector currents. It is easy to verify that for the $Q = \frac{2}{3}, -\frac{1}{3}$ sectors the respective coefficients $v$ and $a$ also coincide with those of the standard electroweak model if $t^2 = 11/6$, as required to maintain the relation $\cos\theta_W M_Z = M_W$.

The same cancellation does not happen with the corresponding currents coupled to the $Z'^0$ boson, each quark having its respective coefficients. Explicitly, we have

$$\mathcal{L}_{Z'Q} = -\frac{g}{4}\frac{M_Z}{M_W}\sum_i[\bar\Psi_i\gamma^\mu(v'^i + a'^i\gamma^5)\Psi_i]Z'_\mu, \tag{33}$$

where

$$v'^u = -(1+8t^2)/\sqrt{3h(t)}, \quad a'^u = 1/\sqrt{3h(t)}, \tag{34a}$$

$$v'^c = v'^t = (1-2t^2)/\sqrt{3h(t)}, \quad a'^c = a'^t = -(1+6t^2)/\sqrt{3h(t)}, \tag{34b}$$

$$v'^d = -(1+2t^2)/\sqrt{3h(t)}, \quad a'^d = -a'^c, \tag{34c}$$

$$v'^s = v'^b = \sqrt{h(t)/3}, \quad a'^s = a'^b = -a'^u, \tag{34d}$$

for the usual quarks, and

$$v'^{J_1} = \frac{2}{\sqrt3}\frac{1-7t^2}{\sqrt{h(t)}}, \quad a'^{J_1} = -\frac{2}{\sqrt3}\frac{1+3t^2}{\sqrt{h(t)}}, \tag{35a}$$

$$v'^{J_2} = v'^{J_3} = -\frac{2}{\sqrt3}\frac{1-5t^2}{\sqrt{h(t)}}, \quad a'^{J_2} = a'^{J_3} = -a'^{J_1}, \tag{35b}$$

for the exotic quarks.

## III. THE SCALAR POTENTIAL

The most general gauge-invariant potential involving the three Higgs triplets is

$$\begin{aligned}V(\eta,\rho,\chi) = {}& \mu_1^2\eta^\dagger\eta + \mu_2^2\rho^\dagger\rho + \mu_3^2\chi^\dagger\chi + \lambda_1(\eta^\dagger\eta)^2 + \lambda_2(\rho^\dagger\rho)^2 + \lambda_3(\chi^\dagger\chi)^2\\ &+ (\eta^\dagger\eta)[\lambda_4\rho^\dagger\rho + \lambda_5\chi^\dagger\chi] + \lambda_6(\rho^\dagger\rho)(\chi^\dagger\chi) + \sum_{ijk}\epsilon^{ijk}(f\eta_i\rho_j\chi_k + \text{H.c.}).\end{aligned} \tag{36}$$

The coupling $f$ has dimension of mass. We can analyze the scalar spectrum defining

$$\eta^0 = v_1 + H_1 + ih_1, \quad \rho^0 = v_2 + H_2 + ih_2, \quad \chi^0 = v_3 + H_3 + ih_3, \tag{37}$$

where we have redefined $v_\eta/\sqrt{2}, v_\rho/\sqrt{2}$, and $v_\chi/\sqrt{2}$ as $v_1, v_2$, and $v_3$ respectively, and for simplicity we are not considering relative phases between the vacuum expectation values. Here we are only interested in the charged scalars spectrum. Requiring that the shifted potential has no linear terms in any of the $H_i$ and $h_i$ fields, $i = 1, 2, 3$, we obtain in the tree approximation the constraint equations

$$\begin{aligned} \mu_1^2 + 2\lambda_1 v_1^2 + \lambda_4 v_2^2 + \lambda_5 v_3^2 + \mathrm{Re}\, f v_1^{-1} v_2 v_3 &= 0, \\ \mu_2^2 + 2\lambda_2 v_2^2 + \lambda_4 v_1^2 + \lambda_6 v_3^2 + \mathrm{Re}\, f v_1 v_2^{-1} v_3 &= 0, \\ \mu_3^2 + 2\lambda_3 v_3^2 + \lambda_5 v_1^2 + \lambda_6 v_2^2 + \mathrm{Re}\, f v_1 v_2 v_3^{-1} &= 0, \\ \mathrm{Im}\, f &= 0. \end{aligned} \tag{38}$$

Then, it is possible to verify that there is a doubly charged Goldstone boson and a doubly charged physical scalar. There are also two singly charged Goldstone bosons,

$$\begin{aligned} G_1^- &= (-v_1\eta_2^- + v_3\chi^-)/(v_1^2 + v_3^2)^{\frac{1}{2}}, \\ G_2^- &= (-v_1\eta_1^- + v_2\rho^-)/(v_1^2 + v_2^2)^{\frac{1}{2}}, \end{aligned} \tag{39}$$

and two singly charged physical scalars,

$$\begin{aligned} \phi^- &= (v_3\eta_2^- + v_1\chi^-)/(v_1^2 + v_3^2)^{\frac{1}{2}}, \\ \varphi^- &= (v_2\eta_1^- + v_1\rho^-)/(v_1^2 + v_2^2)^{\frac{1}{2}}, \end{aligned} \tag{40}$$

with masses $m_1^2 = f v_2(v_1^{-1}v_3 + v_1 v_3^{-1})$ and $m_2^2 = f v_3(v_1^{-1}v_2 + v_2^{-1}v_1)$ respectively. We can see from Eq. (40) that the mixing occurs between $\eta_2^-$ and $\chi^-$, $\eta_1^-$ and $\rho^-$ but not between $\eta_1^-$ and $\eta_2^-$. This implies that the neutrinoless double-$\beta$ decay does not occur in the minimal model. It is necessary to introduce two new Higgs triplets, say $\sigma$ and $\omega$, with the quantum numbers of $\eta$ to have mixing between $\eta_1^-$ and $\eta_2^-$. In this case the potential has terms with $\eta \to \sigma, \omega$ in Eq. (36) and terms which mix $\eta, \sigma$, and $\omega$. In particular the term $\epsilon^{ijk}\eta_i\sigma_j\omega_k$ mixes $\eta_1^-, \sigma_1^-, \omega_1^-$ with $\eta_2^-, \sigma_2^-, \omega_2^-$ [10].

## IV. PHENOMENOLOGICAL CONSEQUENCES

In this model, the lepton number is violated in the heavy charged vector bosons exchange but it is not in the neutral exchange ones, because neutral interactions are diagonal in the lepton sector. However, we have flavor-changing neutral currents in the quark sector coupled to the heavy neutral vector boson $Z'^0$. All these heavy bosons have a mass which depends on $v_\chi$ and this vacuum expectation value is, in principle, arbitrary.

Processes such as $\mu^- \to e^-\nu_e\bar{\nu}_\mu$ are the typical ones, involving leptons, which are induced by lepton-number-violating charged currents in the present model. It is well known that the ratio

$$R = \frac{\Gamma(\mu^- \to e^-\nu_e\bar{\nu}_\mu)}{\Gamma(\mu^- \to \text{all})} \tag{41}$$

tests the nature of the lepton family number conservation, i.e., additive vs multiplicative. Roughly we have

$$R \propto \frac{A(3\mathrm{a})}{A(3\mathrm{b})} \approx \left(\frac{M_W}{M_V}\right)^4$$

where $A(3\mathrm{a})$ and $A(3\mathrm{b})$ are the amplitudes for the processes in Fig. (3a ) and (3b ) respectively. Experimentally $R < 5 \times 10^{-2}$ [11]; then we have that the occurrence of the decay $\mu^- \to e^-\nu_e\bar{\nu}_\mu$ implies that $M_V > 2M_W$.

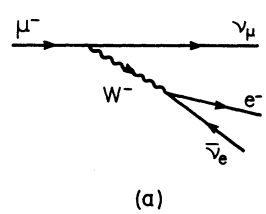
In addition to decays, effects such as $e_L^+ e_R^- \to \nu_{eL}\bar{\nu}_{eR}$ will also occur in accelerators, but these events impose constraints on the masses of the vector bosons which are weaker than those coming from the decays. Notice that the incoming negative charged lepton is right handed because the lepton-number-violating interactions with the $V^+$ vector boson in Eq. (24) is a right-handed current for the electron.

The doubly charged vector boson $U^{--}$ will produce deviations from the pure QED Moller scattering which could be detected at high energies.

Stronger bounds on the masses of the exotic vector bosons come from flavor-changing neutral currents induced by $Z'^0$. The contribution to the $K_L^0$-$K_S^0$ mass difference due to the exchange of a heavy neutral boson $Z'^0$ appears in Fig. 4. From Eq. (33) we have explicitly

$$-\frac{g}{4}\frac{M_Z}{M_W}\cos\theta_C \sin\theta_C [\,\bar{d}\gamma^\mu(v'^d + a'^d\gamma^5)s + \bar{d}\gamma^\mu(v'^s + a'^s\gamma^5)s] Z_\mu^{0'}, \tag{42}$$

with $v'^{d,s}$ and $a'^{d,s}$ given in Eq. (34c,d) respectively, and for simplicity we have assumed only two-family mixing. Then, Eq. (42) produces at low energies the effective interaction



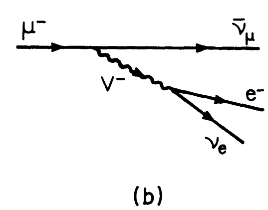




FIG. 3. (a) Lepton-number-conserving process. (b) Lepton-number-violating process.

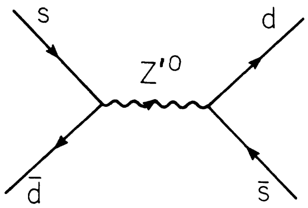



FIG. 4. $Z'^0$ exchange contribution to the effective Lagrangian for $K_S$-$K_L$ mixing.

$$\mathcal{L}_{\rm eff} = \frac{g^2}{16}\left(\frac{M_Z}{M_W}\right)^2 \frac{\cos^2\theta_C \sin^2\theta_C}{M^2_{Z'^0}} \left[\bar{d}\gamma^\mu(c_v + c_a\gamma^5)s\right]^2, \tag{43}$$

where we have defined

$$\begin{aligned} c_v &\equiv v'^d - v'^s = -\tfrac{2}{\sqrt{3}}(1+3t^2)/\sqrt{h(t)}, \\ c_a &\equiv a'^d - a'^s = -c_v. \end{aligned} \tag{44}$$

The contribution of the $c$ quark in the standard model is [12]

$$\begin{aligned} \mathcal{L}^{\rm SM}_{\rm eff} = &-\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi}\frac{m_c^2}{M_W^2\sin^2\theta_W}\cos^2\theta_C \\ &\times \sin^2\theta_C[\bar{d}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)s]^2, \end{aligned} \tag{45}$$

with $g^2/8M_W^2 = G_F/\sqrt{2}$. We can obtain the constraint upon the neutral $Z'^0$ mass assuming, as usual, that any additional contribution to the $K_S^0$-$K_L^0$ mass difference from the $Z'^0$ boson cannot be much bigger than the contribution of the charmed quark [13]. Then, from Eqs. (43) and (45) we get

$$M^2_{Z'^0} > \left(\frac{1}{2}\frac{4\pi}{\alpha}c_a^2\frac{M_W^2}{m_c^2}\tan^2\theta_W\right)M_W^2, \tag{46}$$

which implies the following lower bound on the mass of the $Z'^0$:

$$M_{Z'^0} > 40 \text{ TeV}.$$

From this value and Eq. (19) we see that $v_\chi$ must satisfy

$$v_\chi^2 > \frac{3\sqrt{2}}{8G_F M_W^2(1+3t^2)}(40\text{ TeV})^2,$$

that is, $v_\chi > 12$ TeV. As the vacuum expectation value of the $\chi$ Higgs boson is $<\chi^0>= v_\chi/\sqrt{2}$ then we have that $<\chi^0>> 8.4$ TeV. This also implies, from Eq. (17), that the masses of the charged vector bosons $V^-, U^{--}$ are larger than 4 TeV.

## V. CONCLUSIONS

If we admit lepton-number violation, SU(3) could be a good symmetry at high energies, at least for the lightest leptons $(\nu, e^-, e^+)$. Assuming that this is a local gauge symmetry, the rest of the model follows naturally, including the exotic quarks $J$. To the best of our knowledge, there are no laboratory or cosmological/astrophysical constraints on the masses of the exotic quarks $J_1$ and $J_{2,3}$ with charge $+\frac{5}{3}$ and $-\frac{4}{3}$ respectively, but they must be too massive to be detected by present accelerators. For the case of the heavy vector bosons, charged $U, V$, and the neutral $Z'^0$, rare decays constrain their masses as we have shown before. It is interesting to note that no extremely high-mass scale emerges in this model, making possible its experimental test in future accelerators.

Vertices such as the following appear in the scalar-vector sector:

$$\begin{aligned} \frac{ig}{\sqrt{2}}[\, & W^+_\mu(\eta_1^-\partial^\mu\eta^0 - \partial^\mu\eta_1^-\eta^0) \\ & + V^-_\mu(\eta_2^+\partial^\mu\eta^0 - \partial^\mu\eta_2^+\eta^0)], \end{aligned} \tag{47}$$

and also with $\eta \longrightarrow \sigma, \omega$, when these two new triplets are added to the model. Then we have mixing in the scalar sector which imply 1-loop contributions to the $(\beta\beta)_{0\nu}$ involving the vector bosons $V^-, U^{--}$ but these are less than contributions at the tree level through scalar exchange [10]. On the other hand, this model cannot produce processes such as $K^- \longrightarrow \pi^+ e^-\mu^-$ and $\tau^- \longrightarrow l^+\pi^-\pi^-$ with $l = e, \mu$.

Notice that the definition of the charge-conjugation transformation we have used in this work (see the Appendix) has physical consequences only in the Yukawa interactions and in the currents coupled to the heavy charged gauge bosons where an opposite sign appears with respect to the usual definition of that transformation.

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## APPENDIX

In this appendix we shall treat in more detail how it is possible to get Yukawa interactions from Eq. (11).

In the present model we have put together in the same multiplet the charged leptons and their respective charge-conjugated field. That is, both of them are considered as the two independent fermion degrees of freedom. If we use the usual definition of the charge conjugation transformation $\psi^c = C\bar{\psi}^T$, $\overline{\psi^c} = -\psi^T C^{-1}$ the Yukawa couplings in Eq. (11) vanish, including the mass terms. This is a consequence of the degrees of freedom we have chosen. Notwithstanding, it is possible to define the charge-conjugation operation as

$$\psi^c = \gamma^5 C\bar{\psi}^T, \quad \overline{\psi^c} = \psi^T C^{-1}\gamma^5.$$

This definition is consistent with quantum electrodynamics since its only effect is to change the sign of the mass term in the Dirac equation for the charge conjugated spinor $\psi^c$ with respect to the mass term of the spinor $\psi$, and it is well known that the sign of the mass term in the Dirac equation has no physical meaning. With the negative sign, the upper components of the spinor are the "large" ones, and with the positive sign, the large components are the lower ones [14].

Using this definition it is easy to verify that $\overline{l^c_R}l^c_L = -\bar{l}_R l_L$ instead of $\overline{l^c_R}l^c_L = +\bar{l}_R l_L$, which follows using the usual definition of the charge conjugation transformation.

On the other hand, the definition of charge conjugation we have used in this work produces the same effect as the usual one in bilinear terms for the vector interaction. Then, in the kinetic term and the vector interaction with the photon, it is not possible to distinguish both definitions. For example, the kinetic terms in the model are

$$\sum_l(\bar{l}_L i\partial\!\!\!/ l_L + \overline{\psi^c} i\partial\!\!\!/ l^c_L),$$

with $l = e, \mu, \tau$, and this can be written as

$$\sum_l(\bar{l}_L i\partial\!\!\!/ l_L + \bar{l}_R i\partial\!\!\!/ l_R),$$

where the right-handed electron has been interpreted as the absence of a left-handed positron with $(-E, -\mathbf{p})$.

For charged leptons we have the electromagnetic interaction

$$-e(\bar{l}_L\gamma^\mu l_L - \overline{l^c_L}\gamma^\mu l^c_L)A_\mu;$$

and using $\overline{l^c_L}\gamma^\mu l^c_L = -\bar{l}_R\gamma^\mu l_R$ we obtain the usual vector interaction $-e\bar{l}\gamma^\mu l A_\mu$; but, on the other hand, in the charged currents we have $\overline{\nu^c_{lR}}l_L = -\bar{l}_R\nu_{lL}$.


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