Radiative decay of mesons in an independent-quark potential model

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We investigate in a potential model of independent quarks the $M1$ transitions among the low-lying vector (V) and pseudoscalar (P) mesons. We perform a "static" calculation of the partial decay widths of twelve possible $M1$ transitions such as $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ within the traditional picture of photon emission by a confined quark and/or antiquark. The model accounts well for the observed decay widths.

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I. INTRODUCTION

Ever since the early experiment of 1960 on $\omega \rightarrow \pi\gamma$ and the subsequent pioneering theoretical work of Gell-Mann and others [1], a considerable amount of both theoretical and experimental work has been done on kinematically allowed transitions of the type $V(J^P=1^-) \rightarrow P(J^P=0^-)+\gamma$ and $P(J^P=0^-) \rightarrow V(J^P=1^-)+\gamma$, which proceed via magnetic dipole ($M1$) transitions. By now experimental results [2] are available for almost all such allowed radiative transitions among the ordinary low-lying vector (V) and pseudoscalar (P) mesons displaying a rather broad range of accuracies. A large number of theoretical papers based on the constituent quark model study [3] within the framework of the naive quark model, the phenomenological Lagrangian approach [4] within the SU(3) unitary symmetry scheme, and more recently the bag-model approach [5] as well as the potential model approach [6] incorporating QCD basic features have been written providing a considerable amount of understanding in this area. However there is no single theoretical or phenomenological description of one-photon radiative transitions among the low-lying vector and pseudo-scalar mesons, which can successfully account for all the experimentally observed decay widths. Most of these descriptions usually fail in some specific transitions such as $K^* \rightarrow K\gamma$ and $V \rightarrow \pi\gamma$.

Although quantum chromodynamics has been established as the underlying theory of strong interactions at the basic structural level of hadrons, many low-energy hadronic phenomena such as these $M1$ transitions cannot ordinarily be explained by first principles applications of QCD. Therefore phenomenological models incorporating the basic features of QCD have proven to be quite useful. The so-called relativized quark model of Godfrey and Isgur [6] based on Coulomb and linear potential models motivated by QCD, which provides one such phenomenological framework to describe all mesons in a unified way, is worth mentioning here. Their calculation of radiative decay amplitudes is based on a relativistic modification of the leading nonrelativistic decay amplitude by a smearing factor of the type $(m/E)^f$, where the exponent f has been utilized to obtain reasonably good results. The bag model with its various versions being reasonably successful in its extensive applications to wide-ranging hadronic phenomena also provides a different phenomenological framework. However its application to radiative decays of vector and pseudoscalar mesons by Hays and Ulehla [5] and also by Hackman *et al.* [5] resulted in rather poor agreement with the experimental data. The failure of the MIT bag model in these works can be traced back to be basically due to the underestimation of the static magnetic moments as well as the transition moments of the hadrons. Also in the calculation of the transition moments, the region of the overlap integral is ambiguous because of the different radii of the initial- and final-state bags. Any ansatz to fix the upper limit R_0 in the integral expression of the transition moment by varying R_0 within the range $R_P \leq R_0 \leq R_V$ is subject to a certain amount of uncertainty. The real remedy in fact calls for a significant contribution from the pion cloud effect as rightly accounted for in the cloudy bag model (CBM), a modern hybrid version of the bag model endowed with chiral symmetry. The work of Singer and Miller [7] based on the CBM finds a relatively better agreement with the observed rates using a completely new picture of the radiative decays. They propose three different dynamical mechanisms such as (i) photon emission by quarks, (ii) photon emission by a pion cloud, and (iii) transitions of a vector bag to a photon accompanied by pion emission, which contributes either singly or in combination. The pion involved here is the quantized field pion, where the $(q\bar{q})$ pion state is projected out. Such a picture is certainly quite different from the simple conventional description where photon emission induced by a quark electromagnetic current is the only common mechanism contributing to all the allowed radiative transitions with the pion being treated as a $(q\bar{q})$ bound state. Admittedly though there exists an apparent dichotomy between the $(q\bar{q})$ structure and the Goldstone-boson facet of the pion; a somewhat blended picture [8,9] may not be completely ruled out. Therefore

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until a clear understanding of this intriguing nature of the pion is available through some more complete theoretical formulation, we would like to investigate here these radiative decays within the conventional picture with the help of an alternate scheme of a potential model of relativistic independent quarks. Such a model with a pionic effect appended in the usual manner consistent with the partial conservation of axial-vector-current (PCAC) has proved to provide a successful alternative to the CBM with more simplicity and tractability. It has been found to reproduce quite satisfactorily the mass spectrum as well as the electromagnetic properties of ground-state octet baryons [10]. In the ordinary light meson sector, this model not only accounts well for the (ω,ρ,π) mass difference [9,11], but also leads to the $(q\bar{q})$ pion mass in consistency with that of the PCAC pion and the pion-decay constant [9]. In view of the success of this model in the light hadronic sector, we intend to extend its applicability to the study of radiative transitions of vector and pseudoscalar mesons. Our purpose here is in a way to make a parameter-free calculation of the decay rates by using the potential parameters obtained in our earlier studies [9,10]. The mixing angles appearing in the vector and pseudoscalar meson sectors would be taken in accordance with the quadratic mass formulas.

The assumptions involved in our calculation are mainly as follows. First of all the radiative transitions discussed here are considered to be a predominantly single-vertex process induced solely by the quark electromagnetic current in which a photon is emitted alternately by the confined quark or antiquark. The quark field operators in the electromagnetic current are expanded in terms of the complete set of static solutions of positive and negative energies admissible by the zeroth-order quark dynamics of the model. The decay matrix elements are calculated in the "static" approximation where the ground-state vector and pseudoscalar mesons are represented by the usual spin-flavor SU(6) expressions with the quark and antiquark creation operators corresponding to the lowest eigenmodes. Any momentum dependence, which is likely to arise from the construction of meson-core states from the momentum wave packets, is assumed to be negligible in view of the moderate momentum transfer involved here. Finally the energy δ function arising in the calculation of the S-matrix element includes only the zeroth-order quark and antiquark binding energies of the ground state. The additional correction terms arising out of the residual interaction (such as one-gluon exchange and pion coupling) and the center-of-mass motion, which are important in obtaining the low-lying hadron masses [9] in this model, are inserted by hand for obtaining a phenomenologically correct energy-conservation δ function. These points will be further elaborated on in the text of the paper which is organized as follows. For the sake of completeness, we provide in Sec. II some relevant conventions and consequences of the potential model adopted here. In Sec. III, we describe our framework for obtaining the S-matrix element and the corresponding decay rates for the radiative transitions. Section IV deals with our results and discussions.

II. THE POTENTIAL MODEL

The quark-confining interaction in a hadron, which is believed to be generated by the nonperturbative multi-gluon mechanism, is not possible to calculate theoretically from first-principles QCD. Therefore from a phenomenological point of view the present model assumes that the quark and antiquark in a hadron core are independently confined by an average flavor-independent potential $U(r)$ of the form [9–13]

$$U(r)=\tfrac{1}{2}(1+\gamma^0)(ar^2+V_0)\ , \quad a>0\ . \tag{1}$$

To a first approximation, the confining part of the interaction is believed to provide the zeroth-order quark dynamics inside the hadron core through the quark Lagrangian density

$$\mathcal{L}^0(x)=\bar{q}(x)[\frac{i}{2}\gamma^\mu\overleftrightarrow{\partial}_\mu-U(r)-m_q]q(x)\ . \tag{2}$$

The ensuing Dirac equation admits a static solution of positive and negative energies in the zeroth order given, respectively, in the forms

$$\begin{aligned}\psi_\xi(\mathbf{r})&=\begin{bmatrix} ig_\xi(r)/r \\ \sigma\cdot\hat{\mathbf{r}}f_\xi(r)/r\end{bmatrix}U_\xi(\hat{\mathbf{r}})\ ,\\ \phi_\xi(\mathbf{r})&=\begin{bmatrix} i\sigma\cdot\hat{\mathbf{r}}f_\xi(r)/r \\ g_\xi(r)/r\end{bmatrix}\tilde{U}_\xi(\hat{\mathbf{r}})\ ,\end{aligned} \tag{3}$$

where $\xi\equiv(nljm)$ represents the set of Dirac quantum numbers specifying the eigenmodes. The spin angular parts $U_\xi(\hat{\mathbf{r}})$ and $\tilde{U}_\xi(\hat{\mathbf{r}})$ are described as

$$\begin{aligned}U_{ljm}(\hat{\mathbf{r}})&=\sum_{m_l,m_s}\langle lm_l\tfrac{1}{2}m_s|jm\rangle Y_l^{m_l}(\hat{\mathbf{r}})\chi_{1/2}^{m_s}\ ,\\ \tilde{U}_{ljm}(\hat{\mathbf{r}})&=(-1)^{j+m-l}U_{lj-m}(\hat{\mathbf{r}})\ .\end{aligned} \tag{4}$$

Taking

$$\begin{aligned}E_q'&=\left[E_q-\frac{V_0}{2}\right]\ ,\quad m_q'=\left[m_q+\frac{V_0}{2}\right]\ ,\\ \lambda_q&=(E_q'+m_q')\ ,\quad r_{0q}=(a\lambda_q)^{-1/4}\ ,\end{aligned} \tag{5}$$

one can obtain solutions to the resulting radial equation for $g_\xi(r)$ and $f_\xi(r)$ yielding an independent quark bound-state condition:

$$\left[\frac{\lambda_q}{a}\right]^{1/2}(E_q'-m_q')=(4n+2l-1)\ . \tag{6}$$

The solution of this cubic equation provides the zeroth-order binding energies of the confined quark for various possible eigenmodes. The quark-gluon interaction originating from one-gluon exchange at short distances and the quark-pion interaction in the nonstrange sector required to preserve chiral symmetry and also the center-of-mass motion of independent quarks, when considered perturbatively, provide additional corrections [9–13] to the zeroth-order quark-binding energy which turns out to be crucial for the spectroscopy of light hadrons.

In our present calculation of the S-matrix element for radiative transitions within a "static" approximation, where we expand the quark field operators in terms of the complete set of positive and negative energy solutions, only the lowest eigenmodes corresponding to the ground state of the involved mesons would be relevant. These lowest eigenmodes corresponding to the positive and negative energies find the respective explicit forms as

$$\psi_{\xi(1S_{1/2})}(\mathbf{r})=\frac{1}{\sqrt{4\pi}}\begin{bmatrix} ig(r)/r \\ \sigma\cdot\hat{\mathbf{r}}f(r)/r\end{bmatrix}\chi_m ,$$

$$\phi_{\xi(1S_{1/2})}(\mathbf{r})=\frac{1}{\sqrt{4\pi}}\begin{bmatrix} \sigma\cdot\hat{\mathbf{r}}f(r)/r \\ -ig(r)/r\end{bmatrix}\tilde{\chi}_m , \tag{7}$$

where the two component spinors χ_m and $\tilde{\chi}_m$ denote $\chi_\uparrow=\binom{1}{0}$, $\chi_\downarrow=\binom{0}{1}$, and $\tilde{\chi}_\uparrow=\binom{0}{-i}$, $\tilde{\chi}_\downarrow=\binom{i}{0}$. The reduced radial parts in the upper and lower component solutions corresponding to a quark flavor q are

$$g_q(r)=\mathcal{N}_q\left[\frac{r}{r_{0q}}\right]\exp\left[\frac{-r^2}{2r_{0q}^2}\right] ,$$

$$f_q(r)=-\frac{\mathcal{N}_q}{\lambda_q r_{0q}}\left[\frac{r}{r_{0q}}\right]^2\exp\left[\frac{-r^2}{2r_{0q}^2}\right] , \tag{8}$$

where the normalization factor

$$\mathcal{N}_q^2=\frac{8\lambda_q}{\sqrt{\pi}r_{0q}}\frac{1}{(3E_q'+m_q')} . \tag{9}$$

III. MESONIC $M1$ TRANSITIONS

In this section we obtain the S-matrix elements and the decay widths of energetically allowed mesonic transitions such as $A\rightarrow B+\gamma$ among the vector mesons $V\equiv(\rho,\omega,\phi,K^{0*},K^{+*})$ and pseudoscalar mesons $P\equiv(\pi,\eta,\eta',K^0,K^+)$. Physically the decay reactions occur between the momentum eigenstates of the mesons, which in an exact calculation need to be expressed in terms of the static-core states of the model. However the decay reactions being studied here do have a moderate momentum transfer for which it would be a reasonable approximation to perform a "static" calculation. Any momentum dependence that could arise from the constructions of mesonic-core states from momentum wave packets can be considered negligible. Hence in calculating the S-matrix elements the initial (A) and final (B) mesons in their ground states are described here by the usual spin-flavor SU(6) expressions with the quark and antiquark creation operators corresponding to the lowest eigenmodes.

Assuming that such transitions are predominantly single-vertex processes governed mainly by photon emission from independently confined quark and antiquark inside the meson core [Figs 1(a) and 1(b)], the S-matrix elements in the rest frame of the initial meson A is written in the form

$$S_{BA}=\left\langle B\gamma\left|-ie\int d^4x\, T\left[\sum_q e_q\bar{q}_q(x)\gamma^\mu q_q(x)A_\mu(x)\right]\right|A\right\rangle \tag{10}$$

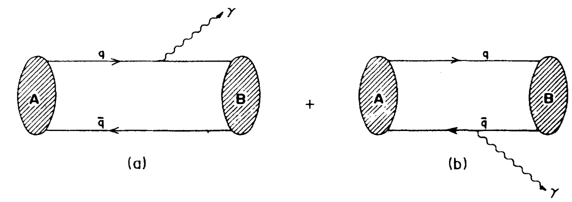

FIG. 1. Lowest-order graphs contributing to mesonic $M1$ transitions.

We make here the usual choice of the photon field $A_\mu(x)$ in the Coulomb gauge $\epsilon(\mathbf{k},\lambda)$ as the polarization vector of the emitted photon having an energy momentum $(k_0=|\mathbf{k}|,\mathbf{k})$ in the rest frame of A. The quark field operators find a possible expansion in terms of the complete set of positive and negative energy static solutions given by Eqs. (3)–(6), so that

$$q_q(x)=\sum_\xi[b_{q\xi}\psi_{q\xi}(\mathbf{r})\exp(-iE_{q\xi}t)$$
$$+\tilde{b}_{q\xi}^\dagger\phi_{q\xi}(\mathbf{r})\exp(iE_{q\xi}t)] . \tag{11}$$

Here the subscript q stands for the quark flavor and ξ represents the set of Dirac quantum numbers specifying all possible eigenmodes. $b_{q\xi}$ and $\tilde{b}_{q\xi}^\dagger$ are the quark annihilation and antiquark creation operators corresponding to the eigenmode ξ. In view of our static calculations, the S-matrix elements in the long run would essentially acquire the contribution of only the lowest eigenmodes present in the field expansion of Eq. (11). Therefore to avoid unnecessary complications, we treat the quark field expansion effectively in terms of the lowest eigenmodes corresponding to the ground-state mesons without affecting the final result in any way. The quark eigenmodes in zeroth order used in our calculation are taken to be the same for the vector and the pseudoscalar mesons belonging to the same SU(6) multiplet. But in that case the energy δ function that would appear in the S-matrix element in Eq. (10) would have only the photon energy as its argument, implying thereby no available energy for the photon emission to take place. However the quark-binding energy $E_{q\xi}$ characteristically appearing in the stationary state harmonic-time dependence can in principle include additional correction terms arising out of the possible residual interactions (such as one-gluon exchange at short distances and pion coupling for restoring chiral symmetry [9,10]) in the Hamiltonian, when treated as constant perturbations in a time-dependent perturbation approach. The gluonic $(\Delta E_{q\xi}^g)$ and pionic $(\Delta E_{q\xi}^\pi)$ correction terms [9,10] in the effective $E_{q\xi}$, being different for vector (V) and pseudoscalar (P) mesons belonging to the same SU(6) multiplet, can restore the energy argument of the δ function. But because of the complications arising out of the center-of-mass motion in an independent quark model such as the present one, it is not possible to generate quantitatively correct energy-conservation through an energy δ function appearing in the static calculation of the S-matrix element. In the absence of any rigorous field-theoretical technique for these decay processes, such difficulties are commonly encountered by any phenomenological model. Therefore following Hays and Ulehla [5], we prefer the usual path of phenomenologically patching up the energy δ function in these reactions. With all these considerations, the S-matrix elements in Eq. (10) can effectively be written as

$$S_{BA}=i\sqrt{(\alpha/k)}\delta(E_B+k-E_A)\sum_{q,m,m'}\langle B|[J^q_{m',m}(k,\lambda)b^\dagger_{qm'}b_{qm}-\tilde{J}^q_{mm'}(k,\lambda)\tilde{b}^\dagger_{qm'}\tilde{b}_{qm}]|A\rangle\ . \quad (12)$$

Here, $E_A=M_A$, $E_B=\sqrt{k^2+M_B^2}$, and (m,m') are the possible spin quantum numbers of the confined quarks corresponding to the ground $(1S_{1/2})$ state of the mesons. Again we have

$$\begin{aligned} J^q_{m'm}(k,\lambda)&=e_q\int d^3r\,\exp(-i\mathbf{k}\cdot\mathbf{r})[\bar{\psi}_{qm'}(\mathbf{r})\gamma\cdot\epsilon(k,\lambda)\psi_{qm}(\mathbf{r})]\ ,\\ \tilde{J}^q_{mm'}(k,\lambda)&=e_q\int d^3r\,\exp(-i\mathbf{k}\cdot\mathbf{r})[\bar{\phi}_{qm}(\mathbf{r})\gamma\cdot\epsilon(k,\lambda)\phi_{qm'}(\mathbf{r})]\ . \end{aligned} \quad (13)$$

Now using $\psi_{qm}(\mathbf{r})$ and $\phi_{qm'}(\mathbf{r})$ as provided in Eqs. (7)–(9) and the integral

$$\int d\Omega\,\exp(-i\mathbf{k}\cdot\mathbf{r})(\sigma\times\hat{r})=-4\pi i j_1(kr)\ , \quad (14)$$

one can reduce Eq. (13) to simpler forms as

$$J^q_{m'm}(k,\lambda)=-i\mu_q(k)[\chi^\dagger_{m'}(\sigma\cdot\mathbf{K})\chi_m]\ ,\quad \tilde{J}^q_{mm'}(k,\lambda)=i\mu_q(k)[\tilde{\chi}^\dagger_m(\sigma\cdot\mathbf{K})\tilde{\chi}_{m'}]\ , \quad (15)$$

where $\mathbf{K}=\mathbf{k}\times\epsilon(k,\lambda)$ and $\mu_q(k)=e_q\mu_q^0(k)$, with

$$\mu_q^0(k)=\frac{2\exp(-k^2r_{0q}^2/4)}{3E_q'+m_q'}\ . \quad (16)$$

In fact $\mu_q(k=0)$ is the magnetic moment of the confined quark in the present model. Then

$$S_{BA}=\sqrt{(\alpha/k)}\delta(E_B+k-E_A)\sum_{q,m,m'}\langle B|\mu_q(k)[\chi^\dagger_{m'}\sigma\cdot\mathbf{K}\chi_m b^\dagger_{qm'}b_{qm}+\tilde{\chi}^\dagger_m\sigma\cdot\mathbf{K}\tilde{\chi}_{m'}\tilde{b}^\dagger_{qm'}\tilde{b}_{qm}]|A\rangle\ . \quad (17)$$

Now in order to calculate the S-matrix elements for various $M1$ transitions, one has to specify the spin-flavor states of initial and final mesons through the usual SU(6) expressions. We must also mention the mixing angle conventions followed here for vector and pseudoscalar mesons. It is well known that for vector meson ω and ϕ, the mixing angle is quite close to the ideal nonet value $\theta_V^0=\arcsin(1/\sqrt{3})\approx 35.3°$ for which the quark-flavor combinations are $\phi^0=-(s\bar{s})$ and $\omega^0=(1/\sqrt{2})(u\bar{u}+d\bar{d})$. But in view of the fact that the nonet-forbidden $\phi\to\rho\pi$ and $\phi\to\pi\gamma$ decays do occur, there is a small departure from the ideal mixing angle θ_V^0. The deviation $\delta_V=(\theta_V^0-\theta_V)$ can be obtained from the requirement $\theta_V=39°$ by the quadratic mass formula [13]. Hence we can express the flavor contents of physical ϕ and ω meson states as

$$\begin{bmatrix}\phi\\ \omega\end{bmatrix}=\begin{bmatrix}\cos\delta_V & \sin\delta_V\\ -\sin\delta_V & \cos\delta_V\end{bmatrix}\begin{bmatrix}\phi^0\\ \omega^0\end{bmatrix}\ . \quad (18)$$

The mixing angle for pseudoscalar η and η' can also be obtained either from the quadratic mass formula [13] as $\theta_P=-(10.1)^0$ or from the two photon decays of η and η' as $\theta_P=-9.5°\pm2.0°$ [14]. If we define the purely strange and nonstrange flavor components in the pseudoscalar (η,η') sector as $\eta_s=-(s\bar{s})$ and $\eta_{\rm ns}=(1/\sqrt{2})(u\bar{u}+d\bar{d})$, respectively, then the flavor content of η and η' can be expressed in terms of $\delta_P=(\arcsin 1/\sqrt{3}-\theta_P)$ as

$$\begin{bmatrix}\eta\\ \eta'\end{bmatrix}=\begin{bmatrix}\cos\delta_P & \sin\delta_P\\ -\sin\delta_P & \cos\delta_P\end{bmatrix}\begin{bmatrix}\eta_s\\ \eta_{\rm ns}\end{bmatrix}\ . \quad (19)$$

The the "perfect mixing" is realized with $\delta_P=45°$, when $\eta=\frac{1}{2}(u\bar{u}+d\bar{d}-\sqrt{2}s\bar{s})$ and $\eta'=\frac{1}{2}(u\bar{u}+d\bar{d}+\sqrt{2}s\bar{s})$ have the same strange flavor content. However, it must be noted that a more recent measurement [15] of $\Gamma_\eta(2\gamma)$ leads to a mixing angle $\theta_P=-17.6°\pm3.6°$. This would therefore imply a possible range of values for δ_P between 45° to 56°. Therefore for convenience we would express the appropriate matrix elements in terms of δ_V and δ_P.

Now with proper specification of pseudoscalar meson states and the vector meson states of different spin projections $S_V\equiv(\pm1,0)$, one can calculate the S-matrix elements for individual processes separately which finds a general expression

$$S_{BA}=\sqrt{(\alpha/k)}\delta(k-\bar{k}_0)\mu_{BA}(k)K_{S_V}\ . \quad (20)$$

Here the kinematically allowed energy of the outgoing photon is given by

$$\bar{k}_0=(M_A^2-M_B^2)/2M_A \quad (21)$$

and for $S_V\equiv(\pm1,0)$; K_{S_V} stands for the following combinations separately for $V\to P\gamma$ and $P\to V\gamma$ type transitions:

$$K_{S_V}(V\to P\gamma)\equiv\left[\pm\frac{K_1\pm iK_2}{\sqrt{2}}\ ,-K_3\right]\ ,$$
$$K_{S_V}(P\to V\gamma)\equiv\left[\pm\frac{K_1\mp iK_2}{\sqrt{2}}\ ,-K_3\right]\ . \tag{22}$$

Finally $\mu_{BA}(k)$ in Eq. (20) are the relevant transition matrix elements obtainable explicitly for different processes separately. Assuming flavor-SU(2) symmetry with $m_u=m_d\neq m_s$ and hence $\mu_u^0(k)=\mu_d^0(k)\neq\mu_s^0(k)$, we obtain them as

$$
\begin{aligned}
&\text{(i)}\ \ \mu_{K^{*+}K^+}(k)=\tfrac{2}{3}\mu_u^0(k)-\tfrac{1}{3}\mu_s^0(k)\ ,\\
&\text{(ii)}\ \ \mu_{K^{*0}K^0}(k)=-\tfrac{1}{3}\mu_u^0(k)-\tfrac{1}{3}\mu_s^0(k)\ ,\\
&\text{(iii)}\ \ \mu_{\rho\pi}(k)=\tfrac{1}{3}\mu_u^0(k)\ ,\\
&\text{(iv)}\ \ \mu_{\omega\pi}(k)=\mu_u^0(k)\cos\delta_V\ ,\\
&\text{(v)}\ \ \mu_{\phi\pi}(k)=\mu_u^0(k)\sin\delta_V\ ,\\
&\text{(vi)}\ \ \mu_{\rho\eta}(k)=\mu_u^0(k)\sin\delta_P\ ,\\
&\text{(vii)}\ \ \mu_{\eta'\rho}(k)=\mu_u^0\cos\delta_P\ ,\\
&\text{(viii)}\ \ \mu_{\omega\eta}(k)=\tfrac{1}{3}\mu_u^0(k)\sin\delta_P\cos\delta_V+\tfrac{2}{3}\mu_s^0(k)\cos\delta_P\sin\delta_V\ ,\\
&\text{(ix)}\ \ \mu_{\eta'\omega}(k)=\tfrac{1}{3}\mu_u^0(k)\cos\delta_P\cos\delta_V-\tfrac{2}{3}\mu_s^0(k)\sin\delta_P\sin\delta_V\ ,\\
&\text{(x)}\ \ \mu_{\phi\eta}(k)=\tfrac{1}{3}\mu_u^0(k)\sin\delta_P\sin\delta_V-\tfrac{2}{3}\mu_s^0(k)\cos\delta_P\cos\delta_V\ ,\\
&\text{(xi)}\ \ \mu_{\phi\eta'}(k)=\tfrac{1}{3}\mu_u^0(k)\cos\delta_P\sin\delta_V+\tfrac{2}{3}\mu_s^0\sin\delta_P\cos\delta_V\ .
\end{aligned} \tag{23}
$$

Now summing over the photon polarization index λ and the final meson spin appropriately while averaging over the initial meson spin when necessary; the partial decay widths of $V\to P\gamma$ and $P\to V\gamma$ processes can be obtained separately from

$$\Gamma_{A\to B\gamma}=\sum_{\lambda,S_V}\int d^3k\frac{|S_{BA}|^2}{T} \tag{24}$$

as follows:

$$\Gamma_{V\to P\gamma}=\tfrac{4}{3}\alpha\bar{k}_0^3[\mu_{VP}(\bar{k}_0)]^2$$
$$\Gamma_{P\to V\gamma}=4\alpha\bar{k}_0^3[\mu_{PV}(\bar{k}_0)]^2\ . \tag{25}$$

Here we have used the spin-polarization sum

$$\sum_{\lambda,S_V}|K_{S_V}|^2=2k^2\ . \tag{26}$$

In fact Eq. (25) thus provides the well-known standard expression for the partial decay widths of mesonic $M1$ transitions.

IV. RESULTS AND DISCUSSION

In this section we use the formalism developed in Secs. II and III to calculate the partial decay widths of several energetically possible $M1$ transitions among the vector and pseudoscalar mesons in a broken SU(3) flavor sector. The calculation involves primarily the potential parameters of the model (a,V_0), the quark masses $(m_u=m_d,m_s)$ and the mixing angles (δ_V,δ_P). The meson masses appearing in the calculation are in fact taken as the observed ones.

Taking (a,V_0) as determined earlier in our application of the present model to baryon and meson sectors [9,10], we have

$$(a;V_0)\equiv(0.017\,166\ \text{GeV}^3;\ -0.1375\ \text{GeV})\ ,$$
$$(m_u=m_d;m_s)\equiv(0.01\ \text{GeV};\ 0.24\ \text{GeV})\ . \tag{27}$$

Then the model dynamics as described in Sec. II, provides the ground-state confined quark energy E_q' and the scale factor r_{0q} relevant for the present calculation in the following manner:

$$(E_u'=E_d';E_s')\equiv(0.52\ \text{GeV};\ 0.6146\ \text{GeV})\ ,$$
$$(r_{0u}=r_{0d};r_{0s})\equiv(3.352\,272\ \text{GeV}^{-1};\ 2.934\,226\,7\ \text{GeV}^{-1})\ . \tag{28}$$

Finally we take the mixing angle parameters in accordance with the quadratic mass mixing formula so as to have

$$(\delta_V;\delta_P)\equiv(-3.7°;\ 45°)\ . \tag{29}$$

In view of such a choice of the parameters, which are all determined from hadron spectroscopy, there is no freedom left in our calculation so that it becomes, in a way, completely parameter-free. The results of our calculation are provided in Table I in comparision with those of the

TABLE I. Partial decay widths $\Gamma(A\to B\gamma)$ in comparison with CBM calculation and experiment with $\delta_V=-3.7°$; $\delta_P=45°(56°)$.

	Present calculation					
$A\to B\gamma$	$\bar{k}_0$ (MeV)	$\mu_u^0(\bar{k}_0)$ (GeV^{-1})	$\mu_s^0(\bar{k}_0)$ (GeV^{-1})	$\Gamma(A\to B\gamma)$ (keV)	CBM [7] results (keV)	Experiment [2] (keV)
$K^{*+}\to K^+\gamma$	309.29	1.018	0.808	48.3	47	50±5
$K^{*0}\to K^0\gamma$	309.85	1.017	0.807	107.1	98	117±10
$\rho^-\to\pi^-\gamma$	374.34	0.899		45.8	124	68±7
$\rho^0\to\pi^0\gamma$	372.13	0.903		45.4	124	121±31
$\omega\to\pi^0\gamma$	379.86	0.888		419	1180	717±50
$\phi\to\pi^0\gamma$	501.07	0.658		2.2	4.7	5.9±2.2
$\rho^0\to\eta\gamma$	187.75	1.206		46.9(64.5)	23	62±17
$\eta'\to\rho^0\gamma$	171.16	1.227		110.2(68.9)	53	62±7
$\omega\to\eta\gamma$	199.03	1.192	0.911	4.9(7.2)	2.3	4.14±1.74
$\eta'\to\omega\gamma$	159.02	1.241	0.940	12.1(8.2)	6	6.2±0.9
$\phi\to\eta\gamma$	362.25	0.921	0.748	62.0(40.2)	43	56.7±2.9
$\phi\to\eta'\gamma$	60.11	1.320	0.985	0.4(0.6)	0.29	<1.84

cloudy bag model [7] and the experiment [2].

We find that for the decays involving K^* mesons; the simple conventional mechanism of photon emission by confined quarks proves to be quite adequate in explaining the experimental data. Although in principle it does not rule out the possible contribution due to pion-exchange currents; it certainly underscores its quantitative significance in the K^* sector contrary to the observation made in CBM calculations. As in most other calculations, our results for decays such as $(\omega,\rho,\phi)\to\pi\gamma$ are not quite satisfactory. This may be partly due to our static approximation based on the assumption of moderate momentum transfer and also partly due to the intriguing nature of the pion itself. Although Geffen and Wilson [3] have obtained very good fits for these decay widths including rest others in their broken SU(3) predictions by allowing arbitrary effective quark moments, they could not completely describe the observed hyperon moments. In the CBM treatment of $V\to\pi\gamma$ as processes involving emission of elementary pions in combination with $V\to\gamma$ transitions induced by quark electromagnetic current, the decay rates are rather overestimated. On the other hand our calculation which underestimates these decay rates calls for the possible additional contribution due to pion-cloud effects. Therefore decays with a pion in the final state require further investigation with a clear understanding of the nature and the role of the pions. Finally the decay widths of $V\to\eta\gamma$ and $V\to\eta'\gamma$ type transitions obtained on the basis of the mixing angles sanctioned by the quadratic mass formula are quite satisfactory. But the decay rates calculated for $\eta'\to V\gamma$ transitions have a noticeable discrepancy with the corresponding experimental data. In fact we observe that by taking $\delta_P=56°$ instead of 45°, one can obtain

$$\begin{aligned}\Gamma(\eta'\to\rho\gamma)&=68.9\ \text{KeV}\ ,\\ \Gamma(\eta'\to\omega\gamma)&=8.2\ \text{KeV}\ ,\end{aligned}\tag{30}$$

which are in good agreement with experiment. Such a choice can find support from the reported [2,15] value $\theta_P\approx-17.6°\pm3.6°$ obtained from $\Gamma_\eta(2\gamma)=0.56\pm0.16$ KeV, which is close to the mixing angle predicted by the linear mass mixing formula. This value of δ_P does not change other predictions very much yielding them well within the experimental error limits. Results for $\delta_P=56°$ are provided with the parentheses in Table I to give an idea about the range of variation of the decay widths within the range $45°\le\delta_P\le56°$.

In general the predictions of the model are in overall agreement with the existing experimental data with wide ranges of accuracies. Thus within the working approximations adopted here, the model provides a simple calculational framework to describe the mesonic $M1$ transitions based on a traditional picture of photon emission by confined quarks. Further improvement on this first-order prediction of the model can be made by including the possible pion-cloud effects through the implementation of chiral symmetry.

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