Self-similarity in particle production in hadron-nucleus interactions at 350 and 200 GeV/c

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Self-similarity in multiparticle production in π^--nucleus interactions at 350 and at 200 GeV/c has been studied in terms of factorial moments F_q as well as fractal moments G_q. Fractal parameters, already developed in the classical chaotic system, have been discussed and those parameters for multiparticle production in π^--nucleus interactions have been evaluated. While the F-moment analysis confirms nonstatistical fluctuations, and in terms of anomalous dimensions, predicts cascading in hadronization, multifractality is indicated through the G-moment analysis. Variation in the nature or values of fractal parameters have been studied in different sizes of phase space and in different energies for the same reactants. Such a phenomenological analysis is essential at the present stage of development in the study of fractality in multiparticle production.

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I. INTRODUCTION

The power-law behavior of variation of moments of multiplicity as a function of phase-space interval size, being studied to identify the dynamical fluctuations in multiparticle production, has gathered growing interest in recent years, in an effort towards understanding the underlying physics of hadronization. The behavior of factorial moments as a function of resolution was first proposed as "intermittency in multiparticle production" by Bialas and Peschanski [1]. Other interpretations with conventional approaches such as cluster production models [2], Bose-Einstein quantum interference [3], short-range correlations [4], etc., also try to describe such behavior of the moments. Though there are contradictions among speculations for a dynamical origin and neither intermittency nor any other interpretation has yet to be universally accepted, experimentally there is qualitative agreement on the power-law behavior of moments of multiplicity for the available sets of data. As a matter of fact, various experimental investigations [5] with reactions initiated either by leptons, hadrons, or nuclei in a wide range of incident energies confirm the power-law dependence of moments on the width of the phase-space interval and therefore suggest a self-similarity property that ultimately leads to the application of fractal analysis of multiparticle production in terms of fluctuations in multiplicity moments. Various methods of investigation of the fractal structure in multiparticle production have so far been suggested [6–9], but the universality of any methodology that reveals the fractal structure is yet to be established. At this stage, an analysis of data at varied energies with different reactants in the framework of various theoretical and phenomenological models is necessary in order to identify the dynamical origin of the power law.

In this article, we present the analyses on our data of π^--Ag/Br at 350 and 200 GeV/c, in terms of the factorial moments F_q to see nonstatistical fluctuations and finally in terms of fractal moments G_q and related fractal theory to extract multifractality, if any, following the methodology proposed by Chiu and Hwa [10]. Although the same methodology of extracting the fractal structure in multiparticle production has been applied to the data of e^+e^- annihilation [11] and nucleus-nucleus interactions [12] and to the data of UA1 and NA35 Collaborations [13], the study of fractality in terms of G_q moments is not yet matured enough to conclude on the acceptability of the methodology.

II. THE FACTORIAL MOMENTS AND SELF-SIMILARITY IN MULTIPARTICLE PRODUCTION

In the study of self-similarity in multiparticle production in terms of "intermittency," the standard normalized moments are not used. Since the number of produced particles at available energies is finite, the statistical fluctuations due to the distribution of an insufficient number of particles among a large number of bins, when the rapidity interval δ is small, are obvious. Using the Bernoulli distribution to eliminate the expected statistical noise from genuine "dynamical" fluctuations, Bialas and Peschanski (Ref. [1]) obtained the normalized factorial moments F_q as the quantities to be studied to find the self-similarity behavior.

If n is the number of particles of an event in the considered phase space of $\Delta\eta$, divided into M bins of width $\delta\eta=\Delta\eta/M$, n_j being the number of particles in the jth bin, so that $\sum_{j=1}^{M} n_j=n$, the normalized factorial moments as defined by Bialas and Peschanski are

$$\langle F_q(\delta\eta)\rangle = M^{q-1}\left\langle \sum_{j=1}^{M} \frac{n_j(n_j-1)\cdots(n_j-q+1)}{n(n-1)\cdots(n-q+1)}\right\rangle , \quad (1)$$

where angular brackets denote the vertical averaging over many events and $M^{-1}\sum_{j=1}^{M}$ is the horizontal averaging over all bins; q being a positive integer, e.g., $q=2,3,4,\ldots,$ gives the order of the moment.

In the case of an analysis of the sample containing events with a range of multiplicities, the normalization factor $M^q/[n(n-1)\cdots(n-q+1)]$ is replaced [14] by $M^q/\langle n\rangle^q$, where $\langle n\rangle$ is the average multiplicity of charged secondaries within the considered phase space.

If the distribution of particles in $\Delta\eta$ has a self-similarity, the power-law dependence of normalized factorial moments on the pseudorapidity width is given by the relation

$$F_q \propto (\delta\eta)^{-a_q} \quad \text{as } \delta n \to 0 \ . \tag{2}$$

a_q, the intermittency exponents, are related to fractal dimensions D_q [15], which are commonly used for the description of fractals and multifractals in the classical chaotic systems, as established in Ref. [16]:

$$D_q = 1 - a_q/(q-1) \ , \tag{3}$$

where

$$a_q/(q-1) = d_q \tag{4}$$

are anomalous dimensions of the spectra. The factorial moments as described above need to be corrected for the nonuniform shape of the pseudorapidity distribution by dividing by the factor [17]

$$R_q = \frac{1}{M}\sum_{j=1}^{M} M^q \frac{\langle n_j\rangle^q}{\langle n\rangle^q} \ . \tag{5}$$

In the case of a flat pseudorapidity distribution, however, the factor R_q becomes equal to unity. Now we have reduced scaled factorial moments:

$$\langle F_{qR}\rangle = \frac{\langle F_q\rangle}{R_q} \ , \tag{6}$$

which are more appropriately used in the study of intermittency.

III. THE FRACTAL MOMENTS

Although the F-moment analysis has been successfully carried out in most of the experimental data sets, available so far, in the range of variable δn up to 0.1, a crucial question that arises in the analysis of data is the limit of resolution of the phase space $\delta\eta\to 0$, precisely beyond $\delta\eta=0.1$ [18]. If the resolution is of the order of the average separation between two neighboring particles in the phase space, then the binning of phase space with that resolution may result in some empty bins. Though, in our analysis in terms of F moments within the range of $\delta\eta$ from 1.0 to 0.1, the empty bins are not a problem; this kind of limitation in the methodology appreciates the introduction of different approaches in the study of self-similarity. A different approach is the analysis in terms of G moments (Ref. [8]). Considering the empty bins as holes, the set of nonempty bins at any stage of binning constitutes one of the fractal sets of the multifractal structure in multiparticle production, in analogy to that in deterministic chaos in nonlinear physics. With this concept of multifractality, the fractal moments G_q have been defined to evaluate parameters which characterize the fractal properties.

When the considered phase space $\Delta\eta$ is divided into M bins of width $\delta\eta=\Delta\eta/M$, there may be bins that have no particles. If M_0 is the number of nonempty bins, which constitute the fractal set, the fractal moments are defined as

$$G_q = \sum_{j=1}^{M_0} p_j^q \ , \tag{7}$$

where $p_j = n_j/n$ with $n = n_i + n_2 + \cdots + n_M$, n_j being the number of particles in the jth bin. The summation is carried over nonempty bins only. q gives the order of the moment. Because of the very nature of formulation of G_q, i.e., summation over nonempty bins only, the fractal moments can be calculated for any positive or negative integral or nonintegral order (value of q) and thus may take a dominant role over other multiplicity moments in revealing the dynamics of multiparticle production through the study of fluctuations in density of produced particles.

The G_q moments as described in Eq. (7) give the horizontal analysis of an event. When it is analyzed vertically over a fixed bin of an ensemble of events, it gives different G_q values except at $q=1$ because of the involvement of different collisional circumstances. While the vertical analysis is done over events of different impact parameters, in the case of the horizontal analysis, there is one collision with one particular impact parameter and one specific process of particle production. To enhance statistics, it is suggested that the horizontally analyzed G_q moments be averaged vertically over the ensemble of events and vertically analyzed bins of ensembles of events be averaged horizontally over many bins [19]. In most of the recent work on the subject, horizontal analysis and vertical averaging have been undertaken. In our analysis also, we calculate the vertically averaged horizontal G_q moments in pseudorapidity (η) phase space to evaluate the fractal parameters.

If the particle production, and hence the pseudorapidity distribution, has multifractal structure, the scaling laws (Ref. [15]),

$$p_j \propto (\delta\eta)^{\alpha}, \quad M_{0\alpha} \propto (\delta\eta)^{-f(\alpha)} \ , \quad G_q \propto (\delta\eta)^{\tau(q)} \quad \text{as } \delta\eta\to 0 \tag{8}$$

are expected. $M_{0\alpha}$ is the number of bins indexed by the same α_q, i.e., multifractal subset, where $\tau(q)$ and $f(\alpha)$ are related to the generalized dimension D_q, introduced by Hentschal and Procaccia (Ref. [15]). By mapping the local fluctuations of pseudorapidity distribution onto $f(\alpha)$, a quantitative description of the distribution by a smooth function is achieved. However, these power laws are not valid in the limit $\delta\eta\to 0$, in the strict mathematical sense, as in that limit of $\delta\eta$, the number of particles in each nonempty bin reaches the lower limit 1, resulting in

G_q equal to n^{q-1}. Thus, at finite energy, where n is not very large, the self-similarity or fractal structure cannot be expected at infinitesimal $\delta\eta$.

IV. THE SPECTRUM $f(\alpha_q)$ AND GENERALIZED DIMENSIONS D_q

Experimentally, $\tau(q)$ in expression (8) is derived from the slope of the plot of $\langle \ln G_q \rangle$ and $-\ln\delta\eta$ as

$$\langle \tau(q) \rangle = -\langle \Delta \ln G_q / \Delta \ln\delta\eta \rangle = -(\ln 2)^{-1}\Delta\langle \ln G_q \rangle / \Delta\nu \ , \qquad (9)$$

where angular brackets denote averaging over events and $M=2^{\nu}$ equals the total number of bins in the considered phase space. Once $\tau(q)$ is determined, the spectrum $f(\alpha_q)$ is obtained through the Legendre transform

$$f(\alpha_q) = q\alpha_q - \tau(q) \ , \qquad (10)$$

where

$$\alpha_q = \frac{d\tau(q)}{dq} \ . \qquad (11)$$

The spectrum $f(\alpha_q)$ is a smooth function, concave downward with its peak at $q=0$, representing fluctuations in a pseudorapidity distribution from event to event. The spectrum $f(\alpha_q)$ actually gives a quantitative description on the multiplicity fluctuations in both the dense and the sparse regions of the pseudorapidity space, corresponding to the left wing and the right wing of the spectrum, respectively. If $f(\alpha_q)$ is not sharply peaked at α_q corresponding to $q=0$, it reveals the fact that the pseudorapidity distribution is not smooth in the phase space from event to event.

The most basic property of any fractal measure is its dimensions. The generalized dimensions D_q, evaluated from $\tau(q)$ by the relation

$$D_q = \tau(q)/(q-1) \ , \qquad (12)$$

play a significant role in fractal theory. More specifically, the set of conventional dimensions for $q=0$, 1, and 2 being

$$D_0 = \text{fractal dimension} = f(\alpha_0) \ , \qquad (13)$$

$$D_1 = \text{information dimension} = f(\alpha_1) \ , \qquad (14)$$

$$D_2 = \text{correlation dimension} = 2\alpha_2 - f(\alpha_2) \ , \qquad (15)$$

could be very sensitive to the dynamics of particle production as these are in the classical chaotic system.

In the framework of the fractal theory already developed for the chaotic dynamical system, physically, the information dimension is defined as

$$D_1 = -\lim_{\delta\eta\to 0} S(\delta\eta)/\ln\delta\eta \ , \qquad (16)$$

where

$$S(\delta\eta) = -\sum_{j=1}^{M} p_j \ln p_j \qquad (17)$$

is the definition of entropy in information theory. The correlation dimension is defined by

$$D_2 = \lim_{\delta\eta\to 0} C(\delta\eta)/\ln\delta\eta \ , \qquad (18)$$

where

$$C(\delta\eta) = \frac{1}{n^2}\sum_{i\neq j}\theta(\delta\eta - |X_i - X_j|) \ , \qquad (19)$$

where $\theta(x)=1$ if $x \geq 0$, 0 if $x<0$. $C(\delta\eta)$ is simply the correlation integral which counts number of pairs of particles with distance in between $|X_i - X_j|$ less than $\delta\eta$.

Geometrically, D_0 represents the peak of the $f(\alpha_q)$ spectrum. For inhomogeneous fractals or multifractals, $D_0 > D_1 > D_2$. When $D_0 = D_1 = D_2$, the fractal is uniform.

As stated in Sec. I, the analysis of G_q moments has already been conducted on different sets of data with different types of reactants and the power-law behavior of G_q moments has been observed. The smooth presentation of chaotic structure in rapidity distribution from event to event, through the spectrum $f(\alpha_q)$, has also been possible for those data sets. D_q have been calculated. Even in other models, the spectrum $f(\alpha_q)$ and the dimensions have been obtained. But, in either model, the detailed characteristics of the spectrum $f(\alpha_q)$ and the generalized dimensions D_q, in the context of multiparticle production, have yet to be cultivated. However, some interesting speculations involving $f(\alpha_q)$ and D_q have already come onto the scene. Hwa speculates (Ref. [13]) a narrowing of $f(\alpha_q)$ with experimental cut in relativistic heavy-ion collisions favoring plasma formation such as at high E_T. As suggested very recently by Bialas and Hwa [20], if a second-order phase transition takes place from a quark-gluon plasma to a hadron phase in the thermodynamic equilibrium, created in high-energy collision, then the produced particles will show intermittency with anomalous dimensions (described in Sec. II) d_q independent of q. On the other hand, if hadronization takes place through the cascading process, d_q is expected to be linear in q. Brax and Peschanski (Ref. [9]), describing multifractal properties of intermittencylike fluctuations in terms of the random cascading model, show a characterization of phase transitions in terms of a "freezing" of the system, with a deviation from the analytic $f(\alpha)$ spectrum when $f(\alpha)$ reaches zero.

At this stage of the study of fractality in multiparticle production, phenomenologically one needs to evaluate different aspects of $f(\alpha_q)$ and D_q: their dependence on energy, type of reactants, impact parameter selection, size and location of the considered phase space, etc. In this article, we attempt to study some of these aspects through a fractal analysis in terms of fractal moments G_q in addition to the analysis of data in terms of factorial moments F_q.

V. EXPERIMENTAL DETAILS

We study the hadron-nucleus interaction data of π^--Ag/Br at 350- and 200-GeV/c incident energies. The stacks of G5 nuclear emulsion plates were exposed horizontally to a π^- beam at CERN with 350 GeV/c and at

Fermilab with 200-GeV/c incident energies. The nuclear emulsion covers 4π geometry and provides very good accuracy, even less than 0.1 mrad, in angle measurements of produced particles with respect to the projectile beam axis due to high spatial resolution and thus is suitable as a detector for the study of fluctuations in fine-resolution intervals of the pseudorapidity phase space. The emulsion plates were area scanned with a Leitz Metalloplan Microscope fitted with a semiautomatic scanning device, having a resolution along the X and Y axes of 1 μm while that along the Z axis is 0.5 μm. A sample of 569 events of π^--Ag/Br at 350 GeV/c and 542 events of π^--Ag/Br at 200 GeV/c was chosen, following the usual emulsion methodology for selection criteria of the events, identification of the tracks and their angle measurement with respect to the beam axis, as described below.

(i) Oil immersion objectives of magnification 10$\times$ and an ocular lens of 25$\times$ were used for area scanning. The primary of each interaction was followed back to the entry point in emulsion to confirm the selection of the primary events.

(ii) The incident beam track of a selected event was less than 3° to the mean direction of the beam axis in the pellicle.

(iii) Interactions within 20 μm from any of the surfaces of the pellicle were not taken in the data sample.

(iv) For identification of tracks and angle measurement, an oil immersion objective of 100$\times$ and ocular lens of 25$\times$ were used.

(v) In nuclear emulsion terminology [21], charged secondaries, i.e., the "shower" tracks, were identified with an ionization of less than $1.4I_0$, where I_0 is the plateau ionization, and with a high range of penetration, generally not confined in the emulsion plates. Tracks with an ionization of more than $1.4I_0$ and with a smaller range of penetration are the "grey" and the "black" tracks.

(vi) Only events with a number of heavy tracks, i.e., the number of black tracks plus the number of grey tracks, $n_h \geq 8$, were selected to ensure $\pi^- +$Ag/Br interactions.

(vii) The emission angle θ of produced secondaries was measured by recording the space coordinates (x,y,z) of a point on the shower track, the point of interaction, and a point on the incident track. For each track, the angle was measured with two different sets of coordinates and averaged. The basic phase-space variable in our analysis is pseudorapidity (η), which is calculated from θ by $\eta = -\ln\tan(\theta/2)$.

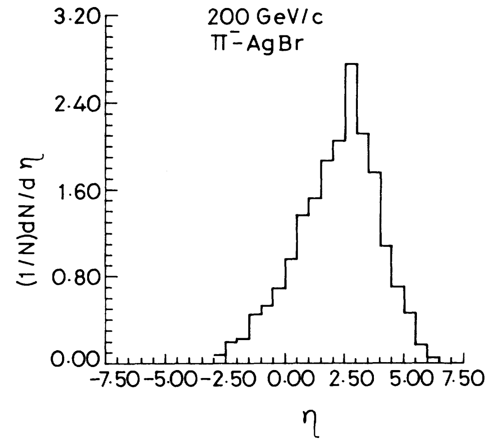

FIG. 2. Pseudorapidity distribution of produced particles in the π^--Ag/Br interaction at 200 GeV/c.

The pseudorapidity distribution for both data sets in the whole phase space for 350- and 200-GeV/c data are presented in Figs. 1 and 2, respectively.

For the F-moment analysis, the pseudorapidity phase space of width $\Delta\eta = 4.0$ around the peak of the pseudorapidity distribution has been chosen. The η distributions for the data samples are not flat within the considered region of phase space. Also, there is a difference, which might be statistical, in the shape of the η distributions at the two energies, with a sharp peak appearing in the η distribution of the data sample of 200 GeV/c. However, in the intermittency study, use of the reduced scaled factorial moments takes care of the nonflat distribution and

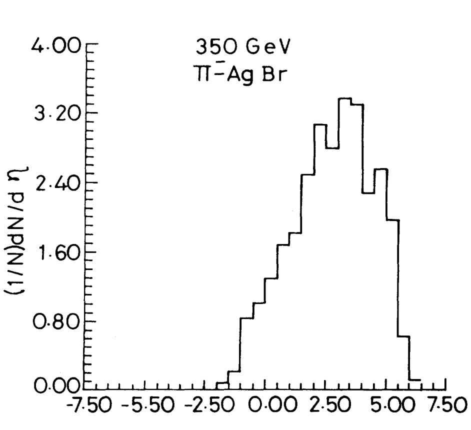

FIG. 1. Pseudorapidity distribution of produced particles in the π^--Ag/Br interaction at 350 GeV/c.

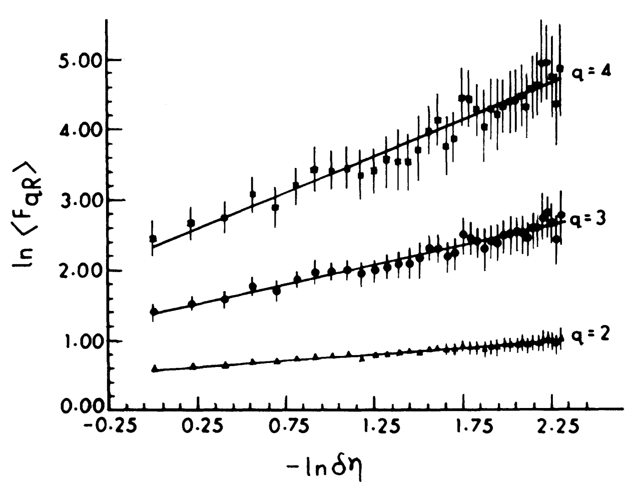

FIG. 3. Plot of $\langle \ln F_{qR} \rangle$ vs $-\ln\delta\eta$ in $\Delta\eta = 4.0$ for the π^--Ag/Br interaction at 350 GeV/c.

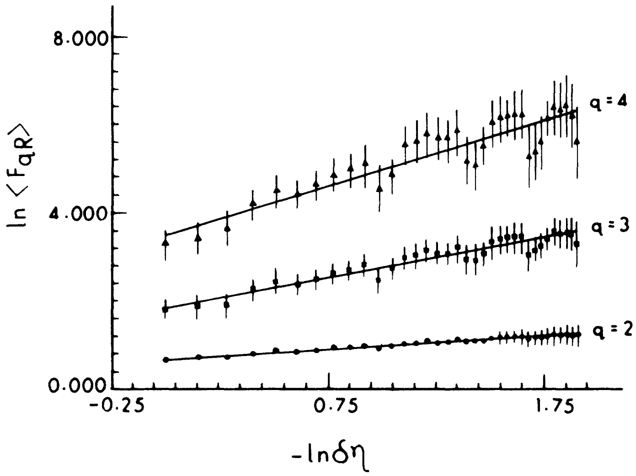

FIG. 4. Plot of $\langle \ln F_{qR} \rangle$ vs $-\ln\delta\eta$ in $\Delta\eta=4.0$ for the π^--Ag/Br interaction at 200 GeV/c.

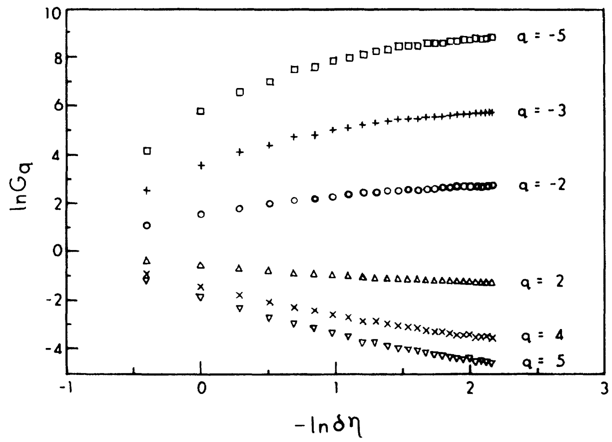

FIG. 6. Plot of $\langle \ln G_q \rangle$ vs $-\ln\delta\eta$ in phase-space (pseudorapidity) size $\Delta\eta=3.0$ for the $\pi^- +$Ag/Br interaction at 350 GeV/c.

for our study also we have necessarily used the reduced scaled factorial moments. The power-law dependence of the reduced scaled factorial moments on the bin size is studied in the region $1 \geq \delta\eta \geq 0.1$ for the order of moment, $q=2$, 3, and 4, by dividing the phase space by a maximum number, $M=40$. In Figs. 3 and 4, the variations in the values of $\ln\langle F_{qR} \rangle$ with $-\ln\delta\eta$ are plotted for 350- and 200-GeV/c data, respectively. The values of generalized dimensions for $q=2$, 3, and 4 for 350-GeV/c data are 0.816 ± 0.007, 0.723 ± 0.012, and 0.659 ± 0.016, respectively, and those for 200-GeV/c data are 0.704 ± 0.009, 0.555 ± 0.015, and 0.507 ± 0.020, respectively. The anomalous dimensions d_q calculated from the plots of $\ln\langle F_{qR} \rangle$ vs $-\ln\delta\eta$ for both the reactions are plotted against q in Fig. 5. We repeat the fractal analysis, as described in Secs. II and IV, in three different overlapping pseudorapidity intervals, viz., $\Delta\eta=3.0$, 2.5, and 2.0, straddling the peak of the pseudorapidity distribution for each data set.

The study of fluctuations in multiplicity moments, or more precisely, the fractal analysis in terms of $f(\alpha_q)$ and D_q aims at the origin of the dynamics of multiparticle production. Fractal analysis in different finite pseudorapidity intervals is important in finding out how the fluctuations in terms of fractal parameters are influenced by different regions of phase space. Such a phenomenological analysis, to study any possible variation in D_q and in the nature of $f(\alpha_q)$ within the size of the considered phase space, is significant due to the physical interpretations of D_q and different speculations involving $f(\alpha_q)$ and D_q already mentioned in Sec. IV.

The variation of G_q moments with the width of the pseudorapidity interval $\delta\eta$ in the considered phase space $\Delta\eta=3.0$ for 350- and 200-GeV/c data is presented in Figs. 6 and 7, respectively, plotting $\langle \ln G_q \rangle$ against

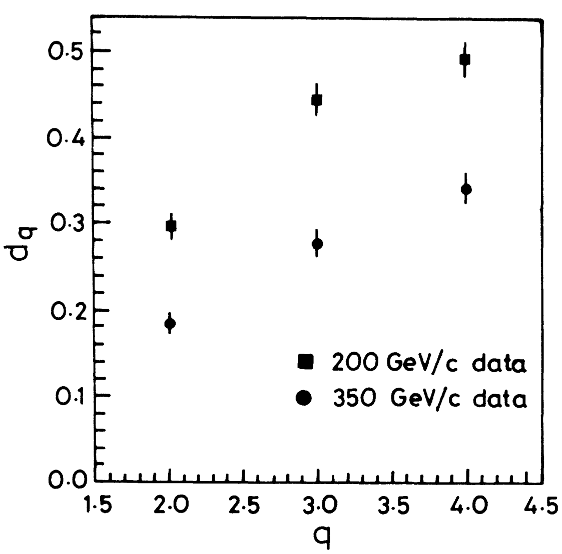

FIG. 5. Anomalous dimensions d_q calculated from F-moment analyses plotted against the order of moment q.

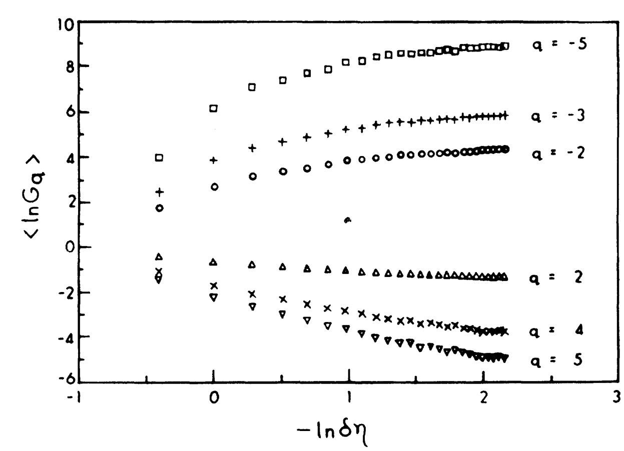

FIG. 7. Plot of $\langle \ln G_q \rangle$ vs $-\ln\delta\eta$ in phase-space (pseudorapidity) size $\Delta\eta=3.0$ for the $\pi^- +$Ag/Br interaction at 200 GeV/c.

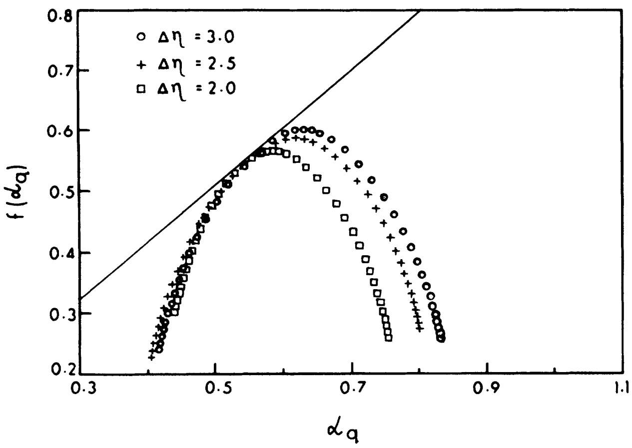

FIG. 8. Plot of spectrum $f(\alpha_q)$ as a function of α_q for the $\pi^- + \mathrm{Ag/Br}$ interaction at 350 GeV/c for $\Delta\eta = 3.0$, 2.5, and 2.0.

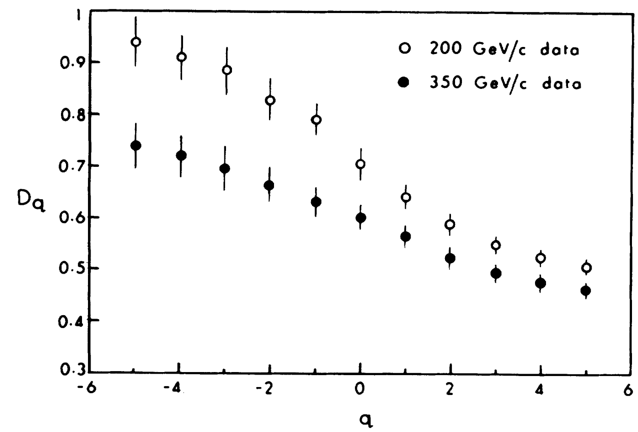

FIG. 10. Comparative plot of D_q vs q for the $\pi^- + \mathrm{Ag/Br}$ interaction data at 350 and 200 GeV/c.

$-\ln\delta\eta$. The moments with positive q values show linearity over a wide range of $\delta\eta$ or M, while the moments with negative q values saturate as $\delta\eta$ decreases or as M becomes larger. In order to calculate the slopes $\tau(q)$ of plots $\langle \ln G_q \rangle$ vs $-\ln\delta\eta$, for all q, positive or negative, the linear region is chosen. We calculate $\tau(q)$ from the first two points, i.e., $M=2$ and 3. Subsequently, from $\tau(q)$, $f(\alpha_q)$ and D_q are obtained. Although the plots for $\langle \ln G_q \rangle$ vs $-\ln\delta\eta$ for all sizes of $\Delta\eta$ are not shown in this paper, $\tau(q)$ and then $f(\alpha_q)$ are obtained for all considered $\Delta\eta$ and are presented against α_q, in Figs. 8 and 9 for 350- and 200-GeV/c data sets, respectively. Plots of D_q vs q for both the data sets at $\Delta\eta=3.0$ are compared in Fig. 10. Values of D_q for $q=0$, 1, and 2 are calculated for both data sets and have been plotted in Fig. 11, against the size of the considered phase space, $\Delta\eta$.

VI. OBSERVATIONS AND DISCUSSION

From the plots of $\ln\langle F_q \rangle$ vs $-\ln\delta\eta$ in Figs. 3 and 4, the existence of intermittency or the self-similarity in produced pions in each of the data sets is obvious. The features, e.g., (1) increase in slope of the intermittency exponents, a_q, with increasing order of moment, (2) decrease in slope with an increase in incident energy for the same reactants, are in qualitative agreement with earlier analyses on the study of intermittency in different sets of data. In terms of speculation by Bialas and Hwa, as described in Sec. IV, the plot of anomalous dimensions d_q vs the order of moment q in Fig. 5 shows a cascading process of hadronization in our data sample.

In terms of the G-moment analysis, plots of $\langle \ln G_q \rangle$ against $-\ln\delta\eta$ in Figs. 6 and 7 are initial indications of the presence of fractal structure in the particle production process in our data sample. Further, these plots are the basis of evaluation of fractal parameters, e.g., $f(\alpha)$, the singularity spectrum, and D_q, the generalized dimensions.

The multifractal singularity spectra $f(\alpha_q)$ for both data sets plotted in Figs. 8 and 9 are smooth curves with a peak at α_0. In none of the cases is $f(\alpha_q)$ sharply

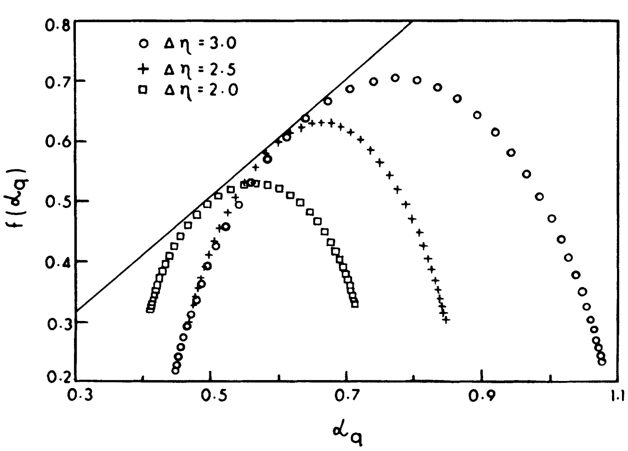

FIG. 9. Plot of spectrum $f(\alpha_q)$ as a function of α_q for the $\pi^- + \mathrm{Ag/Br}$ interaction at 200 GeV/c for $\Delta\eta = 3.0$, 2.5, and 2.0.

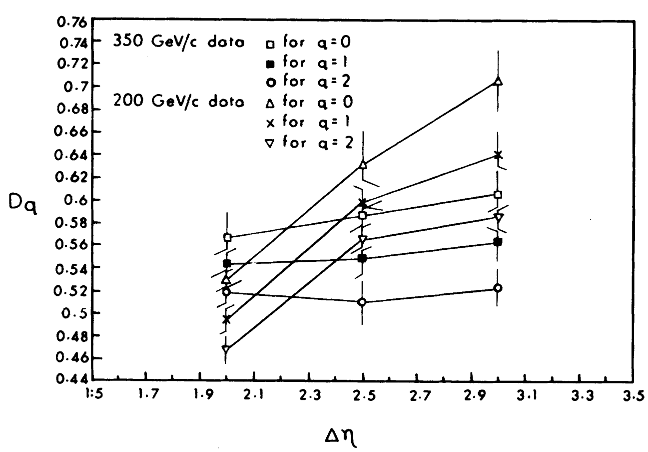

FIG. 11. Plot of conventional dimensions from a G-moment analysis, i,e., D_q at $q=0$, 1, and 2 for $\pi^- + \mathrm{Ag/Br}$ at 350- and 200-GeV/c data sets at different sizes of the considered phase space, $\Delta\eta = 3.0$, 2.5, and 2.0.

peaked, indicating that the distributions of produced particles from event to event are not smooth.

The corresponding values of the set of generalized dimensions D_q for both the data sets for $\Delta\eta = 3.0$ against q values, in Fig. 10, satisfy the condition of multifractality; i.e., D_q is always greater than $D_{q'}$ where $q' > q$.

While comparing the shape of $f(\alpha_q)$ at different energies and the same size of considered phase space $\Delta\eta$, the width of $f(\alpha_q)$ is observed to be narrower with its peak at a lower value of α_q in the case of higher-energy data, showing fewer fluctuations. This phenomenon of energy dependence revealed in terms of D_q has also been presented in Fig. 10.

The dependence of the spectrum $f(\alpha_q)$ and generalized dimensions D_q on the size of the considered phase space, presented in Figs. 8, 9, and 11, is identical for both data sets. The spectrum $f(\alpha_q)$ is broader for a wide range of $\Delta\eta$. For a smaller range of $\Delta\eta$, the width of $f(\alpha_q)$ becomes narrower and $f(\alpha_q)$ shifts towards smaller α_q values. This effect is due to the fact that, the smaller the $\Delta\eta$ range, the finer is the structure in the region investigated for the given number of divisions.

Further, from Figs. 8 and 9 it is clear that for both data sets the broadening of $f(\alpha_q)$ with the wider range of $\Delta\eta$ is mainly due to the variations in the right wings of the spectra, $f(\alpha_q)$, while, except for the spectrum at $\Delta\eta = 2.0$ for the 200-GeV/c data, there is hardly any variation among the left wings, which are sensitive to the peaks of the pseudorapidity distributions. This effect can be understood in terms of the pseudorapidity distributions presented in Figs. 1 and 2. On widening the $\Delta\eta$ range, the edges of the pseudorapidity distributions, mainly corresponding to the valley of the distributions, contribute to the moments with positive q values. This effect of broadening $f(\alpha_q)$ is more prominent in 200-GeV/c data, reflecting the sharper peak of the pseudorapidity distribution. The shifting of the left wing of the spectrum at $\Delta\eta = 2.0$ for the 200-GeV/c data further left is due to the fact that the curve is strongly dominated by the appearance of the sharp peak in the η distribution.

The generalized dimensions calculated from F moments do not match with those obtained from G moments in the same $\Delta\eta$ range. This difference is most likely due to the different range of analysis of bin width $\delta\eta$. While the power-law dependence of F moments is studied within the region $1 \geq \delta\eta \geq 0.1$, for the G moment analysis $\delta\eta$ is necessarily not very small. In fact, D_q's from the G-moment analysis were calculated in the region $\delta\eta > 1$. Moreover, while the F moments are valid correlation functions, the G moments are dominated by a single-particle effect. How the values of D_q, obtained from the G moments, are influenced due to statistical fluctuations is also not yet known. Conclusively, the analysis in terms of F moments reveals a self-similarity property in multiparticle production in the considered energy range of π^--AgBr interaction and shows a cascading process of hadronization.

Although the values of generalized dimensions from G moments do not match with those from F moments and thus may appear to be a nonconvincing methodology of extracting multifractality, the pseudorapidity distribution could be presented in terms of a smooth multifractal singularity spectrum which is sensitive not only to the peaks but also to the valley of rapidity fluctuations. Further, characteristics of the fractal parameters are qualitatively identical with earlier analyses of other data and, within the framework of the G-moment analysis, the data reveals multifractality. However, a quantitative comparison at this level of the study does not appear significant as the values of those parameters are observed to be dependent on the size of the considered phase space, the range of analysis of $\delta\eta$ value, etc.

Since the question of fractality in multiparticle production is yet to be established, the phenomenological analyses, as have been carried out in our study, are essential at the present stage of development of the subject.

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