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Decoupling a fermion whose mass comes from a Yukawa coupling: Nonperturbative considerations

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Perturbative analyses seem to suggest that fermions whose mass comes solely from a Yukawa coupling to a scalar field can be made arbitrarily heavy, while the scalar remains light. The effects of the fermion can be summarized by a local effective Lagrangian for the light degrees of freedom. Using weak coupling and large-N techniques, we present a variety of models in which this conclusion is shown to be false when nonperturbative variations of the scalar field are considered. The heavy fermions contribute nonlocal terms to the effective action for light degrees of freedom. This resolves paradoxes about anomalous and nonanomalous symmetry violation in these models. The application of these results to lattice gauge theory implies that attempts to decouple lattice fermion doublers by the method of Swift and Smit cannot succeed, a result already suggested by lattice calculations.

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I. PRELUDE AND PARADOX

As its title suggests, this paper should be thought of as a continuation of the work of D'Hoker and Farhi [1] on the decoupling of heavy fermions which transform in chiral representations of a spontaneously broken gauge group. This is a phenomenon which is crucial to several ideas in modern particle physics. The conventional wisdom holds that the chirality of the observed fermion representations is a fundamental property of the world, but mirror fermions could be discovered at the next accelerator. Are there theoretical bounds on how large their masses could be? If, indeed, chirality is fundamental and not a low-energy accident, it may have profound implications, for we know of no gauge-invariant regulator for chiral gauge theories, nor any real argument that they are consistent outside the realm of perturbation theory.[1] Attempts to construct chiral gauge theories as continuum limits of honest lattice field theories with short-range couplings are hampered by the Nielsen-Ninomiya theorem. In the naive lattice version of the standard model, this theorem guarantees the existence of mirror partners of quarks and leptons in the continuum limit. One can only hope to decouple them by giving them large Yukawa couplings to the Higgs field, and perhaps masses of the order of the cutoff. The success of this program would imply that there can be no theoretical upper bounds on the masses of mirror fermions. If lattice gauge theorists can send them off to infinity on the computer, God should be able to do the same in the real world. In order to argue against the existence of very heavy mirror fermions one would be reduced to complaining about fine-tuning (the question of how much work we believe God is willing to do) or the failure of perturbation theory (the question of how much work we are willing to do).

In a recent paper [2], one of the authors pointed out a possible problem with most attempts to construct the lattice standard model by these techniques. Any SU(3) lattice gauge theory with no colored Higgs fields and a Lagrangian bilinear in fermions has a global U(1) symmetry that acts on the lattice quark fields like baryon number.[2] This conserved baryon-number symmetry would appear to forbid the nonperturbative baryon-number-violating process discovered by 't Hooft [3] in the semiclassical approximation to the continuum standard model. If baryons are constructed from quark operators in any quasilocal way, lattice Green's functions with nonzero baryon number will vanish identically for all values of the parameters on the lattice.[3] Thus, either the lattice theory

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[1]Among the many pleasing aspects of string theory is the natural cutoff it provides for theories of chiral fermions. Friedan has argued that this should be viewed as a hint that string theory, rather than pointlike field theory, describes the real world. Note that although string theory does not yet give a nonperturbative description of chiral gauge theories, it is a finite gauge-invariant regulator in perturbation theory. All other perturbative regulators break chiral gauge invariance explicitly.

[2]In almost all theories, this symmetry can be gauged on the lattice. The single exception of which we are aware is a theory in which 3 and $\overline{3}$ fields are put on different sites of an Euclidean lattice. This theory has a global baryon-number symmetry which cannot be gauged. It is an interesting example of a lattice gauge theory where the Lagrangian is gauge invariant but the functional measure is not.

[3]We are assuming that in those cases where equal numbers of 3 and $\overline{3}$ fields sit on each site, the U(1) symmetry is not spontaneously broken. We believe this to be the conventional wisdom. If it did suffer spontaneous breakdown the theory would contain a Goldstone boson not observed in nature, and would not converge to the standard model.

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does not succeed in reproducing the conventional continuum model in perturbation theory, or we have discovered a nonperturbative violation of universality.

In an attempt to understand this puzzle without resorting to a computer, we have constructed a model in which heavy-fermion decoupling can be studied entirely within the framework of weakly coupled continuum field theory. The model was motivated by a remark of Kaplan. In order to turn a vectorlike gauge theory into one with a large hierarchy between fermions and their mirror partners, we must make the Yukawa couplings of the mirrors to the Higgs fields which break the gauge symmetry much larger than the gauge coupling. Kaplan remarked that this was possible within the perturbative domain, if we are willing to consider theories with extremely small gauge coupling. While not directly applicable to the real world, such models might prove to be an interesting theoretical laboratory. This is indeed the case as we will see below.

Our model then begins as the standard model with all of the usual couplings scaled down by a factor f. For definiteness we might consider $f \sim 10^{-2}$. We add to this a set of mirror fermions, left-handed Weyl fields that transform in the complex-conjugate representation of the standard-model fermions. The vacuum expectation value (VEV) of the Higgs field is arranged to be ~ 250 GeV as usual, and the mirror fermions are all given Yukawa couplings g_{mirror} of order 1, so that their masses are of order 100 GeV. The gauge bosons and conventional fermions have masses below 1 GeV. We forbid the gauge-invariant mass terms that could be made by pairing conventional fermions with their mirror partners. This is natural, due to a symmetry that will be discussed below. The hierarchy between vector-boson and heavy-fermion masses in this model requires no fine-tuning. Radiative corrections to the squares of gauge-boson masses due to loops of mirror fermions are of order $(e^2_{\text{gauge}} g^2_{\text{mirror}}/4\pi^2)v^2$, and are small compared to the tree-level masses. Note, however, that if we insist that the Higgs-boson mass be as small as the vector-boson mass, the conventional vacuum state becomes metastable. This is a consequence of the familiar unboundedness of the fermionic one-loop correction to the effective potential. In the present model, when the Higgs- and vector-boson masses are a hundred times smaller than the fermion mass, the turnover of the effective potential occurs in a region accessible to perturbation theory, and one might think that the conclusion that the vacuum is only metastable is reliable. If this is the case, then the discussion below can be read as a description of processes going on in this metastable state, and one is confronted with issues of the relative rates of the 't Hooft process and the decay of the false vacuum. We note, however, that Kuti and Shen [4] have argued that in a theory with only bare quartic couplings one cannot attain the renormalized parameter values for which the vacuum is metastable. The Higgs-boson mass remains a finite fraction of the fermion mass for all parameter values. We do not know if this conclusion remains true in the presence of irrelevant couplings in the bare Lagrangian, or when the system is coupled to gauge fields.

We believe that the issue of metastability of the perturbative vacuum as we vary the relative ratio of fermion to Higgs-boson masses is a crucial one, and we will have much more to say about it in Sec. III when we examine a two-dimensional model in the large-N limit. There we will show that by fine-tuning of many parameters we can obtain a model with a stable symmetry-breaking vacuum in which the ratio of the fermion mass to both vector-boson and Higgs-boson masses is extremely large. The puzzle we describe in the next paragraph exists in that model as well. Therefore, we ask the reader to ignore issues of vacuum metastability for the moment.

A bit of thought about nonperturbative baryon-number-violating processes in this model reveals an apparent paradox, whose resolution will be the subject of this paper. The baryon-number current built out of mirror quarks has an SU(2) gauge anomaly which is exactly equal to that of the ordinary baryon-number current. Thus, the difference between ordinary and mirror baryon numbers is an exactly conserved anomaly-free symmetry. Coupled with the fact that all mirror baryons have masses of order 100 GeV, this symmetry forbids the decay of particles with ordinary baryon number and masses of order 1 GeV or below, since any such decay would have to produce mirror baryon number and there are no light particles that carry this quantum number.[4]

Now let us study the same model using conventional ideas of decoupling and low-energy effective field theory. The particle spectrum at 1 GeV and below coincides with that of the conventional standard model rescaled by f. One might conclude then that the physics at this energy scale was well described by the standard model with rescaled couplings. But then, 't Hooft's calculation of the deuteron decay rate could be carried out, giving a result many orders of magnitude below that in the standard model, but still nonzero. This is in blatant contradiction with the exact result demonstrated in the previous paragraph. Note the similarity to the lattice models discussed in [2]. The role of mirror fermions is played by lattice doubles of the continuum fermions. The U(1) symmetry discussed above is continuum baryon number plus double-mode fermion number. If the doubles indeed have masses of the order of the cutoff while the continuum fermions have their observed masses, then we have a paradox very similar to that in the superweakly coupled standard model.

The arguments of D'Hoker and Farhi [1] do not seem to shed much additional light on this situation. These authors work in the limit of a fixed-length Higgs field. They tell us that if we try to compute the mirror baryon-

[4]Note that a very similar argument appears in 't Hooft's original calculation of deuteron decay in the standard model. If first- and second-generation baryon numbers were separately conserved, the deuteron could not decay. Its decay rate vanishes with the Cabibbo angle. By omitting the mass term mixing ordinary and mirror fermions we have eliminated the corresponding "Cabibbo mixing" in our model.

number current in the low-energy theory then it will be equal to the Skyrmion-number current of the nonlinear model representing the unphysical Higgs degrees of freedom. When the model is gauged, this is the same as the Chern-Simons current built out of the gauge-invariant massive gauge fields. It is a gauge-invariant current whose divergence is proportional to the SU(2) topological charge. The difference between it and the ordinary baryon-number current is an anomaly free gauge-invariant conserved current. However, this gauge-invariant Chern-Simons current exists in the standard model even when there are no mirror fermions. Arguments based on it cannot resolve our paradox unless they imply that 't Hooft's calculation is wrong in the unextended standard model. There is always a gauge-invariant conserved current which acts as baryon number when applied to quark fields. Furthermore, the change in the Chern-Simons charge built from massive gauge fields is the integral of a total derivative of a gauge-invariant object constructed out of massive fields. One would expect it to be zero. If the Chern-Simons current were really a well-defined operator in the conventional standard model, this argument would rule out baryon-number violation completely.[5] Thus, if we believe that 't Hooft's calculation is correct in the unextended standard model, the arguments of D'Hoker and Farhi cannot help us to understand the paradoxes of decoupling in the model supplemented with heavy mirror fermions.

This is perhaps the place to discuss the criticisms of the arguments in Ref. [2] made by Dugan and Manohar [5]. These authors claim to show that the conserved lattice current corresponding to the symmetry discussed in [2] is not gauge invariant. As we have stated it, this claim is obviously wrong on the lattice. We can define the current by gauging the U(1) lattice baryon number discussed above (and varying with respect to the background gauge field), and since every term in the Lagrangian is invariant under the standard-model group, so is the current. As the authors of [5] point out in their Eq. (14), the real meaning of their calculation in a model in which gauge invariance of the Wilson term is enforced by introducing a Higgs field, is that the conserved current differs from the light baryon-number current by the Chern-Simons term of the massive gauge fields.[6] Thus, their conclusions are identical to those of D'Hoker and Farhi and do not really shed any more light on the baryon-number paradox.

We will present the resolution of this paradox in the next section. It is, we believe, rather surprising, and shows that the decoupling of a fermion whose mass comes from a Yukawa coupling is profoundly different than ordinary decoupling, even more so than one would have concluded from the work of [1] or from recent work on new parameters arising from loops of heavy chiral fermions in electroweak radiative corrections. In effect, what we will show is that although the *particles* associated with mirror fields are heavy, the mirror fields themselves do not decouple from low-energy physics, as long as the Higgs field is light. Depending on the configuration of low-energy gauge and Higgs fields, an arbitrarily large number of modes of the mirror fields can contribute significantly to low-energy tunneling processes. They completely transform the instanton dynamics of the low-energy gauge system.

As a counterexample to the claim that one can entirely decouple mirror fermions, this weakly coupled model is not completely satisfactory. The nonperturbative effects which exhibit this dramatic violation of decoupling are, in the weak-coupling regime, much smaller than the perturbative effects of nonrenormalizable operators in the baryon-number conserving sector. Thus, there is not a completely clean separation of scales. We cannot reduce the perturbative effects of the heavy mirror particles to arbitrarily small size without leaving the realm of perturbation theory. In addition, and more importantly, if we try to make the Higgs-boson mass much smaller than the fermion mass in this model we are confronted with vacuum instability.[7] Nonetheless, the fact that the zero modes and lack of decoupling are evident for all values of the mass that are amenable to a perturbative analysis, suggests that the phenomenon that we have uncovered persists into the strong-coupling regime. Even if we are able to construct a model with heavy fermions, light Higgs-bosons, and a stable vacuum, we will still find that the fields of the heavy fermions do not decouple from low-energy physics.

To obtain further evidence for this, we examine in Sec. III some two-dimensional chiral gauge theories which are almost soluble. We show that in a model with a fixed-length Higgs field (which is renormalizable in two dimensions), we can indeed decouple heavy mirror fermions. Our paradox about baryon-number conservation is resolved by showing that baryon-number-violating amplitudes vanish in the limit of the fixed-length Higgs field. When the modulus of the Higgs field is allowed to fluctuate this is not the case. We study the fluctuating-length theory in the large-N approximation. In order to keep the radial mode of the Higgs field light, and the classical vacuum stable, we have to fine-tune a number of parameters that grows with the fermion mass. This is a consequence of a general property of decoupling of heavy particles (gauge bosons as well as fermions) whose mass comes solely from the vacuum expectation value (VEV) of a scalar field. It is quite generally true that the effective potential for the scalar induced by virtual heavy particles is large and has curvature of the order of the

[5] In Sec. III we will present a two-dimensional model in which the D'Hoker-Farhi scenario is realized. It is indeed the case that baryon number is not violated in the low-energy effective action of this model.

[6] Dugan and Manohar are clearly working in the fixed-length Higgs model, or ignoring zeros of the Higgs field.

[7] We will see later that this problem of vacuum instability is the real iceberg on which decoupling founders, and that our paradox about baryon number is only the tip of it.

masses of the heavy particles. It is also nonanalytic when the Higgs-field VEV goes to zero, because in this limit the "heavy" particles become massless and the theory contains infrared divergences. Thus, if we perform no fine-tuning, the mass of the Higgs particle itself (the excitation of the radial mode of the Higgs field) is large. Further, because of the nonanalyticity of this potential at the origin, we cannot fine-tune the coefficients of a finite number of analytic functions in the tree-level potential to cancel the large effects of the heavy particles. We will argue below that the Linde-Weinberg lower bound on the Higgs-boson mass [6] is another example of this effect. It can be viewed as a failure of decoupling of heavy vector bosons from an erstwhile effective field theory for light Higgs bosons. We believe that this fundamental obstacle to obtaining a light Higgs boson in the presence of heavy fermions or bosons whose mass is driven by the Higgs VEV is the real reason for the failure of decoupling of chiral fermions. We can obtain a model with a large fermion to scalar mass ratio and a stable vacuum only by fine-tuning many parameters.

If, in our two-dimensional model, we perform the infinite-parameter fine-tuning required to obtain a light Higgs boson and a stable vacuum, we still find problems. Heavy fermions have light modes and do not decouple in the presence of configurations where the Higgs field goes to zero in some regions of spacetime. The effect of these light modes is to drastically change the nonperturbative (in N) physics of the low-energy theory. Baryon-number violation and confinement of fractional charges, which are both present in the model without heavy fermions, disappear in the model with heavy fermions.

To summarize, it appears very difficult to construct a model in which fermions that get their mass from a Yukawa coupling to a scalar field are allowed to have masses much larger than that of the mode which controls fluctuations in the magnitude of the scalar. In two dimensions, using the infinite number of relevant operators at the scalar Gaussian fixed point, it is possible to construct such models. However, when the system is coupled to a gauge field, there are light modes of the heavy fermions in instanton configurations in which the magnitude of the scalar field vanishes locally at certain points in spacetime. These light modes completely change the dynamics of the low-energy theory. The only way to truly decouple the fermions is to freeze the magnitude of the scalar field simultaneously. In this limit, instanton processes have zero amplitude because the instanton action goes to infinity. Thus, all paradoxes related to chiral fermion decoupling are removed, but at the price of "throwing the baby away with the bathwater."

In four dimensions, it seems highly unlikely to us that it is possible to do the fine-tuning necessary to keep the Higgs field light in the presence of extremely massive chiral fermions. The effective potential generated by the heavy fermions naturally has an energy scale of the fermion mass. Furthermore, it is singular at the origin of field space and cannot be well approximated by a quartic polynomial. Renormalizability restricts us to quartic polynomials, so we cannot cancel the effect of the fermions with local counterterms. The Higgs-boson mass would be driven to infinity with the fermion mass. Since there are no sensible continuum theories with fixed-length Higgs fields in four dimensions [7], this argument suggests that it will be impossible to find a four-dimensional model with decoupled chiral fermions, and hence impossible to build a lattice version of the standard model with many of the current local algorithms. In any case, no model built in this way can contain the 't Hooft mechanism for baryon-number violation. If finely tuned models with light Higgs bosons exist, baryon-number conservation will be enforced by confinement of instantons through heavy-fermion zero modes, while in models with fixed-length Higgs fields, instantons will have infinite action. In the penultimate section of this paper we will give a brief survey of attempts to construct lattice standard models and point out those which may evade the difficulties discussed in this paper.

A disturbing possibility raised by our analysis of finely tuned models is the occurrence of important low-energy fields which create only very heavy particle states from the vacuum. This dramatic failure of the association between fields and experimentally accessible particle states would make it difficult to find experimental tests of a theory containing such *phantom* fields. The large-N model of Sec. III certainly contains phantom fields. One is led to ask whether their occurrence is likely in the real world. D'Hoker and Farhi [1] suggested the existence of fermionic solitons in the effective action generated by decoupled chiral fermions. These had the same quantum numbers as the original fermions and masses of the order of the low-energy scale. The solitons of D'Hoker and Farhi are topological excitations in a theory with fixed-length Higgs fields. A related phenomenon[8] is the existence of baglike [8] nontopological solitons in models with a Higgs field of fluctuating magnitude. In these configurations, light states with single fermion quantum numbers are created by deforming the Higgs field from its VEV over a finite region of space. Since the fermion mass is zero in the region where the Higgs field vanishes, these states can be much lighter than fermions propagating in the vacuum if the energy required to deform the Higgs boson is small compared to the fermion mass.

Bagger and Naculich have recently studied these baglike solutions in a strongly coupled large-N model [8].[9] They find that these states have mass comparable to the fermion mass in the strong-coupling region. However, they do not perform the fine-tunings necessary to keep the Higgs-boson mass finite as the fermion mass goes to infinity (their model is four dimensional, and it may not be possible to do this in a consistent way). Thus, it is not surprising that the bag picture, which depends on an easily deformable Higgs field, fails in their model. It seems

[8]We do not really understand the relation between these two types of soliton.

[9]At large N, as we will see in Sec. III, single fermion bags cannot form. Bagger and Naculich study bags containing N fermions.

plausible, however, that baglike solitons with single fermion quantum numbers will exist in most models in which it is possible to fine-tune the Higgs-boson mass to be much smaller than the fermion mass, without destabilizing the vacuum. These are precisely the models in which one might suspect the occurrence of phantom fields. The existence of light bags in such models would eliminate the phenomenon of phantom fields. The phantom would be interpolating fields for the light bag states, and we could ascribe the nonperturbative dynamics associated with them to the action of these particles. Our two-dimensional large-N model is an explicit counterexample to the conjectured existence of light bags in all models with phantom fields. We will argue, however, that this may be a peculiarity of the large-N limit.

We have not really studied the question of the existence of light bag states in much detail. It deserves more attention, for it may be the key to finding a theoretical upper bound on the mass of mirror fermions or other as yet unobserved chiral representations of the standard-model gauge group.

II. MASSLESS MODES OF MASSIVE PARTICLES

Let us then study the weakly coupled version of the standard model introduced in the previous section, ignoring questions of stability of the perturbative vacuum. That is, we will study classical solutions to the Euclidean equations of motion, and the fermion determinant in these backgrounds. The crux of our argument is that the Euclidean Dirac equation for mirror fermions (or ordinary fermions for that matter) in the standard model has such zero modes in the presence of an instanton field. Indeed, if we set the Yukawa couplings to the Higgs field to zero, the existence of such modes is a trivial consequence of the anomaly equations for mirror baryon number and lepton number. Since the zero modes carry baryon number and the Yukawa couplings preserve baryon number, there is no way for the Yukawa couplings to lift these modes to nonzero (Euclidean) energy.

More mathematically, near the center of the instanton, the Higgs field goes to zero and the gauge field approaches that of the instanton solution of pure gauge theory. The solution of the zero-eigenvalue Dirac equation in this region is

$$\psi_L = \psi_0[A] \ , \tag{2.1}$$

$$\psi_R = \psi_R^0 \ , \tag{2.2}$$

where $\psi_0[A]$ is the zero-mode solution of the left-handed Weyl equation in the pure gauge instanton background, and ψ_R^0 is the solution of the right-handed Weyl equation with a source given by the product of the Higgs field and $\psi_0[A]$. Since $\psi_0[A]$ is not singular at the origin, and the Higgs field goes to zero there, no special choice of boundary conditions must be made to make the full solution normalizable at the origin. At infinity, the Higgs field goes to a constant and the gauge field falls off exponentially (in unitary gauge). The Dirac equation becomes that for free massive fermions. There are exponentially increasing as well as exponentially decreasing solutions of this equation, but since we have not used up any parameters making the solutions regular at the origin, we have enough parameters left to eliminate the exponentially increasing solution. Consequently, the zero modes are normalizable despite the fact that asymptotically the fermion fields behave as if they were massive.

The existence of these zero modes means that amplitudes which involve a change of topological charge, and involve only particles which exist in the low-energy theory, vanish identically. The 't Hooft interaction, which describes the effect of instantons on the fermions in the theory, is an operator which changes mirror baryon number. Its form is

$$\mathcal{L}_{\text{'t Hooft}} = \prod \psi_L \prod \psi_H \ , \tag{2.3}$$

where the products run over light- and heavy-fermion zero modes. This interaction connects the heavy sector to the light sector, but has no matrix elements within the light sector itself. Note that this is an exact consequence of the full theory, but it cannot be derived from a low-energy Lagrangian from which the mirror fields are omitted. Thus, 't Hooft's calculation of baryon-number violation is radically altered in the theory with heavy mirror particles. It no longer predicts baryon-number violation in the light sector.

It is worth pointing out that the dramatic violation of decoupling that we have just discussed is actually implicit in 't Hooft's original calculation of baryon-number violation in the standard model. 't Hooft included two generations of quarks and leptons in his calculation of deuteron decay. The second-generation quarks and leptons have instanton zero modes, and if there is no Cabibbo mixing to convert these modes into modes of first-generation fermions, the amplitude for deuteron decay vanishes. It is proportional to $\sin^3\theta_{\text{Cabibbo}}$. This, by the way, is the reason that the deuteron rather than the proton decays by the 't Hooft process. The instanton violates first-generation baryon number by 1 unit, and second-generation baryon number by 1 unit, preserving their difference. Cabibbo mixing violates individual generation baryon numbers by $\frac{1}{3}$, preserving their sum. The final change in baryon number in a process in which no second-generation particles are involved is 2 units. In a three-generation model, the amplitude is further suppressed by mixing angles between the first and third generations, and the total change in baryon number in the minimal instanton process is 3.

The zero modes of the heavy fields have consequences even within the sector of zero topological charge, when we restrict attention to Green's functions containing only the fields of light particles. Indeed, the heavy-fermion determinant in the presence of an instanton–anti-instanton pair factors into the product of the determinants in each individual configuration when the separation between the pair is large. Since the instanton and anti-instanton determinants vanish, the determinant in the pair configuration must go to zero as the separation goes to infinity. We have noted above that the zero-mode wave functions die exponentially. The pair determinant is thus an exponentially vanishing function of separation. In terms of the statistical mechanics of the dilute instanton gas, this is equivalent to an attractive linear confining potential between instantons and anti-instantons:

$$Z_{I+\bar{I}} \sim \int d^4R_I d^4R_{\bar{I}} \exp(-Nm_F|R_I - R_{\bar{I}}|) , \qquad (2.4)$$

where N is the number of heavy-fermion zero modes. Again we see that the dynamics of the low-energy gauge fields is drastically affected by the virtual modes of the heavy mirror fermions.

Is there any kind of effective low-energy field-theoretic description of the system we have studied at energies of order 1 GeV? Certainly the conventional description, in which only fields for the light particles are included, is wrong. Recently a class of models was described in which the low-energy effective theory had to be supplemented by a number of discrete global variables [9].[10] The resulting effective theory violates the clustering axiom. Is a similar description of decoupled mirror fermions available? We suspect that the answer is no. In the nonperturbative regime, the number of heavy-fermion field modes which are important to the low-lying dynamics depends crucially on the configuration of low-energy boson fields. Since the separation between heavy- and light-fermion degrees of freedom is light-field dependent, one should not expect a local effective Lagrangian, unless we keep the fields of the heavy particles in the low-energy effective Lagrangian. There is no way that a local effective Lagrangian for the light fields can produce a linear confining force between instantons.[11]

If the heavy-fermion masses could really be taken to infinity, we would have a somewhat paradoxical situation in which the low-energy theory contained fields which created no particle states from the vacuum. A less radical description is suggested by the work of Refs. [1,8]: soliton states of the combined heavy-fermion–Higgs-boson system survive at low energy even when the elementary fermion masses go to infinity. These solitons have the quantum numbers of the elementary fermions and masses of the order of the vacuum expectation value of the Higgs field. D'Hoker and Farhi were not able to firmly establish the existence of such solitons because they dealt with a theory of fixed-length Higgs fields and relied on the topology of the compact Higgs manifold as well as on hypothetical short-distance corrections to the effective action that could stabilize the soliton configurations of the nonlinear model. However, the existence of such solitons is also suggested by early work on baglike nontopological solitons [8]. In these references, the crucial ingredient is the variable radius of the Higgs field. When a fermion gets its mass from a Yukawa coupling, a single fermion state can exist in which the value of the Higgs field vanishes near the location of the fermion. Fermion modes of low energy exist in which the elementary fermion wave function is trapped in the vicinity of this zero of the Higgs field, avoiding the region of space where the fermion mass is large. If the energy required to locally deform the value of the Higgs field away from its vacuum value is much less than the elementary fermion mass, a light-soliton state with elementary fermion quantum numbers is formed. It is plausible that such states exist in the present model, though we have not investigated the question in detail. If they do, the necessity of keeping the heavy-fermion fields in the low energy Lagrangian would no longer be paradoxical or bizarre. They would be necessary to a description of the light-soliton states.

We should point out that in the model which we have described in this section there is no really tight argument that a purely low-energy description of instanton processes should exist. In a conventional theory with a heavy sector, an effective local theory of the light particles is supposed to describe low-energy physics up to the accuracy $(M_{\text{light}}/M_{\text{heavy}})^p$ for all positive p. In our model $M_{\text{light}}/M_{\text{heavy}} \sim e/g$, where e is the gauge coupling and g is the Yukawa coupling of the heavy fermions. g is required to be of order 1, so these effects are small only when e is very small. On the other hand, the 't Hooft process is parametrically of order $e^{-8\pi^2/e^2}$. Thus, it is smaller than effects of the heavy particles, and we do not have the right to insist that it is described correctly in terms of a low-energy Lagrangian. (On the other hand, in trying to construct a lattice standard model, we really want the lattice fermion doubles to go off to infinite energy, leaving no trace behind. In this case it is crucial that there be a low-energy Lagrangian which correctly describes the symmetries of the model.) We do not believe that this criticism of our analysis is truly substantive. The fermion zero modes and almost zero modes that are crucial to us exist for all values of the Yukawa coupling, and for all configurations in the functional integral that have widely separated lumps of topological charge. There is no indication that anything qualitatively new happens when the Yukawa coupling begins to leave the perturbative regime, other than the fact that the confining force between instanton and anti-instanton gets stronger.

We are, however, deeply disturbed by the potential instabilities of the perturbative vacuum in this model. Thus, in order to confirm and enhance our understanding of the picture of fermion decoupling that we have presented here, we turn in the next section to some two-dimensional models.

[10]Note that the irrational couplings which were the focus of [9] are not necessary to the existence of these global variables. They exist in many perfectly renormalizable four-dimensional field theories.

[11]This situation bears a certain resemblance to that which occurs in theories which have large numbers of degenerate, physically inequivalent vacuum states. In such theories, it is possible for a particle that is massive at a generic point in the vacuum manifold to become massless at certain special points. The effective action, obtained by integrating out this massive particle at a generic point, becomes singular and nonlocal at the special points. The new observation that we are making here is that these nonlocal effects are also important for field configurations which visit special points in the field manifold in a local region of spacetime.

III. TWO-DIMENSIONAL MODELS

Our analysis of the weakly coupled mirror standard model suggests that the failure of decoupling of mirror fermions is related to the existence of configurations in which the Higgs field is equal to zero (or is at least very small) at some point in spacetime. Indeed, the 't Hooft-constrained instanton solution has a vanishing Higgs field at the core of the instanton, and the mechanism for constructing light fermionic bag states also depends on zeros of the Higgs field. If we restrict attention to configurations in which the magnitude of the Higgs field is everywhere bounded from below by a positive constant m_0, it is probably possible to use the methods of Witten and Vafa [10] to prove that the effective action obtained by integrating over heavy fermions contains nonlocality only over some finite scale of order $1/gm_0$. To confirm this intuition, we will study a two-dimensional model with a fixed magnitude Higgs field.

The model that we will study has a U(1) gauge symmetry. It contains two massless left-moving fermions ψ_i, with gauge charges q_i, and a massless right mover ψ with charge q. The charges satisfy the anomaly cancellation condition $q^2=q_1^2+q_2^2$. We will also include gauge singlet partners for each of these particles, in order to describe them in terms of two-component Dirac fields. We use the letter Ψ to denote the triplet of light Dirac fields. In addition, we have mirror fermions P_i and P which are right movers (left movers) and carry charge equal to that of their mirror partners. The mirror fermions also have singlet partners, and we will include Yukawa couplings between mirror fermions and Higgs fields which provide Dirac masses for the mirror fermions in the presence of a Higgs-field vacuum expectation value. The triplet of heavy Dirac fields is denoted P. The Lagrangian is

$$\mathcal{L}=\frac{-1}{4e^2}F_{\mu\nu}^2+|\partial_\mu-iA_\mu\phi|^2-\lambda V(\phi^\dagger\phi)+\bar{\Psi}i\gamma^\mu\left[\partial_\mu-q\left[\frac{1+\epsilon\gamma_3}{2}\right]A_\mu\right]\Psi$$
$$+\bar{P}\left\{i\gamma^\mu\left[\partial_\mu-q\left[\frac{1-\epsilon\gamma_3}{2}\right]A_\mu\right]+g(\phi^\dagger)^q\left[\frac{1-\epsilon\gamma_3}{2}\right]+g(\phi)^q\left[\frac{1+\epsilon\gamma_3}{2}\right]\right\}P\ . \tag{3.1}$$

Here q and ϵ are 3×3 matrices: $q=\mathrm{diag}(q_1,q_2,q)$ and $\epsilon=\mathrm{diag}(\epsilon_1,\epsilon_2,\epsilon_3)=\mathrm{diag}(1,1,-1)$. The gauge coupling e and the Yukawa coupling g both have dimensions of mass, and we take $g\gg e$. The coefficient λ in the Higgs potential has dimensions of mass squared and determines the spacetime scale of fluctuations of the radial mode of the Higgs field. We will first take this scale to be much larger than the Yukawa coupling so that this mode does not participate in the physics at any scale of interest. Thus, the radial mode of the Higgs field is frozen: $\phi=e^{i\theta}$. Instanton configurations of the gauge-Higgs system will then have infinite action.

In this limit it is convenient to transform the heavy-fermion fields by multiplying them by functions of the Higgs fields in such a way as to make them gauge invariant. Let $\mathcal{P}=\exp\{-iq[(1-\epsilon\gamma_3)/2]\theta\}P$ be the gauge-transformed field; then the Lagrangian becomes

$$\mathcal{L}=\frac{-1}{4e^2}F_{\mu\nu}^2+(\partial_\mu\theta-A_\mu)^2$$
$$+\bar{\Psi}i\gamma^\mu\left[\partial_\mu-q\left[\frac{1+\epsilon\gamma_3}{2}\right]A_\mu\right]\Psi$$
$$+\bar{\mathcal{P}}\left\{i\gamma^\mu\left[\partial_\mu-q\left[\frac{1-\epsilon\gamma_3}{2}\right](A_\mu-\partial_\mu\theta)\right]+g\right\}\mathcal{P}\ . \tag{3.2}$$

Note that the mirror fermions now have a constant mass term. In the limit at which the gauge coupling becomes very weak (i.e., is much smaller than the fermion mass) the model evidently reduces to a current-current coupling between a massive fermion and a massless Goldstone boson, a renormalizable Lagrangian. The coupling to the gauge boson is super-renormalizable.

Let us now imagine doing a renormalization-group analysis of the theory, integrating out degrees of freedom above some cutoff scale Λ which is much larger than e and much smaller than the fermion mass. In this integration, the gauge coupling can be treated perturbatively since it is super-renormalizable and the fluctuating degrees of freedom have an infrared cutoff. The result of this integration is an effective field theory for the light bosonic degrees of freedom ϕ and A_μ and the massless fermions. The effective action depends only on the gauge-invariant field $B_\mu=\partial_\mu\theta-A_\mu$, which couples to a chiral current of the heavy fermions. We can classify the possible terms in this effective action according to their dimension. The only term of dimension 2 is quadratic in B_μ. Its coefficient will be logarithmically divergent in the limit $m\to\infty$.[12] All other terms in the action have dimension greater than 2 and their coefficients vanish in the heavy-fermion limit. To lowest order in e, the effective action is obtained from that of the field θ in the theory,

[12]In fact, in the lowest order in the loop expansion this divergence cancels when the contributions of all heavy-fermion loops are summed. It is proportional to the anomaly. This cancellation does not persist in higher orders.

$$\bar{\Psi}i\gamma^{\mu}\left[\partial_{\mu}-q\partial_{\mu}\theta\frac{(1+\gamma_3)}{2}\right]\Psi+\frac{m_V^2}{2e^2}(\partial_{\mu}\theta)^2 , \quad (3.3)$$

by the substitution $\partial_{\mu}\theta \to B_{\mu}$. Higher-order corrections in e vanish in the limit of large mass.

Although the above analysis is motivated by an examination of Feynman diagrams, we believe that it is valid nonperturbatively. When $e=0$, the model from which we obtain the effective action is a version of the two-species Thirring model with one of the fermions made extremely massive. There seems to be no place for unexpected surprises. If this is the case, the decoupling of mirror fermions seems to work in this model. Their effect on the low-energy effective action is simply to introduce an infinite renormalization of the gauge-boson mass term B_{μ}^2. If we are willing to tune parameters to ensure that the gauge boson remains light, then we obtain a chiral gauge theory in the limit $m \to \infty$.

Not surprisingly, in this theory with fixed-length Higgs fields, the D'Hoker-Farhi analysis of the baryon-number current goes through. If we couple an external gauge field a_{μ} to the nonanomalous sum of ordinary and mirror baryon number, it is easy to verify that in the low-energy theory a_{μ} couples to $J_L^{\mu}-(q_1+q_2-q)\epsilon^{\mu\nu}(\partial_{\nu}\theta-A_{\nu})$. However, we can also verify that in this model the result of this low-energy identification is to rule out the 't Hooft process in low-energy physics. As in Sec. II, the D'Hoker-Farhi identification of the divergence of the baryon-number current with the divergence of a current constructed from gauge-invariant, massive fields, suggests that global baryon number is conserved. In the present model the low-energy theory is exactly soluble (when the massless fermions are bosonized the Lagrangian becomes quadratic), and we can verify this conjecture explicitly. The simplest way to see this is to integrate out the massive vectors, to obtain a baryon-number-conserving action for the massless fermions:

$$\bar{\Psi}i\gamma^{\mu}\partial_{\mu}\Psi+C\int dx\, dy\, J^{\mu}(x) \times \left[\frac{g_{\mu\nu}+\partial_{\mu}\partial_{\nu}/M_V^2}{-\partial^2+M_V^2}\right](x,y)J^{\nu}(y) , \quad (3.4)$$

where $J_{\mu}=\bar{\Psi}\gamma_{\mu}q[(1+\epsilon\gamma_3)/2]\Psi$. Although this action is nonlocal on the scale of the vector-boson Compton wavelength, it contains no infrared divergences, and no violation of baryon number.

For completeness, we record the bosonized form of the low-energy action before the vector bosons are integrated out. Each low-energy Dirac fermion is realized in terms of a scalar field whose gradient is the *vector current* of the fermions. We call the scalar corresponding to ψ_i, φ_i. The bare Lagrangian is

$$\mathcal{L}=\frac{-1}{4e^2}F_{\mu\nu}^2+(\partial_{\mu}\varphi_i-q_iA_{\mu})^2+(\partial_{\mu}\theta-A_{\mu})^2 + \sum q_i\epsilon_i\varphi_i\epsilon^{\mu\nu}F_{\mu\nu} . \quad (3.5)$$

After integrating out the heavy fields and rewriting things in terms of the gauge-invariant massive vector-boson field B_{μ}, this becomes

$$\mathcal{L}=\frac{-1}{4e^2}B_{\mu\nu}^2+(\partial_{\mu}\Phi_i-q_iB_{\mu})^2+\alpha(B_{\mu})^2 + \sum q_i\epsilon_i\Phi_i\epsilon^{\mu\nu}B_{\mu\nu} , \quad (3.6)$$

where $\Phi_i=\varphi_i-q_i\theta$. Note that we have had to use the anomaly cancellation condition to show that θ does not appear in this final form of the Lagrangian. The fixed-length Higgs model described above thus realizes the goal that one would like to achieve in constructing the standard model on the lattice. However, it does so at the expense of eliminating the 't Hooft process from low-energy physics. This is perfectly consistent within the framework of the low-energy effective Lagrangian, where the 't Hooft amplitudes clearly vanish in the limit at which the mass of the Higgs boson goes to infinity.[13] In four dimensions we do not know of a consistent version of the standard model with an arbitrarily heavy Higgs particle, so the above scenario cannot be achieved.

In order to study a two-dimensional model with variable length Higgs bosons in a reliable manner, we introduce N copies of both the low-energy and mirror fermions, and take the limit $N\to\infty$ with $e^2N=E^2$, $g^2N=G^2$, and $\lambda N=\kappa$ fixed. In this limit, quantum fluctuations of the boson fields are suppressed, while the ratio of tree-level gauge boson to fermion masses is E/G, and can be as small as we like.

To leading order in N, the theory is solved by finding stationary points of the effective action:

$$S_{\text{eff}}=N\left[|D_{\mu}\phi|^2-\kappa(|\phi|^2-1)^2-\frac{1}{4E^2}F_{\mu\nu}^2 +\text{Tr}\ln\left\{i\gamma^{\mu}\left[\partial_{\mu}-qA_{\mu}\left[\frac{1+\epsilon\gamma_3}{2}\right]\right]\right\} +\text{Tr}\ln\left\{i\gamma^{\mu}\left[\partial_{\mu}-qA_{\mu}\left[\frac{1-\epsilon\gamma_3}{2}\right]\right] -G\phi^{\dagger q}\left[\frac{1-\epsilon\gamma_3}{2}\right]-G\phi^{q}\left[\frac{1+\epsilon\gamma_3}{2}\right]\right\}\right] . \quad (3.7)$$

The large-N vacuum state is determined by stationary points of this effective action with vanishing gauge fields and constant Higgs fields. The heavy-fermion contribution to the effective potential for the Higgs field dominates the classical term for $G^2 \gg \kappa$. It has the form

$$V_{\text{ferm}}(\phi)=\frac{G^2}{4\pi}|\phi|^2\ln(|\phi|^2) . \quad (3.8)$$

[13]In four-dimensional non-Abelian gauge theories, the validity of this claim is not obvious, although it is certain that the conventional instanton action becomes infinite with the Higgs-boson mass. It is hard to discuss the question rigorously, since the entire theory becomes strongly coupled as the Higgs-boson mass gets large, and probably the limiting theory does not exist [7].

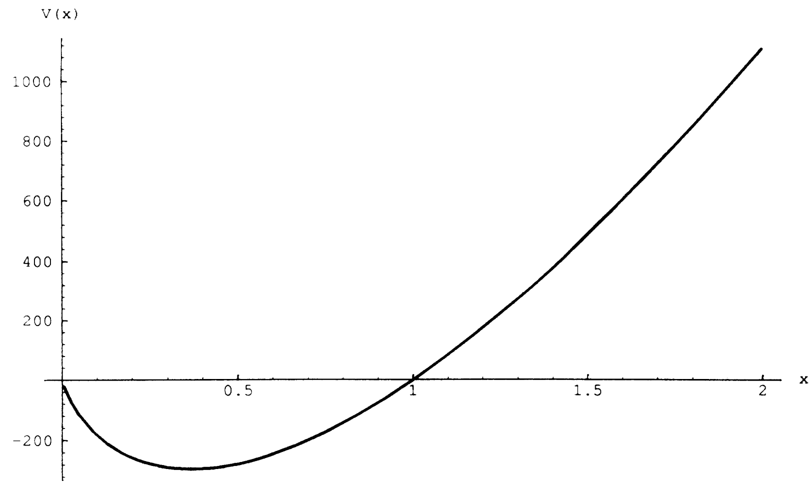

FIG. 1. The one-loop fermion contribution to the effective potential in two spacetime dimensions as a function of $\chi=\phi^2$.

This potential is shown in Fig. 1. It has the typical spontaneous breakdown form, and is bounded from below. It determines the minimum of the Higgs field to lie at $|\phi|^2=e$, and the Higgs-boson mass, determined as the curvature of the potential at its minimum, is of order G^2. Although this seems to be a consistent theory, it is not what we want if we intend to decouple the heavy fermions while keeping the Higgs boson light. In that case we expect to keep the Higgs particle at low mass, and we may attempt to do this by fine-tuning the coefficients of relevant operators in the low-energy theory. In two dimensions there are an infinite number of relevant operators for a scalar field, although conventional renormalization theory leads us to expect only a quadratic term in this leading N approximation. In order to keep the minimum at its classical value $\phi=1$ and keep the Higgs-boson mass of order $\sqrt{\kappa}$, we need to tune at least two parameters. The quartic and quadratic couplings of the classical Lagrangian suffice, but the resulting potential has a negative quartic coupling and is unbounded from below. The addition of a $|\phi|^6$ coupling allows us to keep the potential bounded. There is then one free parameter. For all values of this parameter, the resulting potential has a deeper minimum either much closer to or much further from the origin than $\phi=1$. For a ratio of 100 between the fermion and Higgs-boson masses, the potential typically looks like Fig. 2. The perturbative vacuum with small Higgs-boson mass that we have constructed by fine-tuning three parameters is metastable and rather short lived. One must add higher-order terms to get sensible results. After a while it dawned on us that what we were doing could best be described as follows: *for any value of G^2/κ find a polynomial approximation $P(\phi)$ to $|\phi|^2\ln(|\phi|^2)$ which approximates this function with accuracy κ/G^2 in a range $0\le|\phi|\le\phi_0$ with $\phi_0>1$. Arrange further that $V_{\rm ferm}(\phi)-(G^2/4\pi)P(\phi)$ be positive and monotonically increasing for $\phi>\phi_0$. Then add $-(G^2/4\pi)P(\phi)$ to the classical potential.* The resulting effective potential looks just like the classical potential for $\phi<\phi_0$ and shoots

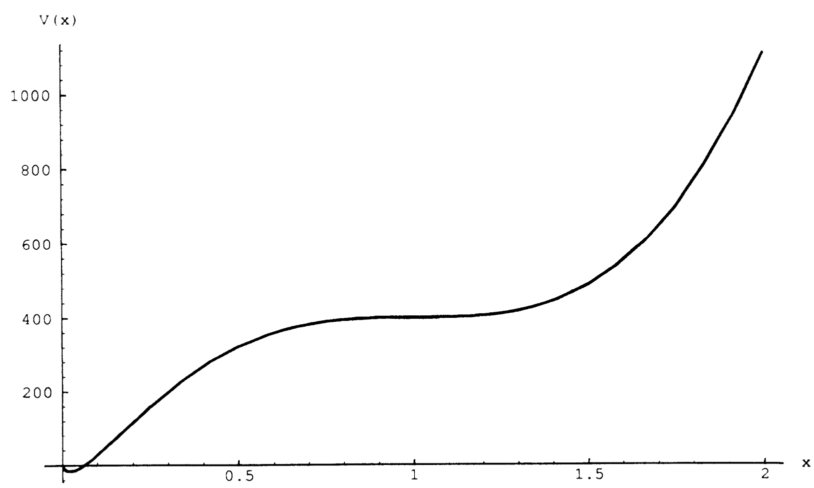

FIG. 2. The effective potential of Fig. 1 with a sixth-order polynomial added to fine-tune the Higgs-boson mass. The figure is scaled by the square of the Yukawa coupling so that any finite curvature implies a mass that goes to infinity.

up dramatically beyond this point. With sufficient fine-tuning we can even make ϕ_0 very large. As the fermion mass gets larger we need to tune more and more parameters to obtain a low-energy effective potential that agrees with that in a theory where the fermions are absent. A similar situation would be found if we tried to use a basis of analytic functions other than polynomials[14] to construct our local counterterms. We would still need a number of parameters which grew with the fermion mass to construct a satisfactory theory with a light Higgs field and a stable vacuum.

The reason for the difficulty we had in obtaining a satisfactory low-energy potential is not hard to find. The fermion contribution to the potential is not analytic at the origin. This is a consequence of infrared divergences which occur because the fermion is massless when the Higgs field vanishes. If the potential had been an entire function, we could have canceled it exactly with a sequence of allowed counterterms. This difficulty is familiar from four dimensions, and was the origin of the instability of the perturbative vacuum in our superweakly coupled standard model with heavy fermions and light Higgs bosons. The two-dimensional example shows that the problem is more general than the unboundedness of the fermion-induced effective potential, for in two dimensions that object is perfectly well behaved. Rather, it is the attempt to make the scalar field, whose VEV was responsible for the fermion mass, much lighter than the fermion itself, which was the cause of the problem. In this way of saying things, it becomes clear that these difficulties are not restricted to the decoupling of fermions. Indeed, the Linde-Weinberg [6] lower bound on the Higgs-boson mass may be viewed as an example of the same phenomenon. Looked at from the point of view of an effective field theory for the conjectural light Higgs boson, the two problems are almost identical. It is only because we have always viewed this problem from the vantage point of the heavy scale (the gauge-boson masses) that it has not caused the same confusion. The statement that the standard-model vacuum is not stable unless the Higgs-boson mass is greater than a certain finite fraction of the gauge-boson mass is equivalent to the statement that one cannot decouple the heavy gauge boson from an effective field theory for the light Higgs boson, despite the fact that the ratio of their masses at tree level appears arbitrary. Again, the problem is caused by the size and nonanalyticity of the effective Higgs potential induced by the heavy particles.

It is also amusing to note that the local terms in the effective Lagrangian which describe the failure of decoupling of chiral fermions in perturbation theory (and in particular, the Peskin-Takeuchi S parameter), are also nonanalytic at vanishing Higgs field. When written in a gauge-invariant manner, they have the typical Higgs dependence[15]

$$\frac{H^{i_1}\cdots H^{i_n}}{|H|^n} . \tag{3.9}$$

This suggests that they may also be viewed as coming from infrared divergences.

Suppose now that we have performed the massive fine-tuning described above and constructed a theory with a stable vacuum and a Higgs boson to heavy-fermion mass ratio which is very small. To all orders in the $1/N$ expansion, the theory will conserve the baryon number of the light fields. To investigate whether this continues to be true nonperturbatively in N we look for solutions of the equations of motion of the large-N effective action which carry nonzero topological charge. There are none. In any configuration of gauge and Higgs fields with nonzero topological charge, the heavy fermions will have normalizable zero modes. The fermion determinant vanishes, and the effective action of instantons is infinite. Note that the polynomial potential $P(\phi)$ cannot change this conclusion. Like the fermion mass, it is finite but large. It cannot cancel an infinity coming from the zero mode.

We would now like to exhibit fermion zero modes in the instanton background in a more explicit manner. To this end, we study a single charged Dirac field in the instanton background, with a Lagrangian

$$\bar{\psi}\gamma^\mu\left[i\partial_\mu - eA_\mu\left[\frac{1+\gamma_3}{2}\right]\right]\psi - g^*\phi^*\bar{\psi}\left[\frac{1+\gamma_3}{2}\right]\psi - g\phi\bar{\psi}\left[\frac{1-\gamma_3}{2}\right]\psi . \tag{3.10}$$

The instanton configuration with winding number n is given by

$$\begin{aligned} eA_\mu &= \epsilon_{\mu\nu}\hat{x}^\nu A(r) , \\ g\phi &= ie^{-in\theta}f(r) . \end{aligned} \tag{3.11}$$

At the core of the instanton, $A(r)\sim 0$ and $f(r)\sim r^{|n|}$, whereas at infinity $A(r)\sim +n/r$ and $f(r)\sim$const. Here we have chosen to work in Landau gauge. The Dirac equation in this background is similar to the one analyzed in [11] for a fermion-vortex system where zero modes of definite chirality were guaranteed by an index theorem [12]. In our problem the Higgs coupling is slightly different, and there are no chiral zero modes. We will, therefore, explicitly solve the equation to find n normalizable zero modes in this sector. Substituting

$$\psi = \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} p\exp\left[+\int A(r)\right] \\ q \end{bmatrix} ,$$

the Dirac equation becomes

$$\exp(i\theta)\left[\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial\theta}\right]q = -f(r)\exp\left[+\int A(r)\right]\exp(+in\theta)p , \tag{3.12}$$

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[14] We might for example use operators of fixed dimension at the Gaussian fixed point, i.e., sines and cosines.

[15] T. B. thanks L. Randall for explaining this point to him.

$$\exp(-i\theta)\left[\frac{\partial}{\partial r}-\frac{i}{r}\frac{\partial}{\partial\theta}\right]p = -f(r)\exp\left[-\int A(r)\right]\exp(-in\theta)q \ .$$

To separate the angular dependence we write $p=e^{-im\theta}p_m(r)$ and $q=e^{i(n-m-1)\theta}q_m(r)$ and obtain the coupled first-order equations

$$\left[\frac{\partial}{\partial r}-\frac{m}{r}\right]p_m=-f(r)\exp\left[\int A(r)\right]q_m \ ,$$
$$\left[\frac{\partial}{\partial r}-\frac{n-m-1}{r}\right]q_m=-f(r)\exp\left[-\int A(r)\right]p_m \ . \quad (3.13)$$

We can turn these into a single second-order differential equation for either p_m or q_m, which will have two linearly independent solutions. For large r, the fermion is massive, so apart from powers of r the two solutions go as $e^{\pm\mu r}$. Only $e^{-\mu r}$ is acceptable as a normalizable solution.

At the origin, from (3.13) and using the asymptotics $f(r)\sim r^{|n|}$ and $\exp[-\int A(r)]\sim$const, we see that the two solutions go as

$$p_m\sim r^m \ , \quad q_m\sim r^{|n|+m+1} \quad (3.14)$$

and

$$p_m\sim r^{|n|+n-m} \ , \quad q_m\sim r^{n-m-1} \ .$$

In general, the solution that is well behaved at infinity will be an arbitrary linear combination of these two solutions, which should also be well behaved at the origin. Thus, from (3.14) we see that, for positive n, there are n normalizable zero-energy solutions for $0\le m\le n-1$. For negative n, the normalizable zero modes come from the Dirac equation for $\bar{\psi}$. (Note that in Euclidean space, ψ and $\bar{\psi}$ are not related by complex conjugation.) As a consequence, the 't Hooft effective action will violate the fermion-number symmetry of these heavy fermions, as required by the anomaly, leading to the physical effects described above.

In particular, as noted above, the large-N effective action will not have finite action instanton solutions with nonzero topological charge. If we consider configurations of zero topological charge, which consist of two widely separated lumps of charge of opposite sign, then, as in four dimensions, the fermion determinant will contribute an effective confining force between instantons and anti-instantons. The confinement of instantons leads to another dramatic effect, which can be studied without the aid of light chiral fermions. The purely bosonic Abelian Higgs model exhibits confinement of external charges which are fractions of the charge on the Higgs field. The mechanism for confinement is a dilute gas of instantons. We now see that the "mere" introduction of very heavy chiral fermions into the theory completely eliminates this nonperturbative and nonlocal effect. The heavy chiral fermions do not decouple as their mass goes to infinity. In this context it is even more apparent that local counterterms cannot mimic these effects. The appearance of a confining force between instantons and the corresponding disappearance of the confining force between fractional charges will not be affected by the inclusion of local gauge-invariant terms in the bosonic action.

IV. APPLICATIONS TO LATTICE GAUGE THEORIES

The results that we have obtained for continuum models suggest analogous problems in any lattice gauge theory which attempts to decouple lattice fermion doubles by using the device of a Wilson-Yukawa coupling to a Higgs field. This includes all of the models studied in [13].

Strictly speaking, our analysis applies only in the spontaneously broken phase of the theory. Lattice analysis had already led to the conclusion that the Wilson-Yukawa method does not work in this phase. Much analysis has been devoted to the symmetric phase of these models. When the Higgs field in the symmetric phase is allowed to have a mass of the order of the cutoff, we can achieve a symmetric phase in which the absolute value of the Higgs field is not small. Symmetry is achieved by making local quantum singlet states by superposing states with the same large magnitude of the Higgs field but different orientations in group space. There can be no analogue of this phase for a continuum Higgs field. The Higgs bilinear which appears in the Wilson-Yukawa coupling is not small in such a phase, and this term in the action can provide a mass to fermion doubles. However, all attempts to utilize this mechanism to construct chiral gauge theories have failed. The fermions always appear in vector representations of the gauge group [14].

With a bit of hindsight and a bit of effective field theory, we can understand why this failure was inevitable. As usual in theories with Wilson terms one must perform fine-tuning in order to make some of the fermions in the theory massless. This means that the erstwhile chiral gauge theory is part of a continuum of theories in which the masses of the massless fermions are nonzero but very small on the scale of the lattice spacing. Now consider an effective field theory for these light, but not exactly massless, fermions. It must be a gauge theory with no spontaneous breakdown, for we are in the symmetric phase. But it must also contain mass terms for the light fermions. This means that the light fermions can have gauge-invariant masses, and are thus in vector representations of the gauge group.

The only lattice gauge theories which can avoid the problems we have described are those which do not use a Higgs field to decouple the fermion doubles. These fall into two categories. The approach of Rossi and coworkers [15] puts a gauge-fixed theory on the lattice. Non-gauge-invariant Wilson terms are added to decouple the doubles, as well as a host of non-gauge-invariant counterterms whose coefficients are supposed to be fine-tuned to achieve Becchi-Rouet-Stora-Tyutin (BRST) invariance in the continuum. As a consequence of the explicit choice of gauge, the theory is not equivalent to a gauge-invariant lattice theory with a Higgs field.[16] In particular, Dugan and Randall [16] have shown that the fermion doublers do not lead to a contribution to the Peskin-Takeuchi S parameter in this model. When applied to the lattice standard model, this approach appears to contain an unwanted baryon-number symmetry that the continuum model does not have. Maiani has argued that the current of this symmetry may not be BRST invariant in the continuum limit. It may indeed be correct that this is the meaning of the Dugan-Manohar calculation in the context of the model of Rossi and co-workers (we have argued that it has quite a different meaning in the Swift-Smit model). Nonetheless, we remain disturbed by the fact that within this model we cannot write down a lattice Green's function which violates baryon number. In order for Maiani's argument to completely resolve the baryon-number paradox in this model, we must demonstrate that the bothersome U(1) symmetry is spontaneously broken on the lattice. Maiani's argument could then be used to show that the corresponding Goldstone boson was an unphysical gauge excitation. It seems that a lot of work must be done to prove that the approach of Rossi and co-workers can really reproduce the continuum standard model. In applications to strongly coupled chiral gauge theories such as the SU(5) model, there does not seem to be a similar problem with the approach of Rossi and co-workers.

We note also that serious questions about the treatment of Gribov ambiguities have been raised in connection with this approach. In addition Parisi [17] has made the very interesting suggestion that conventional renormalization-group arguments about the relevance of operators which break a gauge symmetry may fail in the presence of gauge-field configurations belonging to nontrivial fiber bundles. The corresponding vector potentials are singular somewhere in spacetime (perhaps at infinity), and naive power counting arguments may not be applicable. We do not know whether either of these two potential problems with the approach of Rossi and co-workers is real.

The only model of which we are aware that escapes completely from the problems that we have described is the staggered fermion model [18]. This model has no extra fermion degrees of freedom on the lattice; the doubled modes are identified with known continuum fermions. The only consistent way to do this is to break color SU(3) symmetry on the lattice, or equivalently to introduce colored Higgs fields. This we consider a point in the model's favor, for it destroys the baryon-number symmetry which was the source of all of our worries. It remains to be seen whether enough tuning of parameters can be done in this model to truly reproduce the standard model, but we see no obvious reason for it to fail.

Finally, we should mention the model of Eichten and Preskill [19]. Recently Golterman and Petcher [20] have suggested that this suffers from the same problems as the Smit-Swift models, despite its careful attempt to break all unwanted symmetries by adding multifermion terms to the lattice Lagrangian. We do not understand the physics of either the original model or the recent criticism of it very well. If the criticism is incorrect, the Eichten-Preskill model may also provide a convenient method for simulating the standard model.

V. CONCLUSIONS

We have demonstrated fairly conclusively that the superweakly coupled mirror standard model introduced in Sec. I has a low-energy sector whose nonperturbative physics is not described correctly by a local Lagrangian for the fields of the light particles of the tree-level analysis. This result is confirmed in the two-dimensional model with soft Higgs fields. There we were able to make the mass ratio between the heavy fermions and gauge bosons arbitrarily large by letting the number of fermion multiplets tend to infinity. The zero modes of massive fermions in instanton fields showed up directly as a contribution to the large-N effective action. We also studied the limit of rigid Higgs fields in the two-dimensional model, and showed that although the heavy-fermion fields in this model truly decoupled, the low-energy theory had no baryon-number violation.

It seems to us that phenomena analogous to those we have described would afflict any lattice version of the standard model with an exactly conserved baryon-number current, *if one succeeded in eliminating all unconventional particles from the continuum spectrum.* By analogy with the model studied here, one would suspect that no good continuum limit with such a spectrum could exist, and if a limit were to exist it would certainly not be the standard model. Explicit baryon-number violation must be incorporated into lattice versions of the standard model if they are to converge to the right answer. We caution that it is by no means certain that this necessary condition is a sufficient one. If the conjectures that have been made about Lee-Wick, SLAC, or D'Hoker-Farhi solitons are correct, then one might expect light states with fermion quantum numbers to exist in almost any theory in which fermion masses come solely from the Higgs mechanism. In the limit of large Yukawa coupling, the masses of these states are determined primarily by the dynamics of the Higgs field. Only by sending the renormalized Higgs mass to infinity can we expect to decouple these soliton states. In the two-dimensional model we found that baryon-number violation also vanishes in this limit, in accord with the general argument that in this limit the anomaly is the divergence of a gauge-invariant massive operator.

In four dimensions, it seems unlikely that there will be a sensible continuum limit for any spontaneously broken non-Abelian gauge theory with an infinite Higgs mass. Thus, lattice models with Wilson-Yukawa terms cannot

[16]A formal argument seems to show that the equivalence is reinstated in the continuum limit, but this argument neglects wildly fluctuating lattice Higgs modes. This subtle point was explained in great detail by Smit, Golterman, Petcher, Neuberger, Maiani, and Testa in informal discussions at the Rome conference on chiral lattice gauge theories.

reproduce the spectrum of the standard model. In the previous section we described the class of extant lattice models which may evade this conclusion. Our results also suggest the extra fermions which transform chirally under the standard-model gauge group will be found at scales not too far removed from the weak scale, if they exist at all. The question of the chiral nature of the weak interactions should be settled once and for all by the next generation of accelerators.

ACKNOWLEDGMENTS

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