Remark on the effective action of three-dimensional QED at finite temperature

Won-Tae Kim, Young-Jai Park, Kee Yong Kim, and Yongduk Kim
Department of Physics, Sogang University, C.P.O. Box 1142, Seoul 100-611, Korea
(Received 23 June 1992)

From three-dimensional massive QED, we obtain the temperature-dependent Chern-Simons action by calculating the vacuum current at finite temperature.

PACS number(s): 11.15.−q, 11.30.Er

There has been much attention paid to the parity anomaly of three-dimensional quantum electrodynamics (QED_3) [1,2]. For massless fermionic QED_3, the Chern-Simons action as a quantum effective action is obtained by introducing the Pauli-Villars regularization [2]. Recently many authors have studied the fractional quantum Hall effect or high-T_c superconductivity by using the Chern-Simons term [3]. On the other hand, the finite temperature effect of QED_3 has been studied in terms of the imaginary time formalism [4] by calculating the Feynman diagram of vacuum polarization, or by considering the vacuum expectation value of two-dimensional current [5].

In this Brief Report, we study the generation of a Chern-Simons term by calculating the vacuum expectation value of source current at finite temperature and discuss the parity anomaly of QED_3.

Now let us start with the action [6],

$$S_F[A,\psi]=\int d^3x[i\bar{\psi}\gamma^\mu(\partial_\mu-ieA_\mu)\psi+m\bar{\psi}\psi] \ . \tag{1}$$

Then the effective action $W[A]$ is given by

$$\begin{aligned} {}_A\langle 0^+|0^-\rangle_A &= e^{iW[A]} \\ &= \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{iS_F} \ , \end{aligned} \tag{2}$$

where $|0^\pm\rangle_A$ means the fermion Fock vacuum state under the gauge field background at the time $t\to\pm\infty$. By using the effective action, we obtain the relation

$$\frac{\delta W}{\delta A_\mu}=\frac{{}_A\langle 0^+|J^\mu|0^-\rangle_A}{{}_A\langle 0^+|0^-\rangle_A}\equiv\langle J^\mu\rangle_A \ , \tag{3}$$

where $J^\mu=e\bar{\psi}\gamma^\mu\psi$ is a source current. Then we calculate the vacuum expectation value of this current with the gauge field background to obtain the infinitesimal arbitrary variation of the effective action [2]:

$$\begin{aligned} \langle J^\mu\rangle_A &= ie\,\mathrm{Tr}(\gamma^\mu G) \ , \\ G(x) &= \left\langle x \left| \frac{-1}{i\not{D}+m} \right| x \right\rangle \ , \end{aligned} \tag{4}$$

where G is a Green's function of Dirac equation of original action (1). Rewriting Eq. (4), we obtain

$$\begin{aligned} \langle J^\mu\rangle_A &= ie\,\mathrm{Tr}\gamma^\mu\left\langle x \left| x\frac{i\not{D}}{\not{D}^2+m^2} \right| x \right\rangle \\ &\quad - iem\,\mathrm{Tr}\gamma^\mu\left\langle x \left| \frac{1}{\not{D}^2+m^2} \right| x \right\rangle \ , \end{aligned} \tag{5}$$

where the first term vanishes because the integrand is an odd function with respect to the momentum integral, which will be shown below. Considering the second term in Eq. (5), we rewrite

$$\begin{aligned} \langle J^\mu\rangle_A &= -iem\,\mathrm{Tr}\gamma^\mu\int\frac{d^3k}{(2\pi)^3}\frac{1}{-k^2+m^2-\frac{ie}{4}[\gamma_\nu,\gamma_\rho]F^{\nu\rho}} \\ &= -iem\,\mathrm{Tr}\gamma^\mu\int\frac{d^3k}{(2\pi)^3}\frac{1}{-k^2+m^2} \\ &\qquad\times\left[1+\frac{ie}{4}\frac{ie[\gamma_\nu,\gamma_\rho]F^{\nu\rho}}{k^2-m^2} +O\left[\frac{1}{m^4}\right]\right] \ , \end{aligned} \tag{6}$$

where we use the relation $\not{D}^2+m^2=D^2+m^2-(ie/4)[\gamma_\nu,\gamma_\rho]F^{\nu\rho}$, and the complete set of Dirac operators, and redefined $(ik_\mu-ieA_\mu)\to ik_\mu$ with the fixed background gauge field. At this stage, we know the first term vanishes in Eq. (5) because it is an odd integral after redefining of momentum variable. By integrating out in the imaginary time formalism [4], we obtain the following expression at the large mass limit:

$$\begin{aligned} \langle J^\mu\rangle_A &= -\frac{ie^2m}{16\pi\beta}\sum_{n=-\infty}^{\infty}\frac{1}{\frac{(2n+1)^2\pi^2}{\beta^2}+m^2} \\ &\qquad\times\mathrm{Tr}(\gamma^\mu[\gamma^\nu,\gamma^\rho])F_{\nu\rho} \ , \end{aligned} \tag{7}$$

where β is the inverse of the temperature. By performing the series sum [7] we find the parity-violating term

$$\begin{aligned} \langle J^\mu\rangle_A &= \frac{\kappa_\beta}{8\pi}\epsilon^{\mu\nu\rho}F_{\nu\rho} \ , \\ \kappa_\beta &= e^2\frac{m}{|m|}\tanh\frac{\beta|m|}{2} \ , \end{aligned} \tag{8}$$

which is the well-known result [5], and gives the correct result in the limit of zero temperature [2]. Note that in

Eq. (6), the linearly divergent first integral vanishes due to the traceless property of gamma matrix. Now we integrate out $\langle J^\mu \rangle_A$ with respect to the background gauge field by using the relation (3). Then the Chern-Simons action is obtained as follows:

$$W = \frac{\kappa_\beta}{8\pi} \int d^3x \, \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho \ . \tag{9}$$

As a comment, the parity anomaly seems to be temperature dependent from Eq. (9) in contrast with the even-dimensional chiral anomaly [4]. However, we could not regard the Chern-Simons action as a term of parity anomaly because the original action (1) has the parity-violating fermionic mass term at the outset. If we start with the massless fermionic theory with the Pauli-Villars counterterm, we could obtain the temperature-independent Chern-Simons action in the framework of finite temperature formalism. As a result, the Chern-Simons coefficient is $(e^2/8\pi)(M/|M|)$ when M is an infinite Pauli mass. This calculation is very similar to the massive case and we do not present it here.

The present study was supported in part by the Basic Science Research Institute program, Ministry of Education, 1992, Project No. 236, and by the Korea Science and Engineering Foundation through the Center for Theoretical Physics, Seoul National University.

[1] S. Deser, R. Jackiw, and S. Templeton, Phys. Rev. Lett. **48**, 975 (1982); Ann. Phys. (N.Y.) **140**, 372 (1982).

[2] A. Niemi and G. W. Semenoff, Phys. Rev. Lett. **51**, 2077 (1983); A. N. Redlich, *ibid.* **52**, 18 (1984); Phys. Rev. D **29**, 2366 (1984).

[3] For reviews: *Fractional Statistics and Anyon Superconductivity,* edited by F. Wilczek (World Scientific, Singapore, 1990).

[4] L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974).

[5] E. R. Poppitz, Phys. Lett. B **252**, 417 (1990); L. Moriconi, Phys. Rev. D **44**, R2950 (1991); M. Burgess, *ibid.* **44**, 2552 (1991).

[6] Our conventions: $\gamma^0=\sigma_3$, $\gamma^1=i\sigma_2$, $\gamma^2=i\sigma_3$, $g_{\mu\nu}=\mathrm{diag}(1,-1,-1)$, and $\epsilon^{012}=+1$.

[7] K. Knopp, *Infinite Sequences and Series* (Dover, New York, 1956).