Natural framework for solar and 17-keV neutrinos

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Motivated by recent experimental claims for the existence of a 17-keV neutrino and by the solar-neutrino problem, we consider a class of models which contain in their low-energy spectrum a single light sterile neutrino and one or more Nambu-Goldstone bosons. In these models, the required pattern of small neutrino masses and Nambu-Goldstone-boson couplings are understood as the low-energy residue of the pattern of breaking of lepton-number symmetries near the electroweak scale, and all mass hierarchies are technically natural. The models are compatible with all cosmological and astrophysical constraints, and can solve the solar-neutrino problem either via the Mikheyev-Smirnov-Wolfenstein effect or vacuum oscillations. The deficit in atmospheric muon neutrinos seen in the Kamiokande and IMB detectors can also be explained in these models.

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I. INTRODUCTION

There are presently several reported experimental anomalies which suggest that there is new physics lurking in the neutrino sector, and although any one of these can be incorporated by minimal modifications of the standard model, it is more difficult to incorporate several of these anomalies simultaneously. It is the purpose of this paper to argue that the known indications of new neutrino physics can be naturally understood in terms of the low-energy residue of a particular pattern of lepton-number violation at energies large compared with the weak scale.

The experimental indications of new neutrino physics are the following.

(i) *The solar-neutrino problem:* Recent measurements from Kamiokande [1] and Baksan [2] appear to confirm earlier observations [3] of a deficit in the flux of solar neutrinos as compared to what is predicted by solar models [4]. Although the low event rates make the experiments extremely challenging and uncertainties linger in the theoretical prediction, the recent confirmations of the deficit in different types of detectors, including those that are sensitive to the main *p-p* nuclear cycle in the Sun, have given increased weight to the possibility that new neutrino physics may be responsible. The most popular proposals for the solution of the solar-neutrino problem involve neutrino oscillations between ν_e and some other species.

(ii) *The 17-keV neutrino:* In 1985, Simpson [5] reported experimental evidence for a 17-keV neutrino which mixes with the electron neutrino at the 10% level. This claim was very controversial, since subsequent experiments failed to confirm the effect [6], although Simpson [7] has argued that these experiments were inconclusive. Recently, there has been renewed interest in the subject, with several reports confirming Simpson's results [8], and several others claiming to rule them out [9]. While it is clear that the issue of the existence of the 17-keV neutrino is far from settled, it is striking that the experimental groups which see the effect agree very well on the values of the mass and mixing angle within the experimental uncertainties.

(iii) *The atmospheric-neutrino deficit:* The relative flux of electron- and muon-type neutrinos originating from the decays of pions produced when cosmic rays impinge on the upper atmosphere has been measured in several neutrino detectors. These neutrinos are produced in charged-pion decays through the chain $\pi\to\mu\nu_\mu$ followed by $\mu\to e\nu_e\nu_\mu$. The naive expectation that two ν_μ's should be produced for each ν_e is borne out in detailed simulations which predict $N(\nu_e)/N(\nu_\mu)=0.45$. This ratio as measured by both Kamiokande and IMB [10] is larger than predicted. These observations could be accounted for by near-maximal mixing of ν_μ with another species of neutrino.

None of these experimental results is beyond controversy at present. More experiments are currently underway to determine which (if any) of these effects are real.

Taken separately, each of the neutrino results can be easily accounted for in terms of a particular form for the masses and mixing of the three known neutrino types. However, we wish to argue that the solar-neutrino problem and the existence of the 17-keV neutrino, together with current cosmological and astrophysical bounds, point toward a specific form for the neutrino mass matrix which can arise naturally from new physics at high scales. This same form for the mass matrix can also account for the atmospheric muon-neutrino deficit. While we feel that it is certainly premature to take *all* of these results seriously, we find that it is still interesting to see that they can all be accommodated in a rather simple and natural framework. We will therefore suspend our disbelief and address ourselves to the question of how the neutrino masses and mixing needed to solve the solar-neutrino problem and incorporate the 17-keV neutrino can be added to the standard model.

This paper is organized as follows. In Sec. II, we briefly recount the constraints on the properties of a 17-keV neutrino. We argue that the existence of a 17-keV neutrino, together with a neutrino solution to the solar-neutrino problem, requires the existence of a light sterile neutrino species, and suggests the existence of Nambu-Goldstone bosons (Majorons). In Secs. III and IV, we derive the general form of the interactions of the neutrinos and Majorons at energies below the weak scale, and discuss how these are constrained by laboratory experiments and cosmological and astrophysical arguments. In Secs. V–VII, we explore the implications of this phenomenology at higher energies. Section V gives a statement of the naturalness requirements to which we adhere in our exploration of the candidate models for high-energy physics that might produce the "observed" neutrino spectrum. Sections VI and VII give examples of models which satisfy these criteria. Our conclusions are summarized in Sec. VIII.

II. IMPLICATIONS OF A 17-keV NEUTRINO

There are many constraints on the properties of the 17-keV neutrino, which are usefully reviewed, e.g., in [11]. It cannot be mainly the muon neutrino, since direct bounds on ν_e-ν_μ oscillations already rule out a 10% mixing. In order to avoid conflicting with double-β-decay experiments, the contribution of the 17-keV neutrino to the rate for neutrinoless double-β decay must be accurately canceled by the contributions from other neutrino states. This cancellation arises most naturally if there are two neutrino states of opposite *CP* parity with Majorana masses close to 17 keV, that is, if the 17-keV neutrino is a Dirac (or pseudo Dirac) state. In this case the suppression of the neutrinoless β-decay rate can be understood as being due to the approximate conservation of a quantum number carried by the 17-keV neutrino. It will turn out that this quantum number can be only approximately conserved if the solar-neutrino problem is solved by neutrino oscillations.

This type of neutrino mass spectrum may be obtained using only the three known neutrinos if the ν_μ and ν_τ form the nearly degenerate 17-keV neutrino. If this is the case, then there is no way to solve the solar-neutrino problem by neutrino mixing, since such a solution would require another neutrino state with mass less than 10 eV that can mix with ν_e. The solar-neutrino problem and the 17-keV neutrino taken together therefore require the existence of at least one new neutrino species s beyond the three already observed. This new state must be sterile, i.e., it cannot carry $SU(2)_W \times U(1)_Y$ quantum numbers since it was not observed in the Z width at the CERN e^+e^- collider LEP.

There are therefore two possibilities: either the sterile state forms part of the 17-keV neutrino, or it mixes with the electron neutrino to solve the solar-neutrino problem. Suppose that the first possibility holds [12]. In this case there is a stringent bound coming from the energetics and timing of the observed neutrino pulse from the supernova SN 1987A. The idea is that helicity-flipping processes can produce the sterile state in the core, resulting in rapid core cooling via emission of sterile neutrinos. Early work [13] on this subject gave a bound of $m_D \leq 28$ keV (when corrected for an erroneous factor of 4), but there have been subsequent claims [14] that effects such as neutrino degeneracy will lower the bound to $\simeq 1$ keV. The situation is not yet settled, since there are other competing contributions which have not yet been included in any detailed numerical calculation [15]. Despite the uncertainties in the supernova bound, we will not pursue this possibility here, but concentrate instead on the scenario in which the electron neutrino mixes with the sterile neutrino state [12] to solve the solar-neutrino problem, and ν_μ and ν_τ pair into a pseudo Dirac 17-keV neutrino state that mixes with ν_e at the 10% level.

Cosmological constraints on massive neutrinos suggest that the low-energy spectrum of the theory must be enlarged even further. If a neutrino species with a mass in the range 100 eV $\lesssim m_\nu \lesssim 1$ GeV were absolutely stable and in chemical equilibrium, its present energy density would result in an unacceptably young Universe. A mechanism is therefore required to deplete the number density of the 17-keV neutrino.

It is possible that the 17-keV neutrino is stable and that its number density in the early Universe is depleted by annihilation mediated by some new interactions, but it is far more natural simply to make the 17-keV neutrino unstable. (Standard arguments to this effect are reviewed, e.g., in [11].) The lifetime that is required is shorter than $\sim 10^{12}$ sec. The only standard-model candidates for the decay products of a 17-keV neutrino are $\nu_{17} \to \nu\gamma$ or $\nu_{17} \to 3\nu$. The decay into photons is severely constrained, and exotic interactions are required to make the three-neutrino decay mode sufficiently rapid. We find it simpler to posit another light particle into which the 17-keV neutrino can decay.

In fact, a candidate for such a light particle arises naturally in the class of models we will be considering. In these models, the approximate symmetries that suppress neutrinoless double-β decay are assumed to be broken *spontaneously*. The fact that these symmetries are still approximate is explained by the fact that the symmetry-breaking sector is weakly coupled to observed particles as an automatic consequence of the quantum numbers of the order parameter. In this case, the theory automatically contains massless Nambu-Goldstone bosons (Majorons) which are weakly coupled to the neutrinos and allow the decay mode $\nu_{17} \to \nu' \chi$, where χ is a Majoron.

This mechanism is not the only way to incorporate such Majorons. An alternative would be to consider Nambu-Goldstone bosons arising from the spontaneous breaking of a larger symmetry group which contains the approximate symmetries in our models. Or the broken symmetries could be both explicitly and spontaneously broken in the underlying theory. In this case, the Majoron would be a *pseudo* Nambu-Goldstone boson with a mass and nonderivative interactions whose sizes would be determined by the strength of the explicit breaking of the symmetry. However, we concentrate on the first option because it is more constrained, and because it connects the existence of the Majorons directly with the origin of the approximate symmetries which are, anyhow, already required at low energies.

There is an additional cosmological constraint on neutrino lifetimes which can be derived from considerations of structure formation. In the standard scenario, the structure observed in the Universe today is formed by the gravitational amplification of small density perturbations in the early Universe. This amplification cannot occur during a radiation-dominated epoch, and demanding that the decay products of the 17-keV neutrino do not overly prolong this epoch gives a lower bound on its lifetime. According to Ref. [16] the standard scenario remains undisturbed provided that the lifetime is shorter than $\sim 10^6$ sec. However, some recent studies of large-scale structure [17] indicate that if the 17-keV neutrino lifetime were as large as 10^7-10^8 sec, it might actually improve the status of cold-dark-matter models by enhancing the strength of correlations of density perturbations at long distances. We will find that there are models which satisfy all other bounds but which are in conflict with the structure formation bounds. Since the paradigm for structure formation is not well tested, we do not consider such models to be ruled out.

A final constraint arises if the final state for 17-keV-neutrino decay should include $\bar{\nu}_e$'s. If so, then the light products of heavy neutrinos that decay in flight while in route from SN 1987A can arrive much later than those that are emitted directly from the core. Agreement between the length of the observed pulse and supernova models then requires that the lifetime not lie between 3×10^4 and 2×10^8 sec.

III. NEUTRINO MASSES AND MIXINGS

β-decay experiments, solar-neutrino measurements, and atmospheric neutrinos all probe neutrino properties at energies very low compared to the weak scale. Their implications for the neutrino sector may therefore be most succinctly expressed in terms of the properties of the low-energy theory obtained after integrating out all particles that are heavier than 17 keV. In this section we collect the implications for this low-energy theory of the recent neutrino results. These are used in subsequent sections to infer some of the properties of the underlying physics at higher energies that might be responsible for such an effective theory.

In the standard model the spectrum at extremely low energies contains four exactly massless particles: three left-handed neutrino flavors ν_e, ν_μ, and ν_τ, and the photon. The masslessness of the neutrinos can be explained by the conservation of the three lepton numbers, while the masslessness of the photon is explained by electromagnetic gauge invariance.

Motivated by the arguments of the previous section, we suppose that this spectrum is supplemented by at least two additional states: a single sterile fermion s and a light pseudo Nambu-Goldstone boson χ. χ is kept massless (and, at the renormalizable level, noninteracting) by a symmetry $\chi\to\chi+f$, with f an arbitrary constant.

As for the conserved lepton numbers, all three cannot be symmetries of the low-energy Lagrangian if it is to naturally account for the 17-keV neutrino and to solve the solar-neutrino problem, since the 17-keV neutrino must be unstable and ν_e must oscillate into another light state. Instead, a symmetry is required that can ensure that the 17-keV neutrino is a ν_μ-ν_τ pseudo Dirac state and which allows this state to mix with ν_e at the 10% level. The symmetry must also ensure that the sterile neutrino remains sufficiently light that its mixing with ν_e can deplete the observed solar-neutrino flux.

The pseudo Dirac nature of the 17-keV state and its mixing with ν_e is ensured if the theory approximately preserves the linear combination $e-\mu+\tau$ of the standard-model lepton numbers [18]. The absence of a large Majorana mass for the sterile fermion s suggests a further approximate U(1) chiral symmetry which may be defined so that it rephases only s. We therefore assume that the low-energy Lagrangian approximately preserves the symmetry

$$G_\nu \equiv \mathrm{U}(1)_{e-\mu+\tau}\times \mathrm{U}(1)_s \ , \tag{1}$$

under which the left-handed neutrino fields transform as

$$\nu_e,\nu_\tau \sim (1,0), \quad \nu_\mu\sim(-1,0), \quad s\sim(0,1) \ . \tag{2}$$

Of course, G_ν cannot be an exact symmetry, since it also forbids the ν_e-s oscillations that are to account for the solar-neutrino deficit. G_ν must therefore be only an *approximate* symmetry of the low-energy theory. More will be said about the origins of this symmetry breaking once we discuss explicit models for the underlying physics.

Subject to these assumptions the neutrino mass terms must take the following form when expressed in terms of a weak-interaction basis of left-handed fields:

$$\mathcal{L}_m = -\frac{1}{2}\begin{bmatrix} s \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}^T (M_0+\delta M)\begin{bmatrix} s \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} + \mathrm{H.c.} \tag{3}$$

Here M_0 is G_ν-invariant but $\delta M \ll M_0$ is not. We write

$$M_0 = m_{17}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & s & 0 & c \\ 0 & 0 & c & 0 \end{bmatrix} ,$$
$$\delta M = m_{17}\begin{bmatrix} \gamma & \alpha_1 & \beta & \alpha_2 \\ \alpha_1 & \epsilon_1 & 0 & \epsilon_2 \\ \beta & 0 & \eta & 0 \\ \alpha_2 & \epsilon_2 & 0 & \epsilon_3 \end{bmatrix} , \tag{4}$$

where $s=\sin\theta_{17}$, $c=\cos\theta_{17}$, and θ_{17} is the ν_e-ν_τ mixing angle. For simplicity we choose the elements of δM to be real. The notation is chosen such that matrix elements that are represented by the same greek letters transform identically with respect to the symmetry group G_ν. Since mass-matrix elements that transform in the same way should be of the same order of magnitude, this notation is useful when choosing the symmetry-breaking patterns that are required to produce the "observed" hierarchies in the mass matrix.

It is often convenient to refer to the rotated basis

$$|\nu_e'\rangle = c|\nu_e\rangle - s|\nu_\tau\rangle \ , \qquad |\nu_\tau'\rangle = c|\nu_\tau\rangle + s|\nu_e\rangle \ , \tag{5}$$

in which

$$M_0' = m_{17}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \ , \qquad \delta M' = m_{17}\begin{bmatrix} \gamma & \alpha_1' & \beta & \alpha_2' \\ \alpha_1' & \epsilon_1' & 0 & \epsilon_2' \\ \beta & 0 & \eta & 0 \\ \alpha_2' & \epsilon_2' & 0 & \epsilon_3' \end{bmatrix} \ . \tag{6}$$

(The only relations between the primed and unprimed matrix elements which will be needed in the following are $\alpha_1' = c\alpha_1 - s\alpha_2$ and $\alpha_2' = c\alpha_2 + s\alpha_1$.)

In what follows, we assume that the low-energy theory breaks G_ν via order parameters transforming under G_ν in specified ways. The choice of order parameters determines the hierarchy of the elements of δM. In order to determine the order parameters required, we first discuss the phenomenology which results from the mass matrix of Eq. (4).

In the limit $\delta M \to 0$, the spectrum consists of a massive Dirac state

$$|\nu_{h\pm}\rangle = \frac{1}{\sqrt{2}}(|\nu_\tau'\rangle \pm |\nu_\mu\rangle) \ , \tag{7}$$

with mass $m_{h\pm} = m_{17}$, together with two massless states. We can compute the spectrum for $\delta M \ll M_0$ using standard degenerate perturbation theory. To second order in δM, the heavy states become split with

$$\Delta m_h^2 \equiv m_{h+}^2 - m_{h-}^2 = 2m_{17}^2(\epsilon_3' + \eta + 2\beta\alpha_2') + O((\delta M)^3) \ .$$

To first order in δM, the massless states acquire masses

$$m_{l\pm} = \frac{m_{17}}{2}(\lambda \pm \Delta) \ , \tag{8}$$

where

$$\Delta \equiv \gamma + \epsilon_1' \ , \tag{9}$$

$$\lambda \equiv \sqrt{(\gamma - \epsilon_1')^2 + 4\alpha_1'^2} \ . \tag{10}$$

The mass splitting of the light states is

$$\Delta m_l^2 = m_{17}^2 \lambda \Delta \ . \tag{11}$$

To first order in δM, the light eigenstates are given by

$$|\nu_{l+}\rangle = \cos\theta_l|s\rangle + \sin\theta_l|\nu_e'\rangle \ , \qquad |\nu_{l-}\rangle = \cos\theta_l|\nu_e'\rangle - \sin\theta_l|s\rangle \ , \tag{12}$$

where

$$\tan 2\theta_l = \frac{2\alpha_1'}{\gamma - \epsilon_1'} \ . \tag{13}$$

We now have in hand the physical quantities that arise in neutrino phenomenology in terms of the properties of the neutrino mass matrix. The constraints on the parameters introduced above are as follows.

(i) *Laboratory mass bound:* The present bound $m_{\nu_e} < 9$ eV on the mass of the electron neutrino implies a similar bound on the mass of the light-neutrino state that dominantly overlaps ν_e. In the models we consider, this constraint is easily satisfied.

(ii) *The 17-keV neutrino:* The experiments which see a 17-keV neutrino find that it is produced in approximately 1% of β decays. This requires

$$m_{17} = 17 \text{ keV} \ , \qquad \sin\theta_{17} \simeq 0.1 \ . \tag{14}$$

(iii) *ν_μ-ν_τ oscillations:* Because ν_μ-ν_τ mixing is nearly maximal, the failure to observe ν_μ disappearance at Fréjus implies $\Delta m_h^2 \le 5\times10^{-3}$ eV^2, which gives

$$\delta_h \equiv \epsilon_3' + \eta + 2\beta\alpha_2' \le 2\times10^{-11} \ . \tag{15}$$

(iv) *Atmospheric neutrinos:* The atmospheric-neutrino anomaly reported by Kamiokande and IMB can be explained by near-maximal ν_μ-ν_τ mixing provided that $\Delta m_h^2 \ge 1\times10^{-3}$ eV^2, which gives

$$\delta_h \gtrsim 4\times10^{-12} \ . \tag{16}$$

(v) *The solar-neutrino problem:* ν_e-s oscillations may deplete the solar-neutrino flux observed on Earth either through resonant MSW oscillations in the Sun or through maximal vacuum oscillations. Resonant oscillations are the currently favored mode of solution given the small size of the flux measured by the chlorine experiment. Maximal vacuum oscillations tend to reduce the solar-neutrino flux by an overall factor of 2, unless the oscillation length happens to be close to the Earth-Sun distance, so-called "just-so" oscillations [19]. A factor-of-2 suppression would be in agreement with the Kamiokande measurement, but well outside of the 90%-confidence-level upper bound for the chlorine experiment if we use the theoretical predictions of Ref. [4]. Nonetheless, in what follows, we entertain the idea that maximal mixing with an overall neutrino flux suppression of one half may turn out to be the correct solution of the solar-neutrino problem, and we consider this scenario alongside the more traditional Mikheyev-Smirnov-Wolfenstein (MSW) and "just-so" scenarios. The reader is free to disregard this region of parameter space.

Although we are working with a four-state system, it is clear that the 17-keV neutrino is too massive to be relevant for the solar-neutrino problem. Therefore, we can reduce the problem to that of mixing between the states $|s\rangle$ and $|\nu_e'\rangle$. The parameter regions that are allowed for the different solutions to the solar-neutrino problem are as follows.

(a) *Maximal vacuum oscillations:* Maximal vacuum oscillations can "solve" the solar-neutrino problem provided that $\sin^2 2\theta_l \simeq 1$ and $\Delta m_l^2 \gtrsim 10^{-10}$ eV^2. The lower limit of this mass range corresponds to "just-so" oscillations [19]. In addition, there is a cosmological bound arising from the observation that maximal ν_e-s oscillations can change the number density of ν_e's required for the standard model of big-bang nucleosynthesis. This bound is $\Delta m_l^2 \leq 2\times 10^{-7}$ eV^2. Putting this together, we find the restrictions

$$3\times 10^{-19} \lesssim \lambda\Delta \lesssim 6\times 10^{-16} \tag{17}$$

and

$$\gamma - \epsilon_1' \ll 2\alpha_1' \ . \tag{18}$$

(b) *Resonant oscillations:* Resonant MSW oscillations require $10^{-4} \lesssim \sin^2 2\theta_l \lesssim 0.7$, which gives

$$1 \lesssim \frac{\gamma - \epsilon_1'}{\alpha_1'} \lesssim 200 \ . \tag{19}$$

Because Kamiokande II observes some solar ν_μ's, comparison with SAGE and the ^{37}Cl data can be used to distinguish between $\nu_e \leftrightarrow \nu_\mu$ and $\nu_e \leftrightarrow s$ oscillations. This has been studied in detail [21] with the result that only the nonadiabatic branch of the MSW triangle is allowed, and it is shifted to somewhat lower values of Δm^2 relative to the ordinary MSW effect. Numerically, this branch is specified by

$$\Delta m_l^2 \frac{\sin^2 2\theta_l}{\cos 2\theta_l} \simeq 6.8\times 10^{-8}\ \text{eV}^2 \ , \tag{20}$$

which gives

$$\alpha_1' \simeq 8\times 10^{-9} \ . \tag{21}$$

IV. MAJORON COUPLINGS

The low-energy interactions of Nambu-Goldstone bosons are largely dictated by the symmetry-breaking pattern which give rise to them. This allows us to treat the Majorons which are assumed to appear in our models in our general framework. The lowest-dimension interactions between the neutrinos and Majorons have dimension 5:

$$\mathcal{L}_\chi = \frac{1}{f}\partial_\mu \chi J^\mu \ , \tag{22}$$

where J^μ is the conserved current which is spontaneously broken at the scale f. [If the symmetry is broken by a set of fields Φ_a whose charges and vacuum expectation values (VEV's) are q_a and v_a, respectively, then $f = 2(2\sum_a q_a^2 v_a^2)^{1/2}$.] The coupling to neutrino species ν_j ($j = s, e, \mu, \tau$) is therefore determined by its quantum numbers with respect to the broken symmetry. We write

$$J^\mu = i\bar{\nu}_j \gamma^\mu Q_{jk} \gamma_5 \nu_k \ , \tag{23}$$

where Q_{jk} is the Hermitian matrix which generates the symmetry on a basis of left-handed fields. If the left-handed fermions are rotated to a mass eigenbasis via a unitary matrix U, then the corresponding charge in terms of the mass basis becomes $Q' = U^\dagger Q U$.

As discussed in previous sections, the most economical assumption is that the broken symmetry to which the Majorons couple is G_ν itself. In this case the generators that represent the two factors of this symmetry on the left-handed neutrino fields are both diagonal in the weak-interaction basis:

$$S = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \quad \text{and} \quad L = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix} . \tag{24}$$

In the mass basis, these charges become

$$\begin{aligned} S_{ab}' &= U_{sa}^* U_{sb} \ , \\ L_{ab}' &= U_{ea}^* U_{eb} - U_{\mu a}^* U_{\mu b} + U_{\tau a}^* U_{\tau b} \ . \end{aligned} \tag{25}$$

Here, $a,b = l\pm, h\pm$ label the mass eigenstates.

For both of these generators, the matrix elements that link the heavy with light states vanish at zeroth order in δM. The leading contributions are most conveniently tabulated for the linear combinations $L' \pm S'$:

$$\begin{aligned} (L'+S')_{l\pm,h\pm} &= \sqrt{2}\begin{pmatrix} c_L\alpha_2' + s_L\epsilon_2' & c_L\alpha_2' + s_L\epsilon_2' \\ -s_L\alpha_2' + c_L\epsilon_2' & -s_L\alpha_2' + c_L\epsilon_2' \end{pmatrix} , \\ (L'-S')_{l\pm,h\pm} &= \sqrt{2}\begin{pmatrix} c_L\beta + s_L\epsilon_2' & -c_L\beta + s_L\epsilon_2' \\ -s_L\beta + c_L\epsilon_2' & s_L\beta + c_L\epsilon_2' \end{pmatrix} , \end{aligned} \tag{26}$$

where $c_L = \cos\theta_l$ and $s_L = \sin\theta_l$. For a given choice of symmetry-breaking scalar fields, one can find two orthogonal directions in field space corresponding to the two Majorons. Each couples to its own particular linear combination Q' of the above charges. Then the partial lifetime for the decay of ν_h into that Majoron and one of the two light-neutrino states is

$$\begin{aligned} \tau(\nu_h \to \nu_l \chi) &= \frac{4\pi f^2}{m_h^3 |Q_{hl}'|^2} \\ &= (1.7\times 10^{-5}\ \text{sec}) \left[\frac{f}{100\ \text{GeV}}\right]^2 \frac{1}{|Q_{hl}'|^2} \ . \end{aligned} \tag{27}$$

Satisfaction of the cosmological bound coming from the age of the Universe therefore requires $f/|Q_{hl}'| \lesssim 2\times 10^{10}$ GeV and the structure-formation bound is 100 times smaller.

V. NATURALNESS CRITERIA

We wish to show that the desired neutrino mass pattern can arise in "natural" models. Our criteria for naturalness are supposed to capture the idea that we do not want to give up any of the successes of the standard model, and we do not want to add to its shortcomings.[1] Specifically, our requirements are as follows.

(i) We demand that the model have no new symmetry-breaking scales below the weak scale. The reason for this condition is that all of the known ways of understanding the magnitude of the weak scale (compared, say, to the Planck scale) necessarily involve new particles and interactions not far above the weak scale. If we were to introduce a new symmetry-breaking scale below the weak scale, it would be difficult to imagine how such a hierarchy could be explained without introducing new light particles which should already have been observed.

In fact, in the context of the models we discuss below, the smallness of the neutrino masses compared to the electroweak scale is due to physics at a very large scale, $M \gg M_W$. It is therefore useful to view the standard model as an effective theory which is valid below the scale M. We can summarize the low-energy effects of the physics above the scale M by including all possible higher-dimensional operators in the effective Lagrangian. The coefficients of the higher-dimensional operators in this Lagrangian are proportional to inverse powers of M, and so their effects are typically suppressed at low energies.

(ii) We also demand that the magnitudes of all small parameters be understood in terms of symmetry principles. This criterion comes in two parts. First, supposing that a parameter, such as a neutrino mass, should turn out to be small in the underlying microphysical Lagrangian above the scale M, we require that the smallness of this parameter should be stable under its renormalization to lower energy scales where it is measured. This is ensured if the small parameter satisfies the naturalness criterion of 't Hooft, according to which a parameter is naturally small if additional symmetry arises in the limit at which the parameter in question vanishes. To the extent that the renormalization process preserves this symmetry, the vanishing of the symmetry-breaking parameter must be stable under renormalization, and any deviations from zero that are generated by renormalization are automatically proportional to the original value of the parameter itself. The electron mass is a familiar example of a parameter that is naturally small according to this criterion, since the standard model acquires an extra chiral symmetry in the limit in which the electron mass vanishes.

Of course, naturalness in this technical sense does not address the question of why the parameter is small in the underlying theory in the first place. The second part of our criterion follows from the motivation that we would ultimately like some understanding of the origin of the smallness of a parameter in the underlying high-energy theory. When we turn to models of physics at the scale M, we therefore propose that small parameters such as neutrino masses can be understood in terms of a hierarchy of symmetry-breaking scales, which are themselves protected by a symmetry. An attractive feature of the specific models we will discuss is that they require the introduction of only one new large scale M, and all small parameters are understood in terms of the ratio, v/M, between this and the weak scale v.

(iii) We assume that the only light degrees of freedom that appear in the effective theory at and below electroweak scales are the usual standard-model particles (including a single Higgs doublet), supplemented by the minimal number of additional degrees of freedom that are required to account for the solar-neutrino problem and the 17-keV neutrino. As discussed above, we take these to be a single electroweak-singlet fermion and (at least) one electroweak-singlet Goldstone boson into which the 17-keV neutrino can decay. We do not address the hierarchy problem associated with the standard Higgs field, since we do not expect this to be more difficult to solve here than within the standard model, using supersymmetry, for example.

We next turn to the construction of explicit models which produce the desired low-energy behavior in a natural way.

VI. A VACUUM-OSCILLATION MODEL

The pattern of neutrino masses in our framework is determined by the hierarchies in the mass matrix δM. This, in turn, is predominantly controlled by the quantum numbers of the order parameters that break G_ν. As might be expected, the required quantum numbers differ significantly depending on whether the solar-neutrino problem is solved through resonant or maximal vacuum oscillations, and we treat these cases separately. In this section, we present a model with maximal vacuum oscillations.

In order to systematically build in our naturalness requirements, we begin our analysis at the level of an effective theory valid at the scale at which the symmetry G_ν breaks. This scale will turn out to be near the weak scale.

A. Weak-scale effective theory

The degrees of freedom at the scale at which G_ν is broken are assumed to be the usual standard-model fields, together with the gauge-singlet fermion s and two gauge-singlet scalar fields ϕ_1 and ϕ_2 transforming under G_ν as

$$\phi_1 \sim (\tfrac{1}{2}, -\tfrac{1}{2}) \ \text{and} \ \phi_2 \sim (-\tfrac{1}{2}, -\tfrac{1}{2}) \ . \tag{28}$$

The lowest-dimensional gauge- and G_ν-invariant operators in the effective Lagrangian at this scale that contribute to the neutrino mass matrix are

[1]This succinct formulation follows Barbieri and Hall [22].

$$\text{dimension 5:}\quad \frac{g_e}{M}(L_eH)(L_\mu H),\quad \frac{g_\tau}{M}(L_\mu H)(L_\tau H)\;;$$
$$\text{dimension 6:}\quad \frac{a_j}{M^2}(L_jH)s\phi_2^2,\quad \frac{b}{M^2}(L_\mu H)s\phi_1^2\;;$$
$$\text{dimension 7:}\quad \frac{c}{M^3}ss\phi_1^2\phi_2^2\;; \tag{29}$$
$$\text{dimension 9:}\quad \frac{d_{\mu\mu}}{M^5}(L_\mu H)(L_\mu H)(\phi_1\phi_2^*)^2\;,$$
$$\frac{d_{jk}}{M^5}(L_jH)(L_kH)(\phi_1^*\phi_2)^2\;.$$

Here H is the usual electroweak Higgs doublet, the L's are the standard left-handed lepton doublets, and $j,k=e,\tau$ are generation indices. Explicit factors of a heavy mass scale M have been included so that the coefficients of these operators in the effective Lagrangian are dimensionless. If these operators arise from new physics at the scale M, and there are no symmetries beyond those we have assumed, then all of the coefficients of these operators will be of order unity in the absence of fine tuning.

If we replace the scalars with their vacuum expectation values

$$\langle H\rangle=v=174\ \text{GeV},\quad \langle\phi_1\rangle=w_1,\quad \langle\phi_2\rangle=w_2\;, \tag{30}$$

and define $g=\sqrt{g_e^2+g_\tau^2}$, then the mass-matrix parameters of Eq. (4) consistent with $m_{17}=17$ keV are

$$M=1\times10^7 gv\simeq2\times10^9 g\ \text{GeV}\;,$$
$$\alpha_j=\frac{a_jw_2^2}{gMv}\;,$$
$$\beta=\frac{bw_1^2}{gMv}\;, \tag{31}$$
$$\gamma=\frac{cw_1^2w_2^2}{gM^2v^2}\;,$$
$$\epsilon_j,\eta=\frac{dw_1^2w_2^2}{gM^4}\;.$$

Assuming that $v,w_1,w_2\ll M$, one has the hierarchy $\epsilon,\eta\ll\gamma\ll\alpha,\beta$. In this case, the light-neutrino states form a pseudo Dirac pair with

$$m_l=m_{17}\alpha_1'\;, \tag{32}$$

$$\Delta m_l^2=m_{17}^2\gamma\alpha_1'\;, \tag{33}$$

while the heavy-neutrino states have mass splitting

$$\Delta m_h^2=4m_{17}^2\beta\alpha_2'\;. \tag{34}$$

There are two Majorons in this model, χ_1 and χ_2, which can be thought of as the phases of the fields ϕ_1 and ϕ_2, respectively. χ_1 couples to the charge $Q_1\equiv S-L$, while χ_2 couples to $Q_2\equiv S+L$. The decay constants are related to the corresponding vacuum expectation values by $f=2\sqrt{2}w$. Using Eqs. (26) and (27), the lifetime is

$$\tau=\frac{16\pi}{m_h^3}\left[\frac{\alpha_2'^2}{w_2^2}+\frac{\beta^2}{w_1^2}\right]^{-1}\;. \tag{35}$$

In Sec. II it was noted that in order to satisfy constraints from SN 1987A and cosmology, the lifetime of the 17-keV neutrino should either be $<10^4$ sec or $\sim10^9$ sec. Either possibility can be accommodated in this model.

For the case of long lifetimes, we find that all the phenomenological constraints can be satisfied by choosing $w_1,w_2\sim3v$, $a_j=b=c=g_\tau=1$ and $g_e=0.1$. Then $\alpha,\beta\sim10^{-6}$, $\gamma\sim10^{-12}$, and the lifetime is 10^9 sec. The neutrino masses are given by

$$m_l\sim0.01\ \text{eV}\;,$$
$$\Delta m_l^2\sim10^{-10}\ \text{eV}^2\;, \tag{36}$$
$$\Delta m_h^2\sim10^{-3}\ \text{eV}^2\;.$$

Note that the hierarchies m_h/v and m_l/m_h, as well as $\Delta m_h^2/m_h^2$ and $\Delta m_l^2/m_l^2$, have been explained by the largeness of M relative to v, w_1, and w_2. It is interesting that both w_1 and w_2 preferentially lie near the weak scale because of the ν_μ-ν_τ oscillation bound and our naturalness condition that there be no symmetry-breaking scales below the weak scale. Δm_h^2 is then near the experimental upper limit and in the range required to account for the atmospheric-neutrino anomaly. Also, Δm_l^2 falls naturally into the correct range for "just-so" vacuum oscillations.

Although not a generic prediction of this model, fast 17-keV neutrino decays can be obtained by taking $w_1=100v$, $w_2=v/3$, $a_1=0.1$, $a_2=0.01$, $b=1$, and $c=g=0.3$. Then $\alpha_1=10^{-8}$, $\alpha_2=10^{-9}$, $\beta=10^{-2}$, $\gamma=10^{-10}$, and the lifetime is 2×10^3 sec. The mass splittings are the same as in (36), but m_l itself is now only 10^{-4} eV.

B. Renormalizable model

Here we present a renormalizable model defined at scale M which can give rise to the weak-scale Lagrangian just described. This is done only as an existence proof, since there are clearly many possible models, and it is unlikely that any foreseeable experiment could distinguish among them. In constructing a renormalizable model, we are guided solely by principles of economy.

The model contains the fields previously described with the addition of four gauge-singlet Dirac fermions. In terms of left-handed fields, they transform under G_ν as

$$N_1^\pm\sim\pm(\tfrac{1}{2},\tfrac{1}{2}),\quad N_2^\pm\sim\pm(\tfrac{1}{2},-\tfrac{1}{2}),\quad N_{3,4}^\pm\pm(1,0)\;. \tag{37}$$

Two copies of the last charge assignment are required in order to avoid an accidental symmetry of the neutrino mass matrix which forces two of the light states to be massless.

The renormalizable interactions of this model are the usual standard-model interactions, with the addition of

$$\text{dimension 3:}\quad M_jN_j^+N_j^-\;,$$
$$\text{dimension 4:}\quad (L_\mu H)N_{3,4}^+,\quad (L_jH)N_{3,4}^-,\quad sN_1^-\phi_1\;, \tag{38}$$
$$sN_2^+\phi_2,\quad N_{3,4}^-N_1^+\phi_1,\quad N_{3,4}^+N_1^-\phi_1^*\;,$$
$$N_{3,4}^-N_2^+\phi_2^*,\quad N_{3,4}^+N_2^-\phi_2\;.$$

If we assume that all dimensionless coefficients are of order 0.1–1, then the heavy fermions have masses of order 10^8 GeV. It is easy to check that when the heavy fermions are integrated out, the resulting weak-scale effective theory is exactly the one described above.

VII. AN MSW MODEL

We now turn to the construction of a model which can solve the solar-neutrino problem via resonant MSW oscillations. As in the previous section, we will find that G_ν is preferentially broken near the weak scale.

A. Weak-scale effective theory

The degrees of freedom at the G_ν-breaking scale are assumed to be the usual standard-model fields, together with the gauge-singlet fermion s and two electroweak-singlet scalar fields ϕ_j transforming under G_ν as

$$\phi_1 \sim (-\tfrac{1}{2}, -\tfrac{1}{2}) \text{ and } \phi_2 \sim (0, -\tfrac{2}{3}) . \quad (39)$$

The lowest-dimensional gauge- and G_ν-invariant operators in the effective Lagrangian at this scale that contribute to the neutrino mass matrix are

$$\begin{aligned} &\text{dimension 5: } \frac{g_e}{M}(L_e H)(L_\mu H), \quad \frac{g_\tau}{M}(L_\mu H)(L_\tau H) ; \\ &\text{dimension 6: } \frac{a_j}{M^2}(L_j H) s \phi_1^2, \quad \frac{c}{M^3} s s \phi_2^3 . \end{aligned} \quad (40)$$

Contributions to the remaining terms in the neutrino mass matrix are further suppressed relative to (40) by additional powers of M^{-1}.

Replacing the scalars with their vacuum expectation values $\langle H \rangle = v$ and $\langle \phi_j \rangle = w_j$, and defining $g \equiv \sqrt{g_e^2 + g_\tau^2}$, we find that the heavy scale must be $M = (1 \times 10^7) g v$, and that the dimensionless mass parameters of Eq. (4) are

$$\alpha_j = \frac{a_j w_1^2}{g M v}, \quad \gamma = \frac{c w_2^3}{g M v^2}, \quad \beta, \epsilon, \eta \ll \alpha, \gamma . \quad (41)$$

In this case, the light-neutrino states have masses

$$m_{l\pm} = \frac{m_{17}}{2}\left(\sqrt{\gamma^2 + 4\alpha_1'^2} \pm \gamma\right) , \quad (42)$$

with

$$\Delta m_l^2 = m_{17}^2 \gamma \sqrt{\gamma^2 + 4\alpha_1'^2} , \quad (43)$$

$$\sin^2 2\theta_l = \frac{4\alpha_1'^2}{\gamma^2 + 4\alpha_1'^2} . \quad (44)$$

The splitting of the heavy-neutrino states is negligible in this model.

MSW oscillations of the light states can be accommodated if we choose, e.g., $g=1$, $a_1=0.2$, $a_2=1$, and $c=1$. This gives $\alpha_1' = 1 \times 10^{-8}$, $\alpha_2' = 1 \times 10^{-7}$, $\gamma = 1 \times 10^{-7}$, and

$$\Delta m_l^2 = 3 \times 10^{-6} \text{ eV}^2 , \quad (45)$$

$$\sin^2 2\theta_l = 4 \times 10^{-2} . \quad (46)$$

The two Majorons of this model may be defined to couple to the charges $Q_1 \equiv L - S$, $Q_2 \equiv L$, respectively, giving a lifetime for the 17-keV state of

$$\tau = \frac{16\pi w_1^2}{\alpha_2'^2 m_h^3} . \quad (47)$$

For the choice of parameters given above, the lifetime is $\sim 10^{10}$ sec. This is in conflict with the cosmological structure-formation bounds, but is compatible with all other bounds.

The model considered above can be modified to accommodate the atmospheric-neutrino anomaly by adding a third electroweak-singlet scalar transforming under G_ν as

$$\phi_3 \sim (\tfrac{1}{2}, -\tfrac{1}{2}) . \quad (48)$$

Then there is an additional dimension-6 operator in the weak-scale effective Lagrangian:

$$\frac{b}{M^2}(L_\mu H) s \phi_3^2 , \quad (49)$$

which gives

$$\beta = \frac{b w_3^2}{g M v} . \quad (50)$$

This model is nonminimal, in the sense that there are now more scalar fields than order parameters. However, it can easily accommodate the atmospheric-neutrino anomaly for w_3 near the weak scale. Because of the additional freedom in this model, it can also give rise to very short heavy-neutrino lifetimes. For example, if we choose $w_1 = v/2$, $w_2 = v$, $w_3 = 30v$, $g = 0.1$, $a_1 = 0.1$, $a_2 = 0.01$, $b = 1$, and $c = 0.01$, we find that the model incorporates the MSW effect and atmospheric-neutrino oscillations, and the neutrino lifetime is

$$\tau = \frac{16\pi w_3^2}{m_h^3 \beta^2} \quad (51)$$

$$\simeq 2 \times 10^3 \text{ sec} \quad (52)$$

in the limit with $w_3 \gg w_1, w_2$ and $\beta \gg \alpha_i$.[2]

B. Renormalizable model

As an example of a renormalizable model which can give rise to the two-scalar effective theory discussed above, we add several gauge-singlet Dirac fermions transforming under G_ν as

$$N_1^\pm \sim \pm(\tfrac{1}{2}, -\tfrac{1}{2}), \quad N_2^\pm \sim \pm(0, \tfrac{1}{3}), \quad N_{3,4}^\pm \sim \pm(1, 0) . \quad (53)$$

---

[2]It is also possible to choose scalar quantum numbers so that β is naturally much larger than the other elements of δM, and so to have MSW, and atmospheric oscillations, short lifetimes, and no VEV's below the weak scale. If, for example, $\phi_1 \sim (\frac{1}{3}, -\frac{2}{3})$, $\phi_2 \sim (-\frac{1}{3}, -\frac{1}{3})$, and $\phi_3 \sim (\frac{2}{3}, -\frac{1}{3})$ then $\alpha_i \sim \gamma = O(v^2/M^2)$ while $\beta = O(v/M)$.

(Again, two copies of the last state are required to avoid accidental symmetries of the neutrino mass matrix.)

The most general renormalizable interactions of this model are the usual standard-model interaction, with the addition of

$$\text{dimension 3: } M_j N_j^+ N_j^- ,$$

$$\begin{aligned}\text{dimension 4: } & (L_j H) N_{3,4}^-, \quad (L_\mu H) N_{3,4}^+, \quad s N_1^+ \phi_1 , \\ & s N_2^- \phi_2, \quad N_{3,4}^+ N_1^- \phi_1, \quad N_{3,4}^+ N_1^+ \phi_1^* , \\ & N_2^+ N_2^+ \phi_2, \quad N_2^- N_2^- \phi_2^* . \end{aligned} \qquad (54)$$

If we assume that all dimensionless couplings are of order 0.1–1, then we obtain the effective theory presented above after integrating out the Dirac fermions.

VIII. CONCLUSION

We have shown that several recently reported experimental anomalies in the neutrino sector can be accounted for in a simple class of models with a single light-electroweak-singlet fermion s and an approximate $G_\nu \equiv U(1)_{e-\mu+\tau} \times U(1)_s$ symmetry. All neutrino mass hierarchies are understood in terms of the pattern in which this symmetry is broken. We have examined several models which can give rise to interesting symmetry-breaking patterns, and we always find that G_ν is broken near the weak scale, a feature which we find very attractive. The models are compatible with all astrophysical and cosmological bounds at present.

There are several ways in which the class of models we have discussed will be experimentally probed in the foreseeable future. The first and most obvious is the ongoing effort to confirm or disprove the experimental anomalies which are the motivation for these models. If, in particular, the atmospheric-neutrino effect should persist then it must be due to maximal ν_μ-ν_τ oscillations. Second, solar-neutrino oscillations are into a sterile component, which should be detectable once neutral-current solar-neutrino events are observed, for example at SNO. Third, in models where the Majorons arise from scalar fields, they can contribute a large invisible width to the Higgs boson via its decay to two Majorons. This gives rise to observable missing-energy events at LEP II and the Large Hadron Collider (LHC) at CERN or the Superconducting Super Collider (SSC) for a large portion of parameter space [23]. Fourth, if we are fortunate enough to observe another nearby supernova with detectors that count neutral-current events, and if the 17-keV neutrino lifetime is less than 10^4 sec, then all μ and τ neutrinos can have decayed before reaching the Earth. In addition, depleting the ν_μ and ν_τ fluxes could also prolong the ν_e signal. If, on the other hand, the lifetime should be on the order of 10^9–10^{11} sec, such a neutrino may have interesting applications for galaxy formation. If Simpson's neutrino should be ruled out, we can reduce the ν_e-ν_3 mixing angle with impunity to account for its disappearance. It is worth noting that the framework presented here can still naturally explain both solar and atmospheric neutrino oscillations.

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