Chiral perturbation theory for SU(3) breaking in heavy-meson systems

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The SU(3)-breaking effects due to light-quark masses on heavy-meson masses, decay constants (F_D, F_{D_s}), and the form factor for semileptonic $\overline{B} \to D^{(*)} l \bar{\nu}_l$ transitions are formulated in chiral perturbation theory, using a heavy-meson effective Lagrangian and expanding in inverse powers of the heavy-meson mass. To leading order in this expansion, the leading chiral logarithms and the required counterterms are determined. At this level, a nonanalytic correction to the mass splittings of $O(p^3)$ appears, similar to the one found in light baryons. The correction to F_{D_s}/F_D is roughly estimated to be of the order of 10% and, therefore, experimentally accessible, while the correction to the form factor is likely to be substantially smaller. We explicitly check that the heavy-quark symmetry is preserved by the chiral loops.

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I. INTRODUCTION

In addition to their intrinsic significance for the study of electroweak interactions (quark mixing, rare decays, CP violation), heavy hadrons containing a single heavy quark c or b might also prove to be a useful tool for unveiling new aspects of the strong interactions. The reason for this is the large approximate symmetry available in these systems, which constrains the QCD dynamics and, therefore, substantially simplifies their study. As the mass m_Q of the heavy quark becomes much larger than the characteristic QCD scale (say, m_ρ), in all strong-interaction processes where the relevant scale of momenta is much smaller than the heavy-quark mass, the heavy quark approximately behaves as a static color source in the rest frame of the hadron, with its spin dynamically decoupled. In this limit, the velocity of the heavy quark and its spin become conserved observables. This results in a superselection rule for the velocity [1, 2], and a spin-flavor symmetry [the Isgur-Wise (IW) symmetry] [3, 4] which enjoys all bonafide properties of an internal symmetry. In addition, the light-quark degrees of freedom in the heavy hadron carry information about the chiral $SU_L(3)\times SU_R(3)$ symmetry of QCD. In particular, chiral symmetry dictates the form of the couplings between the heavy mesons and the Goldstone bosons (π, K, η) resulting from the Nambu-Goldstone nature of its realization (see, for instance, [5]).

The corrections to the symmetry limit are naturally obtained by expanding in powers of $1/m_Q$ (more precisely in the present context, in powers of $1/M_H$, where M_H is the heavy hadron mass) and by treating the light-quark masses, which explicitly break chiral invariance, as a perturbation. The expansion in powers of $1/M_H$ has the virtue of enabling a systematic chiral expansion at each order in $1/M_H$, where the chiral power counting is in correspondence with an expansion in loops [6], similarly to chiral perturbation theory in light mesons [7]. This only applies to processes involving only one heavy hadron, otherwise, infrared divergences modify the naive chiral counting [8].

SU(3)-breaking effects induced by the light-quark masses are inherently of low-energy character, and therefore, suited to a systematic study within the chiral expansion. Recently, various groups [9–11] have initiated this field, in which interesting theoretical results are expected to emerge. At present, SU(3) breaking is only observed through the mass splittings in D, D^*, and B mesons and in charmed baryons. In the future, one also expects observation through other quantities (e.g., decay constants). This requires, however, substantial improvement in strange D meson measurements. As for the decay constants, at present only an upper bound exists for nonstrange D mesons: $F_D \leq 200$ MeV ($F_\pi = 93$ MeV). For B mesons, observation of SU(3) breaking in form factors (e.g., in $B_{(s)} \to \overline{D} l \nu$ decays) could only be achieved in a B-meson factory. The corrections to B^0-$\overline{B}^0$ and B_s-$\overline{B}_s$ mixing are also of great interest, and have recently been analyzed [10]. Applications in connection with lattice QCD simulations of heavy-light systems can also be envisaged. For instance, finite-volume effects in the continuum limit, of relevance in this context, can be unambiguously determined using the chiral expansion [5].

The predictive power of the chiral expansion is limited by the counterterms which must be added at each stage. The counterterms are ordered according to chiral power counting and are required as subtractions to UV-divergent chiral loops. To overcome this drawback, a sufficient number of measured observables is needed as input. While this is possible in light mesons, it is not yet clear that it can be achieved in heavy mesons.

In this paper, we study the SU(3)-breaking corrections, in the limit of infinite heavy-quark mass, to the ratio of decay constants $F_{H_s}/F_{H_{u,d}}$ (H denotes a heavy meson), mass splittings, and the Isgur-Wise form factor associated with the charged current in the transitions $B^0 \to D^-$ and $B_s \to \overline{D}_s$. The chiral logarithms and their associated counterterms are determined to one-chiral-loop order. The presence of nonanalytic contributions in the light-quark masses ($\propto m_q^{3/2}$) to the mass splittings is noticed.

II. EFFECTIVE THEORY

In this section, we discuss in detail a formulation of an effective theory for heavy (D, B) mesons coupled to the Goldstone bosons of spontaneously broken chiral symmetry. In this formulation, SU(3)-breaking effects can be consistently studied in a chiral-loop expansion. This is made possible by simultaneously performing an expansion in powers of the inverse of the heavy-meson mass.

The form of the interactions between Goldstone bosons and heavy mesons containing a heavy antiquark and a light quark are determined by the transformation properties of the heavy-meson wave functions under chiral $SU_L(3)\times SU_R(3)$. The transformation law is easily found by the method of Coleman, Wess, and Zumino [12]. In our case, pseudoscalar and vector heavy mesons appear in triplets under flavor SU(3), and this fixes the transformation law under an arbitrary chiral transformation $g = L \otimes R$ to have the following form:

$$g : H = hH, \tag{1}$$

where H denotes the heavy-meson wave function and h is a 3×3 SU(3) matrix which depends on the octet of Goldstone excitations. Although the explicit form of h will not be needed, it can be determined in the following manner: one defines a 3×3 SU(3) matrix $U(x)$ parametrized by the classical Goldstone fields, and whose transformation law is given by $g : U(x) = L\, U(x)\, R^\dagger$, where on the right-hand side ordinary matrix multiplication is meant. By means of $u(x) \equiv \sqrt{U(x)}$, one can determine h in such a way that (1) is a realization of the chiral group. The dependence of h on the Goldstone excitations results from solving the following system of equations:

$$L\, u(x) = u'(x)\, h, \quad R\, u^\dagger(x) = u'^\dagger(x)\, h. \tag{2}$$

In what follows we will use the exponential parametrization for $u(x)$:

$$u(x) = \exp\left(-i\frac{\pi_a(x)\lambda^a}{2F_0}\right), \tag{3}$$

where the Goldstone fields $\pi_a(x)$ are real and identified with the light pseudoscalar octet (π, K, η), $F_0 \sim 93$ MeV is the pion decay constant in the chiral limit, and the Gell-Mann Hermitian matrices are normalized by $\mathrm{Tr}(\lambda^a\lambda^b) = 2\delta^{ab}$.

In order to build the effective Lagrangian invariant under chiral rotations, one needs to introduce a covariant derivative under the transformation law (1). Using the transformation properties of $u(x)$ implied by (2), the explicit form of the covariant derivative is given by

$$\nabla_\mu H = (\partial_\mu + i\Gamma_\mu)H,$$
$$\Gamma_\mu = \Gamma_\mu^\dagger = \frac{i}{2}\,(u\partial_\mu u^\dagger + u^\dagger\partial_\mu u). \tag{4}$$

Besides the covariant derivative, the following Hermitian pseudovector must also be considered:

$$\omega_\mu = \frac{i}{2}\,(u\partial_\mu u^\dagger - u^\dagger\partial_\mu u) \tag{5}$$

whose transformation law under a chiral rotation is given by $g : \omega_\mu = h\omega_\mu h^\dagger$.

Having established the chiral transformation properties for the heavy mesons, we now turn to the important aspects connected with the large mass of the heavy quark. As the mass of the heavy quark tends to infinity, the QCD Lagrangian becomes invariant under a new global symmetry (IW symmetry) [3], which, in the case of a single heavy quark, corresponds to the operation of independently rotating the spins of the heavy quark and antiquark. This becomes an internal symmetry of QCD, which, for N_h heavy quarks is $SU(2N_h)\times SU(2N_h)$ with one factor referring to quarks and the other to antiquarks, as these two sectors become independent in the infinite-mass limit. The lowest-lying pseudoscalar and vector heavy mesons, relevant to this work, belong to a multiplet under the IW symmetry, and must therefore be treated together. Their masses are equal, up to symmetry-breaking corrections of hyperfine origin equal to Λ^2/m_Q (Λ is a typical QCD scale, in this specific case $\Lambda \sim m_\rho/\sqrt{2}$), and their transition amplitudes become related. For all amplitudes where the relevant scale is Λ, the heavy-quark velocity is conserved. These conservation laws obviously extend to the heavy-meson Goldstone-boson interactions relevant in the present context. For this reason, it is convenient to consider as the starting point the effective QCD theory for the heavy-quark sector, defined in terms of heavy quarks and antiquarks separately (i.e., in the effective theory no virtual loops of heavy quarks are required) for each four-velocity v_μ [2]. The corresponding effective theory for the heavy (anti)mesons is obtained in a similar way, by defining nonrelativistic fields for mesons $(\overline{Q}q)$ and antimesons $(\bar{q}Q)$ for a given four-velocity v_μ as follows:

$$H_v^{(+)}(x) = \sqrt{M_H}\, e^{iM_H v\cdot x}\, \Psi_+(x),$$
$$H_v^{(-)}(x) = \sqrt{M_H}\, e^{iM_H v\cdot x}\, \Psi_-^\dagger(x), \tag{6}$$

where $\Psi_{+,-}(x)$ are the positive- and negative-frequency components of the relativistic meson field $\Psi(x)$ and M_H is the heavy-meson mass. Since the meson and antimeson sectors become independent, we will only work with mesons, and the field $H_v^{(+)}(x)$, which annihilates heavy mesons with velocity v_μ, will be simply denoted by $H(x)$.

For heavy mesons, the IW symmetry is elegantly implemented by merging the pseudoscalar and vector mesons into a multiplet using Dirac matrices as follows [13, 14]:

$$\mathcal{H}(x) = \frac{1+\not{v}}{2}\,\left[-\gamma_5 H(x) + \gamma_\mu H^\mu(x)\right], \tag{7}$$

where the vector field H^μ satisfies the constraint $v_\mu H^\mu = 0$. The field conjugated to $\mathcal{H}$ which will create heavy mesons is defined by $\overline{\mathcal{H}} = \gamma_0\mathcal{H}^\dagger\gamma_0$. Under chiral rotations $\mathcal{H}$ transforms as indicated in (1), and under the heavy-quark symmetry rotations its transformation law is the following:

$$\mathcal{H} \to e^{i\theta_j S_j}\, \mathcal{H}, \quad S_j = i\, \epsilon_{jkl}\, [\ \not{e}_k\ ,\ \not{e}_l\ ], \tag{8}$$

where e_i^μ, $i = 1,2,3$ are normalized spacelike vectors orthogonal to v_μ.

Since the definition (6) corresponds in the rest frame of the meson to the subtraction of the rest mass energy, the operator $-i\partial_\mu$ acting on $\mathcal{H}$ gives the residual momentum. In particular, for the purposes of the chiral expansion, this residual momentum will count as a quantity of $O(p)$. Similarly, $\partial_\mu u$ is of $O(p)$ and, therefore, the covariant derivative only contains terms of $O(p)$ as is also the case for ω_μ.

It is now straightforward to write down the lowest-order effective Lagrangian, in both chiral and $1/m_Q$ expansions, which is invariant under chiral and IW transformations as well as under parity and charge conjugation. This Lagrangian is of $O(p)$ and reads as follows:

$$\mathcal{L}^{(1)} = -\frac{1}{2}\, v_\mu\, \mathrm{Tr}_D\, \{\, \overline{\mathcal{H}} \nabla^\mu \mathcal{H}\, \} + \frac{1}{2}\, g\, \mathrm{Tr}_D\, \{\, \overline{\mathcal{H}}\, \omega^\mu\, \mathcal{H}\, \gamma_\mu \gamma_5\, \}, \tag{9}$$

where Tr_D denotes the trace over Dirac indices. The first term contains the kinetic energy and interactions of the heavy mesons with an even number of Goldstone bosons and no change in the spin of the heavy meson. This term is universal and automatically satisfies the IW symmetry. The second term gives rise to virtual transitions $H^* + m\pi \leftrightarrow H + n\pi$ and $H^* + m\pi \leftrightarrow H^* + n\pi$ with $(n+m)$ odd. The IW symmetry imposes that the strength of both types of transitions must be equal. The corresponding dimensionless coupling constant g could be determined from the decay $D^{*+} \to D + \pi$; unfortunately, at present, only an upper bound on the D^{*+} width is available, resulting in $g^2 \leq 4.8$. One may take as a rough estimate for g^2 the corresponding coupling constant in the K-meson system: $g^2_{K^*K\pi} = 0.46$. The quark model result is $g^2 \sim 0.3$ [15], while the QCD sum rules give the substantially smaller value $g^2 \sim 0.08$ [16].

Notice that $\mathcal{L}^{(1)}$ does not contain the heavy-meson masses. This is the key point in the implementation of the power counting of the low-energy expansion as a loop expansion [6]. The Feynman rules are straightforward to derive and the propagators for the heavy mesons are given by

$$\Delta_H(p) = \frac{i}{2\, p \cdot v + i\epsilon}, \qquad \Delta^{\mu\nu}_{H^*}(p) = -i\, \frac{(g^{\mu\nu} - v^\mu v^\nu)}{2\, p \cdot v + i\epsilon}, \tag{10}$$

where p_μ is the residual momentum. The use of these propagators in chiral loop integrals is justified, since the physical cutoff for such integrals is $\Lambda \sim 1$ GeV (actually, since in the effective theory we integrate out resonances, e.g. D_1, the cutoff should be smaller than the mass difference between the resonances and the stable states). The implied change in the UV degree of divergence of the integrals, which occurs at a scale of momenta of the order of the heavy-meson mass, is therefore irrelevant. In calculating the chiral loops it is convenient to use dimensional regularization, as it preserves chiral invariance. Having eliminated the heavy-meson mass, only low-energy scales appear in the loops, thus furnishing the chiral power counting as in the case of light mesons [7].

The most noticeable effect of SU(3) breaking by the quark masses is in the masses of the heavy mesons. The leading contribution to the intramultiplet mass splittings is linear in the light-quark masses [which in chiral power counting are of $O(p^2)$] and can be described by adding the following $O(p^2)$ term to the effective Lagrangian:

$$\mathcal{L}^{(2)}_{\Delta M} = -\frac{C}{4}\, \mathrm{Tr}_D\, \{\, \overline{\mathcal{H}}\, (u^\dagger \mathcal{M} u^\dagger + u \mathcal{M} u)\, \mathcal{H}\, \}, \tag{11}$$

where $\mathcal{M} = \mathrm{diag}(\ m_u, m_d, m_s\ )$ is the light-quark mass matrix [17]. The SU(3) singlet term has been omitted as it is of no interest for our purposes. Under the assumption that higher-order chiral corrections are small (more on this in the next section) and neglecting them, C is estimated from $M_{D_s} - M_{D^+}$ and $M_{B_s} - M_{B^0}$ and the results are $C_D = 99.5 \pm 0.6$ MeV$/(m_s - m_d)$, $C_B = (82 \pm 2.5; 121 \pm 10)$ MeV$/(m_s - m_d)$. For $M_{B_s} - M_{B^0}$ we use the two values quoted as consistent with present data [18]. Establishing this measurement would provide an estimate of the $1/m_Q$ corrections by comparison of C_D with C_B. Notice that $C_K \sim 225$ MeV$/(m_s - m_d)$, as obtained from isospin breaking in the kaon masses and using the ratio $R = (m_s - \hat{m})/(m_d - m_u)$ [$\hat{m} = (m_u + m_d)/2$], which is substantially larger than in heavy mesons. An analysis of isospin breaking effects within the linear approximation has been recently done [19]. The leading corrections to the linear approximation are nonanalytic in the light-quark masses $\sim m^{3/2}$, and they turn out to be proportional to g^2, as we will show in next section. The possibility that these corrections turn out to be important is not yet excluded.

III. DECAY CONSTANTS AND MASSES

In heavy mesons, as in light mesons, the leading SU(3)-breaking correction to decay constants is proportional to the light-quark masses multiplied by a nonanalytic factor, the chiral logarithm, which emerges due to the IR behavior of the one-chiral-loop integrals.

The decay constants are defined in terms of matrix elements of the vector and axial-vector currents $V^i_\mu = \overline{Q}\gamma_\mu q^i$ and $A^i_\mu = \overline{Q}\gamma_\mu\gamma_5 q^i$ between one meson state and the vacuum. In the effective theory, these currents are defined by introducing vector (v_μ) and pseudovector (a_μ) external sources which are triplets under SU(3). These sources couple to the mesons at lowest chiral order [$O(p)$] according to the following effective Lagrangian:

$$\mathcal{L}^{(1)}_{\mathrm{source}} = \frac{1}{2}\, f\, \mathrm{Tr}_D\, \{(\mathrm{v}^\dagger_\mu + \mathrm{a}^\dagger_\mu \gamma_5) \times \gamma_\mu\, \left[(u + u^\dagger) + \gamma_5 (u - u^\dagger)\right] \mathcal{H}\} + \mathrm{H.c.} \tag{12}$$

Clearly the sources defined here in momentum representation will only carry residual momentum and, therefore, they count as quantities of $O(p)$. f is defined in the $m_Q \to \infty$ and chiral limits and related to the usual expressions for the decay constants of the pseudoscalar and heavy mesons in that limit by

$$F_H = f/\sqrt{M_H}, \quad F_{H^*} = f\sqrt{M_H} = F_H M_H. \tag{13}$$

The leading SU(3)-breaking corrections to the ratios of decay constants and the leading nonanalytic (in the light-quark masses) corrections to the mass differences are determined by calculating the two polarizations $\Pi^{A\,ij}_{\mu\nu}(x) = \langle 0 \mid T A^i_\mu(x) A^j_\nu(0) \mid 0\rangle$ and $\Pi^{V\,ij}_{\mu\nu}(x) = \langle 0 \mid T V^i_\mu(x) V^j_\nu(0) \mid 0\rangle$ to one-loop order. At long Euclidean distances, the lowest-lying pseudoscalar and vector heavy-meson poles, respectively, saturate the two-point functions. These pole contributions are given in the effective theory by replacing the currents by the effective currents derived from the source Lagrangian (12). The corresponding Fourier transforms, which are functions of the residual momentum p_μ, are given in the limit $p\cdot v \to 0$ by the following general expressions:

$$\tilde{\Pi}^{A\,ij}_{\mu\nu}(p) = \frac{4}{M_H} F^2_{H_i}\, v_\mu v_\nu\, \frac{i}{2\,(p\cdot v - \delta M_i) + i\epsilon}\, \delta_{ij},$$
$$\tilde{\Pi}^{V\,ij}_{\mu\nu}(p) = 4 M_H F^2_{H^*_i}\, \frac{-i\,(g_{\mu\nu} - v_\mu v_\nu)}{2\,(p\cdot v - \delta M_i) + i\epsilon}\, \delta_{ij}. \tag{14}$$

The diagrams contributing to one-chiral-loop order are shown in Fig. 1. Since these contributions are UV divergent, counterterms are required. It turns out that one only needs to add counterterms which correct the effective currents and which are of $O(p^3)$. Since, as expected, the chiral loops turn out to preserve the heavy-quark symmetry, the counterterms must be invariant under this symmetry as well. The counterterm effective Lagrangian contains three low-energy constants, and is given by

$$\begin{aligned}\mathcal{L}^{(3)}_{\rm source} = 2B_0\, \frac{f}{F_0^2}(\Gamma_1\ {\rm Tr}_D\ &\{(\mathrm{v}^\dagger_\mu + \mathrm{a}^\dagger_\mu\gamma_5)\,\gamma^\mu\,[(U\mathcal{M}u + U^\dagger\mathcal{M}u^\dagger) + \gamma_5(U\mathcal{M}u - U^\dagger\mathcal{M}u^\dagger)]\,\mathcal{H}\}\\ &+ \Gamma_2\ {\rm Tr}_D\ \{(\mathrm{v}^\dagger_\mu + \mathrm{a}^\dagger_\mu\gamma_5)\,\gamma^\mu\,[(\mathcal{M}u + \mathcal{M}u^\dagger) + \gamma_5(\mathcal{M}u^\dagger - \mathcal{M}u)]\,\mathcal{H}\}\\ &+ \Gamma_3\ {\rm Tr}\{\mathcal{M}U^\dagger + \mathcal{M}^\dagger U\}{\rm Tr}_D\ \{(\mathrm{v}^\dagger_\mu + \mathrm{a}^\dagger_\mu\gamma_5)\,\gamma^\mu\,[(u+u^\dagger) + \gamma_5(u - u^\dagger)]\,\mathcal{H}\}) + \text{H.c.},\end{aligned} \tag{15}$$

where $B_0 = \langle\bar{q}q\rangle_0/F_0^2$ is defined in the chiral limit [$M_\pi^2 = B_0(m_u + m_d)$, etc.].

We calculate the loops using dimensional regularization, in which it is convenient to write: $\Gamma_j = \Gamma^r_j(\mu) + \overline{\Gamma}_j\lambda(\mu)$ ($j = 1,2,3$), where μ is the chiral renormalization scale, $\Gamma^r_j(\mu)$ the renormalized effective coupling, and $\lambda(\mu)$ contains the singularity at $d = 4$ and is given by

$$\lambda(\mu) = \frac{1}{16\pi^2}\mu^{4-d}\left(\frac{1}{(d-4)} + \frac{1}{2}[\ln 4\pi + \Gamma'(1) + 1]\right). \tag{16}$$

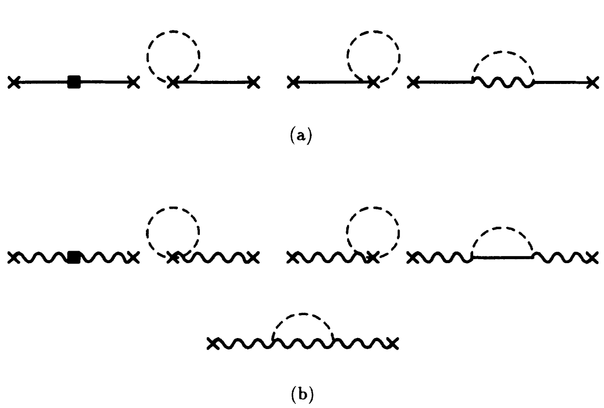

FIG. 1. One-loop contributions to the polarizations (a) $\tilde{\Pi}^{A\,ij}_{\mu\nu}$ and (b) $\tilde{\Pi}^{V\,ij}_{\mu\nu}$. The solid lines correspond to the heavy pseudoscalar, the wavy line to the heavy vector, and the dashed line to the Goldstone bosons. The square dot represents the insertion of $\mathcal{L}^{(2)}_{\Delta M}$. Diagrams not explicitly shown vanish identically.

The following choice leads to an UV finite result for the polarizations:

$$\overline{\Gamma}_1 + \overline{\Gamma}_2 = \frac{5}{24}\ (1 + 3g^2)\,, \quad \overline{\Gamma}_3 = \frac{11}{144}\ (1 + 3g^2)\,. \tag{17}$$

For the sake of convenience we define: $\Gamma^r_{12}(\mu) \equiv \Gamma^r_1(\mu) + \Gamma^r_2(\mu)$.

We first consider $\tilde{\Pi}^{A\,ij}_{\mu\nu}$. One-loop contributions from $\mathcal{L}^{(1)}$ and $\mathcal{L}^{(1)}_{\rm source}$ and a tree-level insertion of $\mathcal{L}^{(2)}_{\Delta M}$ must be included, as shown in Fig. 1(a). The calculation is straightforward, and leads to the following results for the pseudoscalar decay constants in the SU(2) limit:

$$\begin{aligned} F_{H_{u,d}} &= F_H \left\{ 1 - \left(\frac{1+3g^2}{8F_0^2}\right)[3\mu_\pi + 2\mu_K + \tfrac{1}{3}\mu_\eta] + \frac{1}{F_0^2}\,\Gamma^r_{12}(\mu)\,2M_\pi^2 + \frac{4}{F_0^2}\,\Gamma^r_3(\mu)\ (2M_K^2 + M_\pi^2)\right\},\\ F_{H_s} &= F_H \left\{ 1 - \left(\frac{1+3g^2}{8F_0^2}\right)[4\mu_K + \tfrac{4}{3}\mu_\eta] + \frac{2}{F_0^2}\,\Gamma^r_{12}(\mu)\,(2M_K^2 - M_\pi^2) + \frac{4}{F_0^2}\,\Gamma^r_3(\mu)\,(2M_K^2 + M_\pi^2)\right\}, \end{aligned} \tag{18}$$

where the chiral logarithms are given by

$$\mu_P = M_P^2 \frac{1}{16\pi^2} \ln \frac{M_P^2}{\mu^2}. \tag{19}$$

From these expressions one easily finds the SU(3)-breaking corrections to the ratio $F_{H_s}/F_{H_{u,d}}$:

$$\frac{F_{H_s}}{F_{H_{u,d}}} = 1 - \left(\frac{1+3g^2}{8F_0^2}\right)[-3\mu_\pi + 2\mu_K + \mu_\eta] + \frac{4}{F_0^2}\,\Gamma_{12}^r(\mu)\,(M_K^2 - M_\pi^2) \tag{20}$$

This result, which coincides with that of Ref. [10], implies that the decay constant grows with the mass of the light quark in the chiral limit, as also occurs in light mesons. In the case of light mesons, the correction to F_K/F_π is entirely contained in the chiral logarithm term if one takes $\mu \sim 1.5$ GeV [7]. If a similar situation is assumed to hold in the case of the heavy mesons, one obtains $(F_{H_s}/F_{H_{u,d}}) - 1 \simeq 0.13\,(1+3g^2)$ (clearly, this should only be taken as a rough estimate of the size of the effect). A correction of this size should be experimentally accessible in the future in leptonic D-meson decays. A similar direct test is not available for B mesons because B_s is neutral.

The mass shifts δM_i receive an $O(p^2)$ contribution from the insertion of $\mathcal{L}_{\Delta M}^{(2)}$, and a nonanalytic contribution in the quark masses of $O(p^3)$ from the loop diagram proportional to g^2. The latter is similar to the nonanalytic contribution to the mass splittings in the baryon octet identified long ago [20]. The nonanalyticity here, as in the case of the chiral logarithms, is a long distance effect due to the IR behavior of the chiral-loop integral. Since local counterterms must be analytic in the quark masses, these nonanalytic corrections are unambiguous, as one would expect from their long distance nature (for details, see [21]). It is interesting to note that they do not depend on the fact that we are using the leading term in the $1/m_Q$ expansion within the loop integral; exactly the same result is obtained by doing the loop integral in the relativistic theory, and expanding the result. In the SU(2) limit, we obtain the following expression for the mass splitting:

$$\delta M_{H_s} - \delta M_{H_{u,d}}|_{\rm QCD} = C(m_s - \hat{m}) - \frac{3g^2}{128\pi F_0^2}[-3M_\pi^3 + 2M_K^3 + M_\eta^3], \quad \hat{m} = m_u = m_d. \tag{21}$$

Notice that the nonanalytic term has the opposite sign from the leading term and gives a large contribution, unless the effective coupling g^2 is very small. For instance, for its contribution to be less than 20%, $g^2 < 0.06$ is required. For this reason, it is important at some point to determine this coupling constant with a good degree of confidence, since, among other effects, it could lead to sizeable corrections to the linear approximation normally used in the analysis of isospin breaking in heavy mesons. In particular, $C_{D,B}$ will become closer to C_K. A few remarks are in order here. (a) Due to SU(3) breaking the propagators cannot be brought back to the form (10) because chiral invariance demands that only SU(3) singlet redefinitions of the form (6) are admissible. Thus, SU(3) breaking implies that propagators will in general contain an $O(p^2)$ residual mass. (b) The propagator of the heavy meson in the chiral loop could have been taken with the $O(p^2)$ corrections given by the insertion of $\mathcal{L}_{\Delta M}^{(2)}$. This however only produces a correction of $O(p^4)$ while it does not affect the leading nonanalytic term. (c) Clearly, some corrections step by one unit in the chiral power counting, making predictions more difficult as they are only suppressed by a factor $\sim M_K/4\pi F_0$ which is not much smaller than one. For instance, it could well occur that a large leading nonanalytic term such as that in (21) becomes partly compensated by a term of $O(p^4)$. In this particular case, an estimate of the $O(p^4)$ chiral logarithm shows that its contribution will be small if one chooses $\mu = O(1\ {\rm GeV})$, however, the $O(p^4)$ counterterm could eventually lead to the mentioned compensation. If g^2 turns out to be substantially larger than 0.06, one would then have an indication for such a compensation.

The relevant one-loop contributions to $\tilde{\Pi}^{V\,ij}_{\mu\nu}(p)$ are shown in Fig. 1(b), and determine the corrections to the masses and decay constants of the vector mesons. Explicit calculation shows that the heavy-quark symmetry is preserved, as seen in particular by the relation $F_{H^*} = F_H M_H$, which still holds after the chiral-loop corrections are included.

One might wonder about the precision of the chiral-loop corrections in the heavy-quark limit when applied to D and B mesons. Calculations of F_D and F_B in lattice QCD [22] and QCD sum rules [23] have shown that the $1/\sqrt{m_Q}$ scaling characteristic of the heavy-quark limit is not present. On the other hand, there is clear evidence that the scaling violation mainly stems from spin independent effects, and therefore the heavy-quark-symmetry-breaking effects on the ratio $(F_{H^*}/F_H)M_H$ are small: ($\sim$ 10%) for the B mesons [24] and somewhat larger for D mesons ($\sim$ 20%). Analogously, the deviation from unity of the ratio of effective couplings $g_{HH^*\pi}/g_{H^*H^*\pi}$ is expected to be small, since it is also of hyperfine origin. This deviation and the vector-pseudoscalar mass splitting are the main source of departure of the chiral corrections from the $m_Q \to \infty$ limit. We then expect that for D and B mesons their departure from this limit will be small (this involves the reasonable assumption that also the $1/m_Q$ corrections to the counterterms will be small).

IV. $B \to \overline{D}^{(*)}$ FORM FACTOR

In the infinite mass limit, the heavy-quark symmetry permits one to determine the amplitudes for $B \to \overline{D}^{(*)}$ transitions mediated by the charged currents $\bar{c}\gamma_\mu(\gamma_5)b$ in terms of a single real form factor $\xi(v\cdot v')$ [3], where v (v') is the four-velocity of the b (c) quark. At the vanishing recoil point, ξ satisfies the normalization condition $\xi(v\cdot v' = 1) = 1$.

In this section, we will determine the form of the leading SU(3)-breaking corrections to this form factor. To leading order in the chiral and $1/m_Q$ expansions the effective charged currents are obtained from the following source Lagrangian:

$$\mathcal{L}^{(1)}_{b\to c} = \xi(\nu)\ \mathrm{Tr}_D\ \{\overline{\mathcal{D}}\,\mathcal{B}\ (V^\mu + A^\mu\gamma_5)\,\gamma_\mu\} + \mathrm{H.c.}, \tag{22}$$

where $\mathcal{D}$ and $\mathcal{B}$ are the expressions analogous to (7) for D and B mesons, respectively, $\nu = v\cdot v'$, and V^μ and A^μ are SU(3) singlet sources. In particular, (22) shows that the effective currents do not couple to the Goldstone modes at lowest chiral order.

The chiral corrections to the matrix element $\langle \overline{D}_j \mid \bar{c}\gamma_\mu b \mid B_i\rangle$ are calculated by considering the three-point function

$$\Pi^{V\,i,j}_{\rho\mu\nu}(x,y) = \langle 0 \mid T\ \bar{c}\gamma_\rho b(0)\ \bar{q}_j\gamma_\nu\gamma_5 c(y)\ \bar{b}\gamma_\mu\gamma_5 q_i(x) \mid 0\rangle.$$

In the effective meson theory, the corresponding three-point function in the residual momentum representation and in the limit $p\cdot v \sim p'\cdot v' \sim 0$, where p_μ (p'_μ) is the residual momentum associated with the B (D) meson propagator, has the following form:

$$\begin{aligned}\tilde{\Pi}^{V\,i,j}_{\rho\mu\nu}&(p,v,p',v')\\ &= -\frac{4}{\sqrt{M_B M_D}}\ F_{B_i}F_D v_\mu v'_\nu(v+v')_\rho\ \xi_{ij}(\nu)\\ &\quad\times\frac{i}{2\,(p\cdot v - \delta M_i)}\ \frac{i}{2\,(p'\cdot v' - \delta M_j)}.\end{aligned} \tag{23}$$

To one-chiral-loop order, the diagram in Fig. 2(a) gives the correction to the IW form factor, after properly taking into account wave-function renormalization (the latter naturally emerges when explicitly calculating all one-loop diagrams for the three-point function). The result for the form factor is as follows:

$$\xi_{ij}(\nu) = \xi(\nu)\,[\delta_{ij} + \omega_{ij}(\nu)]\,, \tag{24}$$

where $\omega_{ij}(\nu)$ is given by the following expression:

$$\begin{aligned}\omega_{ij}(\nu) &= \Omega(\nu)\ \Delta_{ij} + \text{counterterm},\\ \Delta_{ij} &= \frac{1}{F_0^2}\ \sum_{a=1}^{8}(\lambda_a^2)_{ij}\left[M_a^2\lambda + \frac{1}{2}\mu_a\right],\\ \Omega(\nu) &= g^2[-1 + (2+\nu)\ A(\nu) + B(\nu)],\\ A(\nu) &= \frac{1}{2\sqrt{\nu^2-1}}\ \ln\left(\frac{\nu+1+\sqrt{\nu^2-1}}{\nu+1-\sqrt{\nu^2-1}}\right),\\ B(\nu) &= \frac{\nu}{4\sqrt{\nu^2-1}}\ \ln\left(\frac{\nu-\sqrt{\nu^2-1}}{\nu+\sqrt{\nu^2-1}}\right).\end{aligned} \tag{25}$$

Clearly, Δ_{ij} is diagonal. Expanding at zero recoil ($\nu = 1$), the first few terms are as follows:

$$\Omega(\nu) = g^2\biggl(-\frac{1}{3}(\nu-1) + \frac{2}{15}(\nu-1)^2 - \frac{2}{35}(\nu-1)^3 + \cdots\biggr). \tag{26}$$

As expected from the fact that the effective vector current is conserved at leading order in $1/m_Q$ as a consequence of the IW symmetry (more specifically, the part of the symmetry which corresponds to the flavor rotation between c and b quarks), the corrections vanish at zero recoil. The counterterms needed to render results UV finite only affect the definition of the effective current. They are of $O(p^2)$ and given by

$$\begin{aligned}\mathcal{L}^{(2)}_{b\to c} = 2B_0\ \xi(\nu)\ \frac{\Omega(\nu)}{F_0^2}\bigl(&\eta_1(\nu)\ \mathrm{Tr}_D\ \{\overline{\mathcal{D}}(u\mathcal{M}u + u^\dagger\mathcal{M}u^\dagger)\mathcal{B}(\mathrm{V}_\mu + \mathrm{A}_\mu\gamma_5)\gamma^\mu\}\\ &+\eta_2(\nu)\ \mathrm{Tr}(\mathcal{M}U^\dagger + \mathcal{M}U)\ \mathrm{Tr}_D\ \{\overline{\mathcal{D}}\,\mathcal{B}\,(\mathrm{V}_\mu + \mathrm{A}_\mu\gamma_5)\gamma^\mu\}\bigr),\end{aligned} \tag{27}$$

where the following choice of effective couplings provides a finite result for the three-point function:

$$\begin{aligned}\eta_1(\nu) &= \eta_1^r(\nu;\mu) - \frac{5}{6}\ \lambda(\mu),\\ \eta_2(\nu) &= \eta_2^r(\nu;\mu) - \frac{11}{18}\ \lambda(\mu).\end{aligned} \tag{28}$$

Notice that, in accordance with the normalization condition, the counterterms also have to vanish at zero recoil.

After replacing in ω_{ij} in (25) the contribution of the counterterms, one finds the following SU(3)-breaking correction to the ratio of form factors:

$$\frac{\xi_s(\nu)}{\xi_{u,d}(\nu)} = 1 + \frac{\Omega(\nu)}{F_0^2}\biggl(\mu_K + \frac{1}{2}\mu_\eta - \frac{3}{2}\mu_\pi + 4\ \eta_1^r(\nu;\mu)\ (M_K^2 - M_\pi^2)\biggr). \tag{29}$$

This result shows that for small quark masses the chi-

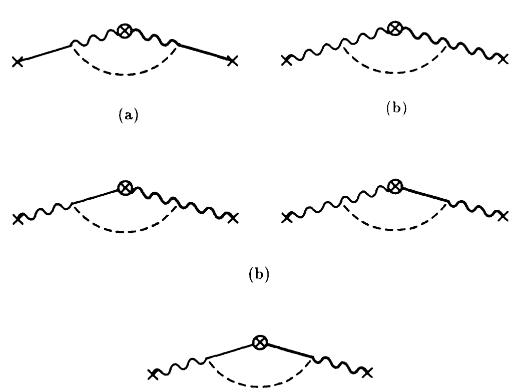

FIG. 2. One-loop correction to the form factor $\xi(v\cdot v')$ as determined from the three-point functions (a) $\tilde{\Pi}^{V\,i,j}_{\rho\mu\nu}$ and (b) $\tilde{\Pi}^{A\,i,j}_{\rho\mu\nu}$. Diagrams that only contribute to decay constants, masses, and wave-function renormalization are not displayed.

ral logarithm tends to increase the value of ξ_s with respect to $\xi_{u,d}$. This result seems to be counterintuitive; one would expect that the heavier the light quark is the faster the form factor will drop with increasing ν. This is certainly true for large enough light-quark mass. As the chiral limit is approached, however, the behavior reverses. Such a behavior is known to occur, for instance, in the quark-antiquark condensates, which near the chiral limit increase with the quark mass, yet start decreasing as the mass becomes large enough. Notice that the correction is algebraically of the same form as the one for the ratio of decay constants. The explicit ν dependence of η_{12} indicates that the counterterm can change the profile of the correction with respect to that given by $\Omega(\nu)$.

It is important to emphasize that the chiral-loop result (24) and (25) holds for any value of the recoil. The behavior of $\Omega(\nu)$ is smooth, and for $\nu \to \infty$ it tends to $-g^2$. In particular, it applies to the whole Dalitz domain of the semileptonic decays $B \to \overline{D}\, l\, \nu_l$, and to nonleptonic decays in the factorization limit (e.g., $B^0 \to D^+\pi^-$ and $B_s \to \overline{D}_s\pi^-$). At the largest available recoil $\nu \sim 1.8$, we have $\Omega \sim -0.2\, g^2$. Choosing values for μ between 1 and 1.5 GeV, the correction to the the ratio (29) contributed by the chiral logarithms is $\simeq -\Omega(\nu) \times (0.3 \text{ to } 0.5)$. This implies that even at the largest recoil this contribution will be small, perhaps, only a few percent. Thus, unless the counterterm contribution is surprisingly large, the SU(3) breaking effects on the form factor will be in the few percent range. Notice that the size of this correction and the nonanalytic contributions to the mass splittings are related, since both are proportional to g^2.

As in the case of the decay constants, we explicitly checked that the heavy-quark symmetry is preserved by the chiral loops. This check was done by considering the three-point function

$$\Pi^{A\ i,j}_{\rho\mu\nu}(x,y) = \langle 0 \mid T\ \bar{c}\gamma_\rho\gamma_5 b(0)\ \ \bar{q}_j\gamma_\nu c(y)\ \ \bar{b}\gamma_\mu\, q_i(x) \mid 0\rangle.$$

In this case, the corrections to the form factor are obtained from the diagrams in Fig. 2(b).

Finally, SU(3) breaking gives rise to direct couplings of the Goldstone bosons to the effective currents, as shown by the counterterm (27). They are proportional to the light-quark masses, and therefore small and unlikely to be of direct physical significance.

V. REMARKS

It is certainly worthwhile to explore possible effects of chiral symmetry in the physics of heavy hadrons. It is likely that they will mainly be restricted to the light-quark-mass-induced SU(3)-breaking corrections as are the ones discussed in this work. Less clear is the accessibility to the predictions of low-energy theorems for the soft-meson emission in decays. At any rate, substantial experimental improvement, especially in strange heavy mesons, is required until effects beyond the mass splittings are accessible. As the predictive power of the chiral expansion is limited by the need of introducing counterterms (as we saw in the cases of the decay constants and the IW form factor), it is not clear that enough experimental information will become available to reach the stage of testing these limited predictions.

We expect that F_{D_s}/F_D, for which the SU(3)-breaking correction might be of the order of 10–20 %, to be a first candidate to be measured. The corrections to the IW form factor will be much harder to observe, since this will require large numbers of B_s mesons, and the effect itself might be very small. The situation could be improved by considering some semi-inclusive decays where the chiral expansion is well defined, for instance, channels for which factorization holds to a good degree. The leading nonanalytic corrections to the mass splittings might be large, depending on the value of g^2, and could, therefore, affect analyses on isospin breaking done within the linear approximation.

Lattice QCD simulations of heavy hadrons might also be an interesting domain of application. For example, chiral symmetry controls the finite-volume effects, which are fully predictable for the unquenched theory. The problem of determining the effective couplings, like those appearing in the counterterms $\mathcal{L}^{(3)}_{\text{source}}$ and $\mathcal{L}^{(2)}_{b\to c}$ might well be first solved on the lattice by looking at the light-quark-mass dependence of the observables we discussed.

Note added. After submission of this work for publication we became aware of a similar calculation of the chiral corrections to the $B \to \overline{D}^{(*)}$ form factor in Ref. [25]. We agree with their result for the chiral logarithms. They do not consider the possible relevance of counterterms. In particular, the counterterms can change the ν dependence of the correction from that predicted by the chiral logarithm with a ν-independent choice of the scale μ, an effect that may be relevant for quark masses of the order of m_s.

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