Weak-scale seesaw model for the 17-keV neutrino

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(Received 31 January 1992)

We present variants of the singlet-Majoron model with the seesaw mechanism effective at the weak scale that accommodate the recently reported 17-keV neutrino (ν_{17}) naturally. First, we show that within the minimal model, by assuming an unbroken global $l_e - l_\mu + l_\tau$ symmetry ($l_i \equiv i$th lepton number), ν_{17} can be identified as a Dirac particle composed of ν_τ and $\bar{\nu}_\mu$. It is then shown that, with the same spectrum, if the $l_e - l_\mu + l_\tau$ symmetry is broken spontaneously below the weak scale, ν_e-ν_s (ν_s stands for sterile neutrino) oscillations can account for the solar-neutrino deficit via the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism. We also derive a recently proposed mass matrix for the 17-keV neutrino, which features ν_e-ν_s MSW oscillations, within the context of the seesaw model. All known constraints from cosmology and astrophysics, including the supernova constraint on the mass of ν_{17}, are satisfied in these variants.

PACS number(s): 12.15.Cc, 12.15.Ff, 14.60.Gh, 95.30.Cq

I. INTRODUCTION

It is well known by now that the properties of the 17-keV neutrino (ν_{17}), reported to be seen [1] as a 10% admixture of ν_e in the β-decay spectra of ^{3}H, ^{35}S, ^{14}C, ^{63}Ni, and ^{76}Ge, are severely constrained by laboratory data on the one hand, and by cosmological and astrophysical considerations on the other hand. In particular, the lower limit on the lifetime for neutrinoless double β-decay [2] implies that it must be a Dirac fermion to a very high precision. Since a Dirac fermion requires two two-component Weyl fermions, the immediate question is what the Dirac partner of ν_{17} is. Assuming $\nu_{17} = \nu_\tau$, in order to be consistent with neutrino oscillation data, its Dirac partner can be either an $SU(2)_L$-singlet neutrino (to be denoted by $\nu_{\tau R}$) or the antiparticle of the familiar muon neutrino ($\bar{\nu}_\mu$). The first possibility may be inconsistent with supernova 1987A observations [3] unless one postulates the existence of new exotic interactions for $\nu_{\tau R}$. It is therefore likely that ν_τ and $\bar{\nu}_\mu$ are Dirac partners and both have a mass of 17 keV. We shall focus here on realizations of this idea in the context of seesaw models.

The possibility that ν_τ and $\bar{\nu}_\mu$ form a Dirac fermion due to an unbroken $l_e - l_\mu + l_\tau$ symmetry was entertained [4] in 1985 after the first report of the 17-keV neutrino. These three neutrino models consisting of ν_e, ν_μ, ν_τ utilized the triplet Majoron [5] to satisfy the cosmological mass density constraints on ν_{17}. The triplet-Majoron model, however, has since been ruled out by precision measurement of the Z^0 width. Moreover, if the solar-neutrino deficit [6] reported by the chlorine, Kamiokande, and SAGE experiments is to be understood in terms of the Mikheyev-Smirnov-Wolfenstein (MSW) resonant neutrino oscillation [7], an $SU(2)_L$-sterile neutrino ν_s has to be introduced into the spectrum. Such a scenario wherein ν_τ-$\bar{\nu}_\mu$ forms a pseudo Dirac pair with mass of 17 keV and where ν_e-ν_s MSW oscillation accounts for the solar-neutrino deficit has been advocated recently by Caldwell and Langacker [8].

The purpose of this paper is threefold. First, we shall point out that the 17-keV neutrino can be accommodated as a ν_τ-$\bar{\nu}_\mu$ Dirac pair within the minimal version of the singlet-Majoron model [9]. (Unlike the triplet or doublet Majorons, the singlet Majoron is compatible with Z^0 width measurement, since it does not couple to Z^0.) Then we show that, within the same model, but with a slightly different assignment of lepton numbers to ν_R's, ν_e-ν_s MSW oscillation could occur, provided that the $l_e - l_\mu + l_\tau$ symmetry is broken below the weak scale. The third purpose of this paper is to derive a recently proposed 4×4 light-neutrino mass matrix [10] that features both the 17-keV neutrino and ν_e-ν_s MSW oscillation, in the context of the seesaw mechanism. All known cosmological and astrophysical constraints including (i) constraints on the lifetime of ν_{17} arising from (a) cosmological mass density and (b) galaxy formation arguments, (ii) the supernova constraint on the mass of ν_{17}, (iii) the limit on the photonic branching ratio from SN 1987A, (iv) the bound on the decay rate $\nu_{17} \to \bar{\nu}_e \chi$ (χ denotes the Majoron), (v) nucleosynthesis limits on the effective number of neutrino species, and (vi) the constraint on the coupling of χ to the neutrinos arising from supernova as well as nucleosynthesis are shown to be satisfied by these variants.

II. THE 17-keV NEUTRINO IN THE SINGLET-MAJORON MODEL

In addition to the standard-model particles, the spectrum of the singlet-Majoron model [9] consists of three right-handed neutrinos $(\nu_{eR}, \nu_{\mu R}, \nu_{\tau R})$ and a complex scalar field σ. Lepton number is assumed to be a global symmetry of the Lagrangian which is broken spontaneously when the field σ acquires a nonzero vacuum expectation value (VEV). This results in a massless Goldstone particle, the Majoron (χ). In addition, the VEV of σ generates large Majorana masses for the right-handed neutrinos, which explains, via the seesaw mechanism, why the light-neutrino masses are so much smaller compared to the charged leptons.

The 17-keV neutrino can be incorporated into the singlet-Majoron model by imposing an unbroken global $l_e - l_\mu + l_\tau$ symmetry ($l_i = i$th lepton number) [11]. The laboratory constraints from neutrino oscillation and neutrinoless double β decay will then be automatically satisfied. If we assign the "normal" $U(1)_{l_e - l_\mu + l_\tau}$ quantum numbers of $(1, -1, 1)$ to $(\nu_{eR}, \nu_{\mu R}, \nu_{\tau R})$, the heavy Majorana mass matrix will have a singular structure [12,13]. In this case, the 17-keV neutrino would be composed of mostly ν_τ and a sterile state. This may be inconsistent with supernova observations [3]. Here we shall present an alternative assignment of $l_e - l_\mu + l_\tau$ charges to the right-handed neutrinos which enables us to identify ν_{17} as a ν_τ-$\bar{\nu}_\mu$ Dirac pair. Both states being $SU(2)_L$ active, the supernova constraint will not be relevant in this case.

We first observe that the quantum numbers of the left-handed lepton doublets under global U(1) symmetries are dictated by their gauge interactions. As far as the right-handed charged fermions are concerned, their Dirac masses determine the U(1) charges to be the same as those of the corresponding left-handed ones. However, as for the right-handed neutrinos, since their masses are *a priori* undetermined and they have no gauge interactions, their charges can be chosen arbitrarily. Making use of this freedom, we assign the $U(1)_{l_e - l_\mu + l_\tau}$ charge of α $(\alpha \neq \pm 1, 0)$ to ν_{eR}, while $\nu_{\mu R}, \nu_{\tau R}$ retain their normal quantum numbers of $(-1, 1)$. Since we desire to leave $l_e - l_\mu + l_\tau$ unbroken even after spontaneous symmetry breaking, we shall assign a zero $l_e - l_\mu + l_\tau$ charge to the Higgs doublet ϕ and the complex singlet σ. As usual, σ carries -2 units of total lepton number which will be broken spontaneously once σ acquires a VEV. The leptonic part of the Yukawa Lagrangian, which is invariant under these global U(1) transformations, is given by

$$L_1 = \sum_{i=e,\mu,\tau} h_i \bar{\psi}_{iL} \phi e_{iR} + f_1 \bar{\psi}_{eL} \tilde{\phi} \nu_{\tau R} + f_2 \bar{\psi}_{\mu L} \tilde{\phi} \nu_{\mu R} + f_3 \bar{\psi}_{\tau L} \tilde{\phi} \nu_{\tau R} + f \nu_{\mu R}^T C^{-1} \nu_{\tau R} \sigma + \text{H.c.} \quad (1)$$

Here, $\tilde{\phi} = i\tau_2 \phi^*$, $\psi_{iL} = (\nu_{iL}, e_{iL})^T$, and C is the charge-conjugation matrix. We have chosen, without loss of generality, the Yukawa couplings of the charged leptons to ϕ, which generates e, μ, τ masses, to be diagonal. Note that the ν_{eR} field, due to its abnormal U(1) charge, does not enter into Eq. (1), which means that it decouples from the rest of the spectrum. Denoting $m_i = f_i v$, $M = f\kappa$, where $\langle \phi^0 \rangle = v$, $\langle \sigma \rangle = \kappa$, the 5×5 neutrino mass matrix can be written down in the basis $(\nu_e, \nu_\mu, \nu_\tau, \nu_\mu^c, \nu_\tau^c)$ as (ν_μ^c and ν_τ^c represent the antiparticles of $\nu_{\mu R}$ and $\nu_{\tau R}$)

$$M_\nu = \begin{bmatrix} 0 & 0 & 0 & 0 & m_1 \\ 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & m_3 \\ 0 & m_2 & 0 & 0 & M \\ m_1 & 0 & m_3 & M & 0 \end{bmatrix} . \quad (2)$$

Since one expects the Dirac mass entries m_1, m_2, m_3 in M_ν to be of the same order as the charged-lepton masses, and the Majorana mass M to be much larger (of the order of weak scale ~ 100 GeV to a few TeV), the above matrix can be diagonalized in the approximation $m_i \ll M$. To leading order in m_i/M, the light 3×3 sector of the mass matrix is then obtained by block diagonalization:

$$M_\nu^{\text{light}} = -\frac{1}{M} \begin{bmatrix} 0 & m_1 m_2 & 0 \\ m_1 m_2 & 0 & m_2 m_3 \\ 0 & m_2 m_3 & 0 \end{bmatrix} + O\left[\frac{m_i^4}{M^3}\right] . \quad (3)$$

Because of the unbroken $l_e - l_\mu + l_\tau$ symmetry, this form of the light-neutrino mass matrix will not be altered even after including higher-order terms in m_i/M.

From Eq. (3), it follows that the ν_e-ν_τ mixing angle is given by $\tan\theta = m_1/m_3$. This angle should be $\sim 10\%$ to explain the β-decay observations. Furthermore, the entry $m_2 m_3/M$ should be ~ 17 keV. Assuming the Dirac mass entries m_2 and m_3 to be of the same order as the μ and τ masses (within a factor of 3 or so), one finds that $M \approx 1-10$ TeV. In other words, the model has a natural explanation for the 17-keV mass if the scale of lepton-number breaking is around the weak scale.

It turns out that due to the unbroken $l_e - l_\mu + l_\tau$ symmetry, the full 5×5 matrix of Eq. (2) can be diagonalized exactly. This yields one two-component massless Weyl fermion ν_1 and two "Dirac" states $\nu_{2,3}$ with masses given by

$$m_{\nu_1}^2 = 0 ,$$
$$m_{\nu_{2,3}}^2 = \tfrac{1}{2}[(m_1^2 + m_2^2 + m_3^2 + M^2) \mp \sqrt{(m_1^2 + m_2^2 + m_3^2 + M^2)^2 - 4m_2^2(m_1^2 + m_3^2)}] . \quad (4)$$

The gauge eigenstates are related to the mass eigenstates through

$$\nu_e = \cos\theta \nu_1 + \sin\theta(\cos\alpha \nu_2 + \sin\alpha \nu_3) ,$$
$$\nu_\mu = \cos\beta \nu_2^c + \sin\beta \nu_3^c ,$$
$$\nu_\tau = -\sin\theta \nu_1 + \cos\theta(\cos\alpha \nu_2 + \sin\alpha \nu_3) , \quad (5)$$
$$\nu_\mu^c = -\sin\alpha \nu_2 + \cos\alpha \nu_3 ,$$
$$\nu_\tau^c = -\sin\beta \nu_2^c + \cos\beta \nu_3^c ,$$

where $\tan\theta = m_1/m_3$,

$$\tan 2\alpha = 2\sqrt{m_1^2 + m_3^2} M/(m_2^2 + M^2 - m_1^2 - m_2^2) ,$$

and

$$\tan 2\beta = 2m_2 M/(m_1^2 + m_3^2 + M^2 - m_2^2) .$$

The angle $\theta \approx 10\%$, while $\alpha \simeq m_3/M$, $\beta \simeq m_2/M \ll 1$. The state ν_2 corresponds to the 17-keV neutrino, ν_1 is exactly massless, and ν_3 is superheavy.

(i) Decay of the 17-keV neutrino. ν_{17} should decay with a lifetime shorter than 10^{12} sec in order to be consistent with cosmological mass density requirements. In the present model, the decay $\nu_{17} \to \nu_e + \chi$ can occur. To estimate the lifetime for this decay, one needs the flavor-changing $\bar{\nu}_\tau \nu_e \chi$ vertex. Since the Majoron χ has a direct coupling only to the sterile states, its coupling to the neutrino gauge eigenstates is given by the matrix of Eq. (2) with m_i set to zero. The same unitary transformation that block diagonalizes M_ν of Eq. (2) to give Eq. (3) should be applied to the Majoron coupling matrix. As shown in Ref. [14], to order (m_ν/M) [or equivalently to order (m_i^2/M^2)], the light 3×3 Majoron coupling matrix turns out to be proportional to M_ν^{light} of Eq. (3). This means that there is no tree-level flavor-changing coupling of χ to order (m_ν/M). Such tree-level couplings are present if corrections of order $(m_\nu/M)^2$ are included. However, it has been noted recently [15] that once one-loop radiative corrections are taken into account, such flavor-changing coupling will be induced even at order (m_ν/M), but now suppressed by a loop factor $1/16\pi^2$. Using this result, we estimate the lifetime of the 17-keV neutrino to be

$$\tau_\nu^{-1} \simeq \frac{m_\nu}{8\pi}\theta^2 \left[\frac{1}{16\pi^2}\frac{m_\nu}{M} \right]^2 . \tag{6}$$

For $m_\nu = 17$ keV, $\theta \simeq 0.1$, and $M = 100$ GeV-1 TeV, we find that the lifetime is in the range 10^2-10^4 sec. This clearly satisfies the cosmological mass density constraint. We note that the somewhat model-dependent constraint on the lifetime arising from galaxy formation arguments ($\tau_{\nu_{17}} \leq 10^7$ sec) is also satisfied by the model [16].

(ii) $\bar{\nu}_{17} \to \bar{\nu}_e + \chi$. Since ν_{17} decays dominantly into $\nu_e + \chi$, the decay $\nu_{17} \to \bar{\nu}_e + \chi$ should also occur. These decays conserve $l_e - l_\mu + l_\tau$ quantum numbers. There are observational constraints on this decay lifetime from IMB and Kamiokande neutrino detectors which could have seen delayed $\bar{\nu}_e$ events from SN 1987A. It has been shown in Ref. [17] that the range of lifetime $\tau_\nu = 10^4-10^8$ sec can be excluded from the nonobservation of delayed events. This constraint is clearly satisfied by our model. Put in another way, if the scale of lepton-number breaking, M, were much larger than 10 TeV, the lifetime of the 17-keV neutrino would have been much longer and would have been inconsistent with supernova observations.

(iii) Photonic branching ratio. The lifetime for the photonic decay $\nu_{17} \to \nu_e + \gamma$ is severely constrained by nonobservation of MeV γ rays from SN 1987A by the Solar Maximum Mission Satellite [18]: $\tau(\nu_{17} \to \nu_e + \gamma) \geq 10^{15}$ sec. In the present model, however, since the only charged particle in the spectrum is the $W^\pm$ gauge boson, such flavor-changing couplings involving the photon are Glashow-Iliopoulos-Maiani (GIM) suppressed and satisfy this constraint easily. We estimate

$$\tau^{-1}(\nu_{17} \to \nu_e + \gamma) \simeq \frac{\alpha}{4\pi}\left[\frac{g^2}{16\pi^2}\frac{m_\tau^2}{m_W^2}\theta\frac{m_{\nu_{17}}}{m_W^2} \right]^2 m_{\nu_{17}}^3 . \tag{7}$$

This corresponds to a photonic lifetime of $\simeq 10^{20}$ sec, which is well above the lower limit.

(iv) Coupling of the Majoron (χ) to neutrinos. There is an upper limit on the diagonal Yukawa coupling [19] of χ to ν_{17}: $h_{\nu\nu\chi} \leq 10^{-6}$. If the Yukawa coupling exceeds this value, the decay $\nu_{17} \to \bar{\nu}_{17} + \chi$ which can occur inside the supernova core will be too fast resulting in rapid cooling of the core via Majoron emission. In the present model, the relevant coupling is given by $(m_{\nu_{17}}/\kappa) \simeq 10^{-7}-10^{-9}$.

(v) Nucleosynthesis. The off-diagonal coupling of χ to neutrinos is constrained by nucleosynthesis arguments. If the off-diagonal Majoron coupling $h'_{\nu_{17}\nu_e\chi}$ is greater than 10^{-6}, the process $\nu_\tau \to \nu_e + \chi$ will be in equilibrium during the epoch of nucleosynthesis [20]. The Majoron contributes as $\frac{4}{7}$ of an effective neutrino species. This would violate the bound $N_\nu \leq 3.3$ that has been derived based on nucleosynthesis [21]. In the present model, the off-diagonal Majoron coupling is of order $(1/16\pi^2)(m_\nu/\kappa) \sim 10^{-10}$, well below the limit. Since there are no extra light species, the model predicts $N_\nu = 3$ for nucleosynthesis.

(vi) Supernova constraint on the 17-keV mass. Since ν_{17} is composed of $SU(2)_L$-active species, both helicity states will be trapped after production inside the supernova core. This does not alter the supernova energy loss or duration of the pulse. Consequently, there is no constraint from SN 1987A on the mass of ν_{17}.

III. SINGLET-MAJORON MODEL FEATURING 17-KEV NEUTRINO AND MSW MECHANISM

In the model presented in Sec. II, because of the unbroken $l_e - l_\mu + l_\tau$ symmetry, ν_e and ν_s ($\nu_s \equiv \nu_e^c$, antiparticle of ν_{eR}) were strictly massless. As a result, there is no oscillation between ν_e and ν_s. In this section we present a variant of the model with the same minimal spectrum which features ν_e-ν_s MSW oscillation that can account for the solar-neutrino deficit in addition to generating the ν_τ-$\bar{\nu}_\mu$ 17-keV state. This is achieved by breaking $l_e - l_\mu + l_\tau$ symmetry spontaneously below the weak scale. This is reminiscent of the triplet-Majoron model where the total lepton number is broken below the weak scale.

The variant assumes the same minimal spectrum of particles (i.e., ν_{eR}, $\nu_{\mu R}$, $\nu_{\tau R}$, and a complex scalar σ in addition to the standard-model particles). Total lepton number is not imposed, but $l_e - l_\mu + l_\tau$ is assumed to be a global symmetry of the Lagrangian. We assign an $l_e - l_\mu + l_\tau$ charge of $\frac{1}{3}$ to ν_{eR} and $-\frac{2}{3}$ to σ. Once σ acquires a VEV, $l_e - l_\mu + l_\tau$ symmetry will be broken spontaneously giving rise to an associated Majoron. We shall assume $\langle \sigma \rangle = \kappa$ to be much smaller than the electroweak scale, of order 10–100 keV. As before, $(\nu_{\mu R}, \nu_{\tau R})$ retain their normal quantum numbers of $(-1, 1)$. The most general leptonic Yukawa Lagrangian is then given by

$$\begin{aligned} L_2 = \sum_{i=e,\mu,\tau} & h_i \bar{\psi}_{iL}\phi e_{iR} + f_1 \bar{\psi}_{eL}\tilde{\phi}\nu_{\tau R} + f_2 \bar{\psi}_{\mu L}\tilde{\phi}\nu_{\mu R} \\ & + f_3 \bar{\psi}_{\tau L}\tilde{\phi}\nu_{\tau R} + M \nu_{\mu R}^T C^{-1}\nu_{\tau R} + f'_1 \nu_{eR}^T C^{-1}\nu_{eR}\sigma \\ & + f'_2 \nu_{eR}^T C^{-1}\nu_{\mu R}\sigma^* + \text{H.c.} \end{aligned} \tag{8}$$

Denoting $f_i v = m_i$ and $f_i' \kappa = \mu_i$, the 6×6 neutrino mass matrix is now [in the basis $(\nu_e, \nu_\mu, \nu_\tau, \nu_s, \nu_\mu^c, \nu_\tau^c)$, $\nu_s \equiv \nu_e^c$]

$$M_\nu = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & m_1 \\ 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_3 \\ 0 & 0 & 0 & \mu_1 & \mu_2 & 0 \\ 0 & m_2 & 0 & \mu_2 & 0 & M \\ m_1 & 0 & m_3 & 0 & M & 0 \end{bmatrix} . \quad (9)$$

Since $\kappa \sim 10-100$ keV, the entries μ_1 and μ_2 of Eq. (9) are much smaller than all the other nonzero entries. M_ν can then be block diagonalized in the approximation $\mu_i \ll m_i \ll M$. The light 4×4 matrix in the basis $(\nu_s, \nu_e, \nu_\mu, \nu_\tau)$ is obtained to be

$$M_\nu^{\text{light}} = -\begin{bmatrix} -\mu_1 & m_1\mu_2/M & 0 & m_3\mu_2/M \\ m_1\mu_2/M & 0 & m_1 m_2/M & 0 \\ 0 & m_1 m_2/M & 0 & m_2 m_3/M \\ m_3\mu_2/M & 0 & m_2 m_3/M & 0 \end{bmatrix} . \quad (10)$$

This matrix is identical to Eq. (3) but for small entries proportional to $\mu_{1,2}$ added along the first row and column. So most of the discussions pertaining to the 17-keV neutrino in the previous section will be valid here too. In particular, ν_{17} is composed of active states, mostly ν_τ and $\bar{\nu}_\mu$. The ν_e-ν_τ mixing angle is given by $\theta \simeq m_1/m_3$. The 17-keV mass $(m_2 m_3/M)$ follows naturally if the mass scale M is of the same order as the weak scale. We should note that a 4×4 light-neutrino matrix of the same form as Eq. (10) has recently been derived based on a different model in Ref. [22].

What is different about the matrix of Eq. (9) compared to Eq. (2) is the small but nonzero elements μ_1 and μ_2. These entries mix the sterile state ν_s with ν_e and ν_τ, which facilitates ν_e-ν_s MSW oscillation. Choosing $\mu_1 \simeq 10^{-3}$ eV, $\mu_2 \simeq$ a few eV and the Dirac mass entries m_i's comparable to the charged-lepton masses as before, one obtains the desired MSW spectrum, viz. $m_{\nu_s} \sim 10^{-3}$ eV, $m_{\nu_e} \sim 10^{-4}$ eV with the ν_e-ν_s mixing angle of a few percent. If the VEV of σ is of order 10–100 keV, the Yukawa couplings f_1' and f_2' should be of order 10^{-7} and 10^{-3}, respectively. These are similar in magnitude to the electron Yukawa coupling in the standard model.

The coupling of the Majoron to the light neutrinos here is different from that of the previous model of Sec. II. The difference arises because σ has primordial couplings here to the ultralight ν_s and not to the superheavy $\nu_{\mu R}$ or $\nu_{\tau R}$, unlike in the previous model. In the basis $(\nu_s, \nu_e, \nu_\mu, \nu_\tau)$, the Majoron coupling matrix to leading order takes the form

$$Y_\chi = -\frac{1}{\kappa M}\begin{bmatrix} -\mu_1 M & m_1\mu_2 & 0 & m_3\mu_2 \\ m_1\mu_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ m_3\mu_2 & 0 & 0 & 0 \end{bmatrix} . \quad (11)$$

The decay $\nu_\tau \to \nu_s \chi$ amplitude is proportional to $(m_3\mu_2/\kappa M)$, which for $(m_3\mu_2/M) \simeq 10^{-3}$ eV and $\kappa \simeq 100$ keV is of order 10^{-8}. The lifetime of ν_{17} is then of order 10^{-2} sec. Note that ν_{17} decays dominantly into a sterile state in this case. The supernova constraint on the lifetime (viz., $\tau_\nu = 10^4 - 10^8$ sec being excluded) is then not applicable. All constraints resulting from cosmology and astrophysics listed previously are seen to be satisfied by the present variant. In particular, supernova and nucleosynthesis bounds on the diagonal and off-diagonal couplings of the Majoron to neutrinos are satisfied since they are of order 10^{-8} or smaller. The estimate of the photonic branching ratio is the same as in Eq. (7).

IV. REALIZATION OF A SIMPLE FORM OF THE NEUTRINO MASS MATRIX

As seen in Sec. III, if the light-neutrino sector involves a sterile state (ν_s) in addition to the usual $(\nu_e, \nu_\mu, \nu_\tau)$, the 17-keV neutrino can be identified as a pseudo Dirac state composed mostly of ν_τ and $\bar{\nu}_\mu$, and still resonant MSW oscillation between ν_e-ν_s could account for the solar neutrino deficit. A simple form of the 4×4 neutrino mass matrix which accommodates these two features was proposed by us in collaboration with Rothstein [10]. In the basis $(\nu_s, \nu_e, \nu_\mu, \nu_\tau)$, the mass matrix was proposed to be

$$M_\nu = \begin{bmatrix} \mu_1 & \mu_2 & 0 & 0 \\ \mu_2 & 0 & m_1 & 0 \\ 0 & m_1 & 0 & m_2 \\ 0 & 0 & m_2 & 0 \end{bmatrix} . \quad (12)$$

If $m_1 \simeq 1.7$ keV, $m_2 \simeq 17$ keV, and $\mu_1 \sim \mu_2 \sim 10^{-3}$ eV, the 17-keV neutrino with the required mixing properties can arise and ν_e-ν_s MSW oscillation can explain the solar-neutrino deficit. In Ref. [10], we constructed an extension of the standard model, where, using $l_e - l_\mu$ and l_τ symmetries, we showed that $m_{1,2}$ arises as a one-loop radiative correction out of the μ and τ masses while $\mu_{1,2}$ arises at the two-loop level, thereby explaining naturally their small values without fine tuning of parameters.

Here we wish to point out that the model of Ref. [10] can be embedded in a more attractive quark-lepton-symmetric extension of the standard model with three right-handed neutrinos (instead of one). In this new scheme, the $\nu_\mu\nu_\tau$ mass term m_2 arises out of a seesaw mechanism at the tree level, and is naturally of order keV. The entry m_1 connecting $\nu_e\nu_\mu$ is induced at the one-loop level, whereas the milli-eV entries μ_1 and μ_2 are generated at the two-loop level. Most of the properties of this model, such as the ν_{17} lifetime, ν_e-ν_s oscillation, etc., are the same as in our previous model.

The model is based on the standard gauge group with the fermion spectrum extended to include three right-handed neutrinos $(\nu_{eR}, \nu_{\mu R}, \nu_{\tau R})$. The Lagrangian of the model is assumed to respect $U(1)_e \times U(1)_{\mu-\tau}$ global symmetry. All fermions, except ν_{eR} are assumed to have their normal quantum numbers under this symmetry. We assume ν_{eR} to transform as (1,1) under this symmetry (see remarks in Sec. II).

The Higgs spectrum of the model along with their global symmetry transformation properties are shown in Table I. As in most two-Higgs-doublet models, we impose a discrete Z_2 symmetry to prevent tree-level flavor-changing neutral currents mediated by the Higgs particles. Under this Z_2, ϕ_2, ν_{iR}, η_2, $\sigma_{1,2}$ are odd, while all other fields are even. The most general gauge-invariant Yukawa couplings of the leptons consistent with these symmetries is given by

$$\begin{aligned} L_3 = \sum_{i=e,\mu,\tau} & h_i \bar{\psi}_{iL}\phi_1 e_{iR} + h'_\mu \bar{\psi}_{\mu L}\tilde{\phi}_2 \nu_{\mu R} + h'_\tau \bar{\psi}_{\tau L}\tilde{\phi}_2 \nu_{\tau R} \\ & + f_1 \psi_{eL}^T C^{-1} i\tau_2 \psi_{\mu L}\eta_1^+ + f_2 e_R^T C^{-1}\nu_{eR}\eta_2^+ \\ & + f_3 e_R^T C^{-1} e_R k_1^{++} + M\nu_{\mu R}^T C^{-1}\nu_{\tau R} + \text{H.c.} \qquad (13) \end{aligned}$$

The most general Higgs potential allowed by the symmetries of the model is

$$\begin{aligned} V = V_0 & + \lambda_1 \phi_1^T \tau_2 \phi_2 \eta_1^- \sigma_1^* + \lambda_2 \eta_1^+ \eta_2^+ k_1^{--} \sigma_2^* \\ & + \lambda_3 k_2^{++} k_1^{--} \sigma_1^2 + M_1 k_2^{++} \eta_2^- \eta_2^- + \text{H.c.} \qquad (14) \end{aligned}$$

Here V_0 consists of terms of the form $(\phi_i^\dagger \phi_i)$, $(\eta^+\eta^-)$, etc. their products, and the term $(\phi_1^\dagger \phi_2)^2$. The $(\text{mass})^2$ terms for ϕ_i and σ_i are chosen to be negative so as to generate VEV's for them parametrized by $\langle \phi_i^0 \rangle = v_i$ and $\langle \sigma_i \rangle = \kappa_i$, $i=1,2$. Since we assume that the only mass scale is the electroweak scale, κ_i will be taken to be around a TeV. The VEV of σ_i breaks the $\mathrm{U}(1)_e \times \mathrm{U}(1)_{\mu-\tau}$ symmetry down to $\mathrm{U}(1)_{e-\mu+\tau}$. While $\langle \sigma_2 \rangle$ breaks this residual symmetry completely, it is transferred into the fermionic sector only at higher order, as shown below. The model has two singlet Majorons, the same as in Ref. [10].

The tree-level neutrino mass matrix involves only $(\nu_\mu, \nu_\tau, \nu_\mu^c, \nu_\tau^c)$ and has the form dictated by $l_\mu - l_\tau$ symmetry:

$$\begin{bmatrix} 0 & 0 & h'_\mu v_2 & 0 \\ 0 & 0 & 0 & h'_\tau v_2 \\ h'_\mu v_2 & 0 & 0 & M \\ 0 & h'_\tau v_2 & M & 0 \end{bmatrix} . \qquad (15)$$

It is reasonable to assume that the neutrino Dirac masses are of the same order as the corresponding charged lepton masses, so that any mechanism that explains the charged fermion masses will also explain these values.

TABLE I. Quantum numbers of the scalar multiplets.

Multiplet	$SU(2)_L \times U(1)_Y$	$U(1)_e \times U(1)_{\mu-\tau}$
ϕ_1, ϕ_2	(2,1)	(0,0)
η_1^+	(1,2)	$(-1,-1)$
η_2^+	(1,2)	$(-2,-1)$
k_1^{++}	(1,4)	$(-2,0)$
k_2^{++}	(1,4)	$(-4,-2)$
σ_1	(1,0)	(1,1)
σ_2	(1,0)	$(-1,-2)$

Block diagonalization of the matrix of Eq. (15) in the approximation $M \gg h'_\mu v_2, h'_\tau v_2$ gives the element m_2 of Eq. (12) to be $m_2 = h'_\mu h'_\tau v_2^2 / M$. For $h'_\mu v_2 \sim m_\mu$ and $h'_\tau v_2 \sim m_\tau$, we obtain m_2 to be of order 17 keV if the mass scale M is of order 1 TeV. Thus, without fine tuning of parameters, a multi-keV mass for the ν_μ-ν_τ pair arises naturally. At the tree level, all other entries of the light 4×4 neutrino mass matrix are zero.

The rest of the mass matrix arises as follows. At the one-loop level, the diagram of Fig. 1 gives rise to m_1, while the entries μ_1 and μ_2 corresponding to ν_s-ν_s and ν_e-ν_s arise at the two-loop level via the diagrams of Fig. 2. We estimate

$$m_1 \simeq \left[\frac{f_1 \lambda_1}{16\pi^2}\right] \left[\frac{v_2}{v_1}\right] \left[\frac{m_\mu^2}{M_H^2}\right] \kappa_1 , \qquad (16)$$

where M_H is a typical Higgs-boson mass. For $f_1 \simeq \lambda_1 \simeq 1.3$, $v_2/v_1 \simeq 5$, and $M_H \simeq \kappa_1/3 \simeq 1$ TeV, m_1 is 1.7 keV, which has the desired magnitude. The two-loop contributions are estimated to be [see Fig. 2(a) and 2(b)]

$$\begin{aligned} \mu_1 &\simeq \frac{f_2^2 f_3 \lambda_3}{(16\pi^2)^2} \left[\frac{\kappa_1^2}{M_H^2}\right] M_1 , \\ \mu_2 &\simeq \frac{f_2 f_3 \lambda_1 \lambda_2}{(16\pi^2)^2} \left[\frac{v_2}{v_1}\right] \left[\frac{\kappa_1 \kappa_2}{M_H^2}\right] m_e . \end{aligned} \qquad (17)$$

For $f_2 \simeq 10^{-3}$, $f_3 \simeq \lambda_3 \simeq 10^{-2}$, and $M_1 \simeq 100$ GeV, we get $\mu_1 \simeq 10^{-3}$ eV. Similarly, with $\lambda_2 \simeq 10^{-1}$, and $\kappa_2 \simeq M_H$, μ_2 is 4×10^{-4} eV. In addition, the ν_e-ν_s mixing angle can be a few %. Needless to say, these values are in the right range for the MSW mechanism to be relevant to resolve the solar-neutrino puzzle.

Without detailed elaboration, we wish to note that the model satisfies the laboratory constraints on neutrino oscillation and neutrinoless double β decay due to the approximate $l_e - l_\mu + l_\tau$ symmetry exhibited by the mass matrix of Eq. (12). All known constraints from cosmology and astrophysics are also met by the model, just as in Ref. [10]. In particular, $\nu_{17} \to \nu_e + \chi$ decay occurs with a lifetime of order 10^{-2}–10^{1} sec. There is no constraint on the mass of ν_{17} since it is made up of two active species. N_ν is predicted to be 3 for nucleosynthesis. The coupling of the Majoron to electrons arises at the one-loop level via the exchange of k^{++} scalar and is estimated to be

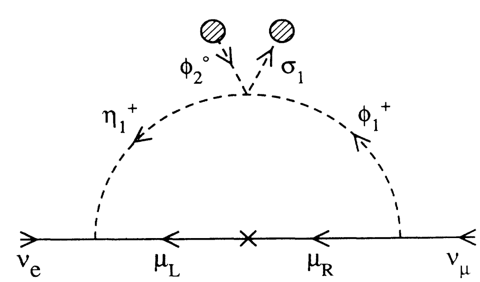

FIG. 1. One-loop diagram responsible for generating the ν_e-ν_μ mass term in the model of Sec. IV.

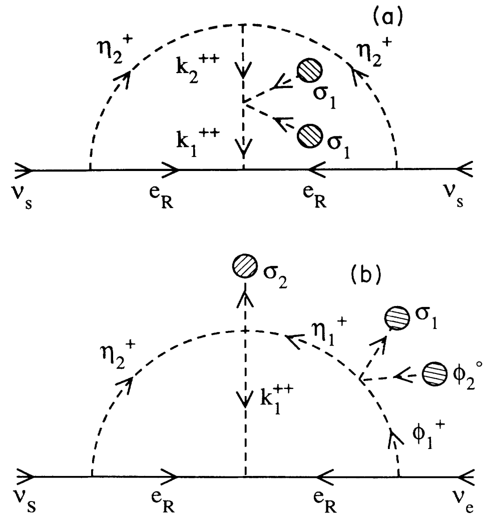

FIG. 2. Two-loop diagrams giving mass to ν_e and ν_s.

$$g_{ee\chi} \sim \frac{f_3^2 \lambda_3^2}{16\pi^2} \left[\frac{\kappa_1^3 m_e}{M_H^4} \right] . \tag{18}$$

For $f_3 \sim \lambda_3 \sim 10^{-2}$, and $M_H \sim \kappa_1/3 \simeq 1$ TeV, this coupling is $\sim 10^{-15}$, well below the astrophysical limit of 10^{-12}. The photonic branching ratio constraint $[\tau(\nu_{17} \to \nu_e + \gamma) \geq 10^{15}$ sec$]$ is also satisfied if $M_H \geq 1$ TeV.

Since the VEV's of σ_1 and σ_2 break the global symmetries of the model completely, one, in principle, expects all vanishing elements of the neutrino mass matrix, Eq. (12), to acquire nonzero values. This is indeed what happens when we go beyond two loops. However, most of these higher loop corrections are vanishingly small and can be neglected. One notable exception is the ν_μ-ν_τ mass splitting. As shown in Ref. [10], the mass splitting arising from the matrix of Eq. (12) is

$$|m_{\nu_\mu}^2 - m_{\nu_\tau}^2| \simeq 2(|\mu_1|\mu_2^2 m_1^2)/(m_1^2 + m_3^2)^{3/2} \sim 10^{-17}\text{eV}^2 .$$

The three-loop corrections to this can be significantly larger. The dominant contribution comes from the graph of Fig. 3, which generates a $\nu_\mu \nu_\mu$ entry in the mass matrix. This can be estimated to be

$$m_{\nu_\mu \nu_\mu} \simeq \left[\frac{f_1^2 f_3 \lambda_2^2 \lambda_3}{(16\pi^2)^3} \right] \left[\frac{\kappa_1^2 \kappa_2^2 M_1 m_e^2}{M_H^6} \right] . \tag{19}$$

For the choice of parameters given above, this entry is of order 10^{-13} eV, corresponding to $|m_{\nu_\mu}^2 - m_{\nu_\tau}^2|$ of order 10^{-9}–10^{-8} eV^2. Furthermore, since ν_μ and ν_τ would have been degenerate in the absence of these small corrections, the mass eigenstates turn out to be equal mixtures of ν_μ and ν_τ corresponding to 45° mixing.

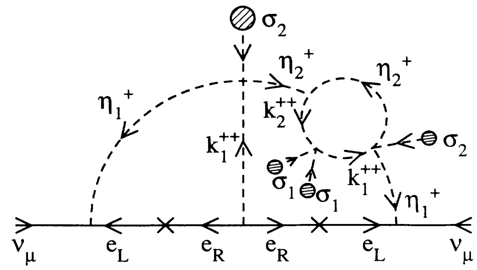

FIG. 3. Three-loop graph which induced $\nu_\mu \nu_\mu$ entry of the neutrino mass matrix.

It will be interesting to see if the model can also account for the reported deficit of atmospheric muon neutrinos [23] via ν_μ-ν_τ oscillations. For large ν_μ-ν_τ mixing, the preferred mass splitting for this to occur is in the range 10^{-4}–10^{-2} eV^2 [24]. Consider the following choice of parameters: $f_1 = \lambda_1 = 1.3$, $f_2 = 3 \times 10^{-5}$, $f_3 = 3 \times 10^{-2}$, $\lambda_2 = 1$, $\lambda_3 = 0.3$, $v_2/v_1 = 5$, $M_H = M_1 = \kappa_1/3 = 1$ TeV. In this case, m_1 of Eq. (16) is 1.7 keV, μ_1, μ_2 of Eq. (17) are of order 10^{-3} eV as before, but $m_{\nu_\mu \nu_\mu}$ of Eq. (19) is now 10^{-8} eV. This corresponds to $|m_{\nu_\mu}^2 - m_{\nu_\tau}^2| \sim 10^{-3}$–$10^{-4}$ eV^2. Clearly then, for a range of parameters which is not unreasonable, the model can account for the atmospheric neutrino deficit.

V. CONCLUSIONS

We have presented in this paper variants of the singlet-Majoron model which can accommodate the 17-keV neutrino naturally. The models presented are consistent with all known cosmological and astrophysical constraints, including the limit on the mass of ν_{17} from supernova. The scale of lepton number violation is required to be of the same order as the electroweak symmetry-breaking scale (a few 100 GeV to a few TeV) so as to be consistent with the various constraints. In Sec. II, we have shown a way to fit ν_{17} into the minimal version of the singlet-Majoron model. This model is characterized by an unbroken $l_e - l_\mu + l_\tau$ symmetry. With the same minimal spectrum, but by allowing for small $l_e - l_\mu + l_\tau$ breaking below the weak scale, we showed that ν_e-ν_s MSW oscillation can also occur which could explain the solar-neutrino deficit. The model of Sec. IV based on the seesaw mechanism seems to be capable of explaining three outstanding puzzles in neutrino physics, viz., the solar-neutrino deficit, the 17-keV neutrino, and the atmospheric muon neutrino deficit.

ACKNOWLEDGMENTS

The work of K.S.B. was supported by a grant from DOE. The work of R.N.M. was supported by a grant from NSF.

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