Smallness of gluon coupling to constituent quarks in baryons and validity of nonrelativistic quark model

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A study of the parametrization of baryon masses and other quantities leads to a ratio $\approx\frac{1}{5}$ for two-gluon-exchange versus one-gluon-exchange terms between two constituent quarks in a baryon. This fact (plus the factor 3 reduction from each additional order in flavor breaking) explains the success, to a part per thousand, of a new baryon mass formula [Phys. Rev. Lett. **68**, 139 (1992)]. It also explains why the results of SU(3) that neglect second-order flavor breaking (such as the Gell-Mann–Okubo baryon mass formula) work much better than expected. Finally the general parametrization, plus the above reduction factors, clarify why the "naive" nonrelativistic quark model (NRQM) is quantitatively fairly good. The reasons are twofold: (a) The structure of the general parametrization (an exact consequence of any QCD-like relativistic field theory) is similar to that of the NRQM; (b) the smallness of the gluon coupling and the reduction factor due to flavor imply that the additive (one-body) terms in the parametrization prevail on the two-body terms and the latter on the three-body ones, the typical feature of the "naive" NRQM.

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I. INTRODUCTION

Recently we have shown [1] how to parametrize rigorously in the spin-flavor space many physical quantities, starting from the underlying field theory. If the basic theory is QCD-like, the general parametrization is rather simple (that is, it consists of a small number of terms) because the Casimir flavor operators are absent. The parametrization has a noncovariant aspect, typical of the nonrelativistic quark model (NRQM) [2], but, being an exact consequence of the basic relativistic field theory, is an exact relativistic result. For instance, the complete parametrization of the octet- and decuplet-baryon masses has the form [1e]

$$\begin{aligned}\tilde{H}=&M_0+B\sum_i P_i^\lambda+C\sum_{i>k}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)+D\sum_{i>k}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)(P_i^\lambda+P_k^\lambda)+E\sum_{\substack{i\neq k\neq j\\(i>k)}}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)P_j^\lambda\\&+a\sum_{i>k}P_i^\lambda P_k^\lambda+b\sum_{i>k}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)P_i^\lambda P_k^\lambda+c\sum_{\substack{i\neq k\neq j\\(i>k)}}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)(P_i^\lambda+P_k^\lambda)P_j^\lambda+dP_1^\lambda P_2^\lambda P_3^\lambda\ .\end{aligned}\tag{1}$$

In Eq. (1) M_0,B,C,D,E,a,b,c,d are real coefficients. The term of Eq. (1) written in Ref. [1e] as $\frac{1}{2}c\sum_{i\neq k\neq j}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_k)(P_i^\lambda+P_k^\lambda)P_j^\lambda$ is rewritten here, equivalently, for uniformly of notation, as $c\sum_{i\neq k\neq j(i>k)}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)(P_i^\lambda+P_k^\lambda)P_j^\lambda$. The $\boldsymbol{\sigma}_i$'s are spin Pauli matrices; $P^\lambda=\frac{1}{3}(1-\lambda_8)$ is the projection operator on the strange quark that arises from the flavor-breaking mass term in the field Lagrangian. The quark fields in the Lagrangian, defined with their masses renormalized at small $q^2(q^2\approx R^{-2}$ where R is the typical hadron radius), are called the constituent quark fields [1a]. In Eq. (1) the sum indices extend from 1 to 3.

Note that in (1) we adopted the rule of writing each term as a sum of all addends obtained permuting 1,2,3; with each different addend appearing once and only once. For instance, the E term is explicitly $E[(\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_1)P_3^\lambda+(\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_1)P_2^\lambda+(\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_2)P_1^\lambda]$.

To obtain the mass of a baryon one takes the expectation value of (1) in its NRQM spin-flavor state [1a]. Because only the combination $(a+b)$ enters in the masses, $(a+b)$ and the other seven coefficients are determined uniquely from the masses. At present the experimental error on the masses of the Δ's is much larger than the error on the other masses; thus we may neglect the d term in Eq. (1) and determine all the remaining coefficients without the use of the Δ mass, obtaining (in MeV)

$$\begin{aligned}&M_0=1085,\quad B=187,\quad C=48.7\ ,\\&D=-16\ ,\\&a+b=-11.6,\quad E=3.4,\quad c=1.17\ .\end{aligned}\tag{2}$$

(The above values differ by decimals from those in Ref. [1a] where the terms a,b,c had not been included.) Alternatively we may neglect both c and d, observing that both c and d multiply three quark terms (terms with three different indices) and that the data indicate a strong decrease of the coefficients of the terms when the number of indices increases. This remark was the basis of the approximation introduced in Ref. [1e]; there, neglecting the c and d terms in (1), we deduced a new relationship between masses of **8** and **10** which is satisfied (after duly taking into account the electromagnetic masses) at the precision of one part per thousand. The same assumption of neglecting three quark terms in the parametrization was also applied in Ref. [1f], obtaining several relationships between electromagnetic masses; they are well satisfied. Also we could understand [1f] why the Coleman-Glashow relationship is satisfied so well.

Here we will clarify, using the underlying field theory, why three-quark terms (terms with three different indices) in the general mass parametrization are so small. First we shall show that in (1) the terms with three indices are due necessarily to Feynman diagrams where at least two gluons are exchanged between the quarks. Second, through an analysis of the various terms in (1) we will show (a) that each additional gluon exchanged reduces the order of magnitude of the term by a factor about 5 and (b) that each additional flavor breaking factor P_i^λ implies a reduction of the order $\frac{1}{3}$.

These conclusions from baryon masses will be confirmed analyzing the magnetic moments. In the next section we will examine in general the correspondence between Feynman diagrams and number of indices in a term.

II. THE CORRESPONDENCE BETWEEN THE TERMS IN THE GENERAL PARAMETRIZATION AND FEYNMAN DIAGRAMS

In the general parametrization method the calculation of the expectation value $\Omega_{\rm av}$ of some field operator Ω in the exact state $|\Psi\rangle$ amounts to that of the expectation value of an effective operator $\tilde{\Omega}$ (acting, for baryons, only in the three-quark sector) on the model state $|\phi\rangle$. Because $|\Psi\rangle = V|\phi\rangle$ we have

$$\Omega_{\rm av} \equiv \langle\Psi|\Omega|\Psi\rangle = \langle\phi|V^\dagger\Omega V|\phi\rangle \ . \tag{3}$$

Because the model state is a three-quark state, only the projection

$$\tilde{\Omega} = \sum |3q'\rangle\langle 3q'|V^\dagger\Omega V|3q\rangle\langle 3q|$$

enters in the calculation of $\langle\phi|V^\dagger\Omega V|\phi\rangle$; finally we can write

$$\Omega_{\rm av} \equiv \langle\Psi|\Omega|\Psi\rangle = \langle\phi|\tilde{\Omega}|\phi\rangle \ .$$

The calculation of $\Omega_{\rm av}$ can be expressed in terms of Feynman diagrams; indeed (see Ref. [1a], Appendix, for the construction of the V transformation in terms of the underlying field theory), V can be related to the U operator of Dyson defined in Ref. [1a]; $\Omega_{\rm av}$ is then expressed in terms of Feynman diagrams

$$\Omega_{\rm av} = \langle\phi|T[\Omega(0)U(+\infty|-\infty)]|\phi\rangle_C \ , \tag{4}$$

where C means "connected." Clearly, because $|\phi\rangle$ is a three-quark state, all diagrams in the calculation of $\Omega_{\rm av}$, Eq. (4), have three quark lines entering and three outgoing. Because ϕ is taken with $L=0$ and is factorizable in a space factor times a spin-flavor factor, $\Omega_{\rm av}$ (after integration on the space coordinates) results in a combination of spin-flavor structures. To construct the spin-flavor structures, we assume that the underlying field theory is, as already stated, a QCD-like theory: The only fields in the Lagrangian are quarks and gluons, the latter neutral and flavor blind. The result of calculating a given Feynman diagram (after contraction of all creation and destruction operators) depends on the Pauli matrices σ_i and the flavor operators (either P_k^λ or Q_k) as follows.

The σ's enter in four ways in the calculation of a Feyman diagram: (1) from quark propagators; (2) from Dirac matrices in the operator Ω; (3) from the Foldy-Wouthuysen-type operator (that enters in V) transforming the Pauli 2-spinors of the model state into Dirac 4-spinors of the relativistic field Lagrangian; it contains $\boldsymbol{\sigma}\cdot\mathbf{p}$ for a quark of Fourier momentum $\mathbf{p}$; (4) from gluon exchange between two quark lines; one-gluon exchange between i and k produces a spin-independent and a spin-dependent part; the latter gives just $(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)$ in the final result, after taking the expectation value on the space $L=0$ model wave function.

As to flavor, a $P_i^\lambda \equiv \frac{1}{3}(1-\lambda_{8i})$ projection operator in flavor space appears in the general parametrization for each strange (λ) quark [recall that $(P_i^\lambda)^n = P_i^\lambda$]; a $Q_i = \frac{1}{2}(\lambda_{3i}+\lambda_{8i}/3) = \frac{2}{3}P_i^{\mathcal{P}} - \frac{1}{3}P_i^{\mathcal{N}} - \frac{1}{3}P_i^\lambda$ appears for each electromagnetic coupling of quark i (in Ref. [1a] Q_i was called P_i^q). Having seen how spin and/or flavor operators arise in each Feynman diagram, we relate the spin-flavor structure of the general parametrization to that of Feynman diagrams.

III. THE ANALYSIS OF THE TERMS IN THE GENERAL PARAMETRIZATION OF THE MASSES

While the above arguments are general, we will refer here for definiteness to the baryon masses; then Ω in Eq. (3) is the exact field Hamiltonian H.

The connected Feynman diagrams that correspond to $V^\dagger HV$ belong to one of the three classes shown in Figs. $1(\alpha)$, $1(\beta)$, and $1(\gamma)$; the diagrams of Fig. $1(\alpha)$ correspond to no interaction among the quarks except for the average interaction expressed by the model Hamiltonian $\mathcal{H}$. Thus the diagrams of Fig. $1(\alpha)$ describe the contribution to the baryon mass with no gluon being exchanged between quarks. From such diagrams one can only get (after taking the expectation value on the space variables), terms that either are spin-flavor independent or are additive (one index) in the spin-flavor space. The diagrams of Fig. $1(\beta)$ imply the exchange of gluons (at least one) between a pair of quarks; from them one gets terms that have either 0 or 1 or 2 indices in the spin-flavor space; finally the diagrams of Fig. $1(\gamma)$, that imply the exchange of at least two gluons among the three different quarks, can give rise to terms with 0,1,2,3 indices in the

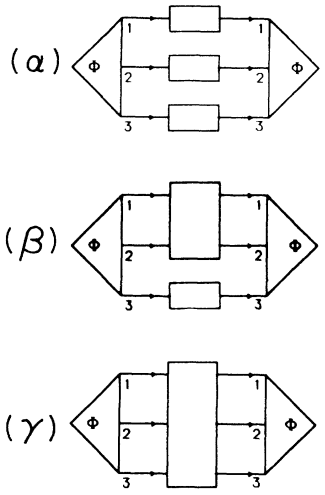

FIG. 1. (α) The class of connected Feynman diagrams giving rise to zero or one-index terms in the spin-flavor space. The diagrams in this class include all diagrams without gluon exchanges but may contain diagrams in which gluon exchange is present without producing factors $(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)$ or $P_i^\lambda P_k^\lambda$. (β) The class of connected Feynman diagrams corresponding to two-index terms in the spin-flavor space. The diagrams in this class imply the exchange of at least one gluon, but may include diagrams with more than one-gluon exchange not producing spin-flavor factors with three indices. (γ) The class of connected Feynman diagrams corresponding to terms with three indices in the spin-flavor space; these diagrams imply the exchange of at least two gluons.

spin-flavor space.

It follows that terms with three different indices in spin-flavor space necessarily imply the exchange of at least two gluons. Terms with two different indices in spin-flavor space necessarily imply the exchange of at least one gluon. Of course the introduction of any collective variable in the spin-flavor space (such as total angular momentum $2\mathbf{J}\equiv\sum_i\boldsymbol{\sigma}_i$ or strangeness $S=-\sum_i P_i^\lambda$) conceals the number of different indices in a term. It is understood that to count the number of different indices, no collective variable in the spin-flavor space has to be introduced in the course of the calculation; so the spin-flavor indices in a term are those that directly result from the original Feynman diagram.

On the basis of the above analysis one can write each coefficient in the mass parametrization (1) as the sum of contributions that correspond (suffix α) to no gluon exchange [Fig. 1(α)], or to the exchange (suffix β) of at least one gluon [Fig. 1(β)] or to the exchange (suffix γ) of at least two gluons [Fig. 1(γ)]. Below we shall also indicate for each coefficient its order in P^λ. We display in the first line of Eq. (5) coefficients of flavor-invariant terms and in the subsequent ones those with, respectively, 1,2,3 factors P^λ's. Because $\sum_i(P_i^\lambda)^n=\sum_i P_i^\lambda$, terms of type α with two or more P^λ's cannot be present in (5); nor can terms of type β with $3P^\lambda$'s exist:

$$
\begin{aligned}
&M_0=M_{0\alpha}+M_{0\beta}+M_{0\gamma}\ , && C=C_\beta+C_\gamma && [\text{no } P^\lambda]\\
&B=B_\alpha+B_\beta+B_\gamma\ , && D=D_\beta+D_\gamma,\ E=E_\gamma && [1P^\lambda]\\
&a=a_\beta+a_\gamma\ , && b=b_\beta+b_\gamma,\ c=c_\gamma && [2P^\lambda]\\
& && d=d_\gamma && [3P^\lambda]\ .
\end{aligned}
\tag{5}
$$

Note for illustration that d, c, and E appear only with the suffix γ; that is, they imply at least two-gluon exchange; C,D,a,b are due to one- (or more) gluon exchange. As to M_0 and B they are dominantly due to no gluon exchange (but may contain, of course, contributions from one or more gluon exchanges that are spin independent). Using the numerical values given in (2), we now first determine the order of magnitude of the ratio of terms with $1P^\lambda$ to flavor-independent ones; or of terms with $2P^\lambda$'s to those with $1P^\lambda$, at equal number of gluons exchanged. Next we shall determine the order of magnitude of the ratio between two- and one-gluon-exchange terms, at equal order in P^λ. We anticipate the result. Each increase by one of the number of P^λ's implies a reduction by a factor about 3; each gluon exchanged costs about 5. These numbers are only order of magnitude; yet they seem to emerge consistently from the analysis below.

We start with the terms C and D in (1). They are both proportional to $(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)$ and, as one sees from (5), they both correspond to the exchange of one gluon or more; the C term is flavor independent and the D term is of the $1P^\lambda$ type; thus the ratio D/C should give the typical reduction due to $1P^\lambda$ flavor breaking for terms that exchange one gluon or more. We write this, in obvious notation [in the evaluation the values (2) of D and C are used], as

$$
\left|\frac{D}{C}\right|=\frac{(\geq 1\text{ gluon exch}\,|1P^\lambda)}{(\geq 1\text{ gluon exch}\,|0P^\lambda)}\cong 0.33\ . \tag{6}
$$

Here the following question might be raised. We might have written equally well the term $D\sum_{i>k}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)(P_i^\lambda+P_k^\lambda)$ in Eq. (1) as $2D\sum_{i>k}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_k)P_i^\lambda$ because only its expectation value on a symmetric wave function is relevant. Why then in giving the order of magnitude for the ratio $1P^\lambda$ to $0P^\lambda$, we used (D/C) rather than $(2D/C)$? The symmetrization rule, adopted in writing (1), implies that the D term must be "normalized" as in Eq. (1); of course, a uniform criterion must be used for all terms and, once adopted, it cannot be changed. Still, because for D/C the factor in play due to this question would be 2, to be compared with an order of magnitude $\cong\frac{1}{3}$ of the ratio (6), we prefer to give a direct check of the correctness of (6). (A check for the term with a coefficient c, that has a structure analogous to that of the D term, will be also given later.)

This check for D is obtained comparing our general parametrization with the explicit results of De Rújula, Georgi, and Glashow (DGG) [3] in a QCD one-gluon ex-

change Fermi-Breit treatment. We require that in that limit our general formula coincides with the DGG one. It will result that in Eq. (6) D/C (as written), and not $2D/C$, is the correct quantity to evaluate.

The explicit QCD calculation by DGG [3] of the one-gluon exchange interaction between two quarks in Fermi-Breit approximation leads to an expression of the same form as the following six terms of Eq. (1)

$$M_0+B\sum_i P_i^\lambda+C\sum_{i>k}(\sigma_i\cdot\sigma_k)+D\sum_{i>k}(\sigma_i\cdot\sigma_k)(P_i^\lambda+P_k^\lambda)+a\sum_{i>k}P_i^\lambda P_k^\lambda+b\sum_{i>k}(\sigma_i\cdot\sigma_k)P_i^\lambda P_k^\lambda\ . \tag{7}$$

But in the DGG formula D and C, as well as b and D, are related by [4]

$$\frac{D}{C}=-\frac{\Delta m}{m};\quad \frac{b}{D}=-\frac{\Delta m}{m}\ . \tag{8}$$

In (8) $m=m_{\mathcal{P}}=m_{\mathcal{N}}$, and m_λ are the quark masses and $\Delta m\equiv(m_\lambda-m)$. Recall that $(\Delta m/m)$ (that we call x) is the coefficient of P^λ in the Hamiltonian of the underlying field theory [5]

$$x=(\Delta m/m)\ . \tag{9}$$

The first equation (8) determines this coefficient of P^λ as

$$x=-D/C\cong(16/48.7)\approx\tfrac{1}{3}\ . \tag{10}$$

Thus we confirm that, as anticipated in Eq. (6), $|D/C|$ is the correct estimate of the ratio between "$1P^\lambda$ flavor breaking" to "no flavor breaking" at an equal number of gluons exchanged. The above value $\approx\frac{1}{3}$ for the coefficient x of P^λ will be reobtained from the baryon magnetic moments and the meson masses [compare Eqs. (22) and (26)].

So far we used only the first equation (8). By the second equation (8) we obtain $|b/D|\approx\frac{1}{3}$, leading to an estimate of

$$b\approx+5.3 \tag{11}$$

that we shall use in a moment. [Recall that the 8 and 10 masses determine only $(a+b)$ in Eq. (1); it is the requirement that our general formula coincides with DGG in their limit that leads to the estimate (11) of b.]

We now evaluate the ratio between the coefficient of a term due to at least two-gluon exchange and that of a term due to at least one-gluon exchange, both at the same level of flavor breaking. For terms exchanging two or more gluons we cannot compare our general result with the DGG limit, because Ref. [3] deals only with one-gluon exchanges [the terms E,c,d of Eq. (1) are not present in the DGG calculation and rightly so; but the "normalization" of the E term does not raise any question]. Consider first in Eq. (1) the terms E and D, both linear in P^λ. The term E implies the exchange of at least two gluons and the term D the exchange of at least one gluon. We get, in the same obvious notation as above, the order of magnitude

$$\left|\frac{E}{D}\right|=\frac{(\geq 2\text{ gluon exch}\,|1P^\lambda)}{(\geq 1\text{ gluon exch}\,|1P^\lambda)}\cong 0.21\ . \tag{12}$$

As anticipated, the exchange of two gluons carries a reduction of about one-fifth with respect to the exchange of one gluon. We confirm the estimate (12) considering another ratio between the coefficients in Eq. (1). From the terms c and b both proportional to $(\sigma_1\cdot\sigma_k)$ and both of order 2 in P^λ we have

$$\left|\frac{c}{b}\right|\frac{(\geq 2\text{ gluon exch}|2P^\lambda)}{(\geq 1\text{ gluon exch}|2P^\lambda)}\cong 0.22\ . \tag{13}$$

Here we assigned to b the value obtained above ($b\cong+5.3$). As to the term c in (1), a check of its "normalization" is obtained from the ratio between c and E; it is

$$\left|\frac{c}{E}\right|=\frac{(\geq 2\text{ gluon exch}|2P^\lambda)}{(\geq 2\text{ gluon exch}|1P^\lambda)}\simeq 0.34 \tag{14}$$

and once more we get $\cong\frac{1}{3}$ for the reduction factor due to one additional P^λ if c is normalized as in Eq. (1).

Thus all determinations point to the reduction factors $\frac{1}{5}$ for one-gluon exchange and $\frac{1}{3}$ for $1P^\lambda$ flavor breaking. In this respect note once more that $1P^\lambda$ takes into account flavor breaking *to all orders on each quark line* $[(P_i^\lambda)^n=P_i^\lambda]$. We stress that terms of the type $P_i^\lambda P_k^\lambda$ arise from at least two different quark lines and, therefore, imply at least one-gluon exchange; thus their contribution is expected to be depressed about 15 times (not just 3 times) with respect to that of $1P^\lambda$ additive (non-gluon-exchange) terms; that is, neglecting them, we neglect terms of order $\frac{1}{45}$ with respect to the gross scale of the mass spectrum. *This is the reason why the "classical" formulas (e.g., Gell-Mann–Okubo formula), derived taking into account flavor breaking to first order [6], work much better than expected.*

A short comment is appropriate on the fact that no one of the ratios considered above refer to the terms M_0 and B in Eq. (1). The point is that M_0 has the meaning of the sum of the quark masses plus their average kinetic energy and average potential energy, and B describes the difference of the above quantities for the λ quark as compared to those of the $\mathcal{P}$ and $\mathcal{N}$ quarks.

Another comment is in order on the question from which we started, namely, why the three quark terms (c and, even more, d) are so small that the mass formula obtained by ignoring them [1e] is correct to a part per thousand. The reason is simple. The magnitude of c is reduced (with respect to the scale of the mass spectrum)

by $\approx\frac{1}{25}\times\frac{1}{9}=[(2$ gluon reduction$)\times(2P^{\lambda}$ reduction$)]$ $\cong 4\times 10^{-3}$. This reduction leads into the region of 1 MeV.

Before confirming the results of this section via the magnetic moments, we digress briefly on the masses of the constituent quark fields.

IV. THE CONSTITUENT QUARK FIELDS AND THEIR MASSES

After the above conclusions on the coupling of the constituent quark fields to gluons, it seems necessary to discuss further how the constituent quark fields are defined. As stated repeatedly we defined such fields as those that enter in the basic Lagrangian of our field theory (say QCD) *when the mass renormalization is performed at a small value of* q^2 (for instance, $q^2\approx R^{-2}$ where R is a typical hadron radius); the field of a bare "current" quark (a parton) is instead, of course, defined, in the limit $q^2\to\infty$.

One may ask if the value of q^2 at which the mass renormalization is done to define the constituent quark field is unique, or if the renormalized masses can be chosen freely in some range. In the general parametrization method the choice of the values of the renormalized masses of the quarks is subjected only to one restriction. That a V transformation exists between the model states and the exact states of the system. Here, when we speak of states, we mean the set of states, the properties of which we intend to calculate [7]. It is likely that a V transformation exists for different choices of the masses of the constituent quarks, in a certain interval, more or less wide. If so, the precise values of these masses would be unimportant except that, as repeatedly stated (see Ref. [1a]), we must require that the flavor-breaking parameter $x\equiv(\Delta m/m)$ that multiplies P^{λ} is decently small. We already stated in Ref. [1a], and reasserted here, that $x\cong\frac{1}{3}$.

There is however a condition that the masses of the constituent quarks must satisfy to agree in a natural way with Eq. (1); we do not call this a consistency condition, because the use of such a name would imply that it is a necessary condition and we are unable to say if this is so.

In Eq. (1) M_0 is just a parameter that the data fix at 1085 MeV. Because M_0 is the spin-flavor-independent part of the mass, it can be interpreted as the sum of the masses $m\equiv m_{\mathcal{P}}=m_{\mathcal{N}}$ of three constituent (nonstrange) quarks, plus their average kinetic energy $T_{\rm av}$, plus their average potential interaction energy $V_{\rm av}$ due to diagrams that exchange gluons *in a spin and flavor independent way:*

$$M_0=3m+T_{\rm av}+V_{\rm av}=1085\ \text{MeV}\ . \tag{15}$$

If we introduce what seems a natural assumption, that the *spin-independent part* $V_{\rm av}$ of the gluon interaction energy in (15) has an order of magnitude not far from that of the spin-dependent part, which is known from the coefficient C of Eq. (1), we obtain $|V_{\rm av}|\approx 200$ MeV. One then gets a natural description if we assign to the nonstrange constituent quarks fields a mass m in a range such as $[(1085/3)\pm 70]$ MeV, that is

$$290<m<430\ \text{MeV}\ . \tag{16}$$

Of course for each m the λ quark is taken to have a correspondingly higher mass so that $(m_{\lambda}-m)/m\approx\frac{1}{3}$. We insist that (16) is only a naturalness condition but it is, we feel, all that can be stated at present on the choice of the renormalization point where to define the constituent quark mass.

Why is it so difficult to say more than this? Probably the basic reason is that quarks are not observable; constituent quark fields can be defined with different masses and be used to describe the same physical results. Indeed provided that a V exists, the general parametrization for a physical quantity has always the same form, independently of the choice of the mass values. One is led, of course, to the same experimental values of the parameters that appear in the general parametrization, no matter what the choice has been. It is possible [7] (but we do not know) that if one could parametrize more data and relate these different parametrizations one would get more limitations on the choice of the constituent quark masses [8]. What looks remarkable is that, in spite of these possible differences in mass, all acceptable constituent quark fields apparently have the same, comparatively small, coupling to the gluon field.

V. THE ANALYSIS OF THE GENERAL PARAMETRIZATION OF MAGNETIC MOMENTS

We now analyze the general parametrization of baryon magnetic moments. The results can be summarized as follows. All coefficients turn out to be in agreement with their expected values except one (g_7) that is apparently larger by a factor 4. However we shall see that g_7 cannot be determined reliably at present. Altogether the orders of magnitude obtained from the masses are confirmed by the magnetic moments.

We start noting that because the magnetic moments of decuplet baryons are largely unknown, the general parametrization of baryon magnetic moments can be developed usefully only up to terms with one P^{λ}, as done in Ref. [1a]; otherwise the number of coefficients exceeds that of the experimental data. As shown in Ref. [1a] the parametrization of the magnetic moments $\mathbf{M}$ including all terms with $1P^{\lambda}$ gives

$$\mathbf{M}=\sum_{\nu=1}^{7} g_{\nu}\mathbf{G}_{\nu} \tag{17}$$

where

$$\begin{aligned}
&\mathbf{G}_1=\sum_i Q_i\boldsymbol{\sigma}_i,\quad \mathbf{G}_2=\sum_i Q_i P_i^{\lambda}\boldsymbol{\sigma}_i\ ,\\
&\mathbf{G}_3=\sum_{i\neq k} Q_i\boldsymbol{\sigma}_k,\quad \mathbf{G}_4=\sum_{i\neq k} Q_i P_i^{\lambda}\boldsymbol{\sigma}_k\ ,\\
&\mathbf{G}_5=\sum_{i\neq k} Q_i P_k^{\lambda}\boldsymbol{\sigma}_k\ ,\\
&\mathbf{G}_6=\sum_{i\neq k} Q_i\boldsymbol{\sigma}_i P_k^{\lambda},\quad \mathbf{G}_7=\sum_{i\neq k\neq j} Q_i P_k^{\lambda}\boldsymbol{\sigma}_j\ .
\end{aligned} \tag{18}$$

With the above definition of the $\mathbf{G}_{\nu}$'s (note that each $\mathbf{G}_{\nu}$ is a sum in which each addend appears once and only once) the values of g_{ν}'s in Eq. (17) derived from the data

(using $p=2.79$, $n=-1.91$, $\Sigma^+=2.48$, $\Sigma^-=-1.16$, $\Lambda=-0.61$, $\Xi^0=-1.25$, $\Xi^-=0.65$ proton magnetons) are

$$g_1=2.79,\ g_2=-0.94,\ g_3=-0.076,\ g_4=0.41\ ,$$
$$g_5=0.097,\ g_6=-0.134,\ g_7=0.155\ . \tag{19a}$$

Using instead $\Sigma^+=2.38$, $\Sigma^-=-1.10$, and all the other magnetic moments as above, the values (19a) are replaced by

$$g_1=2.79,\ g_2=-0.89,\ g_3=-0.076,\ g_4=0.33\ ,$$
$$g_5=0.14,\ g_6=-0.143,\ g_7=0.13\ . \tag{19b}$$

In the real situation the g_i's will be somewhere between the two.

Note that in Ref. [1a] we also wrote $\mathbf{M}$ in the (identical) form

$$\mathbf{M}=\sum_{\nu=1}^{7} g_\nu \mathbf{G}_\nu \equiv \mu\boldsymbol{\Sigma}^q + A\boldsymbol{\Sigma}^\lambda + FQ\cdot(2\mathbf{J}) + HS\cdot(2\mathbf{J})$$
$$+LQ\cdot\boldsymbol{\Sigma}^\lambda + KS\cdot\boldsymbol{\Sigma}^q + GQ\cdot S\cdot(2\mathbf{J})\ , \tag{20}$$

where μ, A, F, H, L, K, G are seven parameters ($\mu=2.869$; $A=+1.005$; $F=-0.076$; $K=+0.289$; $H=+0.086$; $G=-0.155$; $L=-0.175$) that replace $g_1, g_2, g_3, \ldots, g_7$ [the numerical values just listed for them correspond to the choice (19a) for the g_i's]; the quantities in Eq. (20) are the charge Q ($Q\equiv\sum_i Q_i$), strangeness S ($S\equiv-\sum_i P_i^\lambda$), total spin $2\mathbf{J}\equiv\sum_i\boldsymbol{\sigma}_i$ and $\boldsymbol{\Sigma}^q\equiv\sum_i Q_i\boldsymbol{\sigma}_i$ and $\boldsymbol{\Sigma}^\lambda\equiv\frac{1}{3}\sum_i P_i^\lambda\boldsymbol{\sigma}_i$. Here we use the form (17) because its terms are written uniformly according to the general symmetrization prescription. No ambiguity exists in the normalization of the structures $\mathbf{G}_1$ to $\mathbf{G}_7$ as written above in (18). Thus we can extract from the coefficients, g_1 to g_7 [given in Eqs. (19a) or (19b)], the gluon exchange and the flavor factors of interest.

Before examining the values of the coefficients note, however, the following. As stated above, as well as in Ref. [1a], the structures $\mathbf{G}_1$ to $\mathbf{G}_7$ are the only ones containing $1P^\lambda$ (that is, loosely speaking, including flavor breaking to first order). But the result of this paper on the depression factors due to gluon exchange and to P^λ now show that the coefficient of the three-index $1P^\lambda$ structure $\mathbf{G}_7$ might be of the same order of magnitude as that of the two-index $2P^\lambda$ structure $\mathbf{G}_{7b}$ [9]:

$$\mathbf{G}_{7b}=\sum_{i\neq k} Q_i P_i^\lambda \boldsymbol{\sigma}_i P_k^\lambda\ . \tag{21}$$

Indeed $\mathbf{G}_7$ implies the exchange of two gluons (reduction factor $\cong\frac{1}{25}$) and $1P^\lambda$ (reduction factor $\cong\frac{1}{3}$); on the other hand $\mathbf{G}_{7b}$ implies the exchange of one gluon (factor $\frac{1}{5}$) and $2P^\lambda$'s (factor $\cong\frac{1}{9}$); the orders of magnitude are comparable (if anything $\mathbf{G}_{7b}$ is slightly more important).

If we include in (17) also $\mathbf{G}_{7b}$, we have another coefficient (call it g_{7b}); the total number of coefficients becomes 8. Clearly they cannot be determined only from the 7 magnetic moments. It may be possible to determine the 8 coefficients $g_1, \ldots, g_{7b}$ when the matrix element for $\Sigma^0\to\Lambda\gamma$ will be known with an error smaller than the present one (the absolute value of this matrix element is presently 1.61 ± 0.09); indeed the term $\mathbf{G}_{7b}$ determines the main deviations from Okubo's relationship [10] between the magnetic moments and from such deviations one can, in principle, determine g_{7b}.

At present a reanalysis of the magnetic moments to include the term $\mathbf{G}_{7b}$ is not useful. This is due to the comparatively large error in $\Sigma^0\to\Lambda\gamma$ and also to the discrepancies between different measurements for Σ^+ (2.48 ± 0.02 and 2.38 ± 0.02) and, to a smaller extent for Σ^-, noted above in connection with Eqs. (21a) and (21b). We only state that it is possible to reproduce $\Sigma^0\to\Lambda\gamma$ inside its error choosing values of g_7 and g_{7b} both around 0.10; the values of all remaining g_i's stay essentially unmodified with respect to (19a) or (19b).

Consider first the one index (additive) terms in Eq. (17) $\mathbf{G}_1=\sum_i Q_i\boldsymbol{\sigma}_i$ and $\mathbf{G}_2=\sum_i Q_i P_i^\lambda\boldsymbol{\sigma}_i$, that are due (dominantly) to diagrams with no gluon exchange. The ratio of the coefficient g_2 of the $1P^\lambda$ terms to that g_1 of the $0P^\lambda$ term gives the reduction factor due to $1P^\lambda$ flavor breaking. We have, from (19a),

$$\left|\frac{g_2}{g_1}\right|=\frac{(\geq 0\text{ gluon exch}\,|1P^\lambda)}{(\geq 0\text{ gluon exch}\,|0P^\lambda)}=\frac{0.94}{2.79}\simeq 0.33\ , \tag{22}$$

whereas the use of (19b) would give 0.32. Equation (22) confirms the previous determination [Eq. (10)] from the baryon masses.

Now we consider the terms in the parametrization (17) where at least one gluon is necessarily exchanged between two quark lines. We already discussed the coefficient g_7 of $\mathbf{G}_7$. Being due to the exchange of two gluons g_7 should be smaller than g_1 by $\approx(\frac{1}{5})^2\frac{1}{3}\cong 0.013$, about 4 times smaller than it is in reality (taking the average of the two determinations [(19a) and (19b)] we have $|g_7/g_1|=(0.14/2.79)\cong 0.05$). Of course a factor 4 can arise from many reasons, but the arguments on g_{7b} given above show that to be conclusive on this point we must wait for a better determination of $\Sigma^0\to\Lambda\gamma$. As to g_4, g_5, and g_6 they should be smaller than g_1 by a factor 3, due to $1P^\lambda$, and a factor 5 due to one gluon; thus they should be $\approx\frac{1}{15}\cong 0.066$ of g_1. Taking again the average of (21a) and (21b) we get

$$|g_4/g_1|=(0.37/2.79)\cong 0.13\ ,$$
$$|g_5/g_1|=(0.12/2.79)\cong 0.043\ , \tag{23}$$
$$|g_6/g_1|=(0.14/2.79)\cong 0.05\ .$$

The orders of magnitude are as predicted.

A point that must be noted and for which we have no explanation at the moment (though it does not contradict our conclusions) is that g_3 is much smaller than expected, $\mathbf{G}_3$ should be, at first sight, a one-gluon exchange term; thus g_3 should be smaller than g_1 by a factor ≈ 5 while it is ≈ 35 times smaller. Although the smallness of any particular parameter may be due to many reasons, it would be of interest to understand why g_3 is so small, just 3% of g_1; is it just chance (some factors π) or there is more to it? It is amusing to note that the extreme smallness of g_3 is, after all, responsible for the famous ratio

$[M(p)/M(n)] \cong -\frac{3}{2}$, from which a significant part of this whole story came out [11]. Note that the value given for g_3 is exact; being determined only by the nonstrange baryons n and p, g_3 is unaffected by the P^λ terms.

VI. A FEW REMARKS ON THE SEMILEPTONIC BARYON DECAYS AND THE MESON MASSES

A case quite similar to the exceptional smallness of g_3 just discussed (again not in contradiction with our analysis, but to be understood) is the smallness of the coefficient b as compared to a in the semileptonic decays of baryons (Ref. [1b], Eqs. (14) and (15)). The ratio b/a measures the deviation of D/F from the "classical" value $\frac{3}{2}$. Also in that case the ratio b/a is much smaller than expected simply from a one-gluon exchange.

Another comment is in order on the semileptonic decays of baryons. One often finds in the literature the statement that there is no indication for flavor breaking in the data. Because of errors this may be so, at present; but since from time to time some authors insist on this absence of flavor breaking, almost as if it were a basic property, we remark that there is no reason for such exceptional behavior. In Ref. [1b] we noted that the ways of analyzing the data are, *a priori,* too many and the experimental errors too large to arrive at a definite conclusion. However, after the analysis of this paper, it emerges that there is only one flavor-breaking term that does not imply necessarily the exchange of one gluon (and thus a reduction by at least a factor 15). It is the term listed as (1) in Eq. (23) of Ref. [1b]. This fact that just one parameter is expected to be appreciably larger than the others suggests a full reanalysis of the data, on the basis of Eq. (24) of Ref. [1b].

We finally mention, for completeness, the parametrization of the meson masses. As shown in Ref. [1c] the *most general expression* of the mass of a $I \neq 0$ meson $(\pi,K;\rho,K^*)$, *correct to all orders in flavor breaking,* is

$$M_{I\neq 0} = A + B\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + C(P_1^\lambda + P_2^\lambda) + D\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2(P_1^\lambda + P_2^\lambda) , \tag{24}$$

where A,B,C,D are four real coefficients. Here and in what follows the index 1 refers to the quark and 2 to the antiquark. The $(\pi,K;\rho,K^*)$ masses determine univocally the values of the coefficients in Eq. (24). We get (in MeV)

$$A = 612, \quad B = 158, \quad C = 182, \quad D = -58 . \tag{25}$$

The ratio D/B determines the ratio between $1P^\lambda$ and $0P^\lambda$ terms with exchange of at least one gluon; it is, therefore

$$\left|\frac{D}{B}\right| = \frac{(\geq 1 \text{ gluon exch } |1P^\lambda)}{(\geq 1 \text{ gluon exch } |0P^\lambda)} \cong 0.37 . \tag{26}$$

Comparing our general result Eq. (24) with Eq. (17) of DGG [3], one finds again that the terms in Eq. (24) coincide with those of DGG if $(\Delta m/m) = -(D/B)$; in fact the result $\Delta m/m \cong 0.37$ was already obtained from the meson masses by DGG. The new point here is that whereas Eq. (17) of DGG was derived under many approximations, our Eq. (24) is an exact consequence (to all orders in flavor breaking) of any relativistic QCD-like field theory. Once more we find in (26) a value not too far from $\frac{1}{3}$ for the x flavor-breaking parameter.

VII. CONCLUSION. WHY THE NONRELATIVISTIC QUARK MODEL WORKS QUANTITATIVELY FAIRLY WELL FOR LIGHT HADRONS

The issue in the title of this section has been with us since the origin of the NRQM. We feel that the answer, based on the V transformation between quark model states and exact states [1], is now clear at least for the $L=0$ baryon and meson states. It consists of two points.

(a) The structure of the general parametrization (which is an exact consequence of relativistic field theory and thus relativistically correct although noncovariant) has the familiar form typical of the NRQM. If the basic field Lagrangian is QCD-like (the charge is carried only by quarks and only λ_3,λ_8 appear in the Lagrangian of the strong plus electromagnetic interactions), Casimir operators in flavor space are absent in the general parametrization. For this reason and also due to the factorizable choice of the model states with $L=0$, the structure of the general parametrization turns out to be unexpectedly simple. It consists of a rather small number of terms.

(b) The structure of the terms arising in a NRQM calculation is the same as that appearing in the general parametrization. The NRQM often keeps, at least in the simpler calculations, only the simpler terms; we mean by this the additive terms plus two quark terms and so on. Because in most cases the minimum number of exchanged gluons increases with increasing number of different indices in a term, and because each exchanged gluon or each flavor factor P^λ depresses the magnitude of that term, it follows from the results of this paper that the simpler terms are usually the dominant ones.

These two points explain, we believe, why the "naive" NRQM works fairly well.

Before concluding I emphasize the following. As stated repeatedly the fact that only the flavor matrices λ_8 and λ_3 appear in the strong plus electromagnetic QCD Hamiltonian implies, since λ_8,λ_3 form a closed algebra, that in the exact expression of the energy levels derived from the QCD Hamiltonian (that is, the masses of baryons and mesons) the Casimir flavor operators are absent. This remains true no matter how complicated are the steps that are needed to calculate the levels. In particular it remains true also if the ground state of the system does not have the symmetry of the Hamiltonian, that is, if spontaneous symmetry breaking occurs. This shows that any effective Lagrangian that depends explicitly on the pion fields and thus on the $\lambda_1 \pm i\lambda_2$ flavor matrices cannot be in general equivalent to the original Lagrangian; it cannot reproduce correctly the spectrum of the original Hamiltonian, unless in the end result all flavor operators different from λ_8,λ_3 (and in particular all Casimir opera-

tors) disappear; but obviously in that case the introduction of such an effective Lagrangian complicates the description. This argument applies in particular to the Skyrmion model [12], to the various cloudy-bag models with pions as independent degrees of freedom, and, more importantly, to the effective Lagrangian used by Manohar and Georgi [13] to explain the NRQM with chiral quarks, treating the pseudoscalar lowest nonet as quasi Goldstone bosons and the other mesons as $q\bar{q}$ aggregates.

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[1] G. Morpurgo, (a) Phys. Rev. D **40**, 2997 (1989); (b) **40**, 3111 (1989); (c) **41**, 2865 (1990); (d) **42**, 1497 (1990); (e) Phys. Rev. Lett. **68**, 139 (1992); (f) Phys. Rev. D **45**, 1686 (1992). In (a) the method is formulated and applied to magnetic moments, em transition matrix elements, and masses of baryons, in (b) it is applied to the semileptonic baryon decays, in (c) to the meson masses, in (d) to the $V\rightarrow P\gamma$ meson decays; in (e) it is used to derive a new octet-decuplet baryon mass formula and in (f) it is applied to the baryon electromagnetic masses. A list of misprints is given in (e) and (f).

[2] G. Morpurgo, (a) Physics **2**, 95 (1965) [also reproduced in J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969), p. 132]; (b) *Theory and Phenomenology in Particle Physics,* Proceedings of the International School of Physics "Ettore Majorana," Erice, Italy, 1968, edited by A. Zichichi (Academic, New York, 1969), pp. 83–217; (c) in *Proceedings of the 14th International Conference on High Energy Physics,* Vienna, Austria, 1968, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), p. 225.

[3] A. De Rújula, H. Georgi, and S. Glashow, Phys. Rev. D **12**, 147 (1975) (indicated as DGG).

[4] The spin part in Eq. (3) of DGG (the only one of interest here) is $\sum_{i>k}[(\sigma_i\cdot\sigma_k)/m_im_k]$ where m_i and m_k are the quark masses. To obtain Eq. (8) write $m_i=m+(\Delta m)P_i^\lambda$ and expand this term to second order in Δm. In fact the first of equations (8) should be $(-D/C)=(\Delta m/m)-(\Delta m/m)^2+$higher orders in Δm; the second also should contain higher-order corrections in Δm; in writing Eqs. (8) we omitted such corrections; we recall that the only purpose of this comparison with DGG has been the normalizaton question. *Note added in proof.* In fact both Eqs. (8) hold exactly to all orders in Δm; they are not affected by corrections of orders $(\Delta m)^2$ or higher. Indeed from $(m_im_k)^{-1}=m^{-2}[(1+xP_i^\lambda)(1+xP_k^\lambda)]^{-1}$ one gets, with no approximation, after multiplication and division by $(1-xP_i^\lambda)(1-xP_k^\lambda)$: $(m_im_k)^{-1}=m^{-2}(1-x^2)^{-2}[(1-xP_i^\lambda)(1-xP_k^\lambda)]=m^{-2}(1-x^2)^{-2}[1-x(P_i^\lambda+P_k^\lambda)+x^2P_i^\lambda P_k^\lambda]$; thus in the DGG limit Eqs. (8) $[(D/C)=-x$ and $(b/D)=-x]$ are exact.

[5] Writing $m[\Psi^\dagger(\mathbf{x})\gamma_4\Psi(\mathbf{x})+(\Delta m/m)\Psi^\dagger(\mathbf{x})\gamma_4P^\lambda\Psi(\mathbf{x})]$ for the mass part of the quark field Hamiltonian, the coefficient of P^λ is $\Delta m/m\equiv(m_\lambda-m_{\mathcal{P}})/m_{\mathcal{P}}$ rather than $(m_\lambda-m_{\mathcal{P}})/m_\lambda$ that we often used; the difference between the two is second order in $\Delta m/m_\lambda$. But this is irrelevant; the only point that matters is that the coefficient of P^λ is, experimentally, $\approx\frac{1}{3}$.

[6] Compare, e.g., D. B. Lichtenberg, *Unitary Symmetry and Elementary Particles* (Academic, New York, 1970), p. 175.

[7] Recall that so far we have applied the general parametrization method only to a narrow subset of states, the lowest states of baryons and mesons; the question arises if more restrictions on the renormalized masses of the constituent quarks, additional to those to be mentioned here, emerge enlarging the set of states to which the parametrization is applied.

[8] Incidentally defining the constituent quark masses of $\mathcal{P}$ and $\mathcal{N}$ so that the $\mathcal{P}$ quark (for instance) has no anomalous magnetic moment is a possible choice but with no particular justification or privilege over any other choice, at least in the "natural" interval (16) introduced above.

[9] This $2P^\lambda$ structure was inadvertently omitted in Ref. [1a] from the list in Eq. (39); there such omission was irrelevant, because in Ref. [1a] we used only the structures with $1P^\lambda$.

[10] S. Okubo, Phys. Lett. **4**, 14 (1963). The relationship between magnetic moments given by Okubo and rederived in Ref. [1a] is correct to first-order flavor breaking and does not therefore include the term $\mathbf{G}_{7b}$.

[11] M. Beg, B. Lee, and A. Pais, Phys. Rev. Lett. **13**, 514 (1964); Morpurgo (Ref. [2a]).

[12] The Skyrmion model was listed by D. Gross [Phys. Today **40**, No. 12, 112 (1987)] as one of the steps that contributed to clarify the success of the NRQM.

[13] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).