# End point of Hawking radiation

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The formation and semiclassical evaporation of two-dimensional black holes is studied in an exactly solvable model. Above a certain threshold energy flux, collapsing matter forms a singularity inside an apparent horizon. As the black hole evaporates the apparent horizon recedes and meets the singularity in a finite proper time. The singularity emerges naked, and future evolution of the geometry requires boundary conditions to be imposed there. There is a natural choice of boundary conditions which matches the evaporated black hole solution onto the linear dilaton vacuum. Below the threshold energy flux no horizon forms and boundary conditions can be imposed where infalling matter is reflected from a timelike boundary. All information is recovered at spatial infinity in this case.

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## I. INTRODUCTION

Recent months have seen a lot of activity in the study of quantum effects on black holes. For this purpose, Callan, Giddings, Harvey, and Strominger (CGHS) [1] proposed a simple two-dimensional model involving gravity coupled to a dilaton and conformal matter fields. The model has classical solutions which describe the formation of black holes and enables a semiclassical treatment of Hawking radiation and its back reaction on the geometry. This has been developed further by a number of authors [2–10]. The semiclassical equations of the CGHS model have not been solved in closed form, but recently Bilal and Callan [9] and de Alwis [10] have shown how the original model can be modified to allow explicit construction of exact quantum black hole solutions. Astutely chosen field redefinitions allow the modified theory to be written as a Liouville model, and the semiclassical equations are straightforwardly solved in that form.

In this paper we will study a variation on this theme. Rather than modifying the dilaton potential to achieve a solvable field theory, as was done in [8–10], we instead change the kinetic term. This change simplifies somewhat the field redefinitions which take the model into a Liouwlle theory and also allows us to identify the vacuum configuration in a straightforward fashion. While the emphasis in [8–10] was on achieving consistent conformal field theories, our goal here is limited to modifying the CGHS equations to make them exactly solvable and then to study the physics of the resulting solutions. The complete quantization of this system is outside the scope of this paper.

The classical action of the original CGHS model is

$$S_0=\frac{1}{2\pi}\int d^2x\sqrt{-g}\left[e^{-2\phi}(R+4(\nabla\phi)^2+4\lambda^2)-\frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right]. \tag{1.1}$$

In the conformal gauge $g_{++}=g_{--}=0$, $g_{+-}=-\frac{1}{2}e^{2\rho}$, this becomes

$$S_0=\frac{1}{\pi}\int d^2x\left[e^{-2\phi}(2\partial_+\partial_-\rho-4\partial_+\phi\,\partial_-\phi+\lambda^2e^{2\rho})+\frac{1}{2}\sum_{i=1}^{N}\partial_+f_i\partial_-f_i\right]. \tag{1.2}$$

In addition to the equations of motion of $\rho$, $\phi$, and $f_i$, we have to impose two constraints corresponding to the equations of motion of the vanishing metric components:

$$0=e^{-2\phi}(4\partial_\pm\rho\partial_\pm\phi-2\partial_\pm^2\phi)+\frac{1}{2}\sum_{i=1}^{N}\partial_\pm f_i\partial_\pm f_i\ . \tag{1.3}$$

The classical action has a useful symmetry

$$\delta\phi=\delta\rho=\epsilon e^{2\phi}\ , \tag{1.4}$$

where $\epsilon$ is infinitesimal. The associated conserved current is

$$j^\mu=\partial^\mu(\phi-\rho)\ , \tag{1.5}$$

and the conservation equation is

$$\partial_\mu\partial^\mu(\phi-\rho)=0\ . \tag{1.6}$$

This fact allows one to choose a special conformal gauge in which $\rho=\phi$. It turns out to be convenient to preserve this simple form of the current at the one-loop level. This enables us to use the special conformal gauge when solving the semiclassical equations. We can always add covariant terms to the definition of our model for this purpose.

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Let us now consider one-loop quantum corrections. The matter fields contribute the familiar conformal anomaly term, and we also add a local and covariant term to preserve the simple form of the current (1.5).[1] One obtains the effective action

$$S=S_0-\frac{\kappa}{8\pi}\int d^2x\sqrt{-g}\left[R\frac{1}{\nabla^2}R+2\phi R\right]$$
$$=S_0-\frac{\kappa}{\pi}\int d^2x[\partial_+\rho\,\partial_-\rho+\phi\partial_+\partial_-\rho]\ , \qquad (1.7)$$

where $\kappa=N/12$ and the constraints become

$$0=\left[e^{-2\phi}-\frac{\kappa}{4}\right](4\partial_\pm\rho\,\partial_\pm\phi-2\partial_\pm^2\phi)+\frac{1}{2}\sum_{i=1}^{N}\partial_\pm f_i\partial_\pm f_i$$
$$-\kappa(\partial_\pm\rho\,\partial_\pm\rho-\partial_\pm^2\rho+t_\pm)\ . \qquad (1.8)$$

The functions $t_\pm(x^\pm)$ reflect the nonlocal nature of the anomaly and are determined by boundary conditions [1]. The contribution to the constraints from our extra term in (1.7) vanishes on classical solutions, and so it will not affect the rate of Hawking emission from black holes.

It remains to include the one-loop contribution from the reparametrization ghosts, dilaton, and conformal model. If the number of matter fields in the theory is large, this contribution will be insignificant compared with (1.7), which scales with $N$. Therefore our model, as it stands, at the very least provides a good description of large-$N$ black holes. The most straightforward way to include the ghosts, while preserving the symmetry (1.4), is to shift the value of $\kappa$ in (1.7) to $(N-24)/12$. This has the desirable feature that it turns the theory into a conformal field theory with vanishing total central charge and thus makes it one-loop finite [8–10]. However, the flux of Hawking radiation from a black hole is proportional to $\kappa$, and so this shift leads to the undesirable result of unphysical modes contributing to the Hawking radiation [7].

Strominger has suggested an alternate prescription for the one-loop ghost contribution which is designed to decouple the ghosts from the outgoing energy flux [7]. In our model his method boils down to keeping $\kappa=N/12$ in (1.7) and including the following term in the action:

$$S_{\text{ghost}}=\frac{26-2}{12\pi}\int d^2x[\partial_+(\rho-\phi)\partial_-(\rho-\phi)]\ . \qquad (1.9)$$

Adding this term does not violate the symmetry (1.4). In fact, the variation of $S_{\text{ghost}}$ vanishes for all solutions of the semiclassical equations of motion obtained from (1.7). This is as it should be since the role of the ghosts should not be to modify equations of motion, but rather to implement the gauge fixing of the path integral over off-shell geometries. Unfortunately, the theory is no longer a conformal field theory with vanishing central charge if this prescription is used, but exact solutions can still be found. These solutions exhibit reasonable physical behavior for all values of $N$, and in particular the rate of Hawking evaporation is always proportional to $N$.

In this paper we will not attempt to resolve all the issues involved in the quantization of these models, but will focus on exact semiclassical solutions and their physical properties. We will assume that $\kappa$ takes a positive value in the following analysis.[2] Our results therefore apply both to the $N>24$ conformal model (1.7) [with $\kappa=(N-24)/12$] and to the Strominger-type theory with $\kappa=N/12$.

## II. EXACT SOLUTIONS

To solve our model, we follow Bilal and Callan [9] and de Alwis [10] and perform a field redefinition to a Liouville theory. Our new fields are defined as

$$\Omega=\frac{\sqrt{\kappa}}{2}\phi+\frac{e^{-2\phi}}{\sqrt{\kappa}}\ ,$$
$$\chi=\sqrt{\kappa}\rho-\frac{\sqrt{\kappa}}{2}\phi+\frac{e^{-2\phi}}{\sqrt{\kappa}}\ . \qquad (2.1)$$

The effective action (1.7) takes the form

$$S=\frac{1}{\pi}\int d^2x\left[-\partial_+\chi\,\partial_-\chi+\partial_+\Omega\,\partial_-\Omega+\lambda^2e^{(2/\sqrt{\kappa})(\chi-\Omega)}\right.$$
$$\left.+\frac{1}{2}\sum_{i=1}^{N}\partial_+f_i\partial_-f_i\right]\ , \qquad (2.2)$$

and the constraints become

$$\kappa t_\pm=-\partial_\pm\chi\,\partial_\pm\chi+\sqrt{\kappa}\,\partial_\pm^2\chi+\partial_\pm\Omega\,\partial_\pm\Omega$$
$$+\frac{1}{2}\sum_{i=1}^{N}\partial_\pm f_i\partial_\pm f_i\ . \qquad (2.3)$$

Note that $\Omega$ is bounded from below in (2.1) and therefore (2.2) defines a rather unconventional quantum field theory. In this paper we only work with semiclassical equations, and it should be kept in mind that the full quantum theory may well be describe very different physics in regions of strong coupling.

The equations of motion derived from the Liouville action (2.2) can be solved exactly. Let us first consider asymptotically flat static geometries:

$$\Omega=\chi=-\frac{\lambda^2x^+x^-}{\sqrt{\kappa}}+P\sqrt{\kappa}\ln(-\lambda^2x^+x^-)+\frac{M}{\lambda\sqrt{\kappa}}\ , \qquad (2.4)$$

where $P$ and $M$ parametrize different solutions. We are using "Kruskal" coordinates [1,3], which make up the

[1] The symmetry transformation (1.4) will receive quantum corrections $\delta\phi=\delta\rho=\epsilon e^{2\phi}/[1-(\kappa/4)e^{2\phi}]$, but the conserved current (1.5) remains unchanged.

[2] If we do not adopt Strominger's prescription but rather perform the shift of $\kappa$ in (1.7), then $\kappa$ will be negative for $N<24$. In this case no singularity is encountered in gravitational collapse, which might sound attractive, but as pointed out above, the system is unstable and emits negative-energy Hawking radiation.

coordinate system where $\phi=\rho$. Comparing with the definitions (2.1) immediately reveals that the solution with $P=-\frac{1}{4}$ and $M=0$ is the familiar linear dilaton vacuum $e^{-2\phi}=e^{-2\rho}=-\lambda^2x^+x^-$.

Adjusting the value of $P$ corresponds to having a different energy flux at spatial infinity in these solutions. In particular, solutions with $P=-\frac{1}{4}$ have a vanishing asymptotic energy density, whereas a geometry with $P=0$ has a smooth horizon (at $x^+x^-=0$) and describes a quantum black hole in thermal equilibrium with its environment [4]. Solutions with $P\neq-\frac{1}{4}$ have an infinite Arnowitt-Deser-Misner (ADM) mass because in all these solutions there is a nonvanishing energy density at infinity. For the $P=-\frac{1}{4}$ solutions, the ADM mass is $M$. Static solutions with a positive ADM mass are weakly coupled, but singular at $x^+x^-=0$ [4–6]. Solutions with a negative ADM mass have a naked singularity at a finite value of $-x^+x^-$.

The thermal equilibrium solutions (with $P=0$) have an interesting property. Two such static solutions with different values of the parameter $M$ can be continuously matched across an infall line, $x^+=x_0^+$, up to a uniform shift of $x^-$. This corresponds to a black hole absorbing an incoming shock wave with the shift in $M$ being equal to the energy carried by the wave. The fact that the solution remains static indicates that the Hawking temperature of two-dimensional black holes remains independent of their mass, even when our quantum corrections are added. This can also be checked by direct calculation using the geometry (2.4).

Now let us consider a dynamical situation where an incoming shock wave carries energy into the vacuum. The corresponding solution is constructed by patching together across an infall line the linear dilation solution and a time-dependent solution (with $P=-\frac{1}{4}$) which describes the subsequent evolution of the black hole:

$$\Omega=\chi=-\frac{\lambda^2x^+x^-}{\sqrt{\kappa}}-\frac{\sqrt{\kappa}}{4}\ln(-\lambda^2x^+x^-)-\frac{m}{\lambda\sqrt{\kappa}x_0^+}(x^+-x_0^+)\theta(x^+-x_0^+)\ . \qquad (2.5)$$

The matching conditions at $x^+=x_0^+$ are provided by the $++$ constraints in (2.3) with

$$\frac{1}{2}\sum_{i=1}^{N}\partial_+f_i\partial_+f_i=\frac{m}{\lambda x_0^+}\delta(x^+-x_0^+)\ ,$$

where $m$ is the energy carried by the incoming shock wave.

The Liouville fields $\Omega$ and $\chi$ are nonsingular in the solution (2.5), but in terms of the original variables $\phi$ and $\rho$, a singularity forms on the infall line at $\phi=\phi_{\rm cr}=-\frac{1}{2}\ln(\kappa/4)$. This is easy to see by computing, for example, the curvature scalar $R=8e^{-2\rho}\partial_+\partial_-\rho$. Using the transformations (2.1) and the conservation equation (1.6), one finds

$$\partial_+\partial_-\rho=\frac{1}{\Omega'}\left[\partial_+\partial_-\chi-\frac{\Omega''}{\Omega'^2}\partial_+\Omega\,\partial_-\Omega\right]\ . \qquad (2.6)$$

The singularity forms where

$$\Omega'\equiv\frac{d\Omega}{d\phi}=\frac{\sqrt{\kappa}}{2}-\frac{2}{\sqrt{\kappa}}e^{-2\phi}$$

vanishes. It lies on a curve $(\bar{x}^+,\bar{x}^-)$, which is the constant $\phi$ contour of (2.5) at $\phi=\phi_{\rm cr}$ and is defined by the equation

$$1-\ln\frac{\kappa}{4}=-\frac{4\lambda^2}{\kappa}\bar{x}^+\bar{x}^--\ln(-\lambda^2\bar{x}^+\bar{x}^-)-\frac{4m}{\lambda\kappa x_0^+}(\bar{x}^+-x_0^+)\theta(\bar{x}^+-x_0^+)\ . \qquad (2.7)$$

This singularity occurs at the boundary of the range of $\Omega$, which is deep in the quantum-mechanical strong-coupling region. It may therefore well be absent in the full quantum theory.[3]

The singularity forms inside an apparent horizon, which is located where $\partial_+\phi=0$ [3]. The apparent horizon defines another curve $(\hat{x}^+,\hat{x}^-)$ above the infall trajectory:

$$\hat{x}^+=-\frac{\kappa}{4\lambda^2}\frac{1}{\hat{x}^-+m/\lambda^3x_0^+}\ . \qquad (2.8)$$

When a massive black hole begins to evaporate, the apparent horizon recedes at a rate which agrees with calculations in the original CGHS model. The agreement will hold as long as the remaining mass is large compared with $N\lambda$. We were not able to follow the evaporation to completion previously, but this is straightforward now that we have exact solutions. In [3] we conjectured that the singularity would always remain inside the apparent horizon and that the geometry would have a global horizon separating the two. However, the exact solution (2.5) exhibits very different behavior. As suggested by Hawking [5], the singularity and apparent horizon collide in a finite proper time. The intersection point $(x^+,x^-)=(x_s^+,x_s^-)$ of the two curves (2.7) and (2.8) is given by

$$x_s^+=\frac{\kappa\lambda x_0^+}{4m}(e^{4m/\kappa\lambda}-1)\ ,$$
$$x_s^-=-\frac{m}{\lambda^3x_0^+}\frac{1}{(1-e^{-4m/\kappa\lambda})}\ . \qquad (2.9)$$

The singularity goes from being spacelike behind the apparent horizon to being timelike, and therefore naked, after the two have merged. As a result, the future evolution is not uniquely determined unless boundary conditions are imposed at the naked singularity.

We emphasize once again that the naked singularity occurs deep in the quantum-mechanical region where the semiclassical theory is not applicable. The occurrence of

[3]A shock wave sent into a static geometry with positive ADM mass (and $P=-\frac{1}{4}$) will lead to topology change. No singularity is formed on the infall line in this case, but one will form at a later time, splitting the space into two disconnected regions. A topology change of this sort will not occur if the initial state is the linear dilation vacuum.

such singularities means that very quantum-mechanical effects are in causal contact with outside observers. The precise nature of the phenomenon is beyond our present knowledge. It therefore seems appropriate to replace the detailed dynamics of the naked singularity of phenomenological boundary conditions. In particular, we can expect radiation of quantum-mechanical origin out along the null line $x^-=x_s^-$. Hawking has speculated that the emergence of the naked singularity would precipitate a cataclysmic event, a thunderbolt, which propagates outward at the speed of light [11].

## III. FINAL STATE

A possible boundary condition which suggests itself is analytically to continue the solution (2.5) across the null line $x^-=x_s^-$ from region I into region II in Fig. 1. This, however, does not lead to reasonable behavior in the asymptotic future. To see this we observe that all contours of constant $\phi=\phi_0<\phi_{\rm cr}$ enter into region II. These contours are timelike everywhere outside the apparent horizon and can be used to define fiducial observers. One finds that the scalar curvature tends to $-\infty$ as $x^+\to\infty$ along every such a contour:

$$R\sim-\kappa\lambda^2\frac{e^{-4\phi_0}}{(e^{-2\phi_0}-e^{-2\phi_{\rm cr}})^3}\ln\left[\frac{x^+}{x_0^+}\right] . \tag{3.1}$$

Thus all fiducial observers eventually find their way into a region of diverging curvature, no matter how far away from the black hole they set out. Fortunately, this disastrous conclusion is by no means inevitable.

A more reasonable possibility, suggested by Strominger [12], is that the boundary conditions can be chosen in such a way that each fiducial observer eventually tends to a region with vacuum behavior. Miraculously, this type of boundary condition occurs naturally in the exact solution (2.5). Both $\phi$ and $\rho$ take vacuum values on the null line $x^-=x_s^-$ dividing regions I and II. This means we can match the evaporating solution (2.5) in region I onto a linear dilation configuration in region II, which is shifted with respect to the original vacuum:

$$e^{-2\phi}=e^{-2\rho}=-\lambda^2x^+\left[x^-+\frac{m}{\lambda^3x_0^+}\right] . \tag{3.2}$$

The fields are continuous at $x^-=x_s^-$, but their $x^-$ derivatives are not. Evaluating the $--$ constraints at this null line, one finds a $\delta$-function contribution:

$$\frac{1}{2}\sum_{i=1}^{N}\partial_-f_i\partial_-f_i=-\frac{\kappa}{4}\frac{(1-e^{-4m/\kappa\lambda})}{(x^-+m/\lambda^3x_0^+)}\delta(x^--x_s^-) . \tag{3.3}$$

This means that a matter shock wave carries a small amount $(-\kappa\lambda/4)(1-e^{-4m/\kappa\lambda})$ of negative energy out along the line to null infinity. This result can be checked independently as follows. The energy carried by a black hole across null lines of constant $x^-$ is given by

$$m(x^-)=\lim_{x^+\to\infty}\frac{1}{4\lambda}e^{-2\phi}R . \tag{3.4}$$

This gives the correct ADM mass for the $P=-\frac{1}{4}$ static solutions and provides a definition of the remaining mass of an evaporating black hole [6].[4] Using the solution (2.5) to evaluate the mass at $x^-=x_s^-$ gives precisely the same small negative energy as found above. When the negative-energy shock wave reaches null infinity, it brings the energy to zero, the vacuum value, in region II.

The fact that negative energy is carried out from the naked singularity looks strange, but is not very serious. First of all, energy density is not positive definite in quantum theory and global energy positivity is not violated. Second, the amount of negative energy is limited by $-\kappa\lambda/4$ for all values of the original black hole mass $m$ (and vanishes in the $m\to 0$ limit). The limiting value is the analogue of the Planck scale in this theory, and so this negative energy may simply be an artifact of our semiclassical approximation. At any rate, the outgoing shock wave does not represent a violent event on an astronomical scale, and so we will refer to it as a thunderpop rather than a thunderbolt.

We find it rather compelling that it is possible to match the evaporating solution onto the vacuum, with only a Planck mass worth of adjustment needed to the energy. The physical picture this presents is that the black hole evaporates completely, leaving no remnant behind. The region $\phi>\phi_{\rm cr}$ is completely unphysical in this case [3,4,6]. The vacuum should be taken to be the linear dilaton solution for $\phi<\phi_{\rm cr}$ and some boundary conditions supplied at the critical line where it is timelike. Perhaps the appropriate framework for this system is to couple two-dimensional gravity to nontrivial boundary degrees of freedom, as considered in [13] in the context of open string theory.

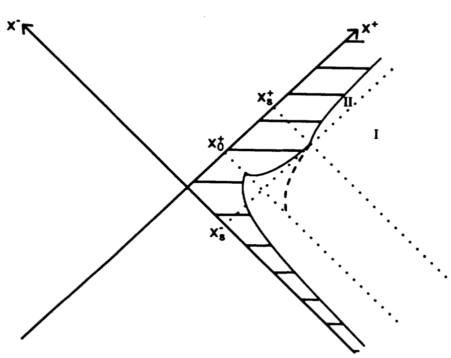


FIG. 1. Black hole formed by an incoming shock wave at $x^+=x_0^+$. A spacelike singularity forms inside an apparent horizon, which recedes until it collides with the singularity at $x_s$. The solution in region II is determined by boundary conditions at the naked singularity.

[4]The definition in [6] included a factor of $[1-(N/12)e^{2\phi}]^{3/2}$, which goes to 1 as $x^+\to\infty$.

## IV. GENERAL DISTRIBUTIONS OF INCOMING MATTER

Finally, we want to consider arbitrary distributions of incoming matter. We will need expressions for the incoming flux of energy in terms of Kruskal coordinates. Let $\lambda\sigma^{\pm}=\pm\ln(\pm\lambda x^{\pm})$. In this coordinate system, the metric asymptotically approaches the Minkowski metric. The energy is by definition conjugate to $\frac{1}{2}(\sigma^{+}+\sigma^{-})$. Transforming to Kruskal coordinates is straightforward, and the total energy of a distribution of incoming matter is given in terms of the Kruskal energy-momentum tensor by

$$M=\lambda\int_0^\infty dx_0^+ x_0^+ T_{++}(x_0^+)\ . \tag{4.1}$$

Another quantity of interest is the total incoming Kruskal momentum conjugate to $x_0^+$. This is given by

$$P_+=\int_0^\infty dx_0^+ T_{++}(x_0^+)\ . \tag{4.2}$$

We will also define $x^+$-dependent truncated versions of (4.1) and (4.2) as

$$\begin{aligned}M(x^+)&=\lambda\int_0^{x^+} dx_0^+ x_0^+ T_{++}(x_0^+)\ ,\\ P_+(x^+)&=\int_0^{x^+} dx_0^+ T_{++}(x_0^+)\ .\end{aligned} \tag{4.3}$$

The exact solution which generalizes (2.5) is

$$\Omega=\chi=-\frac{\lambda^2}{\sqrt{\kappa}}x^+\left[x^-+\frac{P_+(x^+)}{\lambda^2}\right]+\frac{M(x^+)}{\sqrt{\kappa}\lambda}-\frac{\sqrt{\kappa}}{4}\ln(-\lambda^2x^+x^-)\ . \tag{4.4}$$

This solution will typically have naked singularities. In this case (4.4) is only applicable in those regions which are not in the causal future of such singularities and we have to supply additional boundary conditions. We do not know what is the "correct" set of conditions to impose at a naked singularity, but for the purpose of illustration we will adopt a particularly simply boundary condition, demanding that matter energy, carried by the $f$ fields, is totally reflected from the $\phi=\phi_{\rm cr}$ line where it is timelike:

$$f_i=0|_{\phi=\phi_{\rm cr}}\ . \tag{4.5}$$

It is not meaningful to apply boundary conditions where the singularity is spacelike.

We consider first a simple example in which the incoming energy flux is turned on at some finite time and remains steady at smaller rate than the Hawking flux for a two-dimensional black hole. In this case the incoming energy momentum in Kruskal coordinates is

$$T_{++}(x^+)=\frac{\epsilon}{\lambda(x^+)^2}\theta(x^+-x_0^+)\ , \tag{4.6}$$

where $\epsilon<\kappa\lambda/4$ is the constant energy flux and $x^+=x_0^+$ defines the leading edge of the incoming energy. The solution (4.4) reduces to

$$\Omega=\chi=-\frac{\lambda^2}{\sqrt{\kappa}}x^+\left[x^-+\frac{\epsilon}{\lambda^3x_0^+}\right]=\frac{\sqrt{\kappa}}{4}\ln(-\lambda^2x^+x^-)+\frac{\epsilon}{\sqrt{\kappa}\lambda}\left[1+\ln\frac{x^+}{x_0^+}\right]\ . \tag{4.7}$$

The curve $\phi=\phi_{\rm cr}$ is timelike for $\epsilon<\kappa\lambda/4$, as shown in Fig. 2. Region i, where $x^-<-\kappa/4\lambda^2x_0^+$, is not in causal contact with any singularity. On the other hand, region ii, where $x^->-\kappa/4\lambda^2x_0^+$, can receive signals from the singularity and therefore the solution (4.7) is not correct in this region.

The simple reflecting boundary conditions (4.5) imply the following relation between the incoming and outgoing values of the matter energy-momentum:

$$T_{--}^f(x^-)=T_{++}^f(\bar{x}^+)\left[\frac{\partial\bar{x}^+}{\partial x^-}\right]^2\ , \tag{4.8}$$

where $x^+=\bar{x}^+(x^-)$ defines the boundary and $T_{\pm\pm}^f=\frac{1}{2}\sum_{i=1}^N\partial_\pm f_i\partial_\pm f_i$. These boundary conditions along with the equations of motion are sufficient to completely determine the fields as well as the boundary curve itself in region ii in Fig. 2. In our simple example of a small uniform incoming flux, the solution in region ii turns out to be a static configuration (2.4) with $x^-$ shifted by $\epsilon/\lambda^3x_0^+$ and parameters

$$\begin{aligned}P&=-\frac{1}{4}+\frac{\epsilon}{\kappa\lambda}\ ,\\ M&=\epsilon\left[1-\ln\frac{\kappa}{4}\right]+\left[\frac{\kappa\lambda}{4}-\epsilon\right]\ln\left[1-\frac{4\epsilon}{\kappa\lambda}\right]\ .\end{aligned} \tag{4.9}$$

The solutions on either side of the line $x^-=-\kappa/4\lambda^2x_0^+$ match smoothly across it. In particular, there is no outgoing shock wave propagating along this null line, as is easily seen by evaluating the $--$ constraints there.

For a more general incoming flux distribution, which is smaller than $\kappa\lambda/4$ at any given time, the solution in region II is more complicated, but it can be constructed in terms of the incoming flux distribution given the boundary conditions (4.5). In this case all information is reflected off the boundary in the semiclassical approxima-

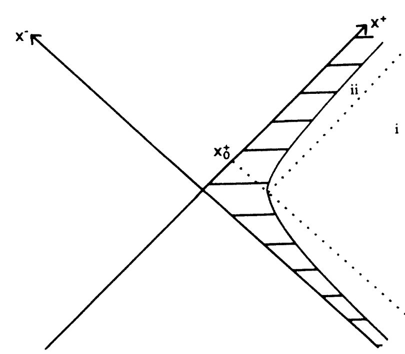


FIG. 2. Small steady incoming energy flux at $x^+>x_0^+$ leads to a timelike singularity. Region i is not in causal contact with the singularity, but the solution in region ii depends on the boundary conditions imposed at $\phi=\phi_{\rm cr}$.

tion. This can be interpreted as saying that information loss does not occur in low-energy (sub-Planckian) physics, at least at the semiclassical level.[5] This does not preclude the possibility of information loss of a more quantum-mechanical nature, e.g., tunneling, in low-energy processes.

The situation is very different when one considers an energy flux $\epsilon > \kappa\lambda/4$ for which the singularity goes spacelike and an actual black hole is formed. This case is qualitatively similar to the incoming shock wave considered in Sec. II. An apparent horizon forms. If the flux is turned off at some point, the apparent horizon will recede and eventually collide with the singularity, sending off a thunderpop. The final state will be the linear dilaton vacuum as before. The boundary condition we applied in Sec. II is in fact a special case of the reflecting boundary conditions we employed in this section.

Let us assume that the incoming flux is large enough from the start for an apparent horizon to form and that it is maintained for a finite length of time. In this case the solution (4.4) is valid until the naked singularity emerges. In particular, the bulk of the outgoing Hawking radiation is found in the region to the right of an outgoing null line analogous to $x^- = x_s^-$ in Fig. 1 and therefore described by (4.4). It is striking that the final ($x^+ \to \infty$) behavior of the fields in that region depends on only two moments $M$ and $P_+$ of the incoming $T_{++}$ and not on the detailed history of the initial state. Evidently, most of the information contained in the initial state is lost in this one-loop semiclassical approximation. It should, however, be noted that the present situation is somewhat better than that without any back reaction because the classical no-hair theorem implies that the final state can only depend on one of these moments, i.e., the total mass $M$. It is tempting to conjecture that a more systematic quantum treatment of the problem (for example, including higher gravitational loops) will introduce a dependence on higher moments:

$$P_+^n = \int_0^\infty dx_0^+ (x_0^+)^{-n+1} T_{++}(x_0^+) \ . \tag{4.10}$$

A simple electrodynamic system where unitarity is restored by quantum corrections was considered in [14].

[5]Note that a classical shock wave cannot provide a good description of an incoming low-energy state because of the uncertainty principle, which ensures that a given energy $\delta$ cannot be localized within a distance less than $1/\delta$.


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