Production of B_c or $\bar{B}_c$ mesons associated with two heavy-quark jets in Z^0 decay

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Computations of the production of B_c or $\bar{B}_c$ mesons and their excited states B_c^* or $\bar{B}_c^*$ associated with two heavy-quark jets in Z^0 decay are presented in detail within the QCD-inspired potential model framework. The results show that the mesons B_c and $\bar{B}_c$ will soon be accessible experimentally in Z^0 decay.

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I. INTRODUCTION

The B_c and $\bar{B}_c$ mesons carry flavors explicitly and are the ground states of the bound $\bar{b}c$ and $b\bar{c}$ heavy-quark–antiquark systems. The study of them as well as their excited states is very interesting for their static properties and for their decay and production characteristics.

It is known that the QCD-inspired nonrelativistic potential model is very successful for describing the heavy quarkonia $c\bar{c}$ and $b\bar{b}$ [1–5], etc. For the $\bar{b}c$ and $b\bar{c}$ systems, with a reduced mass just between those of the $c\bar{c}$ and $b\bar{b}$ systems, the potential framework should be suitable here too, as the nature of the binding force for these systems is induced by gluon exchanges, and, hence, is flavor independent, and the nonrelativistic approximation works well in addition. The spectra of the $b\bar{c}$ and $\bar{b}c$ systems and the properties of their bound states can be predicted by the potential model; thus when the B_c and $\bar{B}_c$ mesons as well as their excited states are discovered in experiments, the experimental data will provide new tests of the potential model. Moreover the decays of the B_c and $\bar{B}_c$ mesons will offer fresh samples for studying the weak decay mechanisms. In particular, some decays contain only those weak-current matrix elements sandwiched between a heavy-quark pair meson such as the B_c or $\bar{B}_c$ meson and a heavy quarkonium such as J/ψ or η_c; i.e., there is no light quark involved at all, which may be calculated in the potential and perturbative QCD framework quite precisely, and may be applied to the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element(s). As for nonleptonic decays, there are a number of factors that need to be clarified; the study of these kinds of decays for the B_c and $\bar{B}_c$ mesons will be able to offer some clue for understanding the decay mechanism so as to clarify certain factors. Because of the difficulty in producing B_c and $\bar{B}_c$ mesons, the experimental study of $b\bar{c}$ and $\bar{b}c$ bound systems has not yet started, although there have been some theoretical discussions and estimations in the literatures [6,7].

Up to now, a large number of Z^0 events have been accumulated at the CERN e^+e^- collider LEP I. The authors of Refs. [6–8] considered a mechanism for producing the bound states $b\bar{c},\bar{b}c$ [6,7] and $c\bar{c}$ and $b\bar{b}$ [8] associated with two heavy-quark jets in the Z^0 decay, respectively. Their results indicated that the concerned processes have a comparatively large branching ratio that hopefully will be observable at LEP I. In the calculation of the process, the amplitude is relevant with the wave function of the bound states, which can be calculated accurately in the framework of a potential model. Furthermore under a reasonable approximation [8,9] this factor may be simplified as the one proportional to the wave function at the origin (or say the decay constant). In fact, in Ref. [6], a conjecture on the decay constant was made in their calculation. Considering the status of the Z^0 events accumulated at LEP I and the indications as well as the theoretical uncertainty of Refs. [6–8], it becomes an urgent task to recalculate the production rate of B_c and $\bar{B}_c$ mesons in Z^0 decay based on a comparatively solid foundation, and to find typical signals. Therefore, in this paper we calculate the wave functions precisely with the QCD-inspired potential, so as to obtain a more solid estimate of the productions. In order to make a comparison, we have tried two kinds of QCD-inspired potentials with different parameters but determined both by fitting the data of the $c\bar{c}$ and $b\bar{b}$ quarkonia. In this paper, we first use the obtained value of $1\,S_0$ state wave function and calculate the partial width of the production of the B_c or $\bar{B}_c$ meson; second, we calculate the partial widths of the productions of the excited states such as $B_c^*(1\,^3S_1,2\,^1S_0,2\,^3S_1)$ or $\bar{B}_c^*(1\,^3S_1,2\,^1S_0,2\,^3S_1)$ in the Z^0 decay, since the wave functions of these states may be calculated accurately too, as they are also interesting, and eventually the productions of the excited states will contribute to that of the ground state B_c or $\bar{B}_c$ meson via decays by emitting π, η, and/or γ(s). Third, we find that the so-called "lowest twist" approximation which was adopted in calculating the distribution and the fragmentation function in Ref. [6] may be improved considerably. Some "high twist" terms have a higher-order singularity so they will give a significant contribution to the con-

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cerned partial width. The precision of the "lowest twist" approximation is therefore enhanced by the inclusion of these "high twist" terms. In this paper we will present a detailed discussion of this point. (It was also mentioned in Ref. [10]).

This paper is organized as follows. In Sec. II the mechanism and the formulation for the B_c or $\bar{B}_c$ meson production associated with two heavy-quark jets in Z^0 decay are presented. In Sec. III, numerical results are presented. Section IV is devoted to discussions, and finally the formulas used for computing the production are put in the Appendix.

II. MECHANISM AND FORMULATION

The production of B_c mesons is very different from that of the flavored mesons containing one light quark. For instance, as for the B (B_s) meson, containing a light quark, one production mechanism is by fragmentation, but this is not suitable for the B_c meson because to "pick up" a light-quark pair from the vacuum is much easier than to produce a heavy-quark one by the soft fragmentation mechanism. According to the successful hadron-string model, the relative possibilities to create a quark pair with various flavors from "vacuum" are [11]

$$u:d:s:c=1.0:1.0:0.3-0.4:10^{-10}-10^{-11} . \tag{1}$$

Second, the meson(s) such as B (B_s) can be produced in e^+e^- colliders at an excited Υ resonance above the threshold of their pair production. But this mechanism is not suitable for the B_c meson either, because first it is an open problem if such a highly excited resonance exists above the B_c pair threshold, which is about 12.6 GeV according to the prediction of the potential models [5], and second, even if such highly excited states exist, the resonance effects are not apparent and the production rate will likely be suppressed by a factor controlled by Eq. (1).

The third possibility is to produce the B_c or $\bar{B}_c$ meson by directly coupling to a virtual W^+ or W^- boson, whereas it also has a small branching ratio, as there is a strong suppression from the CKM matrix element $V_{cb}\sim 0.04$, compared with the other dominant channels.

A production mechanism for producing the ground states of the $c\bar{b}$, and $b\bar{c}$ systems, i.e., B_c and $\bar{B}_c$ mesons, was proposed first in Ref. [6]. Recently the authors of Ref. [8] considered the same mechanism not only for the ground-state production but also for the excited ones, but of the $c\bar{c}$ and $b\bar{b}$ systems only. The mechanism is for Z^0 decay to produce the meson associated with two heavy-quark jets. It is so important that the concerned channels are comparatively large. The mechanism includes two steps: first of all, two pairs of the relevant heavy-quark–antiquarks pairs are produced by a hard process (where at least one of them is produced by exchanging one hard virtual gluon) with a quite significant possibility; then in the second step, one quark from one of the two pairs and one anti-quark from the other pair are bound together with nonzero possibility to form the bound state. Here the first step can be calculated well by the perturbative QCD theory because q, the momentum of the virtual gluon, satisfies $q^2>4m_c^2\gg\Lambda_{\rm QCD}^2$ (m_c is the c-quark mass and $\Lambda_{\rm QCD}$ is the QCD scale parameter) while the second step can be described with a wave function of the bound state in potential model framework; furthermore, under a reasonable approximation [6,8,9] the amplitude of the process may become proportional to the decay constant of the meson, or say, the wave function at origin in the bound-state language. In Ref. [6] the value of the decay constant of the meson was not taken very carefully, only based on some consideration, and the excited state (B_c^* even $1\,^3S_1$) productions were not calculated at all. In this paper, we calculate all the productions of these states (the ground and excited ones) based on the mechanism and with the well-calculated wave functions in the QCD-inspired potential model framework, and present the calculations in detail.

The lowest Feynman diagrams for the process $Z^0\to B_c(B_c^*)+b+\bar{c}$ are shown in Fig. 1. As for $Z^0\to\bar{B}_c(\bar{B}_c^*)+c+\bar{b}$, the diagrams are similar but with opposite direction for each fermion line, and due to the symmetry the width of $Z^0\to B_c(B_c^*)+b+\bar{c}$ and $Z^0\to\bar{B}_c(\bar{B}_c^*)+c+\bar{b}$ are the same, so it is enough to focus on the decay $Z^0\to B_c(B_c^*)+b+\bar{c}$ only later on. Based on the Mandelstam formalism [12] the decay $Z^0\to B_c+b+\bar{c}$ amplitude corresponding to the diagrams in Fig. 1 is written as

$$M=\bar{u}_s(q_2)\int\frac{d^4q}{(2\pi)^4}\{\Lambda_1+\Lambda_2+\Lambda_3+\Lambda_4\}g_s^2 \times\frac{e}{\sin\theta_W\cos\theta_W}\frac{4}{3\sqrt{3}}v_{s'}(q_1) , \tag{2}$$

where s,s' are the spin projections of the quark and the antiquark; $\sin\theta_W$ is the electroweak mixing angle; g_s is the coupling constant of QCD; e is the charge of a posi-

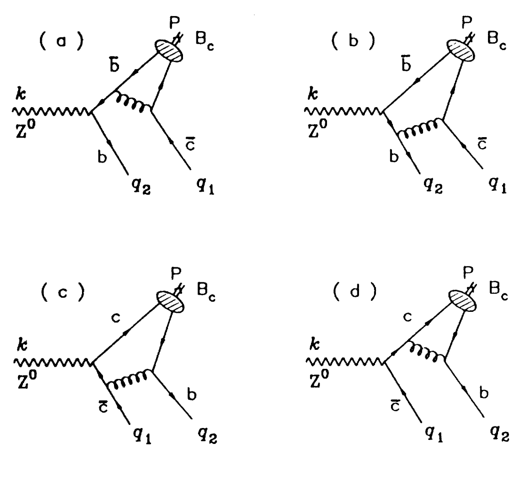

FIG. 1. The Feynman diagrams for $Z^0\to B_c+b+\bar{c}$ decay. (a) The virtual gluon is emitted by the antiquark $\bar{b}$ and creates a pair of $c\bar{c}$ quarks. (b) The virtual gluon is emitted by the quark b and creates a pair of $c\bar{c}$ quarks. (c) The virtual gluon is emitted by the anitquark $\bar{c}$ and creates a pair of $b\bar{b}$ quarks. (d) The virtual gluon is emitted by the quark c and creates a pair of $b\bar{b}$ quarks.

tron; $\Lambda_n (n=1,2,3,4)$ are the factors

$$
\begin{aligned}
\Lambda_1 &= \not{\epsilon}_z \Gamma_{z\bar{b}} \frac{\not{q}_2 - \not{k} + m_b}{(q_2-k)^2 - m_b^2} \gamma^\mu \frac{\chi(q)}{(p_1+q_1)^2} \gamma_\mu \,, \\
\Lambda_2 &= \gamma^\mu \frac{\not{k} - \not{p}_2 + m_b}{(k-p_2)^2 - m_b^2} \not{\epsilon}_z \Gamma_{z\bar{b}} \frac{\chi(q)}{(p_1+q_1)^2} \gamma^\mu \,, \\
\Lambda_3 &= \gamma^\mu \frac{\chi(q)}{(p_2+q_2)^2} \not{\epsilon}_z \Gamma_{zc} \frac{\not{p}_1 - \not{k} + m_c}{(p_1-k)^2 - m_c^2} \gamma_\mu \,, \\
\Lambda_4 &= \gamma^\mu \frac{\chi(q)}{(p_2+q_2)^2} \gamma_\mu \frac{\not{P} + \not{q}_2 + m_c}{(P+q_2)^2 - m_c^2} \not{\epsilon}_z \Gamma_{zc} \,;
\end{aligned}
\tag{3}
$$

here $\chi(q)$ is the Bethe-Salpeter (BS) wave function of the B_c meson, ϵ_z is the polarization vector of Z^0 boson, and the coupling matrices $\Gamma_{z\bar{b}} \equiv \frac{1}{4} - \frac{1}{3}\sin^2\theta_W - \frac{1}{4}\gamma_5$ and $\Gamma_{zc} \equiv \frac{1}{4} - \frac{2}{3}\sin^2\theta_W - \frac{1}{4}\gamma_5$ correspond to those of Z^0 to $\bar{b}$ quark and to c quark, respectively. The vertex $\Gamma_{z\bar{b}}$, but not Γ_{zb}, should be employed, due to a $\bar{b}$ but no b quark being inside the B_c meson. However, with respect to the decay $Z^0 \to \bar{B}_c + c + \bar{b}$, the situation is reversed, i.e., Γ_{zb} and $\Gamma_{z\bar{c}}$ will replace $\Gamma_{z\bar{b}}$ and Γ_{zc}, and it is easy to check the general relation $\Gamma_{z\bar{q}} = -\Gamma_{zq}$. In Eq. (3), k, P, q_1 and q_2 are the momenta of the Z^0 boson, $B_c(\bar{B}_c)$ meson, $\bar{c}(c)$ quark and $b(\bar{b})$ quark, respectively; q is the relative momentum between the two heavy quarks inside the B_c (or $\bar{B}_c$) meson. As the two quarks have different masses inside the bound state, their momenta have the relations $p_1 = [m_c/(m_b+m_c)]P - q$ and $p_2 = [m_b/(m_b+m_c)]P + q$; here p_1 and p_2 are the momenta of c and $\bar{b}$ quarks in the bound state. Similarly, the above formulas are also suitable for the B_c^* or $\bar{B}_c^*(J^P=1^-, {}^3S_1)$ state and other excited states. Under the instantaneous and nonrelativistic approximation [13] the BS wave function $\chi(q)$ of the heavy-quark pair system may be written as follows [in the c.m. system (c.m.s) of the meson, $\mathbf{P}=0$]:

$$
\chi(q) = \frac{a-b}{(q_0-a_+)(q_0-b_-)} [\gamma_5 \alpha + \not{\epsilon}(\mathbf{P})\beta] \times \frac{1+\gamma_0}{2\sqrt{2}} \varphi(|\mathbf{q}|)\sqrt{2M} \,,
\tag{4}
$$

where $\alpha=1$, $\beta=0$ for the pseudoscalar one (${}^1S_0$), or $\alpha=0$, $\beta=1$ for the vector ones (${}^3S_1$); $\epsilon_\mu(\mathbf{P})$ is the polarization of the meson;

$$
\begin{aligned}
a_+ &= m_c - \sqrt{\mathbf{q}^2 + m_c^2} + i\epsilon \,, \\
b_- &= -m_b - \sqrt{\mathbf{q}^2 + m_b^2} - i\epsilon \,,
\end{aligned}
\tag{5}
$$

are the poles of the BS wave function with respect to the zeroth component of the relative momentum; $\varphi(|\mathbf{q}|)$ is the solution of the Schrödinger equation induced by the BS one under the adopted approximation; M is the mass of the bound state.

In this paper, as the authors of Refs. [8,9] did, we ignore the dependence on the relative momentum q in the expression of Λ_n $(n=1,\ldots,4)$. It may be considered as the lowest approximation as pointed out in Ref. [14], but we will treat it more carefully elsewhere [15]. Thus the integration over the relative momentum q may be carried out first with the formula

$$
\int \chi(q) \frac{d^4 q}{(2\pi)^4} = (\gamma_5 \alpha + \not{\epsilon}\beta) \frac{\not{P} + M}{2\sqrt{M}} \varphi(0) \,.
\tag{6}
$$

Therefore, the amplitude of the decay turns out to be proportional to $\varphi(0)$, the wave function at the origin. To calculate it and the energies of the bound states for the $\bar{b}c$ (or $b\bar{c}$) system, we have tried two kinds of different parametrized potentials [3,5], each of which has correct QCD properties: a Coulomb-like one at short distance coinciding with two-loop asymptotic QCD behaviors, and a linear confinement one at long distance coinciding with the lattice calculations, but they have slightly different behavior from each other at "intermediate" distances and quite different fitting parameters $\Lambda_{\overline{\rm MS}}$ of QCD, where $\overline{\rm MS}$ denotes the modified minimal subtraction scheme. Both of these potentials fit the $c\bar{c}$ and $b\bar{b}$ data very well. According to a semi-classical estimation, under the threshold there are two S-wave bound states only (the ground one and the first radius excited one). By solving the Schrödinger equation numerically we obtain the energies of the bound states and their wave functions at origin, respectively (see Table I).

One may see that the energies $E(1S)$ and $E(2S)$ of the bound states are insensitive to the adopted potentials, but the wave functions at the origin are slightly sensitive to it within about ten percent. Note here that when fitting the experimental $c\bar{c}$ and $b\bar{b}$ spectra, the quark masses $m_c = 1.478$ GeV and $m_b = 4.878$ GeV are determined in potential II.

The calculation of the process, involving production of a pair of heavy quarks and a bound state, is very complicated and lengthy, if a common method, such as calculating the squared absoluted amplitude, is used. To shorten the calculations, we propose a method to calculate the polarized amplitude first, and then obtain the square of the absolute value of the amplitude. In fact, our method is similar to the one proposed by Kleiss and Stirling [16]. The difference is that in our method the amplitude is calculated by taking the trace of the Dirac γ matrix string while in Ref. [16] the so-called spin product needs to be

TABLE I. Prediction of the energies and wave functions at origin of the $b\bar{c}$ bound states.

The Model	Potential I [3]	Potential II [5]	
$\Lambda_{\overline{\rm MS}}$ (MeV)	510	100	200
$E(1S)$ (GeV)	6.34	6.34	6.35
$\varphi_{1s}(0)$ (GeV$^{3/2}$)	.369	.344	.346
$E(2S)$ (GeV)	6.91	6.91	6.91
$\varphi_{2S}(0)$ (GeV$^{3/2}$)	.281	.258	.258

calculated. The method and the formulas are presented in the Appendix explicitly.

III. NUMERICAL CALCULATIONS AND RESULTS

Using the formulas given in the Appendix, we can carry out the numerical calculation of the decay width of the process. In the numerical calculation, we take the parameters as follows:

$$\alpha_s=0.15\ ,\quad \sin^2\theta_W=0.225\ ,\quad M_Z=91.18\ \text{GeV}\ . \tag{7}$$

Finally, we obtain the partial width of the decay $Z^0\to B_c+b+\bar{c}$:

$$\Gamma(Z^0\to B_c+b+\bar{c})=50.9\ \text{keV}\ . \tag{8}$$

Similarly, we also calculate the partial widths for Z^0 decay into $1\,^3S_1$, $2\,^1S_0$, and $2\,^3S_1$ of the $b\bar{c}$ bound states associated with the same two jets. All of the results are listed in Table II [20].

Our results confirm that of the "exact" one obtained by the algebraic manipulation program REDUCE and the Monte Carlo integration routine VEGAS in Ref. [6] when taking the same values of the parameters in the "exact" numerical calculation. However, the authors of Ref. [6] evaluated one kind of fragmentation function under the so-called "lowest twist" approximation. Nevertheless, in the procedure of the calculations, we have found that some "high twist" terms proportional to M^2 or $d\equiv M^2/s\ll 1$ (here $s\equiv M_z^2$) have a higher-order singularity when $z\equiv(2q_1\cdot k)/M_z^2$ approaches 1, and after completing the phase space integration once (over the variable z), they contribute significantly. To illustrate this point in detail, let us write down the double differential width of the process:

$$\frac{d^2\Gamma}{dx\,dz}=\frac{4\alpha\alpha_s^2 g_b^2|\psi(0)|^2}{27M_zM\sin^2\theta_W\cos^2\theta_W}\left[\frac{W_0^{(p)}}{(1-z)^2}+\frac{W_1^{(p)}d}{(1-z)^3}+\frac{W_2^{(p)}d^2}{(1-z)^4}+\cdots\right], \tag{9}$$

where

$$W_0^{(p)}=\frac{(1+a_1x)^2(1-x)}{a_1^2(1-a_2x)^2}\ ,$$
$$W_1^{(p)}=\frac{(a_2-a_1)(1-a_2x)(2-x)-4a_1x}{a_1^2(1-a_2x)}\ , \tag{10}$$
$$W_2^{(p)}=\frac{4a_2}{a_1}\ ,$$

with $a_1=m_c/M$, $a_2=m_b/M$, $x=(2P\cdot k)/M_z^2$, and $g_b^2\equiv g_v^2+g_a^2=(\frac{1}{4}-\frac{1}{3}\sin^2\theta_W)^2+(\frac{1}{4})^2$. In Eq. (9), we have ignored the terms of which the "index" [the power of d plus that of $(1-z)$] is larger than -2. Note that the contribution from Figs. 1(c) and 1(d) is much smaller because to create a pair of b quarks as in Figs. 1(c) and 1(d) is much more difficult than creation of c quarks as in Figs. 1(a) and 1(b) due to the exchange of one virtual gluon, so we neglect Figs. 1(c) and 1(d) altogether in the calculations. The variables x, z are bounded by $0<x,z<1$ and in the limit of $d\ll 1$, the factor $(1-z)_{\min}\sim d$. After a straight- forward calculation, up to the first order of d for a fixed x, the upper and lower integration bounds over z may be written down:

$$z_1=1-\frac{(1-a_2x)^2d}{x(1-x)}+\cdots\ ,$$
$$z_2=1-x+\frac{(1-a_1x)^2d}{x(1-x)}+\cdots\ . \tag{11}$$

Having integrated over the variable z, each of the three terms in the large parenthesis of Eq. (9) will contribute the same order of magnitude when $d\ll 1$. Namely, using Eqs. (9), (10), and (11) and completing the integration over z, the distribution of the B_c meson versus x,

$$\frac{d\Gamma}{dx}=\frac{4\alpha\alpha_s^2 g_b^2|\psi(0)|^2}{27MM_z\sin^2\theta_W\cos^2\theta_W}(F_0^{(p)}+F_1^{(p)}+F_2^{(p)})\ , \tag{12}$$

where

$$F_0^{(p)}=\frac{(1+a_1x)^2(1-x)^2x}{(1-a_2x)^4d}\ ,$$
$$F_1^{(p)}=\frac{[(a_2-a_1)(1-a_2x)(2-x)-4a_1x](1-x)^2x^2}{2(1-xa_2)^5d}\ . \tag{13}$$
$$F_2^{(p)}=\frac{-4a_1a_2(1-x)^3x^3}{3(1-a_2x)^6d}\ ,$$

is obtained; here $F_0^{(p)}$, $F_1^{(p)}$ and $F_2^{(p)}$ correspond to the contributions from $W_0^{(p)}$, $W_1^{(p)}$, and $W_2^{(p)}$, respectively, and these three terms have the same order of magnitude as mentioned above when $d\ll 1$. The authors of Ref. [6] referred to $W_1^{(p)}, W_2^{(p)}$ as the "high twist" terms and ignored them so that only $F_0^{(p)}$ is contained in their results.

We plot the partial width of the process $Z^0\to B_c+\bar{b}+c$ versus x with various approaches in Fig. 2, respectively, for comparison. These curves correspond to the "exact result" (the solid line) [in the numerical calculation with no approximation applying to the phase space integration and the amplitude calculation in addition to that of the wave function at origin as in Eq. (6)], the "approximate one" given by Eqs. (12), (13) (the dash-dotted line), and the "lowest twist approximate one" in

TABLE II. The decay widths for the Z^0 to the S-wave bound states of $\bar{b}c$ associated with two jets.

Process	Width (keV)
$Z^0\to(\bar{b}c)_{1\,^1S_0}+\bar{c}+b$	50.9
$Z^0\to(\bar{b}c)_{1\,^3S_1}+\bar{c}+b$	66.5
$Z^0\to(\bar{b}c)_{2\,^1S_0}+\bar{c}+b$	28.5
$Z^0\to(\bar{b}c)_{2\,^3S_1}+\bar{c}+b$	37.1

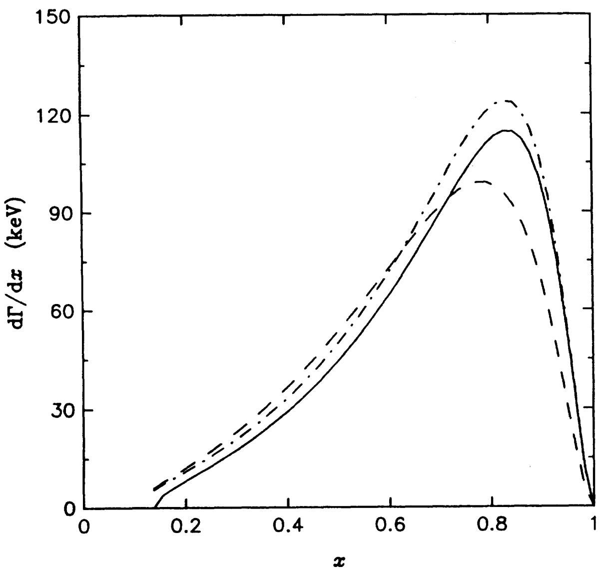

FIG. 2. The distribution of producing B_c (or $\bar{B}_c$) meson versus x for the cases of the "exact result" (solid line); the "approximate result" (dash-dotted-line) and the "lowest twist result" (dashed line).

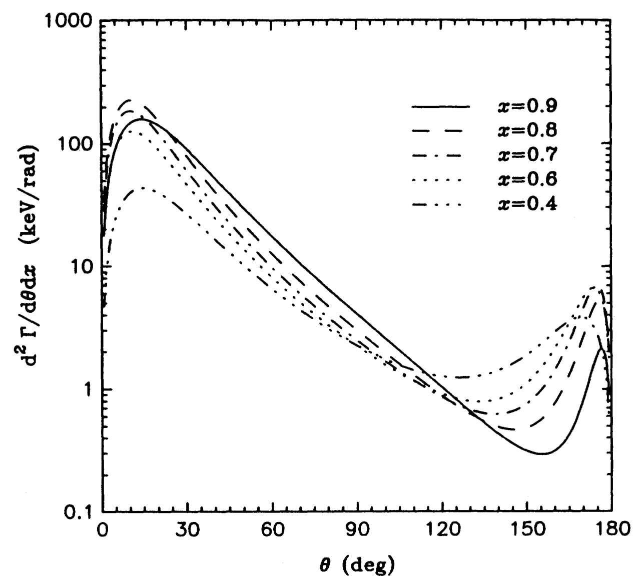

FIG. 3. The distribution for the process $Z^0 \rightarrow B_c + b + \bar{c}$ versus the angle θ (see the definition in text).

Ref. [6] (the dashed line); here we have taken the same decay constant as obtained by the potential model (according to the convention of Ref. [6], it happened to take $f_{B_c} \simeq 0.28$ GeV, but here in this paper $f_{B_c} \simeq 0.24$ GeV when translating the wave function at origin into the decay constant). From Fig. 2, it is easy to see the approximate result here is closer to the "exact one" than that of the lowest twist one in Ref. [6] (the precision is expected to be within 5% under this approximation, while it is about 20% in the "lowest twist" approximation [6]). Denoting θ as the angle between the B_c meson and the $\bar{c}$ quark jet in the c.m.s. of the boson Z^0, the distribution of the B_c production versus θ at given x is also calculated and shown in Fig. 3, and one may see that the main contribution comes from the small angle θ.

Following Ref. [6], we can extract the hadronization (fragmentation) function. The hadronization function $B_{\bar{b}}^{B_c}(x)$ for the $\bar{b}$ quark into the meson B_c may be defined

$$D_{\bar{b}}^{B_c}(x) \equiv \frac{1}{\Gamma} \frac{d\Gamma}{dx} .$$

From Eqs. (12), (13), it follows that

$$D_{\bar{b}}^{B_c}(x) \propto \frac{x(1-x)^2}{(xa_2-1)^6} \{3[(a_2-a_1)(1-a_2x)(2-x)-12a_1x](1-a_2x)x+6(1+xa_1)^2(1-a_2x)^2-8a_1a_2x^2(1-x)\} , \tag{14}$$

whereas when only $W_0^{(p)}$ was taken as in Ref. [6], the hadronization function was

$$D_{\bar{b}}^{B_c}(x) \propto \frac{x(1-x)^2(1+a_1x)^2}{(1-a_2x)^4} . \tag{15}$$

In the same way, we can evaluate the distribution and the hadronization function for the production of B_c^*, the vector state ${}^3S_1$. We can also write down the differential width of the process as

$$\frac{d^2\Gamma}{dx\,dz} = \frac{4\alpha\alpha_s^2|\psi(0)|^2}{27M_zM\sin^2\theta_W\cos^2\theta_W} \times \left[\frac{W_0^{(v)}}{(1-z)^2} + \frac{W_1^{(v)}d}{(1-z)^3} + \frac{W_2^{(v)}d^2}{(1-z)^4} + \cdots \right] , \tag{16}$$

where

$$W_0^{(v)} = \frac{[(1+xa_1)^2+2x^2](1-x)}{(1-xa_2)^2a_1^2} ,$$

$$W_1^{(v)} = -\frac{2x^2a_2^2-3x^2a_2+4xa_2^2+4xa_2-9x-4a_2+6}{(1-xa_2)a_1^2} , \tag{17}$$

$$W_2^{(v)} = -12\frac{a_2}{a_1} .$$

In Eq. (16) we once more keep the only terms of which the index is not larger than -2. Furthermore, in the following the upper and lower bounds of the integration over z are cut approximately; i.e., we will take only the first-order terms of d for given x, just as in Eq. (11).

Similarly, having integrated over the variable z, the three terms in large parentheses of Eq. (16) will contribute the same order of magnitude in d in the present case $d \ll 1$, i.e., with Eqs. (11), (16), and (17) the partial width is

$$\frac{d\Gamma}{dx}=\frac{4\alpha\alpha_s^2|\psi(0)|^2}{27MM_z\sin^2\theta_W\cos^2\theta_W}(F_0^{(v)}+F_1^{(v)}+F_2^{(v)}) , \tag{18}$$

where

$$\begin{aligned} F_0^{(v)}&=\frac{[(1+a_1x)^2+2x^2]x(1-x)^2}{r^2(1-xa_2)^4a_1^2d} , \\ F_1^{(v)}&=\frac{(2x^2a_2^2-3x^2a_2+4xa_2^2+4xa_2-9x-4a_2+6)x^2(x-1)^2}{2r^2(1-a_2x)^5a_1^2d} \\ F_2^{(v)}&=-\frac{4a_2x^3(1-x)^3}{(xa_2-1)^6a_1d} . \end{aligned} \tag{19}$$

Here $F_0^{(v)}$, $F_1^{(v)}$ and $F_2^{(v)}$ correspond to the contributions from $W_0^{(v)}$, $W_1^{(v)}$, and $W_2^{(v)}$, respectively. These three terms have the same order contributions for $d \ll 1$ as the case of $^1S_0$ and again it is not so accurate when only the "lowest twist" term $W_0^{(v)}$ remains.

For comparison, we draw the partial width of the process $Z^0 \to B_c^*(^3S_1)+\bar{b}+c$ versus x in Fig. 4 with various approaches. The curves correspond to the "exact result" (the solid line), the "approximate one" by Eqs. (18) and (19) (the dashed-dotted line), and the "lowest twist one" (the dashed line), respectively. From Fig. 4, it is easy to see that the present approximate result is closer to the accurate one.

Using Eqs (18) and (19) we can also extract the behavior of the hadronization function $D_{\bar{b}}^{B_c^*}(x)$ for the production of the vector meson B_c^*:

$$\begin{aligned} D_{\bar{b}}^{B_c^*}(x) \propto \frac{x(1-x)^2}{(1-xa_2)^6}\{&(2x^2a_2^2-3x^2a_2+4xa_2^2+4xa_2-9x-4a_2+6) \\ &\times(xa_2-1)x+2[(1+a_1x)^2+2x^2](1-xa_2)^2-8(1-x)a_1a_2x^2\} . \end{aligned} \tag{20}$$

In this paper, we also calculate the process $e^++e^- \to B_c+b+\bar{c}$ (or $e^++e^- \to \bar{B}_c+c+\bar{b}$) off the mass shell of the Z^0 boson, by taking into account one photon and one Z^0-boson exchange diagrams. The total cross section σ ($\sigma \le 2.0\times10^{-3}$ pb), is shown in Fig. 5 within an energy range $\sqrt{s}=15-60$ GeV (corresponding to the energies such as that of the collider TRISTAN), and the result shows that is too small to be observed. The cross section of the production at $\sqrt{s}\sim 200$ GeV (LEP II energy) is also calculated, which is about 10^{-2}

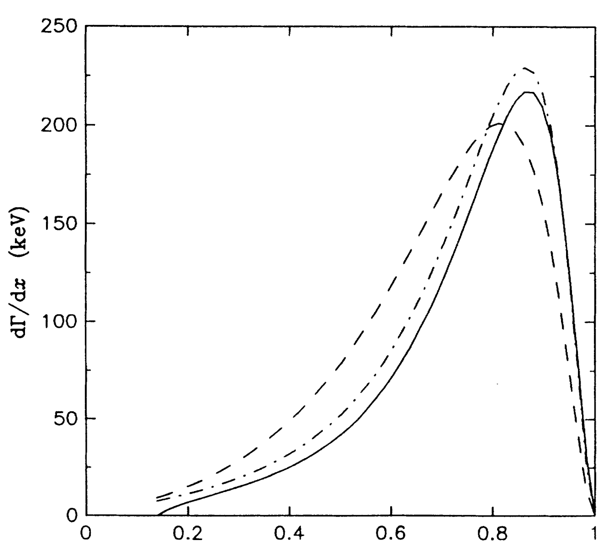

FIG. 4. The distribution of producing B_c^* (or $\bar{B}_c^*$) meson versus x for the cases of the "exact result" (solid line); the "approximate result" (dash-dotted-line) and the "lowest twist result" (dashed line).

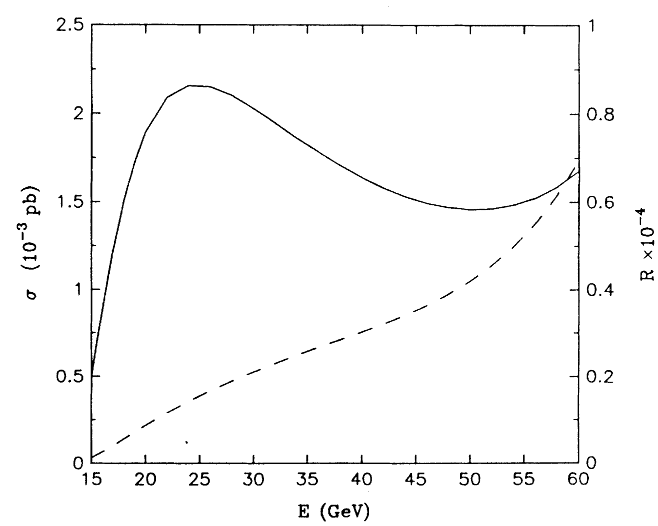

FIG. 5. The total cross section $\sigma(e^-e^+ \to B_c+b+\bar{c})$ (solid line) and $R \equiv \sigma(e^-e^+ \to B_c+b+\bar{c})/\sigma(e^-e^+ \to \mu^-\mu^+)$ (dashed line) versus E (GeV). The energy is in the center of mass system.

pb, so it is also too small to produce the mesons numerously at LEP II.

IV. DISCUSSIONS

The B_c and $\bar{B}_c$ mesons, being the ground state and carrying two different kinds of heavy flavors, are stable to the strong and electromagnetic interactions but not to weak interactions, i.e., they may decay to a lighter flavor by weak interaction only so that they happen to have a typical (quite long) lifetime. As for the excited bound states in $1\,^3S_1$, $2\,^1S_0$, and $2\,^3S_1$ of the $(b\bar{c})$ or $(\bar{b}c)$ systems, they are not stable to strong and electromagnetic interactions so that they can decay into the ground state, B_c or $\bar{B}_c$ meson, by emitting π, η, etc. mesons and/or by emitting photon(s) quickly and dominantly (with a branching ratio almost 100%) by means of the interactions. Therefore we should take into account the indirect production of the B_c or $\bar{B}_c$ meson when we estimate the total production rate. As for the P-wave production in Z^0 decay, the amplitude is suppressed due to a factor of the derivative of the P-wave function at origin over its mass instead of the wave function at origin for an S-wave production, which is much smaller; hence, we ignore the P-wave excited state contributions here.

Based on the above argument, the total number of B_c or $\bar{B}_c$ produced in Z^0 decay should be the number of the direct and indirect (from the decay of excited states) productions summed up. Thus from Table II, the full "branching ratio" for B_c production (direct plus indirect) in Z^0 decay should be $R^* \simeq 7.2\times 10^{-5}$ (here Γ_Z, the width of Z^0, $\simeq 2.53$ GeV, is used). This also applies to the $\bar{B}_c$ meson. As about 10^6 Z^0 events have been accumulated at the LEP collider, this means that ideally about 140 events involving at least one B_c or $\bar{B}_c$ have been accumulated already. In the near future there is the possibility to accumulate 10^7 Z^0 events at LEP I before shifting to LEP II. Thus about one thousand and four hundred of B_c and $\bar{B}_c$ meson samples may be accumulated; this may make it possible to discover the mesons B_c and $\bar{B}_c$ and even to study their main properties experimentally. We also study the properties of the B_c and $\bar{B}_c$ mesons, and will present them in detail elsewhere [15].

At LEP I, if vertex detectors are installed, it should not be very difficult to identify the B_c or $\bar{B}_c$ mesons for the following reasons: (a) The B_c or $\bar{B}_c$ meson is charged. (b) The lifetime of the meson B_c ($\bar{B}_c$) is not too short. One may estimate the lifetime of the meson based on the present understanding of the heavy-quark meson decay, such as for D and B meson decays; i.e., one may think that the spectator mechanism is dominant and may ignore the contributions from annihilation, internal W-emission, horizontal W loop (penguin), W exchange, and vertical W loop etc. Thus for a rough estimate on the B_c or $\bar{B}_c$ meson, according to the mechanism, each of the two components of the meson c or $\bar{c}$ and $\bar{b}$ or b may play a role in the decay and the other act as a spectator, so the lifetime should be

$$\frac{1}{\tau_{B_c}} \simeq \frac{1}{\tau_b} + \frac{1}{\tau_c} \,, \tag{21}$$

where τ_b and τ_c are the lifetime for b quark and c quark respectively. The lifetime τ_b and τ_c may be estimated based on the same mechanism and from the measurements on the lifetime of the mesons B^+ and D^0, τ_B and τ_D, respectively [17]. Note here that we ignore the contribution from the annihilation mechanism, however, by considering it the lifetime will be slightly shortened by less than ten percent [15]. Based on Eq. (21) and the latest measurements of τ_B and τ_D [18,19],

$$\tau_B \simeq 1.28\times 10^{12}\ \mathrm{s}\,, \quad \tau_D \simeq 0.421\times 10^{-12}\ \mathrm{s}\,,$$

we have

$$\tau_{B_c} \simeq 0.32\times 10^{-12}\ \mathrm{s}\,.$$

Because of a large Lorentz delay effect of the production in Z^0 decay, it is probably practical to measure the lifetime with "flight" method. (c) The decays of the B_c and $\bar{B}_c$ mesons, produced in Z^0 decay, have a very special characteristic. First, one possible feature is that one component of the $B_c(\bar{B}_c)$ meson, $\bar{b}(b)$ or $c(\bar{c})$, decays and it may form a decay vertex first; then when the other component $c(\bar{c})$ or $\bar{b}(b)$ decays, it will form the second decay vertex; finally the decay product of the $\bar{b}(b)$ probably is $\bar{c}(c)$ and it may form the third decay vertex; thus, the B_c (or $\bar{B}_c$) event manifests three decay vertices: two of them present a cascade one, and the other a single one in the vertex detector. Another feature is that the B_c ($\bar{B}_c$) meson decays into J/ψ [7,15] plus something else with a quite large branching ratio, and the produced J/ψ may be used as a trigger signal in experiments to identify the mesons B_c and $\bar{B}_c$. Obviously, in this feature the event manifests one vertex in the vertex detector. (d) The invariant mass of B_c or $\bar{B}_c$ is about 6.34 GeV, if the invariant mass is measured with enough accuracy, and it also may be considered as an experimental characteristic feature.

In conclusion we confirm that the mesons B_c and $\bar{B}_c$ are accessible to experiments at LEP soon.

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APPENDIX

Now let us illustrate the method for calculating the squared amplitude under the spirit of Ref. [16].

In general, the amplitude for the process in which a massive fermion with spin projection s and anti-fermion with spin projection s' are involved can be written as

$$M_{ss'} = \bar{u}_s(q_1) A v_{s'}(q_2) \,, \tag{A1}$$

where q_1 and q_2 denote the momenta of the fermion and the anti-fermion with masses m_1 and m_2, respectively, and A is an explicit string of Dirac γ matrices for the process; it can read out from Eqs. (2) and (3) in our process $Z^0 \to B_c + b + \bar{c}$.

Let us introduce a massless spinor $u_-(k_0)$ with a lightlike momentum k_0 and negative helicity first. Thus $u_-(k_0)$ is satisfied with the following projection relation:

$$u_-(k_0)\bar{u}_-(k_0) = \omega_- \not{k}_0 \ , \tag{A2}$$

where $\omega_- = (1-\gamma_5)/2$. By introducing another spacelike vector k_1 which satisfies the relations

$$k_1 \cdot k_1 = -1 \ , \quad k_0 \cdot k_1 = 0 \ , \tag{A3}$$

then the other, massless, independent and positive helicity spinor $u_+(k_0)$ may be constructed:

$$u_+(k_0) = \not{k}_1 u_-(k_0) \ , \tag{A4}$$

It is easy to check that $u_+(k_0)$ is satisfied with the projection

$$u_+(k_0)\bar{u}_+(k_0) = \omega_+ \not{k}_0 \ , \tag{A5}$$

where $\omega_+ = (1+\gamma_5)/2$.

Using these massless spinors, one can construct the massive spinors for the fermion and antifermion as follows:

$$\begin{aligned}
u_s(q) &= (\not{q} + m)u_-(k_0)/\sqrt{2k_0 \cdot q} \ , \\
u_{-s}(q) &= (\not{q} + m)u_+(k_0)/\sqrt{2k_0 \cdot q} \ , \\
v_s(q) &= (\not{q} - m)u_-(k_0)/\sqrt{2k_0 \cdot q} \ , \\
v_{-s}(q) &= (\not{q} - m)u_+(k_0)/\sqrt{2k_0 \cdot q} \ ,
\end{aligned} \tag{A6}$$

with the spin vector s_μ:

$$s_\mu = \frac{q_\mu}{m} - \frac{m}{q \cdot k_0} k_{0\mu} \ . \tag{A7}$$

Using the above identities, we can write down the amplitude $M_{\pm s \pm s'}$ with four possible spin projections in the trace form for the γ matrices:

$$\begin{aligned}
M_{ss'} &= N \,\mathrm{tr}[\not{q}_2 - m_2)\omega_- \not{k}_0(\not{q}_1 + m_1)A] \ , \\
M_{-s-s'} &= N \,\mathrm{tr}[(\not{q}_2 - m_2)\omega_+ \not{k}_0(\not{q}_1 + m_1)A] \ , \\
M_{-ss'} &= N \,\mathrm{tr}[(\not{q}_2 - m_2)\omega_- \not{k}_0 \not{k}_1(\not{q}_1 + m_1)A] \ , \\
M_{s-s'} &= N \,\mathrm{tr}[(\not{q}_2 - m_2)\not{k}_1 \omega_- \not{k}_0(\not{q}_1 + m_1)A] \ ,
\end{aligned} \tag{A8}$$

with the normalization constant

$$N = 1/\sqrt{4(k_0 \cdot q_1)(k_0 \cdot q_2)} \ .$$

Thus the unpolarized of the matrix elements squared, $|M|^2$ can be written as

$$|M|^2 = |M_{s,s'}|^2 + |M_{-s,-s'}|^2 + |M_{-s,s'}|^2 + |M_{s,-s'}|^2 \ . \tag{A9}$$

From now on we are to simplify the calculation. First we recombine these $M_{\pm s \pm s'}$ into M_n $(n=1,\ldots,4)$ as follows:

$$\begin{aligned}
M_1 &= \frac{1}{\sqrt{2}}(M_{ss'} + M_{-s-s'}) \ , \\
M_2 &= \frac{1}{\sqrt{2}}(M_{ss'} - M_{-s-s'}) \ , \\
M_3 &= \frac{1}{\sqrt{2}}(M_{s-s'} - M_{-ss'}) \ , \\
M_4 &= \frac{1}{\sqrt{2}}(M_{s-s'} - M_{-ss'}) \ ,
\end{aligned} \tag{A10}$$

Using Eqs. (A9), (A10), and (A8), $|M|^2$ can be rewritten as

$$|M|^2 = |M_1|^2 + |M_2|^2 + |M_3|^2 + |M_4|^2 \ , \tag{A11}$$

and M_n $(n=1,\ldots,4)$ may be expressed as

$$\begin{aligned}
M_1 &= \frac{1}{\sqrt{2}} N \,\mathrm{tr}[(\not{q} - m_2)\not{k}_0(\not{q}_1 + m_1)A] \ , \\
M_2 &= \frac{1}{\sqrt{2}} N \,\mathrm{tr}[(\not{q}_2 - m_2)\gamma_5 \not{k}_0(\not{q}_1 + m_1)A] \ , \\
M_3 &= \frac{1}{\sqrt{2}} N \,\mathrm{tr}[(\not{q}_2 - m_2)\not{k}_0 \not{k}_1(\not{q}_1 + m_1)A] \ , \\
M_4 &= \frac{1}{\sqrt{2}} N \,\mathrm{tr}[(\not{q}_2 - m_2)\gamma_5 \not{k}_1 \not{k}_0(\not{q}_1 + m_1)A] \ .
\end{aligned} \tag{A12}$$

In order to write down A as explicitly and simply as possible, let us construct the vector k_0:

$$k_0 = q_1 - \alpha q_2 \ , \tag{A13}$$

where the coefficient α is determined by the requirement that k_0 be a lightlike vector:

$$\alpha = \frac{q_1 \cdot q_2}{m_2^2} \pm \frac{\Delta}{m_2^2} \ , \tag{A14}$$

with $\Delta = \sqrt{(q_1 \cdot q_2)^2 - m_1^2 m_2^2}$.

Furthermore, if choosing k_1,

$$k_1 \cdot q_1 = 0 \ , \quad k_1 \cdot q_2 = 0 \ , \tag{A15}$$

Eq. (A3) is satisfied precisely. With Eqs. (A13) and (A14), the resultant M_n $(n=1,\ldots,4)$ can be simplified as

$$\begin{aligned}
M_1 &= L_1 \mathrm{tr}[(\not{q}_2 - m_2)(\not{q}_1 + m_1)A] \ , \\
M_2 &= L_2 \mathrm{tr}[(\not{q}_2 - m_2)\gamma_5(\not{q}_1 + m_1)A] \ , \\
M_3 &= -L_2 \mathrm{tr}[(\not{q}_2 - m_2)\not{k}_1(\not{q}_1 + m_1)A] \ , \\
M_4 &= -L_1 \mathrm{tr}[(\not{q}_2 - m_2)\gamma_5 \not{k}_1(\not{q}_1 + m_1)A] \ ,
\end{aligned} \tag{A16}$$

where

$$2L_{1,2} = \frac{1}{\sqrt{q_1 \cdot q_2 \mp m_1 m_2}} \ .$$

Generally speaking, an arbitrary vector k_1 will cause the Lorentz structure of the amplitude extra complication. To avoid this, one may choose k_1 explicitly as follows:

$$k_1^\mu = i N_0 \varepsilon^{\mu\nu\rho\sigma} q_{2\nu} k_\rho q_{1\sigma} \ , \tag{A17}$$

where N_0 is a suitable normalization constant and k a reference momentum in general, but here the momentum of the Z^0 boson. According to the definition of Eq. (A17), $\not{k}_1$ can also be expressed as

$$\not{k}_1=N_0\gamma_5(q_2\cdot k\not{q}_1+q_1\cdot k\not{q}_2-q_1\cdot q_2\not{k}-\not{q}_2\not{k}\not{q}_1)\ . \quad \text{(A18)}$$

Putting Eq. (A18) into Eq. (A16), M_3 and M_4 can be rewritten as

$$\begin{aligned} M_3=&-N_0(q_2\cdot km_1+q_1\cdot km_2)M_2\\ &+N_0L_2^{-1}\mathrm{tr}[(\not{q}_2-m_2)\gamma_5\not{k}(\not{q}_1+m_1)A]\ ,\\ M_4=&-N_0(q_2\cdot km_1-q_1\cdot km_2)M_1\\ &+N_0L_1^{-1}\mathrm{tr}[(\not{q}_2-m_2)\not{k}(\not{q}_1+m_1)A]\ . \end{aligned} \quad \text{(A19)}$$

In this way, it is easy to calculate the amplitude involving massive fermions by taking the trace of the γ matrices as given by Eqs. (A16) and (A19) rather than to calculate the square of the amplitude matrix element directly, especially when the process involves many Feynman diagrams to be calculated.

Now let us write down the amplitude and its square in absolute values for the process $Z^0\to B_c+b+\bar{c}$. Denoting ϵ as the polarization vector of Z^0, and k,P,q_2 the independent momenta as illustrated in the text, the general Lorentz structure of the amplitude M_n, $(n=1,2)$ may be written as

$$M_n=\text{const}\times\left[\frac{A_n}{M_z}P\cdot\epsilon+\frac{B_n}{M_z}q_2\cdot\epsilon+\frac{C_n}{M_z^3}i\varepsilon(k,P,q_2,\epsilon)\right]\ , \quad \text{(A20)}$$

where

$$\text{const}\equiv\frac{4}{3}\frac{g_s^2e}{\cos\theta_W\sin\theta_W}\frac{\varphi(0)}{2M_Z\sqrt{M}}\ ,$$

and with the notation

$$\varepsilon(k,P,q_2,\epsilon)\equiv\varepsilon^{\mu\nu\rho\sigma}k_\mu P_\nu q_{2\rho}\epsilon_\sigma\ .$$

So $|M_n|^2$ $(n=1,2)$ can be written as

$$\begin{aligned} |M_n|^2=\tfrac{1}{4}N_n(\text{const})^2\{&U_0C_n^2-(4r^2-x^2)A_n^2\\ &+2(xz-2u)A_nB_n\\ &+(z^2-4r_2^2)B_n^2\}\ , \end{aligned} \quad \text{(A21)}$$

where

$$\begin{aligned} &U_0=(x^2/4-r^2)(z^2/4-r_2^2)-(xz/4-u/2)^2\ ,\\ &u=x+z-1-2rr_1\ ,\quad N_1=8M_z/(1-x)\ ,\\ &N_2=4M_z/[(1-x)/2+2r_1r_2]\ ,\\ &r=M/M_Z\ ,\quad r_1=m_1/M_Z\ ,\quad r_2=m_2/M_Z\ , \end{aligned}$$

and x,y,z have been defined in the text. For M_n $(n=3,4)$, the Lorentz structure corresponding to Eq. (A19) is

$$\begin{aligned} M_n=\text{const}\times\Big\{&i\frac{A_n}{M_z^4}U_0(P\cdot\epsilon)\varepsilon(k_1,P,q_2,k)+i\frac{B_n}{M_z^4}U_0(q_2\cdot\epsilon)\varepsilon(k_1,P,q_2,k)\\ &+\frac{C_n}{M_z}(k_1\cdot\epsilon)+i\frac{D_n}{M_Z^2}\varepsilon(k,q_2,k_1,\epsilon)+i\frac{F_n}{M_Z^2}\varepsilon(P,q_2,k_1,\epsilon)+i\frac{H_n}{M_Z^2}\varepsilon(k,P,k_1,\epsilon)\Big\}\ . \end{aligned} \quad \text{(A22)}$$

Also $|M_n|^2$ $(n=3,4)$ can be written as

$$\begin{aligned} |M_n|^2=\tfrac{1}{4}N_{5-n}(\text{const})^2\{&2[(xz-2u)B_n-4D_n-2F_nx]A_nU_0+2[(xz-2u)H_n-(2xr_2^2-zu)F_n]D_i\\ &-4(F_nz-2H_n)B_nU_0+2(2r^2z-xu)F_nH_n-(4r^2r_2^2-u^2-4U_0)F_n^2\\ &-(4r^2-x^2)(A_n^2U_0+H_n^2)+(z^2-4r_2^2)(B_n^2U_0+D_n^2)+4C_n^2\}\ . \end{aligned} \quad \text{(A23)}$$

Defining f_n $(n=1,\ldots,4)$ as

$$\begin{aligned} &f_1=r/[r_1(1-z)^2]\ ,\quad f_2=r/[r_1(1-z)(1-xr_2/r)]\ ,\\ &f_3=-r[r_2(1-y)(1-xr_1/r)]\ ,\quad f_4=-r/[r_2(1-y)^2]\ , \end{aligned}$$

starting with Eqs. (2) and (3), and if the result of the squared amplitude for the concerned process is written down in the form as Eqs. (A20) and (A22), the coefficients $A_n, B_n,\ldots$ $(n=1,\ldots,4)$ are found as follows:

$$\begin{aligned} A_1=(&[(f_2+f_3-f_4+f_1)r_1-(f_4-3f_1-f_2-f_3)r_2]z\\ &+2[(f_4r_1^2-f_1-f_2-f_3)r_2+2(f_4-f_1)r_2^2r_1+(f_4-2f_1)r_2^3+f_4ur_1]r\\ &-2\{2[2(f_4-f_1)r_2^2+f_4u]-(f_4-f_1)z\}r^2-2(ru-xr_2)(r_2f_2+r_1f_3)-2r^3r_2f_4)g_a/r\ , \end{aligned}$$

$$B_1=[2(2\{[(u+1)f_1-f_4r_1^2-r_1^2f_3]+(f_4-f_2)r_2^2-(f_2+f_3)r_2r_1\}+(f_4-f_1)x-2zf_1)r$$
$$-[(f_4-f_1-f_2-f_3)x-2(f_4-f_1)u-2r^2f_4]r]g_a\ ,$$
$$C_1=-2r(f_4g_v+f_1g_v'+f_2g_v'+f_3g_v)\ ,$$
$$A_2=[\{[(f_4g_v+f_1g_v'+5f_2g_v'-f_3g_v)r_2-(f_2g_v'+3f_3g_v)r_1+f_4r_1g_v-f_1r_1g_v']z$$
$$+2(f_4r_1^2g_v-f_1g_v'-2uf_2g_v'-4r_1^2f_3g_v-f_2g_v'+f_3g_v)r_2-2(f_4g_v+4f_2g_v')r_2^3$$
$$+8(f_2g_v'+f_3g_v)r_2^2r_1-2f_4ur_1g_v+4ur_1f_3g_v\}r$$
$$+2[2(2f_4g_v-f_2g_v')r_2^2-(f_4g_v-f_1g_v')z-2(2f_4-f_3)r_2r_1g_v+2f_4ug_v]r^2$$
$$+2(r_2f_2g_v'-r_1f_3g_v)xr_2+2(f_2g_v'+f_3g_v)r_2ur_1+2r^3r_2f_4g_v-2r_2^2uf_2g_v'-2ur_1^2f_3g_v]/r\ ,$$
$$B_2=-\{[(f_4g_v+3f_1g_v'+f_2g_v'-f_3g_v)r_2-(f_2g_v'-f_3g_v)r_1+f_4r_1g_v-f_1r_1g_v']x$$
$$-2[2(f_4g_v+2f_1g_v')r_2^2-4(f_4g_v+f_1g_v')r_2r_1+(f_4g_v-f_1g_v')x+2(u+1)f_1g_v'-2zf_1g_v'+2f_4r_1^2g_v]r$$
$$-2(2r^2+u)(f_4g_v+f_1g_v')r_2+4r^3f_4g_v+2f_4ur_1g_v+2f_1ur_1g_v'\}\ ,$$
$$C_2=-2[(f_2-f_3)(r_1-r_2)+(f_4+f_1)r]g_a\ ,$$
$$A_3=4(g_v'f_2+g_vf_3)r_1/r\ ,$$
$$B_3=4g_vf_4\ ,$$
$$C_3=(2\{[(f_3+f_4-f_1)r_1-(f_4-3f_1)r_2+f_2r_2]z-[(2r_2^2+u)f_3+4(f_4-f_1)r_2^2+2f_4u]r_1$$
$$-2[(u+1)f_1-f_4u]r_2+(2r_1r_2-2r_2^2-u)f_2r_2+2(f_3+f_4)r_1^2r_2-(f_4-f_1)xr_2+2(f_4-2f_1)r_2^3\}r$$
$$+[(f_4-f_1-f_3)r_1r_2-(r_1r_2-r_2^2-u)f_2-(f_3+f_1)z+(f_4+3f_1)r_2^2+f_3r_2^2]x$$
$$-[(f_2-f_3+f_4-f_1)z+4r_1r_2f_4+r_2^2f_1-f_4u]r^2-2(f_4-f_1)r_1r_2u$$
$$+(f_4-2f_1)r_2^2u+4r^3r_2f_4-zf_4u-f_2u+r_1^2f_4u+f_3u+f_4u^2+f_1u)g_a\ ,$$
$$H_3=2\{(r_1-r_2)[f_2g_v'+(f_3-f_4)g_v]+(g_v'f_1+2g_vf_4)r\}r_2\ ,$$
$$D_3=-2[(f_2-f_1)g_v'+(f_3-f_4)g_vr^2+(g_v'f_1-g_vf_3)x+2(g_v'f_1+g_vf_4)(r_1-r_2)r-g_v'f_1u]\ ,$$
$$F_3=-2[(g_v'f_1-g_vf_4)z+(f_2-f_1)g_v'+(f_3+r_2^2f_4+f_4u)g_v+r^2g_vf_4-xg_v'f_2-r_1^2g_vf_4]\ ,$$
$$A_4=4r_1(f_3-f_2)g_a/r\ ,$$
$$B_4=4f_4g_a\ ,$$
$$C_4=\{[(f_2-f_1)g_v'+(f_3-f_4)g_v]z+g_vf_4u\}r^2$$
$$-\{[(r_1r_2+r_2^2+u)f_2+r_1r_2f_1+r_2^2f_1]g_v'-(g_v'-g_vf_3)z-r(f_3+f_4)g_vr_2\}x$$
$$+2[(rg_v'f_1+r_1g_vf_4-g_vr_2f_4)z+(g_v'f_1-g_vf_4)xr_2+2(r_2^2+u)g_vr_2f_4-2g_v'r_2f_1-2r_1^2g_vr_2f_4]r$$
$$+(f_2-f_1)g_v'u+(f_3+r_2^2f_4+f_4u)g_vu+4r^3g_vr_2f_4-zg_vf_4u-r_1^2g_vf_4u\ ,$$
$$H_4=-2[(f_4+f_1+f_3)r_1-rf_2-2(f_4+f_1)r-(f_4-f_1)r_2+f_3r_2]r_2g_a\ ,$$
$$D_4=-2\{2[(f_3+f_4-f_1)r_1-(f_4-f_1)r_2+f_2r_2]r-(f_2-f_3+f_4-f_1)r^2-(f_3+f_1)x+f_1u\}g_a\ ,$$
$$F_4=2\{2[(f_3+2f_4)r_1+f_2r_2+2r_2f_1]r+(z-2r_1r_2)(f_4+f_1)$$
$$-(f_4+2f_1)r_2^2-r^2f_4-xf_2+f_2-r_1^2f_4-f_3-f_4u-f_1\}g_a\ .$$

Thus we have the formulas of the unpolarized squared amplitude by putting the coefficients A_n,B_n,C_n $(n=1,\ldots,4)$ and H_n,D_n,F_n, $(n=3,4)$ into Eqs. (A21) and (A23). Note that in the above formulas we have kept the contributions from Figs. 1(c) and 1(d) so that the vector couplings of Z^0 to the c quark and $\bar{b}$ quark are $g_v=\frac{1}{4}-\frac{2}{3}\sin^2\theta_W$ and $g_{v'}=\frac{1}{4}-\frac{1}{3}\sin^2\theta_W$ respectively with the same $g_a=-\frac{1}{4}$ occurring in the coefficients. It is a straightforward calculation from the squared amplitude to the decay width by evaluating the phase space integrations numerically, and the results are shown in Figs. 2 and 3. As for the $^3S_1$ excited state productions, the formulas are more lengthy; we would not write down them here because to deal with the change from $^1S_0$ to $^3S_1$ is straightforward for REDUCE and other algebra symbolic programs such as MACSYMA, MAPLE, MATHEMATICA, etc.

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[17] In fact there is a coherence effect that makes the lifetime of D^+ twice as long as that of D^0 and D_s. Here approximately we ignore the effect for B_c altogether, although it may play some role in those channels such as $B_c^+ \to J/\psi + D^+$, etc., so we think Eq. (9) is roughly correct when the lifetime of D^0 or D_s but not D^+ to abstract τ_c is adopted. However, if we do take the effect into account here, due to the fact that it plays a role only in the case of the $\bar{b}$-quark decay inside the B_c meson and only a half of the $\bar{b}$-quark decay diagrams have the effect, it cannot make a significant change as that in D^+ meson decay. A rough estimate on the effect shows it may make τ_{B_c} longer about within ten percent.

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[20] We should point out here that with the numerical calculation checked, we have found a misprint in Ref. [8]; i.e., the width of the decay $Z^0 \to \eta_c + c + \bar{c}$ should be about 45 keV not 145 keV [8].