# Covariant formalism for $F$-wave quarkonium production and annihilation: Application to $^3F_J \to gg$ decays

R. W. Robinett and L. Weinkauf
*Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802*
(Received 6 April 1992)

We derive a covariant formalism to describe $F$-wave quarkonium production and annihilation processes in the context of nonrelativistic potential models. We apply it to evaluate the decay rates for the processes $^3F_J \to gg$ (where $J=2,3,4$). Such a calculation may be useful in evaluating the prospects of producing the predicted narrow $F$ states in the $\Upsilon$ system. Related predictions for the two-photon widths of such $F$ states could also be used in testing the validity of nonrelativistic potential models in the light quark system. We also explicitly calculate the two-photon decay width of the $^1G_4$ state using similar techniques to compare to a recent general calculation.

PACS number(s): 13.25.+m, 12.40.Qq, 13.40.Hq, 14.40.Gx

The study of the spectroscopy of the charmonium and $\Upsilon$ systems [1] in terms of QCD-motivated potential models now has a long history. While there are many new avenues open for study in the $\psi$ and $\Upsilon$ systems (ranging from searches for exotics at the Beijing Electron-Positron Collider [2] to $CP$ violation studies on the $\Upsilon(4S)$ [3]), the search for the additional level structure predicted by otherwise successful potential models will no doubt continue to be an important aspect of heavy quarkonium physics. The recent high-precision study of the $\chi_1$ and $\chi_2$ charmonium [4] states and the apparent discovery of the $^1P_1$ state in exclusive $p\bar{p}$ collisions by the E760 Collaboration [5] suggest that searches for charmonium $D$ states in this manner may be possible and predictions for their production rates using the methods of exclusive QCD have recently appeared [6]. Other detailed studies of the production prospects of $\Upsilon$ $D$ states also exist [7].

Nonrelativistic potential models (perhaps supplemented by relativistic corrections and coupled-channel mixing effects [7,8]) can be used to predict masses and radiative decay rates for such quarkonium states are important for investigating the prospects for producing new states. Other important features which can be addressed are predictions for the annihilation decays and related processes in quarkonium production. The two-and three-gluon decays of $S$- and $P$-state quarkonia (both at the tree level [1] and including higher-order corrections [9,10]) have been used to describe the hadronic decay widths of quarkonium states while the $gg\gamma$ decays of the $\Upsilon$ [11] have been used to extract a value of $\alpha_s$. In this context, the calculation of the hadronic decay widths of $D$-state quarkonia are more recent than earlier calculations for $S$- and $P$-wave states with the rate for $^1D_2 \to gg$ appearing sometime ago [12] while the calculations for $^3D_J \to ggg$ ($J=0,1,2$) have appeared more recently [13,14].

Potential model studies indicate [7] that there should be one set of narrow (i.e., below threshold) $F$ states in the $\Upsilon$ system and it would be useful to extend the studies of Ref. [7] to investigate the production prospects of these states via radiative decays. As indicated above, a similarly important aspect of such a study would be the calculation of the hadronic decay widths of the $^1F_3$ and $^3F_J$ ($J=2,3,4$) states via their $ggg$ and $gg$ decay modes, respectively, to assess the extent to which their annihilation widths compete with their radiative decays. The calculation of $\Gamma(^3F_J \to gg)$ would also provide a first estimate of the production prospects for triplet $F$ states in hadronic collisions where the dominant subprocess would likely be $gg \to {}^3F_J$ (similarly to the case of hadronic production of $P$-wave states [15]).

Another possible motivation would be the study of light quark mesons via two-photon production. Nonrelativistic potential model predictions for the $S$-, $P$- and $D$-states two-photon widths have been compared with experimental data [16], but it seems that large relativistic corrections and/or mass effects are needed to fully reproduce the data [17]. A calculation of the two-photon widths of the triplet $F$ states would add further information. Finally, changes in trivial color factors yield the two-photon decay widths of triplet $F$-state positronium states.

Because of the computational difficulty of such calculations involving $D$ and $F$ states, a covariant formalism for describing such decays (which can then be used with popular algebraic manipulation programs) is of value and an extension of the formalism by Kühn, Kaplan, and Safiani [18] (KKS) to $D$ states exists [19] and has been used for several calculations involving $D$-state annihilation decays [14] and production processes [20]. In this paper, we extend this formalism to $F$ states and use it to calculate the two-photon decay widths of $^3F_{2,3,4}$ quarkonium states (which are then trivially related to the two gluon widths by well-known color factors). We also extend the formalism to singlet $G$ states in an attempt to explicitly reproduce a recent general result for the two-photon decay widths of spin-singlet states [17]. We begin by briefly reviewing the formalism of Ref. [18] and then extend it to $L=3$ quarkonia.

We write the amplitude for the annihilation interaction of a free fermion-antifermion pair as

$$\mathcal{A} = \bar{v}(\bar{f},\bar{s})\mathcal{O}u(f,s) , \tag{1}$$

where $f,\bar{f}$ and $s,\bar{s}$ are the particle and antiparticle momenta and spins and $\mathcal{O}$ is the relevant Dirac operator. The amplitude for a bound pair can then be described in a nonrelativistic approximation by

$$A=\left[\frac{1}{m}\right]^{1/2}\int\frac{dk}{(2\pi)^{3/2}}\psi_{s\bar{s}}(k)\bar{v}(\bar{f},\bar{s})\mathcal{O}u(f,s)\ ,\qquad(2)$$

where

$$f+\bar{f}=P=(M,0),\quad f-\bar{f}=2k=(0,2k)\ ,\qquad(3)$$

and

$$\sum_{s\bar{s}}\int dk|\psi_{s\bar{s}}|^2=1\ .\qquad(4)$$

$M$ and $m$ are the masses of the bound state and quark, respectively, and we work in the rest frame of the bound state described by the momentum space wave function $\psi_{s\bar{s}}(k)$.

In Eq. (2) we may consider spin-singlet and -triplet states. This leads to appropriate sums on $s$ and $\bar{s}$ which can be expressed in terms of traces as

$$\sum_{s\bar{s}}\bar{v}(\bar{f},\bar{s})\mathcal{O}u(f,s)\langle\tfrac{1}{2},s;\tfrac{1}{2},\bar{s}|S,S_Z\rangle=\frac{1}{\sqrt{E_{\bar{f}}+m}}\frac{1}{\sqrt{E_f+m}}\mathrm{Tr}\left[(m-\bar{f})\mathcal{O}(f+m)\frac{1+\gamma_0}{2\sqrt{2}}\Pi_{SS_Z}\right]\ ,\qquad(5)$$

with

$$\Pi_{00}=-\gamma_5,\quad \Pi_{1,S_Z}=-\not{\epsilon}_{(S_Z)}\qquad(6)$$

and $\not{\epsilon}_{(S_Z)}$ is the $S=1$ polarization vector.

Then, following Ref. [18], we expand $A$ in powers of $k/m$ to the desired order, which in the case of $F$ waves is third order in $k$. We find that Eq. (5) then reduces to

$$\frac{1}{2\sqrt{2}}\left[\frac{1}{M^2}\mathrm{Tr}[\not{k}\mathcal{O}_1^a k_a\not{k}(\not{P}+M)\Pi_{SS_Z}]+\frac{1}{2M}\mathrm{Tr}[\{\mathcal{O}_2^{ab}k_ak_b,\not{k}\}_+(\not{P}+M)\Pi_{SS_Z}]+\tfrac{1}{6}\mathrm{Tr}[\mathcal{O}_3^{abc}k_ak_bk_c(\not{P}+M)\Pi_{SS_Z}]\right]\qquad(7)$$

where $\mathcal{O}_1^a,\mathcal{O}_2^{ab},\mathcal{O}_3^{abc}$ are the first, second, and third derivatives of $\mathcal{O}$ with respect to $k^a$, respectively (with $k=0$). In Eq. (7), all the terms are of the same (leading) order in $1/M$.

For the orbital part, the integral of $k_ak_bk_c$ over $d^3k$ [the analogue of Eq. (6) of KKS] is then given by

$$\left[\frac{1}{m}\right]^{1/2}\int\frac{d^3k}{(2\pi)^{3/2}}k_ak_bk_c\psi_3^{(m)}(\mathbf{k})=be_{abc}^{(m)}\qquad(8)$$

where

$$b=\frac{i}{2}\left[\frac{35}{\pi M}\right]^{1/2}\phi'''(0)\ .\qquad(9)$$

In the above, $e_{abc}^{(m)}$ is the symmetric spin-3 polarization tensor which satisfies

$$g^{ab}e_{abc}^{(m)}=0,\quad P^ae_{abc}^{(m)}=0\qquad(10)$$

and $\phi'''(0)$ is the third derivative of the $F$-state radial wave function evaluated at the origin.

For the $S=0$ case where $J=L=3$, we directly associate $e_{abc}^{(m)}$ with $e_{abc}^{(M_J)}$ while for the spin-triplet case where $J=2$, 3, or 4, using explicit Clebsch-Gordan coefficients, we find

$J=2$:

$$e_{abc}^{(m)}e_d^{(S_Z)}\langle 31;mS_Z|2M_J\rangle=-\frac{2}{3\sqrt{35}}\{(\tilde{g}_{bc}e_{ad}^{(M_J)}+\tilde{g}_{ac}e_{bd}^{(M_J)}+\tilde{g}_{ab}e_{cd}^{(M_J)})-\tfrac{5}{2}(\tilde{g}_{cd}e_{ab}^{(M_J)}+\tilde{g}_{bd}e_{ac}^{(M_J)}+\tilde{g}_{ad}e_{bc}^{(M_J)})\}$$

where $e_{ab}$ is the symmetric spin-2 polarization tensor which satisfies

$$g^{ab}e_{ab}^{(M_J)}=0,\quad P^ae_{ab}^{(M_J)}=0\ ,\qquad(11)$$

and where the sum over polarization states is given by

$$\sum_{M_J=-2}^{2}e_{ab}^{(M_J)}e_{xy}^{(M_J)*}=\tfrac{1}{2}(\mathcal{P}_{ax}\mathcal{P}_{by}+\mathcal{P}_{ay}\mathcal{P}_{bx})-\tfrac{1}{3}\mathcal{P}_{ab}\mathcal{P}_{xy}\qquad(12)$$

where

$$\mathcal{P}_{ab}=-g_{ab}+\frac{P_aP_b}{M^2}=-\tilde{g}_{ab}\ .\qquad(13)$$

$J=3$:

$$e_{abc}^{(m)}e_d^{(S_Z)}\langle 31;mS_Z|3M_J\rangle=\frac{i}{M\sqrt{12}}(e_{abz}^{(M_J)}\epsilon_{ucdz}P^u+e_{acz}^{(M_J)}\epsilon_{ubdz}P^u+e_{bcz}^{(M_J)}\epsilon_{uadz}P^u)\ ,\qquad(14)$$

where $e_{abc}^{(M_J)}$ is the symmetric spin-3 polarization tensor. The required sum over polarization states is given by (see, e.g., Ref. [21])

$$\sum_{M_J=-3}^{3} e_{abc}^{(M_J)} e_{xyz}^{(M_J)*} = \tfrac{1}{6}\Omega_{abc;xyz}^{(1)} - \tfrac{1}{15}\Omega_{abc;xyz}^{(2)} \quad (15)$$

where

$$\Omega_{abc;xyz}^{(1)} = \mathcal{P}_{ax}\mathcal{P}_{by}\mathcal{P}_{cz} + \mathcal{P}_{ax}\mathcal{P}_{bz}\mathcal{P}_{cy} + \mathcal{P}_{ay}\mathcal{P}_{bx}\mathcal{P}_{cz} + \mathcal{P}_{ay}\mathcal{P}_{bz}\mathcal{P}_{cx} + \mathcal{P}_{az}\mathcal{P}_{by}\mathcal{P}_{cx} + \mathcal{P}_{az}\mathcal{P}_{bx}\mathcal{P}_{cy} \quad (16)$$

and

$$\begin{aligned}\Omega_{abc;xyz}^{(2)} = {} & \mathcal{P}_{ab}\mathcal{P}_{cz}\mathcal{P}_{xy} + \mathcal{P}_{ab}\mathcal{P}_{cy}\mathcal{P}_{xz} + \mathcal{P}_{ab}\mathcal{P}_{cx}\mathcal{P}_{yz} \\ & + \mathcal{P}_{ac}\mathcal{P}_{bz}\mathcal{P}_{xy} + \mathcal{P}_{ac}\mathcal{P}_{by}\mathcal{P}_{xz} + \mathcal{P}_{ac}\mathcal{P}_{bx}\mathcal{P}_{yz} \\ & + \mathcal{P}_{bc}\mathcal{P}_{az}\mathcal{P}_{xy} + \mathcal{P}_{bc}\mathcal{P}_{ay}\mathcal{P}_{xz} + \mathcal{P}_{bc}\mathcal{P}_{ax}\mathcal{P}_{yz} \; . \end{aligned} \quad (17)$$

Finally, we have

$J=4$:

$$e_{abc}^{(m)} e_{d}^{(S_Z)} \langle 31; mS_Z | 4M_J \rangle = e_{abcd}^{(M_J)} \quad (18)$$

where $e_{abcd}^{(M_J)}$ is the completely symmetric spin-4 polarization tensor which satisfies

$$g^{ab} d_{abcd}^{(M_J)} = 0, \quad P^a e_{abcd}^{(M_J)} = 0 \; . \quad (19)$$

The polarization sum is given by

$$\sum_{M_J=-4}^{4} e_{abcd}^{(M_J)} e_{wxyz}^{(M_J)*} = \frac{1}{24}\tilde{\Omega}_{abcd;wxyz}^{(1)} - \tfrac{1}{84}\tilde{\Omega}_{abcd;wxyz}^{(2)} + \tfrac{1}{105}\tilde{\Omega}_{abcd;wxyz}^{(3)} \quad (20)$$

where

$$\begin{aligned} \tilde{\Omega}_{abcd;wxyz}^{(1)} &= \mathcal{P}_{aw}\mathcal{P}_{bx}\mathcal{P}_{cy}\mathcal{P}_{dz} + \text{permutations} \; , \\ \tilde{\Omega}_{abcd;wxyz}^{(2)} &= \mathcal{P}_{ab}\mathcal{P}_{wx}\mathcal{P}_{cy}\mathcal{P}_{dz} + \text{permutations} \; , \\ \tilde{\Omega}_{abcd;wxyz}^{(3)} &= \mathcal{P}_{ab}\mathcal{P}_{cd}\mathcal{P}_{wx}\mathcal{P}_{yz} + \text{permutations} \; . \end{aligned} \quad (21)$$

For quarkonium decays to color-singlet final states, the appropriate color factor in Eq. (7) is simply $\sqrt{3}$ (as in Ref. [18]) in the amplitude.

We note that the formalism employed here (and in Refs. [18] and [19]) does give rise to infrared logarithms in various decay processes. These logarithms, which are in general nonperturbative in nature, can be more suitably treated by a new formalism developed by Bodwin, Braaten, and Lepage [22] who illustrate their method with an improved calculation of $P$-wave decay rates.

We can now make use of this formalism to evaluate the previously uncalculated two-gluon decay rates for the triplet $F$-wave quarkonium states, i.e., $\Gamma(^3F_J \to gg)$ where $J=2, 3, 4$. As the $gg$ decay widths are trivially related to those for two photon decay, we present the results for $\Gamma(^3F_J \to \gamma\gamma)$. The basic operator $\mathcal{O}(k)$ can be written as

$$\mathcal{O}(k) = ie^2 Q_f^2 \left[ \not{\epsilon}_2 \frac{1}{(\not{f} - \not{q}_1 - m)} \not{\epsilon}_1 + \not{\epsilon}_1 \frac{1}{(\not{q}_1 - \bar{\not{f}} - m)} \not{\epsilon}_2 \right] \quad (22)$$

and all of the appropriate derivatives in Eq. (7) can be easily calculated from this. The amplitudes for $^3F_{2,3,4} \leftrightarrow \gamma\gamma$ are then derived using the algebraic manipulation package FORM making heavy use of the symmetry and transversity properties of the various polarization tensors and the resulting amplitudes are squared and summed over polarization states using the expressions in Eqs. (11), (15), and (20). The results can then be written as

$$\Gamma(^3F_4 \to \gamma\gamma) = \frac{2560}{3} \frac{\alpha^2 Q^4 |\phi'''(0)|^2}{M^8} \; , \quad (23)$$

$$\Gamma(^3F_3 \to \gamma\gamma) = \frac{2560}{3} \frac{\alpha^2 Q^4 |\phi'''(0)|^2}{M^8} \; , \quad (24)$$

$$\Gamma(3F_2 \to \gamma\gamma) = \frac{117632}{15} \frac{\alpha^2 Q^4 |\phi'''(0)|^2}{M^8} \; . \quad (25)$$

These results can then be multiplied by the color factor $2\alpha_s^2/9\alpha^2 Q^4$ to yield the expressions for their two-gluon decay widths. (After the submission of this paper, we learned of recent work by Ackleh, Barnes, and Close [23] who derive the two-photon widths for positronium and quarkonium states with arbitrary angular momentum. Our results for the two-photon decays widths of the $^3F_J$ states are in agreement with their expressions.)

These calculations for $^3F_J \to \gamma\gamma$ can be trivially modified to yield results for the radiative decays of the $Z^0$ to $^3F_J$ states and we find that

$$\frac{\Gamma(Z^0 \to {}^3F_2 + \gamma)}{\Gamma(Z^0 \to {}^1S_0 + \gamma)} = \frac{64}{27} \tilde{R} \frac{9 + 912\mu + 9983\mu^2 - 17218\mu^3 + 5514\mu^4}{(1-\mu)^4} \; , \quad (26)$$

$$\frac{\Gamma(Z^0 \to {}^3F_3 + \gamma)}{\Gamma(Z^0 \to {}^1S_0 + \gamma)} = \frac{3584\tilde{R}}{9} \frac{6 + \mu + 5\mu^2}{(1-\mu)^2} \; , \quad (27)$$

$$\frac{\Gamma(Z^0 \to {}^3F_4 + \gamma)}{\Gamma(Z^0 \to {}^1S_0 + \gamma)} = 512\tilde{R} \frac{2 + 5\mu + 5\mu^2}{(1-\mu)^2} \; , \quad (28)$$

where

$$\tilde{R} \equiv \frac{|\phi'''(0)|^2}{M^6 |\phi(0)|^2} \quad (29)$$

and $\mu \equiv M^2/M_Z^2$. The dissimilarity in form between the $J=2$ and $J=3,4$ cases is easily understood as arising from angular momentum barrier effects. The $J=3,4$ $F$ states can only be produced in a relative state of orbital angular momentum leading to a suppression factor of $(1-\mu)^2$ while the $J=2$ state can proceed with no such suppression. Thus, the complicated numerator in Eq. (26) need not factor to yield additional powers of $(1-\mu)$ as the corresponding expressions in Eqs. (27) and (28) do. [A similar phenomena is noted in Ref. [19] for $Z^0 \to {}^3D_j + \gamma$. The differences in numerical factors in Eqs. (23)–(25) are due to similar arguments.]

Extensions to even higher-orbital angular momentum states are conceptually trivial but computationally difficult. One relatively easy calculation involving $G$ states (i.e., $L=4$) is the two-photon width of the $^1G_4$ state. Recently, Ackleh and Barnes [17] have derived a remarkably simple form for the two-photon decay widths of singlet positronium or quarkonium states in the nonrelativistic approximation for arbitrary orbital angular momentum which for positronium reads

$$\Gamma^J_{\rm NR}(e^+e^- \to \gamma\gamma) = \frac{\alpha^2|\phi^{(J)}(0)|^2}{m^{2J+2}} \tag{30}$$

where $m$ is the electron-positron mass. One identifies $m$ with half the quarkonium mass and adds the appropriate color factors for spin singlet quarkonia.

We can extend our formalism to singlet $G$ states and find

$$A = \frac{1}{2\sqrt{2}} \left[ \frac{1}{2M^2}\mathrm{Tr}[\not{k}\mathcal{O}_2^{ab}k_a k_b \not{k}(\not{P}+M)\Pi_{SS_z}] + \frac{1}{6M}\mathrm{Tr}[\{\mathcal{O}_3^{abc}k_a k_b k_c, \not{k}\}_+(\not{P}+M)\Pi_{SS_z}] + \tfrac{1}{24}\mathrm{Tr}[\mathcal{O}_4^{abcd}k_a k_b k_c k_d(\not{P}+M)\Pi_{SS_z}] \right] \tag{31}$$

and we restrict ourselves to $\Pi_{00} = -\gamma_5$. For the orbital part, the integral of $k_a k_b k_c k_d$ over $d^3k$ is then given by

$$\left[\frac{1}{m}\right]^{1/2} \int \frac{d^3k}{(2\pi)^{3/2}} k_a k_b k_c k_d \psi_4^{(m)}(\mathbf{k}) = b e^{(m)}_{abcd} \tag{32}$$

where

$$b = \frac{3}{4}\left[\frac{35}{\pi M}\right]^{1/2} \phi^{(4)}(0) . \tag{33}$$

For the spin-singlet case, we use $e^{(m)}_{abcd} = e^{(M_J)}_{abcd}$. With these expressions we can calculate the two-photon width of the $^1G_4$ state directly and (after a long FORM calculation) find agreement with Eq. (30) for $L=J=4$.

In conclusion, we have set up a general covariant formalism, well suited for calculations using computer algebra, for the coupling of $F$-wave quarkonium states and we have applied this formalism to the previously uncalculated two-gluon decays of $^3F_J$ quarkonium states which should describe their hadronic decays and production cross sections in hadronic collisions as well as their two-photon widths. A similar extension to singlet $G$ states reproduces a known general result for the $^1G_4 \to \gamma\gamma$ decay width as well. While most applicable to heavy quark systems, our results can, perhaps, be used to analyze future results in two-photon production of light quark resonances.

We thank O. Jedlinska and Z. Kolodziejski for collaboration on some aspects of this work. This work was supported in part by the National Science Foundation under Grant No. PHY-9001744 (R.R.) and by the Texas National Research Laboratory Commission (R.R.).

[1] W. Kwong, J. L. Rosner, and C. Quigg, Annu. Rev. Nucl. Part. Sci. **37**, 325 (1987).
[2] Y. Minghan and Z. Zhipeng, in *Proceedings of the XIVth International Symposium on Lepton and Photon Interactions,* Stanford, California, 1989, edited by Michael Riordan (World Scientific, Singapore 1990), p. 122.
[3] E. Nordberg, in *The Vancouver Meeting—Particles and Fields '91,* Proceedings of the Joint Meeting of the Division of Particles and Fields of the American Physical Society and the Particle Physics Division of the Canadian Association of Physicists, Vancouver, 1991, edited by D. Axen, D. Bryman, and M. Comyn (World Scientific, Singapore, 1992), p. 1090.
[4] E760 Collaboration, T. Armstrong *et al.*, Phys. Rev. Lett. **68**, 1468 (1992).
[5] G. Smith and R. Lewis (private communications).
[6] R. W. Robinett and L. Weinkauf, Phys. Lett. B **271**, 231 (1991); L. Weinkauf, Phys. Rev. D **45**, 4333 (1992).
[7] W. Kwong and J. L. Rosner, Phys. Rev. D **38**, 279 (1988).
[8] H. Grotch, X. Zhang, and K. J. Sebastian, Phys. Rev. D **35**, 2900 (1987).
[9] R. Barbieri, M. Caffo, R. Gatto, and E. Remiddi, Phys. Lett. **95B**, 93 (1980); R. Barbieri, G. Curci, E. d'Emilio, and E. Remiddi, *ibid.* **154B** 535 (1979); R. Barbieri, M. Caffo, R. Gatto, and E. Remiddi, Nucl. Phys. **B192**, 61 (1981).
[10] P. Mackenzie and G. P. Lepage, Phys. Rev. Lett. **47**, 1244 (1981).
[11] See D. M. Photiadis, Phys. Lett. **164B**, 160 (1985), for a discussion of the radiative corrections to $\Upsilon \to gg\gamma$.
[12] V. A. Novikov, L. B. Okun, M. A. Shifman, A. I. Vainshtain, M. B. Voloshin, and V. I. Zakharov, Phys. Rep. **41C**, 1 (1978).
[13] G. Bélanger and P. Moxhay, Phys. Lett. B **199**, 575 (1987).
[14] L. Bergström and P. Ernström, Phys. Lett. B **267**, 111 (1991).
[15] See, e.g., V. Barger and A. D. Martin, Phys. Rev. D **31**, 1051 (1985), for a discussion of hadronic production of $P$-wave quarkonium states.
[16] J. D. Anderson, M. H. Austern, and R. N. Cahn, Phys. Rev. D **43**, 2094 (1991). We note that this paper reproduces the earlier result of Ref. [12] for the $^1D_2 \to \gamma\gamma$ decay rate.
[17] E. S. Ackleh and T. Barnes, Phys. Rev. D **45**, 232 (1992).
[18] J. H. Kühn, J. Kaplan, and E. G. O. Safiani, Nucl. Phys. **B157**, 125 (1979); see also R. Barbieri, G. Gatto, and E. Remiddi, Phys. Lett. **61B**, 465 (1979), for a similar formalism.
[19] L. Bergström, H. Grotch, and R. W. Robinett, Phys. Rev. D **43**, 2157 (1991).
[20] L. Bergström and R. W. Robinett, Phys. Rev. D **45**, 116 (1992).
[21] L. Durand III, Phys. Lett. **18**, 58 (1967); see also J. P. Harnard, Nucl. Phys. **B38**, 350 (1972), for a thorough discussion of the polarization sums for massive spin $J$ bosons.
[22] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **46**, R1914 (1992).
[23] E. S. Ackleh, T. Barnes, and F. E. Close, Phys. Rev. D **46**, 2257 (1992).