Bremsstrahlung and Fulling-Davies-Unruh thermal bath

A. Higuchi, G. E. A. Matsas,* and D. Sudarsky†

The Enrico Fermi Institute, The University of Chicago, 5640 Ellis Avenue, Chicago, Illinois 60637-1433

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The bremsstrahlung effect associated with a point charge with constant proper acceleration is discussed from the point of view of the frame coaccelerating with the charge. In this frame the charge is immersed in the so-called Fulling-Davies-Unruh thermal bath. It is shown that the emission of a photon from the source as described by the inertial observer can be interpreted in the accelerated frame as *either* the emission *or* the absorption of a zero-energy Rindler photon in the thermal bath. It is shown by explicit calculations that the emission rate of photons with fixed transverse momentum in the inertial frame agrees with the combined rate of emission and absorption of zero-energy Rindler photons with the same transverse momentum in the accelerated frame. A discussion on the issue of the detectability of these zero-energy Rindler particles is provided.

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I. INTRODUCTION

At present it is widely believed that the study of quantum field theory in curved spacetime can provide some insight into quantum gravity effects, while the full theory is not available. Despite the fact that this semiclassical theory of gravity is not appropriate to be applied in the Planck scale, it has already provided some enlightening results. Undoubtedly, one of the most important developments in this field was the discovery by Hawking [1] that quantum effects can lead to thermal evaporation of black holes. This nontrivial effect was soon realized to be closely associated with the existence of an event horizon in Schwarzschild spacetime. Among all the different background spacetimes in which quantum field theories have been analyzed, the Rindler wedge associated with accelerated observers in Minkowski spacetime has been given special attention. This is so because in addition to possessing a horizon the spacetime is also flat. In fact it is well known [2] that a detector accelerated in the standard Minkowski vacuum perceives a thermal bath of particles with temperature directly proportional to its proper acceleration. This effect has already been the subject of comprehensive discussions (see, e.g., [3]). Specific calculations of emission rates can also be found in the recent literature [4].

In this paper, we discuss in detail some results previously announced [5] concerning the radiation emitted by accelerated classical charges (i.e., the bremsstrahlung effect), as described in the source's rest frame. In order to make a meaningful comparison of the results in the source's rest frame with those obtained in the inertial frame, we must take into account the Fulling-Davies-Unruh (FDU) thermal bath. (See Ref. [6] for a related work computing the response rate of a classical source in two different frames of reference in de Sitter spacetime.) The problem of detectability of the relevant Rindler modes, namely, the zero-energy Rindler particles, is also addressed.

The paper is organized as follows. In Sec. II we discuss the appropriate classical current to be considered in this problem. Section III is devoted to the quantization of the Maxwell field in the Rindler wedge. In Sec. IV we compute the combined rate of absorption and emission of zero-energy Rindler photons in the thermal bath by the charged source. In Sec. V we compare the results obtained in Sec. IV with the usual bremsstrahlung emission of photons as measured in the inertial frame. Finally, in Sec. VI we summarize the results. We also discuss here the issue of the detectability of zero-energy Rindler photons. We will use natural units $k_B = c = \hbar = 1$ throughout this paper.

II. THE CHARGED SOURCE

The Rindler wedge can be described by the line element (see, e.g., Ref. [7])

$$ds^2 = e^{2a\xi}(d\tau^2 - d\xi^2) - dx^2 - dy^2 \ . \tag{2.1}$$

The standard line element $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$ of Minkowski spacetime is obtained by letting

$$t = \frac{e^{a\xi}}{a}\sinh a\tau, \quad z = \frac{e^{a\xi}}{a}\cosh a\tau \ . \tag{2.2}$$

Hence, the metric (2.1) covers the portion of Minkowski spacetime with $z > |t|$ (the Rindler wedge). The boundary planes $z \pm t = 0$ ($\xi = +\infty$) constitute the Killing horizon of $\partial/\partial\tau$, i.e., the null hypersurface which is orthogonal to the Killing field $\partial/\partial\tau$.

The world line with constant ξ, x, and y has a constant proper acceleration $ae^{-a\xi}$. Thus, a point charge q placed at $\xi = x = y = 0$ has a constant acceleration a. The corresponding conserved current is

$$j^\tau = q\delta(\xi)\delta(x)\delta(y) \ , \quad j^\xi = j^x = j^y = 0 \ . \tag{2.3}$$

Note that τ coincides with the proper time of the charge.

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*Present address: Universidade de São Paulo, Instituto de Física, Departamento de Física-Matemática, Caixa Postal 20516, 01498 São Paulo, SP, Brazil.

†Present address: Departamento de Matematicas y Fisica, PUCMM, Apartado Postal 2748, Santo Domingo, Dominican Republic.

We will need to evaluate the response rate of this current in the FDU thermal bath in the Rindler frame. The response will consist of emission and absorption of photons to and from the FDU thermal bath. It is clear that the rate of spontaneous emission is zero because the current (2.3) is static. However, it does not imply that the rates of induced emission and absorption vanish as well. This is because these rates are proportional to the number of photons present in the thermal bath which couple to the current (2.3). Since the number of *zero-energy* (Rindler) photons in the (FDU) thermal bath, which are the relevant ones in this case, is infinite, the rates of induced emission and absorption are *indefinite.* Hence one needs to "regularize" the current (2.3) to make both its strength of coupling to the field and the relevant photon number finite. (The "regulator" will be removed in the end.)

Let us discuss our "regularization" procedure in two steps. First we modify (2.3) by considering a charge oscillating with frequency E,

$$j^\tau=\sqrt{2}q\cos E\tau\,\delta(\xi)\delta(x)\delta(y)\ ,\quad j^\xi=j^x=j^y=0\ , \tag{2.4}$$

and take the limit $E\to 0$ in the end. The factor $\sqrt{2}$ appears because of the following reason: Note first that the radiation rate, in first order of perturbation, is proportional to the square of the charge. When the oscillation is slow, i.e., when $E\ll a,k_\perp$, the charge is expected to interact with the field as if it were a constant charge at each τ. (We assume continuity of the rate in the limit $E\to 0$.) Hence, the τ average of the square of the charge must be set equal to q^2, and therefore, the factor $\sqrt{2}$ is necessary.

Now, the current (2.4) does not satisfy electric charge conservation. For this reason we replace this current by an oscillating dipole arrangement described by

$$j^\tau=\sqrt{2}q\cos(E\tau)[\delta(\xi)-e^{-2aL}\delta(\xi-L)]\delta(x)\delta(y)\ , \tag{2.5}$$

$$j^\xi=\sqrt{2}qE\sin(E\tau)e^{-2a\xi}\theta(\xi)\theta(L-\xi)\delta(x)\delta(y)\ , \tag{2.6}$$

$$j^x=j^y=0\ . \tag{2.7}$$

We will take the limit $E\to 0$ and $L\to+\infty$ in the end. Neither the extra charge introduced at $\xi=L$ in the dipole (2.5) nor the current flow j^ξ between the two charges will contribute to the final results. They are added here only to keep the condition $\nabla^\mu j_\mu=0$ valid and make the computation gauge independent even before taking the limit $E\to 0$.

III. QUANTIZATION OF THE MAXWELL FIELD IN THE RINDLER WEDGE

We will analyze the interaction of the source (2.5)–(2.7) with the Maxwell field in the Rindler wedge. For this purpose we need to quantize the electromagnetic field with the positive-frequency modes defined with respect to the Rindler time τ. We start with the standard Langrangian

$$\mathcal{L}=-\sqrt{-g}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2\alpha}(\nabla^\mu A_\mu)^2\right]\ . \tag{3.1}$$

The corresponding field equations in the Feynman gauge ($\alpha=1$) are

$$\nabla_\mu\nabla^\mu A_\nu=0\ . \tag{3.2}$$

The presence of ∂_τ, ∂_x, and ∂_y as Killing fields makes it sufficient to look for solutions of (3.2) of the form

$$A_\mu^{(\lambda,\omega,k_x,k_y)}(x^\nu)=f_\mu^{(\lambda,\omega,k_x,k_y)}(\xi)e^{i(k_x x+k_y y-\omega\tau)}\ , \tag{3.3}$$

where λ labels the mode polarization. The physical modes are defined as those which are not pure gauge (the gradient of a scalar) and which satisfy the Lorenz condition

$$\nabla_\mu A^\mu=0\ , \tag{3.4}$$

in addition to (3.2).

Next, we expand the electromagnetic quantum field in terms of annihilation and creation operators as

$$\hat{A}_\mu(x^\nu)=\int_{-\infty}^{+\infty}dk_x\int_{-\infty}^{+\infty}dk_y\int_0^{+\infty}d\omega\sum_{\lambda=1}^{4}\{a_{(\lambda,\omega,k_x,k_y)}A_\mu^{(\lambda,\omega,k_x,k_y)}(x^\nu)+\text{H.c.}\}\ , \tag{3.5}$$

where $A_\mu^{(\lambda,\omega,k_x,k_y)}(x^\nu)$ are solutions of (3.2) of the form given in (3.3). These modes are conveniently expressed in terms of the solutions of the scalar field equation $\Box\phi=0$, or, equivalently,

$$\left[e^{-2a\xi}\left[\frac{\partial^2}{\partial\tau^2}-\frac{\partial^2}{\partial\xi^2}\right]-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right]\phi=0 \tag{3.6}$$

(see Ref. [8]). For each set of quantum numbers the solution which does not diverge for $\xi\to+\infty$ is given (up to an arbitrary multiplicative constant) by

$$\phi^{(\omega,k_x,k_y)}=K_{i\omega/a}\left[\frac{k_\perp}{a}e^{a\xi}\right]e^{ik_x x+ik_y y-i\omega\tau}\ , \tag{3.7}$$

where $K_\nu(z)$ is the Bessel function of imaginary argument [9], and $k_\perp=\sqrt{k_x^2+k_y^2}$. (We will not treat the case $k_\perp=0$ because its measure is zero in the solution space.) The solutions $\phi^{(\omega,k_x,k_y)}$ go to zero very rapidly for $\xi\to+\infty$.

One can choose a set of independent normal modes as

$$A_\mu^{(\mathrm{I},\omega,k_x,k_y)}=C^{(\mathrm{I},\omega,k_x,k_y)}(0,0,k_y\phi,-k_x\phi)\ , \tag{3.8}$$

$$A_\mu^{(\mathrm{II},\omega,k_x,k_y)}=C^{(\mathrm{II},\omega,k_x,k_y)}(\partial_\xi\phi,-i\omega\phi,0,0)\ , \tag{3.9}$$

$$A_\mu^{(G,\omega,k_x,k_y)}=C^{(G,\omega,k_x,k_y)}(-i\omega\phi,\partial_\xi\phi,ik_x\phi,ik_y\phi)\ , \tag{3.10}$$

$$A_\mu^{(L,\omega,k_x,k_y)}=C^{(L,\omega,k_x,k_y)}(0,0,k_x\phi,k_y\phi)\ , \tag{3.11}$$

where $A_\mu=(A_\tau,A_\xi,A_x,A_y)$, $C^{(\lambda,\omega,k_x,k_y)}$ are normalization constants, and $\phi\equiv\phi^{(\omega,k_x,k_y)}$. The mode $A_\mu^{(\mathrm{I},\omega,k_x,k_y)}$ is clearly a solution of Eq. (3.2) because the equations for A_x and A_y become the massless scalar field equation if

$A_\tau = A_\xi = 0$. (This is because there are no nonzero Christoffel symbols with indices x or y.) This mode also satisfies the Lorenz condition (3.4). Thus it is a physical mode. Next, note that the mode $A_\mu^{(\mathrm{II},\omega,k_x,k_y)}$ is proportional to $\epsilon_{\mu\nu}\nabla^\nu\phi$ where $\epsilon_{\mu\nu}$ is the antisymmetric tensor $\epsilon_{tz} = -\epsilon_{zt} = 1$ (in the standard Minkowski coordinate system) with all other components being zero. (Note that $\epsilon_{\tau\xi} = -\epsilon_{\xi\tau} = e^{2a\xi}$ in Rindler coordinates.) Since $\nabla^\alpha\nabla_\alpha(\epsilon_{\mu\nu}\nabla^\nu\phi) = \epsilon_{\mu\nu}\nabla^\nu\nabla^\alpha\nabla_\alpha\phi$, the mode $A_\mu^{(\mathrm{II},\omega,k_x,k_y)}$ satisfies Eq. (3.2). It also satisfies the Lorenz condition (3.4) because $\nabla^\mu(\epsilon_{\mu\nu}\nabla^\nu\phi) = \epsilon_{\mu\nu}\nabla^\mu\nabla^\nu\phi = 0$. Hence it is also a physical mode. The mode $A_\mu^{(G,\omega,k_x,k_y)}$ is proportional to $\nabla_\mu\phi$ and therefore is a pure gauge mode. [It is a solution of equation (3.2) since $\nabla^\alpha\nabla_\alpha\nabla_\mu\phi = \nabla_\mu\nabla^\alpha\nabla_\alpha\phi = 0$.] Finally the mode $A_\mu^{(L,\omega,k_x,k_y)}$ satisfies Eq. (3.2) because the x and y components are proportional to ϕ while the τ and ξ components are zero. But it does not satisfy the Lorenz condition.

The normalization constants $C^{(i)}$ can be determined from the canonical commutation relations of the fields by requiring suitable commutation relations for the operators $a_{(i)}$ and $a_{(i)}^\dagger$. [Here the label i represents (λ,ω,k_x,k_y).] In this context, it is convenient to introduce the generalized Klein-Gordon inner product

$$(A^{(i)}, A^{(j)}) \equiv \int_\Sigma d\Sigma_\mu W^\mu[A^{(i)}, A^{(j)}] \tag{3.12}$$

between any two modes $A^{(i)}$ and $A^{(j)}$. The integration in (3.12) is performed on some Cauchy surface Σ for the Rindler wedge, e.g., any hypersurface $\tau = $const, and

$$W^\mu[A^{(i)}, A^{(j)}] \equiv \frac{i}{\sqrt{-g}}(A_\nu^{(i)*}\pi^{(j)\mu\nu} - A_\nu^{(j)}\pi^{(i)\mu\nu*}) \tag{3.13}$$

with $\pi^{(i)\mu\nu} \equiv \partial\mathcal{L}/\partial_\mu A_\nu|_{A_{(i)\mu}}$. The $\pi^{(i)\mu\nu}$ are calculated in the Feynman gauge to be

$$\pi^{(i)\mu\nu} = \sqrt{-g}\,[\nabla^\nu A^{(i)\mu} - \nabla^\mu A^{(i)\nu} - g^{\mu\nu}\nabla_\alpha A^{(i)\alpha}] \; . \tag{3.14}$$

It can be seen [10] that the field equations ensure conservation of current (3.13), and thus the inner product (3.12) is independent of the choice of the Cauchy surface Σ.

Using the inner product (3.12) for the normal modes (3.8)–(3.11), we can verify the orthogonality properties

$$(A^{(\lambda,\omega,k_x,k_y)}, A^{(\lambda',\omega',k_x',k_y')}) = 0 \tag{3.15}$$

for $\lambda = $I or II, with λ' being any polarization other than λ. In other words, the physical modes are orthogonal to the *pure gauge* mode G and to the *Lorenz-condition-violating* mode L and to each other.

Now, from the canonical commutation relations one finds

$$[(A^{(i)}, \hat{A}), (\hat{A}, A^{(j)})] = (A^{(i)}, A^{(j)}) \; . \tag{3.16}$$

This equation and Eq. (3.5) imply that

$$(A^{(i)}, A^{(l)})[a_{(l)}, a_{(l')}^\dagger](A^{(l')}, A^{(j)}) = (A^{(i)}, A^{(j)}) \; , \tag{3.17}$$

where we have used the fact that positive- and negative-frequency modes are orthogonal to each other. The schematic summation over l represents integrations over ω, k_x, and k_y, as well as the summation over λ. Next, define the matrix $M^{(i)(j)}$ by

$$M^{(i)(j)} \equiv (A^{(i)}, A^{(j)}) \; . \tag{3.18}$$

Then Eq. (3.17) implies (see Ref. [11])

$$[a_{(i)}, a_{(j)}^\dagger] = (M^{-1})_{(i)(j)} \; , \tag{3.19}$$

where $(M^{-1})_{(i)(j)}$ is defined by

$$(M^{-1})_{(i)(l)} M^{(l)(j)} = \delta^{\lambda\lambda'}\delta(\omega-\omega')\delta(k_x-k_x')\delta(k_y-k_y') \; . \tag{3.20}$$

Since the physical modes are orthogonal to the other modes [see Eq. (3.15)], it is sufficient to know the restriction of the matrix $M^{(i)(j)}$ to the physical subspace (i.e., to $\lambda = $I,II) in order to derive the commutators among the physical annihilation and creation operators according to Eq. (3.19). Thus, by requiring the commutators of annihilation and creation operators associated with the physical modes (i.e., with λ and λ' being I or II) to be

$$[a_{(\lambda,\omega,k_x,k_y)}, a_{(\lambda',\omega',k_x',k_y')}^\dagger] = \delta_{\lambda\lambda'}\delta(\omega-\omega')\delta(k_x-k_x')\delta(k_y-k_y') \; , \tag{3.21}$$

we find the normalization condition

$$(A^{(\lambda,\omega,k_x,k_y)}, A^{(\lambda',\omega',k_x',k_y')}) = \delta^{\lambda\lambda'}\delta(\omega-\omega')\delta(k_x-k_x')\delta(k_y-k_y') \; . \tag{3.22}$$

We will use Eq. (3.22) to determine the normalization constant $C^{(\mathrm{II},\omega,k_x,k_y)}$. We do not need any other normalization constants because the current (2.5)–(2.7) will not excite the physical mode I [see Eq. (3.8)] nor the unphysical modes G or L via the appropriate interaction Lagrangian

$$\mathcal{L}_{\mathrm{int}} = \sqrt{-g}\, j^\mu \hat{A}_\mu \; . \tag{3.23}$$

Using (3.22) for $\lambda = \lambda' = $II, we obtain

$$|C^{(\mathrm{II},\omega,k_x,k_y)}|^2 I(\omega,\omega') = k_\perp^{-2}\delta(\omega-\omega')\delta(k_x-k_x')\delta(k_y-k_y') \; , \tag{3.24}$$

where

$$\mathrm{I}(\omega,\omega') \equiv i\int_\Sigma d\xi\, dx\, dy\, \phi^{(\omega,k_x,k_y)*}\overleftrightarrow{\partial_\tau}\phi^{(\omega',k_x',k_y')} \; , \tag{3.25}$$

and $\phi^{(\omega,k_x,k_y)}$ is defined in (3.7). [We have used the field equation for $\phi^{(\omega,k_x,k_y)}$ in deriving (3.24).] We can express (3.25) as

$$I(\omega,\omega')=4\pi^2(\omega+\omega')e^{i(\omega-\omega')\tau}S(\omega,\omega')\delta(k_x-k_x')\delta(k_y-k_y')\ , \tag{3.26}$$

where

$$S(\omega,\omega')=\int_{-\infty}^{+\infty}d\xi\, K_{i\omega/a}\left[\frac{k_\perp}{a}e^{a\xi}\right]K_{i\omega'/a}\left[\frac{k_\perp}{a}e^{a\xi}\right]\ . \tag{3.27}$$

We have used the fact that $K_{i\alpha}(x)$ is real for real α if x is real and positive. In order to compute (3.27), let us define

$$S_A(\omega,\omega')\equiv\int_{-A}^{+\infty}d\xi\, K_{i\omega/a}\left[\frac{k_\perp}{a}e^{a\xi}\right]K_{i\omega'/a}\left[\frac{k_\perp}{a}e^{a\xi}\right]\ , \tag{3.28}$$

which can be shown to satisfy

$$(\omega^2-\omega'^2)S_A(\omega,\omega')=-K_{i\omega/a}\left[\frac{k_\perp}{a}e^{a\xi}\right]\overleftrightarrow{\partial}_{\xi}K_{i\omega'/a}\left[\frac{k_\perp}{a}e^{a\xi}\right]\Bigg|_{\xi=-A}\ , \tag{3.29}$$

by using

$$\left[e^{-2a\xi}\left[\frac{\partial^2}{\partial\xi^2}+\omega^2\right]-k_\perp^2\right]K_{i\omega/a}\left[\frac{k_\perp}{a}e^{a\xi}\right]=0\ . \tag{3.30}$$

Next, applying the well-known formula for the Bessel function $J_\nu(z)$ for small z and the definition of $K_\nu(z)$ [9], one finds

$$K_{i\alpha}(z)\approx\frac{i\pi}{2\sinh\pi\alpha}\left\{\frac{(z/2)^{i\alpha}}{\Gamma(1+i\alpha)}-\frac{(z/2)^{-i\alpha}}{\Gamma(1-i\alpha)}\right\}\quad(z\ll 1)\ . \tag{3.31}$$

It is straightforward to evaluate $S(\omega,\omega')$ as the $A\to+\infty$ limit of $S_A(\omega,\omega')$ by using this formula and the relation

$$\lim_{A\to+\infty}\frac{\sin Ax}{x}=\pi\delta(x)\ . \tag{3.32}$$

We obtain

$$S(\omega,\omega')=\frac{\pi^2 a}{2\omega\sinh(\pi\omega/a)}\delta(\omega-\omega')\ . \tag{3.33}$$

As a result, we can use (3.24), (3.26), and (3.33) to find the absolute value of the normalization constant:

$$|C^{(\mathrm{II},\omega,k_x,k_y)}|=\frac{1}{2\pi^2k_\perp}\left[\frac{\sinh(\pi\omega/a)}{a}\right]^{1/2}\ . \tag{3.34}$$

Hence, the physical mode of interest (3.9) can be written up to a phase as

$$A_\mu^{(\mathrm{II},\omega,k_x,k_y)}=\frac{1}{2\pi^2k_\perp}\left[\frac{\sinh(\pi\omega/a)}{a}\right]^{1/2}(\partial_\xi\phi,-i\omega\phi,0,0)\ . \tag{3.35}$$

IV. EMISSION AND ABSORPTION RATES OF ZERO-ENERGY RINDLER PHOTONS

We first use the interaction Lagrangian (3.23) to determine, at the tree level, the amplitude $\mathcal{A}^{\mathrm{em}}_{(\omega,k_x,k_y)}$ of emission of a Rindler photon, with quantum numbers $(\mathrm{II},\omega,k_x,k_y)$, from the charge into the Rindler vacuum state $|0\rangle_R$.

Recall that the Rindler vacuum is defined by

$$a_{(\lambda,\omega,k_x,k_y)}|0\rangle_R=0 \tag{4.1}$$

for all (λ,ω,k_x,k_y). In lowest order in perturbation theory this amplitude is given by

$$\mathcal{A}^{\mathrm{em}}_{(\omega,k_x,k_y)}={}_R\langle\mathrm{II},\omega,k_x,k_y|\times i\int d^4x\sqrt{-g}\,j^\mu(x)\hat{A}_\mu(x)|0\rangle_R\ , \tag{4.2}$$

where

$$|\mathrm{II},\omega,k_x,k_y\rangle_R\equiv a^\dagger_{(\mathrm{II},\omega,k_x,k_y)}|0\rangle_R\ . \tag{4.3}$$

It is straightforward to compute $\mathcal{A}^{\mathrm{em}}_{(\omega,k_x,k_y)}$ for the current (2.5)–(2.7) using (3.21) and (3.5) with (3.35). We obtain

$$\mathcal{A}^{\mathrm{em}}_{(\omega,k_x,k_y)}=iq\left[\frac{\sinh(\pi E/a)}{2\pi^2 a}\right]^{1/2}\delta(E-\omega)\left\{K'_{iE/a}(k_\perp/a)-e^{aL}K'_{iE/a}(k_\perp e^{aL}/a)-\frac{E^2}{ak_\perp}\int_{k_\perp/a}^{(k_\perp/a)e^{aL}}\frac{dz}{z}K_{iE/a}(z)\right\}\ , \tag{4.4}$$

where derivatives with respect to the argument are denoted by primes.

We are interested in the differential probability of emission per unit time and transverse momentum squared, for fixed transverse momentum (k_x,k_y), given by

$$dW_0^{\rm em}(\omega,k_x,k_y)=|\mathcal{A}^{\rm em}_{(\omega,k_x,k_y)}|^2 d\omega/T\ , \tag{4.5}$$

where T is the length of the time interval during which the interaction remains turned on. At this point we take the limit $L\to+\infty$ in (4.4) to eliminate the influence of the extra charge; thus, we obtain

$$dW_0^{\rm em}(\omega,k_x,k_y)=\frac{q^2}{4\pi^3 a}\sinh(\pi E/a)\left|K'_{iE/a}(k_\perp/a)-\frac{E^2}{ak_\perp}\int_{k_\perp/a}^{+\infty}\frac{dz}{z}K_{iE/a}(z)\right|^2\delta(E-\omega)d\omega\ , \tag{4.6}$$

where we have let $\delta(0)=2\pi T$, following the standard interpretation [12].

Now, it is well known that the Minkowski vacuum described in Rindler coordinates is not the Rindler vacuum, but a thermal state characterized by the temperature $\beta^{-1}=a/(2\pi)$, i.e., an incoherent mixture of states with the weight of a state containing n photons with the energy ω being

$$p_n(\omega)=Z^{-1}e^{-\beta n\omega}\ , \tag{4.7}$$

where Z is a normalization factor. The probability $dW_n^{\rm em}$ of emission of an extra photon into an n-photon state is related to the probability $dW_0^{\rm em}$ of emission of a photon into the vacuum by $dW_n^{\rm em}=(n+1)dW_0^{\rm em}$. Therefore, the total differential rate (per unit transverse momentum squared) of emission of photons with given transverse momentum (k_x,k_y) into the thermal bath is

$$P^{\rm em}_{(k_x,k_y)}=\int_0^{+\infty}\sum_n p_n(\omega)dW_n^{\rm em}(\omega,k_x,k_y)\ , \tag{4.8}$$

which can be written as

$$P^{\rm em}_{(k_x,k_y)}=\int_0^{+\infty}dW_0^{\rm em}(\omega,k_x,k_y)\left[\frac{1}{e^{2\pi\omega/a}-1}+1\right]\ . \tag{4.9}$$

The two terms inside the parentheses are associated with induced and spontaneous emissions, respectively. Evaluating the integral (4.9) and taking the limit $E\to 0$ (thus removing the "regulator") we obtain

$$P^{\rm em}_{(k_x,k_y)}dk_x dk_y=\frac{q^2}{8\pi^3 a}|K_1(k_\perp/a)|^2 dk_x dk_y\ . \tag{4.10}$$

Analogously, the total absorption rate of photons with fixed (k_x,k_y) is

$$P^{\rm abs}_{(k_x,k_y)}=\int_0^{+\infty}dW_0^{\rm abs}(\omega,k_x,k_y)\frac{1}{e^{2\pi\omega/a}-1}\ . \tag{4.11}$$

On unitarity grounds we have

$$dW_0^{\rm abs}(\omega,k_x,k_y)=dW_0^{\rm em}(\omega,k_x,k_y)\ , \tag{4.12}$$

and one can evaluate (4.11) using (4.6). We obtain, in the limit $E\to 0$,

$$P^{\rm abs}_{(k_x,k_y)}dk_x dk_y=\frac{q^2}{8\pi^3 a}|K_1(k_\perp/a)|^2 dk_x dk_y\ . \tag{4.13}$$

The reason for the equality of $P^{\rm em}_{(k_x,k_y)}$ and $P^{\rm abs}_{(k_x,k_y)}$ is that the spontaneous emission becomes negligible in comparison to the induced emission as E approaches zero. It is also interesting to note that it is the existence of an infinite number of zero-energy Rindler photons in the thermal bath that prevents $P^{\rm em}_{(k_x,k_y)}$ and $P^{\rm abs}_{(k_x,k_y)}$ from vanishing. In the absence of the thermal bath, the emission and absorption rates would vanish.

Next, we note that since there is no interference between the processes of emission and absorption of Rindler photons at the tree level, the total response rate will be given by adding (4.10) and (4.13); thus we find

$${}^{\rm ac}P^{\rm tot}_{(k_x,k_y)}dk_x dk_y=\frac{q^2}{4\pi^3 a}|K_1(k_\perp/a)|^2 dk_x dk_y\ . \tag{4.14}$$

In the next section, we compute the emission rate of photons from an accelerated charge in the inertial frame, and compare it with the results obtained here.

V. BREMSSTRAHLUNG EFFECT IN THE INERTIAL FRAME

In this section, we study the bremsstrahlung effect, i.e., the emission of photons from an accelerated charge, as seen in the inertial frame. In particular we compute the emission rate of photons with fixed transverse momentum. (Some related results including the energy-momentum spectrum of radiation have been obtained using classical electrodynamics by Nikishov and Ritus [13].) We find that this rate is equal to the combined rate of emission and absorption in the FDU thermal bath obtained in the preceding section. The fact that the transverse momentum (k_x,k_y) is invariant under boosts in the z direction allows us to compare the emission and absorption rates corresponding to Minkowski and Rindler photons with the same transverse momentum. We will adopt the notation of Itzykson and Zuber [12].

The amplitude of emission of a photon with momentum $\mathbf{k}$ and polarization λ by the accelerated charge in the Minkowski vacuum is

$$\mathcal{A}^{(\lambda,\mathbf{k})}={}_M\langle\mathbf{k},\lambda|i\int d^4x\, j^\mu(x)\hat{A}_\mu(x)|0\rangle_M\ , \tag{5.1}$$

where the subscript M indicates Minkowski states. The current (2.3) can be written in Minkowski coordinates as

$$\begin{aligned} j^t&=qaz\delta(\xi)\delta(x)\delta(y)\ ,\\ j^x&=j^y=0\ ,\\ j^z&=qat\delta(\xi)\delta(x)\delta(y)\ , \end{aligned} \tag{5.2}$$

where

$$\delta(\xi)=\frac{\delta(z-\sqrt{t^2+a^{-2}})}{a\sqrt{t^2+a^{-2}}}\ . \tag{5.3}$$

Using for $\hat{A}_\mu(x)$ the standard Fourier expansion,

$$\hat{A}_\mu(x)=\int \frac{d^3\mathbf{k}}{2(2\pi)^3k_0} \times \sum_{\lambda=1}^{4}[a^{(\lambda)}(\mathbf{k})\epsilon_\mu^{(\lambda)}(\mathbf{k})e^{ik_\nu x^\nu}+\text{H.c.}] , \tag{5.4}$$

with $k_0\equiv\sqrt{k_z^2+k_\perp^2}$, we obtain, for (5.1),

$$\mathcal{A}^{(\lambda,\mathbf{k})}=i\int d^4x\, j^\nu(x)\epsilon_\nu^{(\lambda)}(\mathbf{k})e^{i(\omega t-\mathbf{k}\cdot\mathbf{x})} , \tag{5.5}$$

where $\epsilon_\mu^{(\lambda)}$ are polarization vectors that can be chosen as

$$\epsilon^{(0)\mu}=\frac{1}{\sqrt{2}}(-1,0,0,1) , \tag{5.6}$$

$$\epsilon^{(1)\mu}=(0,1,0,0) , \tag{5.7}$$

$$\epsilon^{(2)\mu}=(0,0,1,0) , \tag{5.8}$$

$$\epsilon^{(3)\mu}=\frac{1}{\sqrt{2}}(1,0,0,1) . \tag{5.9}$$

Here, the Cartesian frame is chosen such that $k^\mu=(|\mathbf{k}|,0,0,|\mathbf{k}|)$ (where the first component is the time component). Next, we can express the total rate of emission of photons with fixed transverse momentum (k_x,k_y), divided by the total *proper* time T of the accelerated charge during which the interaction remains turned on, as

$$^{\text{in}}P^{\text{tot}}_{(k_x,k_y)}=\sum_{\lambda=1}^{2}\int_{-\infty}^{+\infty}d\tilde{k}_z|\mathcal{A}^{(\lambda,\mathbf{k})}|^2/T , \tag{5.10}$$

where $d\tilde{k}_z\equiv dk_z/[(2\pi)^3 2k_0]$, and the sum runs only over the physical polarizations $\lambda=1,2$. Using (5.5) in (5.10), one has

$$^{\text{in}}P^{\text{tot}}_{(k_x,k_y)}=\int_{-\infty}^{+\infty}d\tilde{k}_z\int d^4x\, d^4x'\left[\sum_{\lambda=1}^{2}\epsilon_\mu^{(\lambda)}(\mathbf{k})\epsilon_\nu^{(\lambda)}(\mathbf{k})\right]j^\mu(x)j^\nu(x')e^{i\omega(t-t')-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} . \tag{5.11}$$

Now we note the identity

$$\sum_{\lambda=1}^{2}\epsilon_\mu^{(\lambda)}\epsilon_\nu^{(\lambda)}=-\epsilon_\mu^{(0)}\epsilon_\nu^{(3)}-\epsilon_\mu^{(3)}\epsilon_\nu^{(0)}-\eta_{\mu\nu} , \tag{5.12}$$

where $\eta_{\mu\nu}$ is the metric of Minkowski spacetime. Because of current conservation $\partial_\mu j^\mu=0$ and due to the fact that $\epsilon_\mu^{(3)}$ is proportional to k_μ, the first two terms in (5.12) do not contribute when one substitutes it in (5.11). Hence,

$$^{\text{in}}P^{\text{tot}}_{(k_x,k_y)}=-\frac{1}{T}\int d\tilde{k}_z\int d^4x\int d^4x' j^\mu(x)j_\mu(x')e^{i\omega(t-t')-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} . \tag{5.13}$$

Next, substituting the current (5.2) in this formula, we obtain

$$^{\text{in}}P^{\text{tot}}_{(k_x,k_y)}=-\frac{q^2}{T}\int_{-\infty}^{+\infty}d\tilde{k}_z\int_{-\infty}^{+\infty}d\tau'\int_{-\infty}^{+\infty}d\tau''\cosh a(\tau'-\tau'')\exp\left[-i\frac{k_z}{a}(\cosh a\tau'-\cosh a\tau'')+i\frac{k_0}{a}(\sinh a\tau'-\sinh a\tau'')\right] , \tag{5.14}$$

where we have made the transformation of coordinate $t=a^{-1}\sinh a\tau$. This integral is infinite due to the fact that interaction is turned on for an infinite amount of time. To obtain the rate per unit time it is necessary to factor out the total proper time $T=\int_{-\infty}^{+\infty}d\tau$, where $\tau=(\tau'+\tau'')/2$. To this end, we first note that the momentum of the emitted photon is boosted due to the nonzero velocity of the source, which is τ dependent. Hence, it is expected that the integrand can be made τ independent by boosting back the momentum variables. Motivated by this physical picture, we introduce new momentum variables as

$$k_z'=k_z\cosh a\tau-k_0\sinh a\tau , \tag{5.15}$$

$$k_0'=k_0\cosh a\tau-k_z\sinh a\tau . \tag{5.16}$$

[Equation (5.16) follows from (5.15) and the definition $k_0'\equiv\sqrt{k_z'^2+k_\perp^2}$.] Then we indeed find that the integrand becomes τ independent and

$$^{\text{in}}P^{\text{tot}}_{(k_x,k_y)}=-q^2\int_{-\infty}^{+\infty}d\tilde{k}_z'\int_{-\infty}^{+\infty}d\sigma\,\cosh a\sigma\,\exp\left[\frac{2ik_0'}{a}\sinh\frac{a\sigma}{2}\right] , \tag{5.17}$$

where $d\tilde{k}_z'\equiv dk_z'/[(2\pi)^3 2k_0']$ and $\sigma\equiv\tau'-\tau''$. To evaluate this integral we cut off the contribution from large $|\sigma|$ smoothly by letting $\sigma\to\sigma+2i\epsilon$ (where ϵ is an infinitesimal positive number) in the exponent, and then taking the limit $\epsilon\to 0$ in the end. (Otherwise this integral would be indefinite.) With this change, the exponential in (5.17) becomes

$$\exp\left[\frac{2ik_0'}{a}\sinh\frac{a\sigma}{2}\right]\to\exp\left[\frac{ie^{ia\epsilon}k_0'}{a}e^{a\sigma/2}-\frac{ie^{-ia\epsilon}k_0'}{a}e^{-a\sigma/2}\right] . \tag{5.18}$$

Then by introducing change of variables as

$$s_{\pm}=\frac{k_0'+k_z'}{k_\perp}e^{\pm a\sigma/2} , \quad (5.19)$$

and, using the formula [9]

$$\int_0^{+\infty}dx\, x^{\nu-1}\exp\left[\frac{i\mu}{2}\left[x-\frac{\beta^2}{x}\right]\right]=2\beta^\nu e^{i\nu\pi/2}K_\nu(\beta\mu) , \quad (5.20)$$

where $\mathrm{Im}\mu>0$, $\mathrm{Im}(\beta^2\mu)<0$, we obtain

$$^{\mathrm{in}}P^{\mathrm{tot}}_{(k_x,k_y)}dk_x dk_y=\frac{q^2}{4\pi^3 a}|K_1(k_\perp/a)|^2 dk_x dk_y . \quad (5.21)$$

By comparing this equation and Eq. (4.14) we find $^{\mathrm{ac}}P^{\mathrm{tot}}_{(k_x,k_y)}={}^{\mathrm{in}}P^{\mathrm{tot}}_{(k_x,k_y)}$. Thus, we have established by explicit calculations that the rate of photon emission from a uniformly accelerated charge can be reproduced by summing the rates of emission and absorption of zero-energy Rindler photons in the FDU thermal bath.

VI. DISCUSSIONS

In this paper we studied the QED bremsstrahlung rate from a classical charge with a constant proper acceleration at the tree level from the point of view of an observer coaccelerating with the charge. We showed by explicit calculations that it is reproduced as the sum of emission and absorption rates of zero-energy Rindler photons in the Fulling-Davies-Unruh thermal bath. This result is consistent with the observation that each photon emitted in the inertial frame must correspond in the accelerated frame to *either* the emission *or* the absorption of a Rindler photon [3] since both observers must agree concerning changes in the state of the quantum field.

Finally, we address the issue of the detectability of these zero-energy Rindler photons. One might be tempted to conclude immediately that they could not be detected by a Rindler observer, i.e., an accelerated observer whose horizon is the same as that of the charged source, because their energy is zero. However, it might also appear that one could use their nonzero transverse momentum to detect them.

Let us first ask whether or not a Rindler observer sees any difference in the thermal bath due to the emission and absorption of these photons. The answer is negative. To see this, note that the source not only emits and absorbs these extra photons at the same rate, but also *leaves the thermal bath undisrupted,* since the transition rate from an n-photon state to an $(n+1)$-photon state and that of the inverse process become equal in the limit $E\to 0$. That is, the source is in thermal equilibrium with the quantum field. In addition, since the expected number of photons in a mode with energy E is $1/(e^{\beta E}-1)$, which diverges as $E\to 0$, it becomes increasingly difficult to distinguish an extra photon from those already in the thermal bath as E approaches zero.

In fact we will find that zero-energy Rindler photons are not detectable *by a Rindler observer* even in the Rindler vacuum. Roughly speaking, this is because a state with small Rindler energy is concentrated near the horizon. (Thus, one cannot utilize the nonzero transverse momentum for detection unless one goes arbitrarily close to the horizon.) To see this, we consider a wave packet $|\phi\rangle$ given by

$$|\phi\rangle=\int d\omega\, dk_x dk_y c(\omega,k_x,k_y)a^\dagger_{(\mathrm{II},\omega,k_x,k_y)}|0\rangle_R , \quad (6.1)$$

where

$$\int d\omega\, dk_x dk_y |c(\omega,k_x,k_y)|^2=1 . \quad (6.2)$$

The corresponding one-particle wave function is

$$_R\langle 0|A_\mu|\phi\rangle=\int d\omega\, dk_x dk_y c(\omega,k_x,k_y)A_\mu^{(\mathrm{II},\omega,k_x,k_y)} . \quad (6.3)$$

Now, consider a wave packet with small average Rindler energy E and $\Delta E\sim E$ and with the support of $c(\omega,k_x,k_y)$ bounded away from $k_\perp=0$. Then the maximum value of $\int dk_x dk_y c(\omega,k_x,k_y)$ (as a function of ω) will be of order $1/\sqrt{E}$. Now, for a fixed value of ξ, the mode function $A_\mu^{(\mathrm{II},\omega,k_x,k_y)}$ goes to zero like $\sqrt{E}$ because of the normalization factor $\sinh^{1/2}(\pi\omega/a)$ in (3.35). Hence we find $_R\langle 0|A_\mu|\phi\rangle\sim E$. Thus the one-particle wave function goes to zero away from the horizon for $E\to 0$. (This is true also for derivatives of the one-particle wave function.) Another measure of probability distribution in space is given by the expectation value of the energy density $:T^\tau{}_\tau:$ which is normal ordered using the particle notion in Rindler spacetime. We find that the quantity $\langle\phi|:T^\tau{}_\tau:|\phi\rangle$ will concentrate more and more near the horizon as E approaches zero. Thus, although the state has nonzero transverse momentum, one cannot use it to detect the photon as long as one stays away from the horizon.

Note, however, that wave packets which approximate zero-energy Rindler photons are all detectable. Hence it is helpful to clarify in what sense these photons, defined as the "limit" of wave packets, are not detectable. In the spirit of the standard method of Cauchy completion we *identify* a zero-energy Rindler photon with a sequence of wave packets $\{|\phi_i\rangle\}_{i=1}^\infty$ with the property

$$\lim_{i\to\infty}\int d\xi\, dx\, dy\, e^{2a\xi}\langle\phi_i|:T^\tau{}_\tau:|\phi_i\rangle=0 . \quad (6.4)$$

Next, we define the detectability of a sequence of wave packets. We specialize to the cases where particles are defined through a timelike Killing field [14] as is the case for Minkowski as well as Rindler particles. Consider a localized detector D and a device R which carries it. We require that the proper acceleration a of R integrated over the proper time be less than a fixed value F, which cannot be varied once the device R is given. (For example, the device R can be a rocket with a finite amount of fuel represented by the quantity F.) We do not need to put any restriction on the detector D other than that its detection probability be proportional to the square of one-particle wave function of the state where the detector D is located. The detector is allowed to be turned on only when it is following an orbit of the Killing field so that the particle concept under consideration agrees with the detector response. (We assume that the whole detector can approximately follow the orbits of the Killing field [3].) We say that a sequence of a wave packets $\{|\phi_i\rangle\}_{i=1}^{\infty}$ is detectable if there exists a detector D and a device R as above and a sequence of trajectories for the detector, each one compatible with the restrictions placed on R, such that the greatest lower bound of the corresponding sequence of probabilities P_i, for detecting the wave packet $|\phi_i\rangle$, is nonzero. This definition of detectability is natural in the sense that ordinary (normalizable) Minkowski and Rindler photons turn out to be detectable while zero-energy Minkowski photons do not. (In this context, an ordinary wave packet state will be represented by the sequence whose elements are all identical to this packet.)

We find that zero-energy Rindler photons are not detectable according to this definition. This is because the motion needed for the detection along the Killing vector approaches the horizon as the energy of the wave packet goes to zero [15]. This requires the proper acceleration of the device R to be increased without bound. Hence the proper time of detection must approach zero due to the restriction on the device R as $i \to +\infty$. Hence, the greatest lower bound of $\{P_i\}_{i=1}^{+\infty}$ is zero. (We are using the assumption that the probability per unit proper time does not diverge *in this case* as the limit $E \to 0$ is taken. This assumption can be shown to be valid for some model detectors.)

Boulware [16] has shown in *classical* electrodynamics that all electromagnetic radiation emitted by an accelerated charge goes into a region of spacetime inaccessible to the coaccelerating observers. His analysis is in agreement with our observation here that the emission and absorption of zero-energy Rindler photons by the charge cannot be detected by a coaccelerating observer inside the Rindler wedge.

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