# Intermediate scales of symmetry breaking in Calabi-Yau models

M. Masip
*Institute for Fundamental Theory, Department of Physics, University of Florida, Gainesville, Florida 32611*
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We discuss the generation of large intermediate scales of symmetry breaking in grand unification models suggested by the heterotic string. We analyze on dimensional grounds a particular scenario where one flat direction in the effective potential defines two different scales of gauge-symmetry breaking at very high energies ($\approx 10^{15}$ GeV). This mechanism implies the presence of one light ($\sim 1$ TeV) nonchiral neutrino. The size of the observable low-energy effects seems, however, quite model dependent.

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## I. INTRODUCTION

The ten-dimensional $E_8\times E_8$ heterotic string [1] contains, near the Planck scale, more dimensions, symmetry, and matter than that observed at low energies. To obtain realistic models, it is necessary to assume that the (unknown) dynamics of the theory favors a vacuum that compactifies the six extra dimensions, breaks the excess of symmetry, and makes the extra fields heavy enough.

In the framework of compactification on a three-generation Calabi-Yau manifold [2], one may distinguish several processes that decouple from the model its exotic ingredients. The identification of the spin connection of the manifold in the gauge fields leaves in the observable sector $N=1$ supersymmetry (SUSY) and $E_6$ gauge symmetry, with chiral superfields in the 27, $\overline{27}$, and 1 representations of this group. In the known cases, the three-family manifold is constructed from a nonsimply connected one, where nontrivial gauge configurations provide a breaking of the remaining $E_6$ symmetry to a smaller rank-6 group [3]. This process will introduce a convenient asymmetry (not present in the $E_6$ model) between the quark and lepton sectors. The effective theory must also break SUSY. In the most promising scenario [4], SUSY is broken at large scales in a sector connected just through gravitational interactions with quarks and leptons and one obtains the usual (order 1 TeV and universal at the Planck scale) soft-breaking terms. Finally, in order to reduce the gauge symmetry to the rank-four group $SU(3)_C\times SU(2)_L\times U(1)_Y$, the evolution of the model down to low energies must define intermediate scales (IS's) of symmetry breaking. Below these scales the exotic fields in the three chiral 27's of $E_6$ (two down-type quarks, two neutrinos, and two Higgs doublets) will combine into nonchiral representations of the standard-model symmetry and will (possibly) become massive. The vectorlike $27+\overline{27}$ multiplets will also acquire masses through effective nonrenormalizable interactions. The IS must be very large (typically $10^{15}$ GeV), since a slow proton decay and an acceptable value of the low-energy gauge couplings require that we are left at these energies with essentially a minimal supersymmetric extension of the standard model [7].

Although the IS's represent another source of uncertainty in the connection of the string with its observable limit, it has been argued [8] that they have model-independent implications that may be accessible to experiment. In particular, the IS's would imply the existence of fields in the TeV region producing neutrino masses and lepton-number-violating processes. We would like here briefly to review the mechanism that generates these IS's and also introduce a particular structure in the superpotential ($P$) with some interesting properties. This structure requires just one flat direction to produce the two stages of symmetry breaking required in rank-six models. It provides differences of several orders of magnitude between the IS's. In a realistic low-energy model, the Higgs sector (whose lightness is not protected by chirality [7]) and the neutrino sector (where seesaw masses [5] would reflect the physics in the *desert* ) are, in principle, sensitive to the splitting of the scales. This particular structure is not contained in the analysis done by Arnowitt and Nath [8], where two independent flat directions are implicitly assumed. In contrast with them, we find only one nonchiral neutrino which is necessarily light ($\sim 1$ TeV) and other quite model-dependent observable effects.

When we need to be definite we will refer to the Tian-Yau three-generation manifold [6-14] with

$$SU(3)_C\times SU(3)_L\times SU(3)_R\subset E_6 \tag{1.1}$$

gauge symmetry, which is the first and most extensively studied case. However, our arguments will also be relevant for the other three-generation models (Schimmrigk and *bicubic* in $P^2\times P^2$ [15]). In the Tian-Yau model the nonsinglet-gauge matter consists of nine families of leptons $\lambda$, six of $\bar\lambda$, seven of quarks $q$ and antiquarks $Q$, and four of $\bar q$ and $\bar Q$, where

$$27\to\lambda\equiv(1,\bar 3,3)+q\equiv(3,3,1)+Q\equiv(\bar 3,1,\bar 3)\ , \tag{1.2}$$

and the assignment of standard quantum numbers is

$$\lambda\sim\begin{bmatrix} h^0 & h'^- & e\\ h^+ & h'^0 & \nu\\ e^c & \nu_4 & \nu_5\end{bmatrix}, \qquad q\sim\begin{bmatrix} u\\ d\times 3\text{ colors}\\ d'\end{bmatrix}, \tag{1.3}$$

 

$$Q \sim \begin{bmatrix} u^c \\ d^c \times 3 \text{ colors} \\ d'^c \end{bmatrix} .$$

The $E_6$-singlet sector of this model has been recently calculated [15], resulting in a minimum of 60 fields (this number jumps for especially symmetric choices of the manifold). The masses that these fields receive through nonperturbative effects may be consistently small, and some of them could survive (protected by the discrete symmetries) below the IS. In any case, their $27\times\overline{27}\times 1$ couplings with the nonsinglet sector make them an essential part of the model at the compactification scale $M_c$.

## II. GENERATION OF INTERMEDIATE SCALES

Our starting point is the effective model at $M_c(<M_{\text{Planck}})$. We suppose that the residual $N=1$ SUSY is broken in the effective potential by scalar masses and by a term proportional to $P$. The SUSY-breaking effects would also fix the vacuum expectation values (VEV's) of the moduli fields (whose potential is, in principle, flat). These VEV's determine the complex structure of the manifold and then the value of all the couplings in $P$. We will assume that $P$ incorporates a group of discrete symmetries acting in a definite way over each multiplet of chiral superfields. In Calabi-Yau models these symmetries are necessary to achieve the hierarchies $M_Z/M_c$ or $m_{\text{up}}/m_{\text{top}}$ [9], and also to define a low-energy matter parity [10].

Since the supersymmetric part of the effective potential is positive semidefinite, the nontrivial minimum that defines the IS's should be favored by the SUSY-breaking terms. Although these effective terms are, in principle, flavor independent (they are generated via gravitational interactions), their universality is broken by Yukawa radiative corrections. In Ref. [11] it is shown via the renormalization group that the mass coefficients evolve fast (in 1 or 2 orders of magnitude) to negative values when one scales down the effective model, triggering nonzero VEV's. Then if the potential contains a flat direction (a direction in the field space without self-interactions) the VEV's will grow up to values as large as the scale where these mass coefficients become negative. It is remarkable that the Tian-Yau model mentioned above contains more families and couplings of leptons $\lambda$ than of quarks $q$ and $Q$, a fact that favors VEV's for fields in lepton multiplets and preserves the color symmetry. (A larger number of $\lambda$'s above the IS's is also necessary for a correct unification, since the gauge couplings verify $\alpha_C > \alpha_{L,R}$.)

The flatness of the potential is a necessary ingredient to define large IS's. The supersymmetric scalar potential includes the $D$ and $F$ terms

$$D^\alpha = g\phi_i^\dagger T_j^{\alpha i}\phi^j , \quad F_{\Phi_i} = \frac{\partial P}{\partial \Phi_i} , \tag{2.1}$$

$$V(\phi) = \sum_i |F_{\Phi_i}|^2 + \sum_\alpha |D^\alpha|^2 . \tag{2.2}$$

$D$ flatness is obtained by requiring that, for each VEV along a gauge flavor, there is an identical VEV along a field in the conjugate representation; since these fields have opposite charges, their $D$ contributions cancel. $F$ flatness is obtained if all the terms in $P$ contain more than one field with a zero VEV. When only $\nu_5$ and $\nu_4$ [see (1.3)] develop VEV's, the gauge symmetry provides flatness with respect to terms of type $27^3$ and $\overline{27}^3$ in $P$. However, including trilinears $27\times\overline{27}\times 1$ and effective $(27\times\overline{27})^n$ nonrenormalizable terms (which we assume are suppressed by inverse powers of $M_c$), the nonflat contributions can be forbidden only with the help of the discrete symmetries of the model.

In general, it will be difficult to find models with completely flat directions. Suppose that the VEV's along $\nu_5$ involve $n$ families; there are $n+1$ equations ($D=0$ and $F_i=0$, with $i=1,\ldots,n$) but just $n$ VEV's, and then a flat direction would require two dependent equations. The case with only one dependent equation (such as in Ref. [12]) correspond to a discrete set of zeros, with cancellations between terms of different dimension that would imply VEV's of order $M_c$. Actually, completely flat directions have been found only in models that incorporate an $R$ symmetry [13,14]. Experience shows, however, that when flatness is protected by an unbroken symmetry, one also gets a large amount of extra massless fields, which tend to move the electroweak angle and the proton lifetime to nonacceptable values (this is the case, for instance, in Ref. [13]).

Another possibility, easier to realize in models without $R$ symmetries, is the generation of IS's along noncompletely flat directions. If flatness is broken just by terms of high dimension (very suppressed by powers of $M_c^{-1}$), the scales may be large enough. Suppose that the effective potential receives contributions from terms $(27\times\overline{27})^n$ in $P$. On dimensional grounds, the minimum will result from the balance between order $m_s^2 \approx 1$ TeV$^2$ (negative) bilinears and order $M_c^{-4n+6}$ (positive) $F$ contributions of dimension $4n-2$. The VEV's, order $(m_s/M_c)^{1/(2n-2)}M_c$, will be 1 or 2 orders of magnitude below $M_c$ (as required by perturbative unification of the gauge couplings) for $n=4-6$. (The soft contribution proportional to $P$, of order $m_s M_c^{2n-3}$, is unnecessary in this argument, because it is not dominant for any value of the fields).

We note that, in breaking along an *almost* flat direction, the nonzero value of the $F$ terms introduces a new source of SUSY breaking that will involve the scalars interacting with the fields developing VEV's. In contrast with the ones coming from gravitational interactions, these soft-breaking terms are not universal and do not affect the three chiral families (massless after the IS's), although they could affect the Higgs fields. We also observe that the fermionic partners of the scalars developing a VEV (nonweakly interacting neutrinos) receive only order $m_s$ mass contributions from the terms $(27\times\overline{27})^n$ in $P$. However, the Yukawa couplings of these neutrinos with the low-energy fields are necessarily very small (order $m_s/\text{IS} \approx 10^{12}$); sizable trilinears [of type $\nu_5(h^0h^+)(h'^-h'^0)$] in $P$ would give large masses to the low-energy fields.

Finally, we would like to mention the cosmological scenario that may accommodate this type of phase transitions [12,19]. The finite-temperature effective potential would contain a quadratic term proportional to $T^2$, where $T$ is the temperature of the thermal bath in contact with the scalar field. For the field with the flat potential that we are considering, this term prevents the transition (from zero to large VEV's) until the temperature of the Universe is $O(m_s \approx 1\,\text{TeV})$. Then, the entropy generated would dilute to nonacceptable values any existing baryon asymmetry. There are several ways to avoid this problem: to suppose a period of inflation (where the Universe is extremely cool before reheating and baryogenesis), to recreate the baryon number at $T \sim 1\,\text{TeV}$ (transferring energy to heavy fields via anharmonic couplings), or simply to assume that the initial values of the field [at $T = O(M_c)$] are likely to be large (and the field would be trapped when $T$ drops below the IS).

## III. A MODEL WITH TWO INTERMEDIATE SCALES

The rank-six models under study here require two IS's of gauge-symmetry breaking, defined by VEV's of two independent combinations of $\nu_4$ and $\nu_5$ (with identical $\bar{\nu}_4$ and $\bar{\nu}_5$ VEV's) in two different $\lambda + \bar{\lambda}$ vectorlike multiplets. It will be possible to make a gauge transformation and leave the multiplet containing bigger VEV's with $\langle {}^1\nu_4 \rangle = 0$ (the superscripts label the two $\lambda$ and $\bar{\lambda}$). The smaller VEV's in the second family will, in principle, involve both gauge flavors. However, the fields ${}^2\nu_5$ and ${}^2\bar{\nu}_5$ are not neutral with respect to the possible matter parities one can define in the Tian-Yau manifold (see Table II). Since a low-energy matter parity is a necessary ingredient of any realistic SUSY model, these fields should not develop VEV's. We will consider models where this possibility is favored by the discrete symmetries, and the VEV's in ${}^{1,2}\lambda$ and ${}^{1,2}\bar{\lambda}$ satisfy

$$\langle {}^1\nu_5 \rangle = \langle {}^1\overline{\nu_5} \rangle = \phi_1, \quad \langle {}^1\nu_4 \rangle = \langle {}^1\overline{\nu_4} \rangle = 0 , \tag{3.1a}$$

$$\langle {}^2\nu_5 \rangle = \langle {}^2\overline{\nu_5} \rangle = 0 \quad \langle {}^2\nu_4 \rangle - \langle {}^2\overline{\nu_4} \rangle = \phi_2 . \tag{3.1b}$$

We define $x \equiv \phi_1 / M_c$ and $y \equiv \phi_2 / M_c$ $(1 > x > y)$.

In the Tian-Yau model with $SU(3)_C \times SU(3)_L \times SU(3)_R$ gauge symmetry, the first VEV's [in Eq. (3.1a)] break the symmetry to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, nine gauginos combine with the fermions ${}^1e$, ${}^1\bar{e}$, ${}^1e^c$, ${}^1\bar{e}^c$, ${}^1\nu$, ${}^1\bar{\nu}$, ${}^1\nu_4$, ${}^1\bar{\nu}_4$, and $1/\sqrt{2}({}^1\nu_5 - {}^1\bar{\nu}_5)$ in the first supermultiplets, defining Dirac fields. At the second scale [$\phi_2$ in Eq. (3.1b)], three other gauginos combine with ${}^2e^c$, ${}^2\bar{e}^c$, and $1/\sqrt{2}({}^2\nu_5 - {}^1\bar{\nu}_5)$ leaving unbroken just the standard-model symmetry, $SU(3)_C \times SU(2)_L \times U(1)_Y$.

In principle, the $\nu_5$ and $\nu_4$ VEV's in Eq. (3.1) should grow along two independent flat directions of the scalar potential. We are going to discuss a different and more economic scenario where just one, *almost* flat, direction is sufficient to generate the two IS's. This is achieved for a specific form of the ${}^1\nu_5 - {}^2\nu_4$ interactions that will fix a *small* $\langle {}^2\nu_4 \rangle$ component when $\langle {}^1\nu_5 \rangle$ grows along the flat direction and will define a hierarchy of type $y = O(x^a)$.

To discuss the orders of magnitude, we will consider the model with a superpotential,

$$P = \frac{\alpha}{M_c^{2n-3}} ({}^1\nu_5\, {}^1\bar{\nu}_5)^n + \frac{\beta}{M_c^{2m-3}} ({}^2\nu_4\, {}^2\bar{\nu}_4)^m + \frac{\gamma}{M_c^{2(i+j)-3}} ({}^1\nu_5\, {}^1\bar{\nu}_5)^i ({}^2\nu_4\, {}^2\bar{\nu}_4)^j , \tag{3.2}$$

and effective scalar masses destabilizing the potential along ${}^1\nu_5$ (i.e., $m^2_{{}^1\nu_5} + m^2_{{}^1\bar{\nu}_5} \equiv -m_s^2 < 0$). The actual value of the exponents in Eq. (3.2) would be fixed by the discrete symmetries of the particular manifold, while the parameters $\alpha$, $\beta$, and $\gamma$ are arbitrary complex coefficients that we assume to be of order 1. As shown in the previous section, if the ${}^1\nu_5$ self-interactions (first term in $P$) define an *almost* flat direction [$n = 4-6$ in Eq. (3.2)] these fields will develop large VEV's, $\langle {}^1\nu_5 \rangle = \langle {}^1\bar{\nu}_5 \rangle = x M_c$, with $F$ contributions in the effective potential given by

$$F_{{}^1\nu_5} = F_{{}^1\bar{\nu}_5} = n\alpha M_c^2 x^{2n-1} \approx m_s M_c x , \tag{3.3a}$$

$$F_{{}^2\nu_4} = F_{{}^2\bar{\nu}_4} = 0 . \tag{3.3b}$$

Now we study the possibility of VEV's along the nonflat [$m = 2-3$ in (3.2)]${}^2\nu_4$ direction. If $\langle {}^2\nu_4 \rangle = \langle {}^2\bar{\nu}_4 \rangle = y M_c$, the $F$ contributions become

$$F_{{}^1\nu_5} = F_{{}^1\bar{\nu}_5} = n\alpha M_c^2 x^{2n-1} + i\gamma M_c^2 x^{2i-1} y^{2j} , \tag{3.4a}$$

$$F_{{}^2\nu_4} = F_{{}^2\bar{\nu}_4} = m\beta M_c^2 y^{2m-1} + j\gamma M_c^2 x^{2i} y^{2j-1} . \tag{3.4b}$$

The dimensionally less suppressed contribution is the first term in $F_{{}^2\nu_4}$. We find, however, that if the discrete symmetries of the model fix the exponents in (3.2) such that

$$i + \frac{n}{m} j = n , \tag{3.5}$$

there will be a value of $\phi_2$ (small respect to $\phi_1$) that exactly cancels the nonflat contributions in $F_{{}^2\nu_4}$ while just adding to $F_{{}^1\nu_5}$ *almost* flat contributions (of the same order as the one introduced by $\phi_1$). This value of $\phi_2 (= y M_c)$ is

$$y = \left[ \frac{-j\gamma}{m\beta} \right]^{1/2(m-j)} x^{n/m} , \tag{3.6}$$

and the $F_{{}^1\nu_5}$ term fixing the minimum becomes

$$F_{{}^1\nu_5} = \left[ \alpha + i\gamma \left[ \frac{-j\gamma}{m\beta} \right]^{j/(m-j)} \right] M_c^2 x^{2n-1} . \tag{3.7}$$

Depending on the specific values of $\alpha$, $\beta$, and $\gamma$, but without need of fine-tuning, this combination of two terms in $F_{{}^1\nu_5}$ may be more flat than the direction with $\phi_2 = 0$, and then may correspond to a deeper minimum. Although both minima will be of the same order [$O(|F_{{}^1\nu_5}|^2)$], they are separated (when $\phi_2$ varies from zero to the value that cancels $F_{{}^2\nu_4}$) by the barrier $|F_{{}^2\nu_4}|^2$ in Eq. (3.4b).

We should emphasize that the ingredients to generate

TABLE I. Order of magnitude of the IS and the masses of the neutrinos in the multiplet defining $\phi_2$

| $n$ | $\phi_1/M_c$ | $m$ | $\phi_2/M_c$ | $(i,j)$ | $m_2$ (GeV) |
|---|---|---|---|---|---|
| 6 | $10^{-1.3}$ | 2 | $10^{-4.0}$ | (3,1) | $10^9$ |
| | | 3 | $10^{-2.7}$ | (2,2) | $10^7$ |
| | | 3 | $10^{-2.7}$ | (4,1) | $10^7$ |
| | | 4 | $10^{-2.0}$ | (3,2) | $10^5$ |
| 4 | $10^{-2.7}$ | 2 | $10^{-5.4}$ | (2,1) | $10^6$ |

such a minimum [a flat direction favored by the effective masses and interacting with another direction as specified in Eq. (3.5)] are not rare in Calabi-Yau models, where typically one is left at $M_c$ with many different families and large groups of discrete symmetries. In Ref. [9], for example, a model that satisfies certain phenomenological requirements (unique within its class) is singled out. We find that in this case the IS's have to be generated through the discussed mechanism, with $n=6$, $m=2$, $i=3$, and $j=1$; for $m_s=1$ TeV and $M_c=10^{16.5}$ GeV we obtain

$$\phi_1=O\left[\left[\frac{m_s}{M_c}\right]^{1/10}\right]M_c\approx 10^{15.2}\text{ GeV },$$
$$\phi_2=O(x^3)M_c\approx 10^{12.5}\text{ GeV }. \tag{3.8}$$

Other possibilities are tabulated in Table I.

## IV. LOW-ENERGY IMPLICATIONS

As pointed out by Arnowitt and Nath [8], the flatness arguments have implications over the low-energy matter content. In particular, they found that in the Tian-Yau model the symmetry breaking along two *almost* flat directions leaves a minimum of two (and possibly four) nonchiral neutrinos with light masses (order $m_s$). We are going to show that our way of generating the IS has different phenomenological implications.

The right-handed neutrinos in the two vectorlike $\lambda+\bar{\lambda}$ multiplets will receive mass contributions from gauge and chiral interactions. The first type of interaction will combine them with gauginos associated with lost symmetries, as stated in the previous section. Chiral interactions will affect in a different way the fields involved in each of the IS's. While ${}^1\nu_5$, ${}^1\bar{\nu}_5$, ${}^1\nu_4$ and ${}^1\bar{\nu}_4$ will receive masses of order 1 TeV from terms $({}^1\lambda{}^1\bar{\lambda})^n$ in $P$, the neutrinos in the second family will receive from $({}^2\lambda{}^2\bar{\lambda})^m$ and $({}^1\lambda{}^1\bar{\lambda})^i({}^2\lambda{}^2\bar{\lambda})^j$ heavy mass contributions [in the case corresponding to (3.8), the masses come from terms $({}^2\lambda{}^2\bar{\lambda})^2$ and are of order $y^2M_c\approx 10^9$ GeV]. Considering all these mass contributions we obtain (a) one neutrino with mass $m_1\sim$ TeV, $(1/\sqrt{2})({}^1\nu_5+{}^1\bar{\nu}_5)\equiv N_1$, (b) three neutrinos with intermediate mass $m_2$ [$m_2\sim 10^9$ GeV for the case in (3.8)], $(1/\sqrt{2})({}^2\nu_4+{}^2\bar{\nu}_4)\equiv N_2$, $[{}^2\nu_5+O(y/x)^1\bar{\nu}_4]\equiv N_3$, and $[{}^2\bar{\nu}_5+O(y/x)^1\nu_4]\equiv N_4$.

The neutrinos whose scaler superpartner develops a VEV cannot get further masses through mixing with $E_6$ singlets or other vectorlike fields without spoiling the flatness arguments. Note that a trilinear of type ${}^1\nu_5\,{}^1\bar{\nu}_5 s$ in $P$ (where $s$ is a singlet field), giving a mass of order $xM_c$ to $N_1$, would also introduce nonflat contributions in $F_s$, and then it should be forbidden by the discrete symmetries of the model. For this reason $N_1$ is necessarily light (if the flatness were exact, it would be massless) and $N_2$ has a mass $m_2$. On the contrary, the two neutrinos $N_3$ and $N_4$ may mix with the $E_6$ singlets (terms $\langle{}^1\nu_5\rangle{}^2\bar{\nu}_5 s$ and $\langle{}^1\bar{\nu}_5\rangle{}^2\nu_5 s$) and acquire heavy masses of order $xM_c$.

Although the presence of $N_1$ below 1 TeV is a model-independent feature, we find that definite predictions over its observable effects would require a specific model. This neutrino does not couple in $P$ with the rest of the low-energy fields (see Sec. II). However, relevant couplings may appear in the gauge sector. After the first stage of breaking the gauginos $(\lambda^-\lambda^0)_L$ mix with the three chiral lepton doublets $(e\nu)_i$ $(i=1,\ldots,3)$, through mass terms of type

$$\langle{}^1\bar{\nu}_5^\dagger\rangle\begin{bmatrix}{}^1\bar{e}\\{}^1\bar{\nu}\end{bmatrix}\begin{bmatrix}\lambda^-\\\lambda^0\end{bmatrix}_L,$$
$$\langle({}^1\nu_5\,{}^1\bar{\nu}_5)^a\rangle\begin{bmatrix}{}^1\bar{e}\\{}^1\bar{\nu}\end{bmatrix}\begin{bmatrix}e\\\nu\end{bmatrix}_i. \tag{4.1}$$

Then, the gauge interactions of these gauginos with $N_1$ generate Yukawa couplings of type $({}^1\bar{e}\,{}^1\bar{\nu})^\dagger\overline{N_1}(e\nu)_i$. The sleptons $({}^1\tilde{e}\,{}^1\tilde{\nu})$ will have an order $\phi_2/\phi_1\equiv\sin\theta$ component along $({}^2\tilde{h}'^-\,{}^2\tilde{h}'^0)$, which may have sizable components along the space of the three low-energy sleptons. It results in

$$L_Y=\lambda_{ij}\sin\theta\begin{bmatrix}\tilde{e}\\\tilde{\nu}\end{bmatrix}_i^\dagger\overline{N_1}\begin{bmatrix}e\\\nu\end{bmatrix}_j+\text{H.c.} \tag{4.2}$$

All the mixings are allowed by the $Z_2$ and $Z_3$ matter parities [10] defined from discrete symmetries of the Tian-Yau manifold (see Table II). The Yukawas couplings in Eq. (4.2) would induce at the one-loop level effective operators of anomalous magnetic moment type ($l_i$, with $i=1$, 2, and 3, stands for the charged leptons $e$, $\mu$, and $\tau$),

$$L_{\text{eff}}=\frac{e}{4m_j}F^{\mu\nu}\bar{l}_i\sigma_{\mu\nu}(a_L^{ij}P_L+a_R^{ij}P_R)l_j+\text{H.c.}\ , \tag{4.3}$$

producing lepton-number violation. For instance, in the process $\mu\to\gamma e$ one obtains an amplitude proportional to $m_\mu/\overline{m}^2$, where $\overline{m}=O(m_{\text{slepton}}+m_1)$, with $a_L=(m_e/m_\mu)a_R$ and then an emitted electron almost

TABLE II. $Z_2$ and $Z_3$ matter parities defined from the discrete symmetries of the Tian-Yau manifold [10].

| Field | $Z_2$ | $Z_3$ | Field | $Z_2$ | $Z_3$ |
|---|---|---|---|---|---|
| $(e,\nu)_i$ | $-1$ | 1 | $\langle{}^1\nu_5\rangle,\langle{}^1\bar{\nu}_5\rangle$ | $+1$ | 1 |
| $e^c{}_i$ | $-1$ | $\alpha^2$ | $\langle{}^2\nu_4\rangle,\langle{}^2\bar{\nu}_4\rangle$ | $+1$ | 1 |
| $(ud)_i$ | $-1$ | 1 | ${}^1\nu_4$ | $-1$ | $\alpha$ |
| $u^c{}_i$ | $-1$ | $\alpha$ | ${}^1\bar{\nu}_4$ | $-1$ | $\alpha^2$ |
| $d^c{}_i$ | $-1$ | $\alpha^2$ | ${}^2\nu_5$ | $-1$ | $\alpha^2$ |
| $(h^0h^+)$ | $+1$ | $\alpha^2$ | ${}^2\bar{\nu}_5$ | $-1$ | $\alpha$ |
| $(h'^-h'^0)$ | $+1$ | $\alpha$ | $({}^1e\,{}^1\nu),\overline{({}^1e\,{}^1\nu)}$ | $-1$ | 1 |
| $N_1$ | $+1$ | 1 | $({}^2h'^-\,{}^2h'^0),\overline{({}^2h'^-\,{}^2h'^0)}$ | $-1$ | 1 |

100% left-handed polarized. Although this is an interesting effect, very constrained experimentally ($|a_L^{12}|+|a_R^{12}|<10^{-13}$) [16,17], its size here depends on ratios of nonrenormalizable terms and appears to be model dependent.

The fields in the intermediate region $m_2$ could be relevant for the masses of the three chiral neutrinos ($\nu_i$). Since the scalars ${}^2\nu_5$ and ${}^2\bar{\nu}_5$ do not develop VEV's, their fermionic partners can couple to the low-energy fields without making them super heavy. (Note, however, that trilinears of $N_{3,4}$ with the two Higgs doublets are forbidden by the matter parities in Table II.) Then, terms of type $h^0\nu_i N_{3,4}$ would mix these heavy fields ($N_3$ and $N_4$) with the chiral neutrinos and would make two of them massive via the seesaw mechanism. In addition, one expects the couplings of $N_4$ (mainly in a $\overline{27}$ of $E_6$) to be smaller than those of $N_3$, which suggests one chiral neutrino ($\nu_h$) heavier than the other two. In this scenario, cosmological constraints [18] imply $m_{\nu_h} \lesssim 100\,\text{eV}$. Assuming typical leptonic couplings [$O(m_l/\langle h^0\rangle)$, with $m_l \sim 1\,\text{GeV}$], this translates into a condition for the heavy neutrinos, $m_2 > O(m_l^2/100\ \text{eV}) \sim 10^7$ GeV, which is satisfied in just some of the models in Table I.

We should emphasize that although the neutrinos $N_3$ and $N_4$ may have a relevant phenomenological impact, they appear in a model-dependent way. From the flatness arguments required to generate the IS's one can only infer that their mass may consistently vary from zero (the case with two exact flat directions) to $\sim 10^5-10^9\,\text{GeV}$ (the case discussed here), and that in any case their mixing with $E_6$ singlets could give them a mass $\sim 10^{15}\,\text{GeV}$. Even if the discrete symmetries protect these two neutrinos and they are present at low energies, their couplings with the rest of fields are also model dependent. Actually, the $Z_3$ matter parity forbids the simplified scheme of seesaw masses discussed above. In that case, after diagonalizing the $5\times 5$ mass matrix corresponding to $\nu_i$, $N_3$, and $N_4$, we find two heavy neutrinos [$N_4$ and $N_3+O(m_l/m_2)\nu_i$] defining a Dirac field plus three massless neutrinos [$\nu_i+O(m_l/m_2)N_3$]. For these reasons, although the scenario suggests masses for the standard-model neutrinos, even the estimate of orders of magnitude requires a more specific model.

## V. CONCLUSIONS

We have analyzed a mechanism that requires just one *almost* flat direction in the scalar potential to generate the two stages of gauge-symmetry breaking needed in rank-six Calabi-Yau models. This mechanism is not contained in the general analysis done by Arnowitt and Nath (where two flat directions are assumed), and consequently its implications are different from those. We find the second scale of breaking several orders of magnitude smaller than the first. We also find only one nonchiral neutrino (the one along the flat direction) necessarily light, with mass contributions in the intermediate region for the neutrinos in the multiplet defining the second scale. This scenario has nonstandard low-energy implications, such as seesaw masses for the chiral neutrinos and lepton-number-violating processes. However, their size appears to be quite model dependent. An analysis of an explicit model where this mechanism is realized will be done elsewhere.

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