Influence of the electric coupling strength in current-carrying cosmic strings

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The fully coupled Witten model for superconducting cosmic strings is analyzed in detail, with special emphasis on the effects due to a nonzero value of the electromagnetic coupling constant in order to check the accuracy of the neutral-limit treatment of the equation of state. Qualitative differences for the influence of the coupling constant between timelike and spacelike situations are examined in order to obtain a new understanding of the mechanical (as opposed to electromagnetic) properties of a cosmic string. Longitudinal- and transverse-perturbation velocities are estimated and related to stability criteria. It transpires that superconducting cosmic-string loops are generally not strictly stable against nonaxisymmetric perturbations.

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INTRODUCTION

In a previous article [1], the most important properties of current-carrying cosmic strings, which are of purely mechanical origin, were investigated by studying the limiting case of strictly neutral currents for which ambiguities resulting from divergent integrals are entirely absent. The equation of state was investigated for timelike as well as spacelike currents and it was shown (contrary to what occurs in the commonly used linear approximation [2,3]) the longitudinal-perturbation velocity is systematically lower than that of transverse perturbation [4] while the effective tension remains strictly positive everywhere.

The purpose of the present work is to examine the extent to which these results are affected by the allowance for a small but nonzero coupling constant e such as was included in the original Witten [5] model with the assumption that its numerical value is given by the usual formula $e^2=\frac{1}{137}$ (in unrationalized units with $\hbar=c=1$). The results of the work described below confirm that the neutral-limit treatment will, in fact, be a very good first approximation for a wide range of realistic applications with the implication that qualitatively and quantitatively correct results can be obtained from the use of the macroscopic formalism [3,4,6–8]. This will be useful as a basis for more reliable investigations of electromagnetic effects of potential cosmological interest such as formation of galaxies [9,10] (including voids [11,12]), and the interaction of strings with charged particles (including fermionic interactions [13,14] and the Aharonov-Bohm effect [15]) and plasmas [16]. (Many of the pioneering studies of such phenomena were flawed by the use of a treatment that took electromagnetic effects to be not just a correction but dominant, while neglecting more important mechanical effects whose significance was first realized by Davis and Shellard [17,18].)

An important effect that was pointed out in the neutral-limit model [1] concerning the "electric" part of the equation of state, i.e., that for which the current is timelike (as opposed to "magnetic" states for which a spacelike current is considered) is the existence of a phase-frequency threshold beyond which no stationary solution exists. The meaning of this threshold is clear: if the phase gradient of the trapped boson exceeds its mass, then it becomes energetically favored for the current carrier to flow out of the string. It is found that the lifetime of such configurations may be estimated as being roughly of order m_σ/M_ϕ^2, where m_σ is the mass of the current carrier and M_ϕ the mass of the Higgs boson, the resulting value (typically $\lesssim 10^{-30}$ s) being so small that such states are of no practical relevance for any physical purpose.

In principle, when a charge coupling constant is introduced, strictly stationary "electric" string states do not exist at all because the phase-frequency threshold for particle emission is moved to zero due to the well-known logarithmic divergence of the electrostatic potential surrounding an infinite charged string: as the energy density increases with increasing distances, it will eventually become sufficient for there to be a finite Coulomb-tunneling probability of a charged particle to go out of the core, so that if the surrounding space is large enough, any electrically charged configuration is, in principle, unstable. In practice, however, as discussed in Sec. II, there always exists a natural cutoff scale (determined by the characteristic distance between neighboring string segments, or, at most, the cosmological-horizon radius), which is sufficient to keep the energy per unit length, the tension, and the charge (or current) density finite (with values insensitive to the particular cutoff length that is actually chosen) both in electric and magnetic states. Due to this cutoff, the effective phase-frequency threshold for particle emission is only moved to a smaller (nonzero) value, and "electric" string states survive with what, in practice, is usually a very long lifetime. Indeed, the validity of the stationarity hypothesis is recovered since the lifetime we obtain already exceeds 10^{10^6} s for a symmetry-breaking scale of order 10 TeV, while for a symmetry-breaking scale of order 10^{15} GeV [grand unified theory (GUT)] one obtains $10^{10^{26}}$ s.

A different and more practically relevant mechanism for charged-particle emission by a charged cosmic string is that of ordinary pair creation since this effect has been shown to be of purely electric origin (neglecting gravitational effects or direct coupling with the Higgs field [19]). Assuming the trapped particles are much more massive than an electron, the electric field surrounding a charged cosmic string will indeed be generally sufficient to yield a pair-creation probability per unit time such that the initial charge of the string is "instantaneously" screened (considering the stationarity hypothesis) until it decreases to a maximum value depending only on the mass of the electron (i.e., the lightest charged particle), and the age of the Universe. As a result, the long-range electromagnetic effects are much smaller than would otherwise have been the case, a consideration which provides further justification for using the neutral-limit approximation for the treatment of "electric" string states in many applications.

I. BASIC EQUATIONS

The aim of this section is to recapitulate the essentials of the model used throughout the rest of this work together with the indispensable notation. We shall consider a Witten-type theory in which a U(1) symmetry is spontaneously broken by means of a Higgs field Φ which acquires a vacuum expectation value η. The Higgs field is coupled with a gauge vector B_μ by means of a coupling constant q, and with a charged scalar boson Σ. Electromagnetism is described in the usual way by the photon A_μ and the coupling constant e. The most general Lagrangian one can build with such fields is therefore

$$\mathcal{L}=-\tfrac{1}{2}(D_\mu\Phi)(D^\mu\Phi)^*-\tfrac{1}{2}(D_\mu\Sigma)(D^\mu\Sigma)^* \\ -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}-\frac{1}{16\pi}H_{\mu\nu}H^{\mu\nu}-V(\Phi,\Sigma)\ , \tag{1}$$

$$V(\Phi,\Sigma)=\frac{\lambda_\phi}{8}(|\Phi|^2-\eta^2)^2+f(|\Phi|^2-\eta^2)|\Sigma|^2 \\ +\frac{\lambda_\sigma}{4}|\Sigma|^4+\frac{m_\sigma^2}{2}|\Sigma|^2\ , \tag{2}$$

$$D_\mu\Phi=(\nabla_\mu+iqB_\mu)\Phi,\quad D_\mu\Sigma=(\nabla_\mu+ieA_\mu)\Sigma\ , \tag{3}$$

$$F_{\mu\nu}=\nabla_\mu A_\nu-\nabla_\nu A_\mu,\quad H_{\mu\nu}=\nabla_\mu B_\nu-\nabla_\nu B_\mu\ . \tag{4}$$

This kind of theory allows the formation of vortex defects of the vacuum by the Kibble mechanism. Such a vortex—the subject of the present study—is defined as a stationary configuration of cylindrical symmetry which we may choose to be aligned with the z axis. This means that quantities having a clear physical significance (e.g., the scalar-boson amplitudes and all the integrated functions of these) may depend neither on the internal string coordinates (z and t) nor on the polar angle θ. Consequently, the scalar-boson amplitudes can depend only on the radial coordinate r. The phase of the Higgs field has the form $n\theta$ for some integral winding number n, and the phase of the current carrier is a function only of the internal string coordinates. Thus, we can set

$$\Phi=\varphi(r)e^{in\theta}=\varphi e^{i\alpha}$$

and (5)

$$\Sigma=\sigma(r)e^{i\psi(z,t)}=\sigma e^{i\psi}\ ,$$

and find the equations of motion to be

$$\nabla_\mu[\varphi^2(\nabla^\mu\alpha+qB^\mu)]=0\ , \tag{6}$$

$$\nabla_\mu[\sigma^2(\nabla^\mu\psi+eA^\mu)]=0\ , \tag{7}$$

$$\nabla_\mu\nabla^\mu\varphi=\varphi(\nabla_\mu\alpha+qB_\mu)(\nabla^\mu\alpha+qB^\mu) \\ +\frac{\lambda_\phi}{2}\varphi(\varphi^2-\eta^2)+2f\varphi\sigma^2\ , \tag{8}$$

$$\nabla_\mu\nabla^\mu\sigma=\sigma(\nabla_\mu\psi+eA_\mu)(\nabla^\mu\psi+eA^\mu)+2f\varphi^2\sigma \\ +\lambda_\sigma\sigma^3+(m_\sigma^2-2f\eta^2)\sigma\ , \tag{9}$$

$$\nabla_\mu H^{\mu\nu}=4\pi q\varphi^2(\nabla^\nu\alpha+qB^\nu)\ , \tag{10}$$

$$\nabla_\mu F^{\mu\nu}=4\pi e\sigma^2(\nabla^\nu\psi+eA^\nu)\ . \tag{11}$$

Once the equations of motion are solved, we can use the solutions to compute conserved quantities such as the energy-momentum tensor (which we shall examine in detail later) and the Noether current whose conservation is expressed by (7), and which is associated with the invariance of the Lagrangian (1) with respect to local changes in the phase of the current carrier Σ, namely,

$$\mathcal{T}^\mu=\sigma^2(\nabla^\mu\psi+eA^\mu)\ . \tag{12}$$

This current is well defined even in the neutral limit [1], and when a nonzero electromagnetic coupling e is allowed for, it determines a corresponding electric current given by $\delta\mathcal{L}/\delta A_\mu=e\mathcal{T}^\mu$.

Inserting (5) into (10) and (11) and using the cylindrical symmetry, we see that the only nonzero components of the gauge fields are $A_z(r)$, $A_t(r)$, and $B_\theta(r)$. Now, again invoking axisymmetry, one finds that the current defined by (12) cannot depend on the internal string coordinates z and t. Since only this current is required to be gauge invariant (with respect to electromagnetism), we see that the most general form for ψ is, up to the addition of an unphysical constant,

$$\psi=\omega t-kz\ . \tag{13}$$

Setting

$$Q(r)=n+qC_\theta\ ,\\ P_z(r)=\partial_z\psi+eA_z\ ,\\ P_t(r)=\partial_t\psi+eA_t\ , \tag{14}$$

and using a prime to denote differentiation with respect to r, we obtain the following set of radial equations:

$$\begin{aligned}
&\varphi''+r^{-1}\varphi'=\frac{1}{r^2}\varphi Q^2+\frac{1}{2}\lambda_\phi\varphi(\varphi^2-\eta^2)+2f\varphi\sigma^2 ,\\
&\sigma''+r^{-1}\sigma'=\sigma(P_z^2-P_t^2)+2f\varphi^2\sigma+\lambda_\sigma\sigma^3\\
&\qquad\qquad+(m_\sigma^2-2f\eta^2)\sigma ,\\
&Q''-r^{-1}Q'=4\pi q^2\varphi^2 Q ,\\
&P_z''+r^{-1}P_z'=4\pi e^2\sigma^2 P_z ,\\
&P_t''+r^{-1}P_t'=4\pi e^2\sigma^2 P_t .
\end{aligned}\tag{15}$$

Since the last two equations are linear in the quantities P_z and P_t, and since regularity on the axis requires that the derivatives of both these quantities should vanish at $r=0$, it follows that we shall have

$$P_z(r)=P_z(0)P_*(r),\quad P_t(r)=P_t(0)P_*(r) ,\tag{16}$$

for some function $P_*(r)$ satisfying

$$P_*''+r^{-1}P_*'=4\pi e^2\sigma^2 P_*$$

with the boundary condition $P_*(0)=1$, so we may set $w=P_z^2(0)-P_t^2(0)$ to get

$$P_z^2-P_t^2=wP_*^2 ,\tag{17}$$

where

$$\begin{aligned}
&w>0 \text{ in the magnetic case} ,\\
&w<0 \text{ in the electric case} ,\\
&w=0 \text{ in the null case} ,
\end{aligned}\tag{18}$$

this parameter being strictly identical to the one previously defined in a previous article [1]. This classification corresponds closely to that of Carter [6,7] in the sense that, for $w>0$, there exists a referential in which the current (12) is purely spatial, so that there is no charge density, while if $w<0$, the preferred frame yields a four-current having only one (timelike) component, i.e., a number (or charge) density.

Due to the requirement of local regularity on the axis and the requirement that the medium surrounding the string is the vacuum as defined after the symmetry breaking, we have the physical boundary conditions

$$\begin{aligned}
&\varphi(0)=0,\quad \varphi(\infty)=\eta ,\\
&\frac{d\sigma}{dr}(0)=0,\quad \sigma(\infty)=0 ,\\
&Q(0)=n,\quad Q(\infty)=0 ,\\
&P_*(0)=1,\quad \frac{dP_*}{dr}(0)=0 .
\end{aligned}\tag{19}$$

This set of conditions uniquely determines the solution of Eq. (15).

We recall briefly the rescaling (necessary for numerical purposes) whereby one obtains a set of dimensionless variables given by

$$\varphi=\eta X,\quad \sigma=\frac{m_\sigma}{\sqrt{\lambda_\sigma}}Y,\quad r=\frac{\rho}{\sqrt{\lambda_\phi}\eta} ,\tag{20}$$

$$w=\frac{\lambda_\phi\lambda_\sigma}{m_\sigma^2}\eta^4\tilde{w},\quad q^2=\lambda_\phi\tilde{q}^{\,2},\quad e^2=\frac{\lambda_\phi\lambda_\sigma\eta^2\tilde{e}^{\,2}}{m_\sigma^2} ,\tag{21}$$

and we define the free parameters of the theory as

$$\alpha_1=\frac{m_\sigma^2}{\lambda_\sigma\eta^2} ,\quad \alpha_2=\frac{fm_\sigma^2}{\lambda_\phi\lambda_\sigma\eta^2} ,\quad \text{and}\quad \alpha_3=\frac{m_\sigma^4}{\lambda_\phi\lambda_\sigma\eta^4} .\tag{22}$$

Since we require that the charged boson be trapped in the string, we have to impose that the potential for Σ in the vortex should have a minimum for $\sigma\neq0$, which is achieved by imposing that $2f\eta^2-m_\sigma^2$ be positive (we shall see later on that this is a quite weak requirement), and as the vacuum is observed to be neither superconducting nor charged, the potential should be greater in the core of the string than in the vacuum. This condition is restated in terms of the α parameters as

$$(\alpha_3-2\alpha_2)^2<\frac{\alpha_3}{2} .\tag{23}$$

The equations that are now to be satisfied with the boundary conditions (19) read

$$\begin{aligned}
&X''+\frac{1}{\rho}X'=\frac{1}{\rho^2}XQ^2+\frac{1}{2}X(X^2-1)+2\alpha_2XY^2 ,\\
&Y''+\frac{1}{\rho}Y'=\frac{\tilde{w}}{\alpha_1}P_*^2Y+2\frac{\alpha_2}{\alpha_1}(X^2-1)Y+\frac{\alpha_3}{\alpha_1}Y(Y^2+1) ,\\
&Q''-\frac{1}{\rho}Q'=4\pi\tilde{q}^{\,2}X^2Q ,\\
&P_*''+\frac{1}{\rho}P_*'=4\pi\tilde{e}^{\,2}Y^2P_* ,
\end{aligned}\tag{24}$$

These equations have been solved numerically [20] in electric (as well as magnetic) configurations (the magnetic case has already been studied in detail, see, for instance, the work of Babul, Piran, and Spergel [21]) and a typical solution may be seen in Fig. 1.

II. PERTURBATIVE EXPANSION

Assuming the mass of the Σ field to be less (and even much less) than that of the φ field [22], we can make a perturbative expansion in the parameter $\tilde{e}^{\,2}$, i.e., as usual in electromagnetism $X=X_0+\tilde{e}^{\,2}X_1+\cdots$, and so on, with $P_*=1+\tilde{e}^{\,2}P_1+\cdots$. It has to be emphasized that this perturbative expansion is strongly justified because, according to Eq. (21), $\tilde{e}^{\,2}$ is proportional to the ratio of the carrier mass m_σ to the Higgs-boson mass $M_\varphi=\sqrt{\lambda_\varphi}\eta$. If the latter is at the GUT level, the result for $\tilde{e}$ will be microscopically small, while even for the more moderate value required, for instance, if the superconducting cosmic strings are to be used for Universe-filling purposes, it has been plausibly argued [23] that the symmetry-breaking scale should not be much less than roughly 10 TeV, whereas the current carrier might be the W intermediate vector boson say, of mass ~100 GeV, so

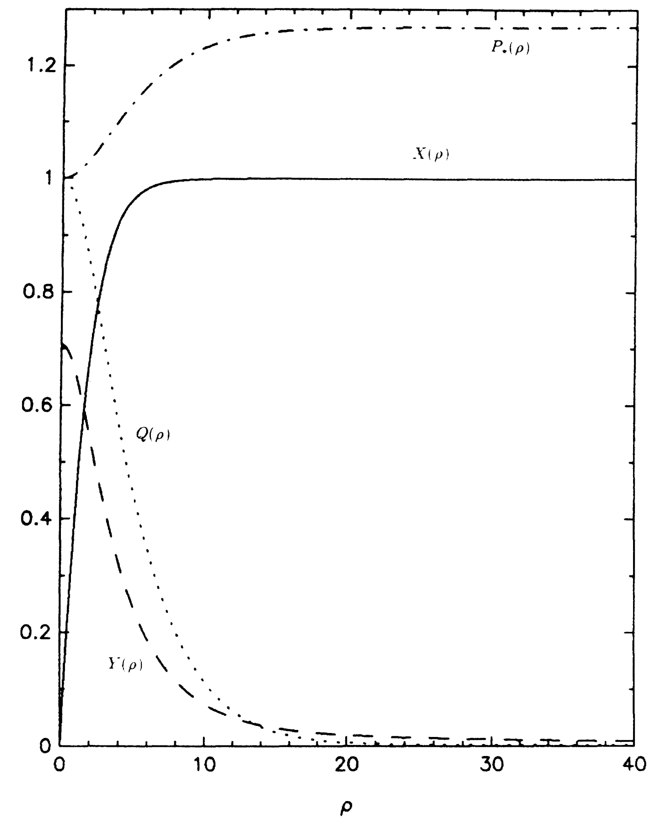

FIG. 1. A typical solution of the system (24) for an electric situation: X (solid line), Y (dashed), Q (dotted), and P_* (dash-dotted) whose nearly constant behavior for large ρ is clearly shown.

one obtains a maximum numerical value for $\tilde{e}^2$ of order

$$4\pi\tilde{e}^2 \lesssim 10^{-6} \tag{25}$$

(with the fine-structure constant $\alpha \sim 10^{-2}$). The effect of this coupling has been exaggerated on our curves by setting the value $4\pi\tilde{e}^2=0.1$ (with other parameters fixed to the values they had in a previous article [1]). Since even in this exaggerated case the corrections turn out to be rather small, it follows that, in realistic cases subject to (25), the effect of the coupling constant can be neglected altogether as a first approximation, which gives the zeroth-order set of differential equations (i.e., the previously studied neutral limit):

$$X_0''+\frac{1}{\rho}X_0'=\frac{1}{\rho^2}X_0Q_0^2+\frac{1}{2}X_0(X_0^2-1)+2\alpha_2X_0Y_0^2\ ,$$

$$Y_0''+\frac{1}{\rho}Y_0'=\frac{\tilde{\omega}+2\alpha_2(X_0^2-1)}{\alpha_1}Y_0+\frac{\alpha_3}{\alpha_1}Y_0(Y_0^2+1)\ , \tag{26}$$

$$Q_0''-\frac{1}{\rho}Q_0'=4\pi\tilde{q}^2X_0^2Q_0\ .$$

Let us recall briefly the most useful results concerning this limit (for which we use a subscript 0): integration of the current (12) leads to a charge number density $\mathcal{C}_0$ which is expressible (see also Sec. III) as

$$\mathcal{C}_0=\frac{2\pi v m_\sigma^2}{\lambda_\sigma M_\varphi^2}B\ , \tag{27}$$

where B is a numerical factor of order unity given by

$$B=\int_0^\infty \rho\, d\rho\, Y_0^2(\rho)\ . \tag{28}$$

The system (26) also yields an asymptotic behavior for the function Y_0 given by

$$Y_0(\rho)\sim C\times\begin{cases}K_0(\kappa\rho) & \text{if } w>-m_\sigma^2\ ,\\ J_0(\kappa\rho) & \text{if } w<-m_\sigma^2\ ,\end{cases} \tag{29}$$

where $\kappa^2=|\tilde{\omega}-\alpha_3|/\alpha_1=|\tilde{\omega}-\tilde{\omega}_c|/\alpha_1$, K_0 and J_0 are modified Bessel functions of zeroth order (for which we use standard notations [24]), and C is a constant which is (numerically) found to be approximately independent of κ. This may be understood as being due to the normalization of the Bessel functions which is more or less that of the function Y so that a matching at the origin with the asymptotic solution is possible without any further dependence on the parameters. The mass of the current carrier is then seen to represent a phase-frequency threshold beyond which no stationary solution exists since the integral (28) diverges with the solution J_0. Due to the fact that $\mathcal{C}$ is independent of κ, one finds the behavior of B when ω approaches m_σ as

$$B\propto\frac{\omega}{\omega^2-m_\sigma^2}\ . \tag{30}$$

Now we can obtain a lot of information concerning the first-order term without actually solving the corresponding system of equations. The first thing we see is that the equation for P_1, namely,

$$P_1''+\frac{1}{\rho}P_1'=4\pi Y_0^2 \tag{31}$$

is exactly soluble in closed form by means of

$$P_1(\rho)=4\pi\left[\ln\rho\int_0^\rho x\, Y_0^2(x)dx-\int_0^\rho x\ln x\, Y_0^2(x)dx\right]\ , \tag{32}$$

in which the integrals are convergent as $\rho\to\infty$ with limits given respectively by B as defined in (28) and by

$$A=\int_0^\infty x\ln x\, Y_0^2(x)dx\ , \tag{33}$$

so that, as $\rho\to\infty$, one obtains the asymptotic expression

$$P_1(\rho)\sim 4\pi\tilde{e}^2(B\ln\rho+A)\ , \tag{34}$$

expressing the well-known logarithmic divergence. However, the region of space around the string where this divergence becomes important is $\rho\gtrsim\rho_\infty$, where ρ_∞ is such that $\tilde{e}^2P_1(\rho_\infty)\simeq 1$, which gives

$$\rho_\infty\simeq\exp\left\{(4\pi\tilde{e}^2B)^{-1}+\frac{A}{B}\right\}\ . \tag{35}$$

Due to the definition of $\tilde{e}^2$, we find, expressed in units of the inverse of the energy scale (or in any standard unit of length, as numerical application reveals), a divergence radius of order

$$\rho_\infty \sim \exp\left[\frac{1}{4\pi\tilde{e}^2 B}\right], \quad \frac{1}{4\pi\tilde{e}^2 B} = \frac{M_\phi^2 \lambda_\sigma}{e^2 B m_\sigma^2}, \tag{36}$$

because the ratio A/B is found numerically of order unity and therefore effectively negligible in practice. Equation (36) corresponds to 10^{10^6} for a symmetry-breaking scale of order 10 TeV, and $10^{10^{26}}$ for the GUT energy scale, with, as before, a W current trapped in the string; these numbers greatly exceed the horizon size. This is very important because it is the reason why such strings can be considered to be effectively stable with respect to Coulomb tunneling: the characteristic time for the charged particles to flow out of the string is found to be of order ρ_∞ (it is evaluated in Sec. III B 2) and is therefore very much greater than the Hubble time.

In view of the finite, albeit large, value of the divergence radius, all the quantities of physical interest need to be regularized. When numerically solving the system (15), we have to choose an appropriate cutoff length Λ depending on the various parameters which are used. In order to fix this length more precisely, we require as a consistency prescription that the zeroth-order current density (27) integrated up to this cutoff should yield an approximation of the fully integrated function having an error less than $4\pi\tilde{e}^2$. This means that we require

$$\left|\frac{B_\Lambda - B}{B}\right| \lesssim 4\pi\tilde{e}^2, \tag{37}$$

a subscript Λ having the meaning throughout the rest of this work that the corresponding integral is computed with its upper limit fixed at $\rho = \Lambda$. Using Eqs. (28) and (29), one can develop (37) explicitly and thereby conclude that the effective cutoff has to be chosen at least to satisfy

$$\kappa\Lambda \gtrsim \ln\rho_\infty . \tag{38}$$

The behavior of P_* allows us to define a structure around the string: let ε be a small numerical constant (which might be fixed by computer precision) and let ρ_ε be the value of ρ beyond which Y_0 is negligible so that

$$|P_*(\rho) - [1 + 4\pi\tilde{e}^2(A + B\ln\rho)]| < \varepsilon, \tag{39}$$

this behavior being true at any order of perturbation (if A and B are arbitrary constants) since it satisfies Eqs. (24) asymptotically with $Y \to 0$ for $\rho \to \infty$. Then, due to the very slow variation of the logarithm, for $\rho_\varepsilon < \rho \ll \rho_\infty$, assuming this region exists, which we do because of Eq. (36), one can write

$$P_*(\rho) \simeq 1 + 4\pi\tilde{e}^2(A + B\ln\rho_\varepsilon), \tag{40}$$

i.e., $P_*(\rho)$ has a value different from unity because of $e \neq 0$, but still approximately constant. The validity of this approximation is "demonstrated" in Fig. 1 where a typical (stable) electric configuration is shown, obtained by numerical resolution of the system (24), that effectively includes all orders in e^2 since we have chosen a large value for $4\pi\tilde{e}^2$. The flatness of the behavior of P_* that one sees in this figure is not due to any special values of the α parameters but is really a quite "standard" comportment except in regions for ν in which the integrals that appear in (32) become divergent, in which case the expansion itself does not have any meaning and the string is unstable.

Thus, as shown in Fig. 2, there exists three qualitatively distinct regions surrounding the string, namely, the core (region [I]), defined by

$$0 \le \rho < \rho_\varepsilon, \tag{41}$$

in which the Higgs field differs significantly from its vacuum expectation value, a second region which we call "physical infinity" in which the electromagnetic potential is nearly constant (region [II]), defined by

$$\rho_\varepsilon \le \rho \ll \rho_\infty, \tag{42}$$

and finally "mathematical infinity," having no physical relevance in most of the astrophysical applications such as formation of galaxies or dark matter (region [II]), defined by

$$\rho \gtrsim \rho_\infty . \tag{43}$$

The existence of these distinct regions gives insight on the cutoff needed to regularize the theory. As Λ is

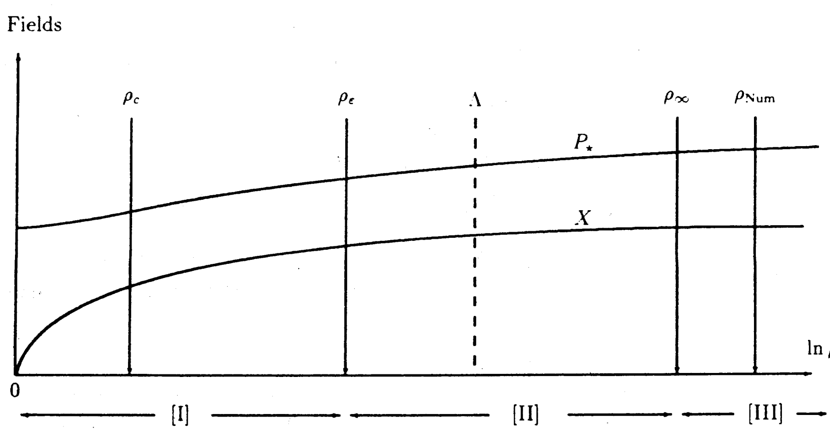

FIG. 2. The various characteristic distances and cutoff needed to define regions [I], [II], and [III] (see text).

chosen so that Y_0 is negligible in practical cases [for which $4\pi\tilde{e}^2$ satisfies Eq. (25)], it seems clear that Λ should be in region [II] (see Fig. 2), so that it is possible to compute (numerically) the constants that appear in the asymptotic behaviors of the fields and then to integrate until ρ goes to another (fixed) cutoff ρ_{num}, which is chosen in region [III]. It turns out that even if choosing Λ in region [II] is crucial in order to obtain physically correct results, these are quite insensitive to the actual value chosen for ρ_{num} as long as it is in region [III] in the sense that it can vary by many orders of magnitude without changing the numerical results. We have chosen this fixed cutoff at e^{100} over the mass of the Higgs field for numerical computations, and we checked that the results obtained were not dependent on this latter value. In examining the integrals with a cutoff in the range $e^{10}\to e^{1000}$, consistency was always within the limits of the computer precision.

To conclude this section, let us mention a phenomenon which is qualitatively (and in most cases also quantitatively) not affected by the inclusion of a charge coupling constant, namely, the existence of the phase-frequency threshold obtained in the framework of the neutral limit as a consequence of the behavior (29). Indeed, in the "physical-infinity" region [II], Eq. (24) for Y becomes

$$Y''+\frac{1}{\rho}Y'\sim\frac{\tilde{w}}{\alpha_1}[1+4\pi\tilde{e}^2(A+B\ln\rho_\varepsilon)]^2Y+\frac{\alpha_3}{\alpha_1}Y(Y^2+1)\ , \tag{44}$$

which implies that w_c is moved to the right (assuming $B\ln\rho_\varepsilon>A$, a relation which is verified numerically): the solution of the asymptotic equation (44) in the charge coupled case also has form (29) with w_c replaced by

$$\tilde{w}_c=\frac{-\alpha_3}{[1+4\pi\tilde{e}^2(A+B\ln\rho_\varepsilon)]^2} \tag{45}$$

because of the nonzero value of the electromagnetic coupling constant. If no cutoff was introduced, then, as P_* diverges, we would get $w_c=0$, but, in practice, we have seen that the cutoff is always physically justifiable (the Hubble radius of the universe being more than sufficient) and in any case necessary for the existence of strictly stationary current-carrying string states. We shall see later (Sec. III B 2), however, that such a shift in the phase-frequency threshold correspond to an (exponentially suppressed) emission of charged particles. It may be seen in Eq. (45) that, unfortunately, the new threshold is only defined implicitly since the coefficients A and B, as integrals over the zeroth-order fields, do depend on w.

As emphasized before, one expects "electric" strings to have a (qualitatively) finite lifetime (see Sec. III B 2). The typical lifetime τ as a function of the fundamental parameter $\nu=\text{sgn}(w)\sqrt{|w|}$ may be qualitatively evaluated as shown in Fig. 3: we see that, in the neutral-limit case, its inverse τ^{-1} is actually zero (so that the lifetime is infinite) for $\nu>-m_\sigma$ but that τ^{-1} rapidly increases as ν gets more strongly negative, so that the corresponding states are no longer effectively stationary. The presence of a

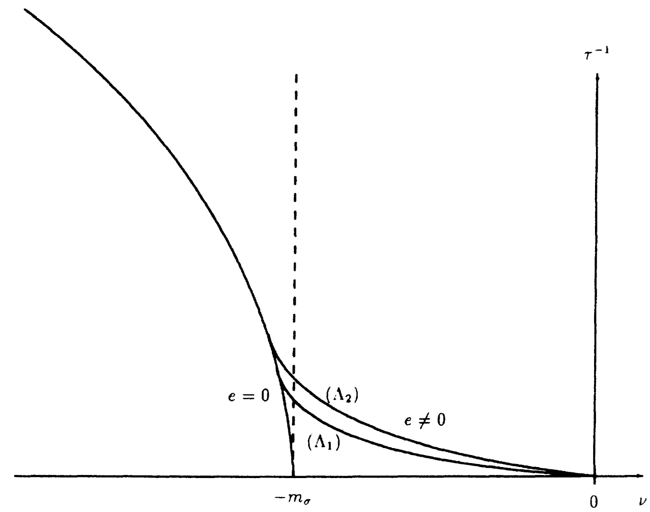

FIG. 3. The inverse (τ^{-1}) of the typical lifetime of a charged cosmic string as a function of ν. This is a qualitative picture so the axes are arbitrarily labeled. Typical behavior with the regularization cutoff Λ is shown with $\Lambda_1>\Lambda_2$.

charge coupling constant somewhat modifies this qualitative picture since τ^{-1} is nonzero as soon as $\nu<0$, but it is nevertheless found that quantitatively the dynamical behavior is not changed much, since the lifetimes that are obtained typically exceed the age of the universe unless ν is close to the threshold ν_c for particle emission in the neutral case.

III. INTEGRATED VARIABLES

A. Equation of state

We now turn back to the conserved quantities resulting from the invariances of the Lagrangian (1) such as the energy-momentum tensor

$$T^\mu_\nu=-2g^{\mu\alpha}\frac{\delta\mathcal{L}}{\delta g^{\alpha\nu}}+\delta^\mu{}_\nu\mathcal{L}\ , \tag{46}$$

and the electromagnetic four-current is given by Eq. (12). Introducing Latin indices $a,b=1,2$ to label the intrinsic coordinates, we then define the macroscopic tensors internal to the string by means of

$$\overline{T}^{ab}=2\pi\int r\,dr\,T^{ab}\ , \tag{47}$$

and

$$\overline{\mathcal{T}}^{a}=2\pi\int r\,dr\,\mathcal{T}^{a}\ , \tag{48}$$

with the numerical density of charge or the intensity of current (depending on the case)

$$\mathcal{C}=\sqrt{|\overline{\mathcal{T}}_t^2-\overline{\mathcal{T}}_z^2|}=2\pi\sqrt{|w|}\int r\,dr\,\sigma^2P_*\ . \tag{49}$$

The energy per unit length U and the tension T are defined in general situations by diagonalizing $\overline{\mathbf{T}}$ in the form $\overline{T}^{ab}=Uu^au^b-Tv^av^b$, where $\mathbf{u}$ and $\mathbf{v}$ are, respectively, a preferred timelike and spacelike unit vector tangent to the string [6]. The preferred frame for which we get

$$U=\overline{T}^{tt}$$

and (50)

$$T=-\overline{T}^{zz}$$

is obtained by choosing a reference frame in which P_μ has only one component. In the null case, with $w=0$ as in an ordinary nonconducting cosmic string, we simply obtain

$$\overline{T}^{tt}+\overline{T}^{zz}=0\ ,$$

i.e., (51)

$$U=T\ .$$

For a magnetic state, with $w>0$, we have

$$U=\pi\int r\,dr\left[\varphi'^2+\sigma'^2+\frac{Q'^2}{4\pi q^2r^2}+\frac{wP'^2}{4\pi e^2}+\frac{\varphi^2Q^2}{r^2}+wP_*^2\sigma^2+2V\right]=-L\ , \tag{52}$$

$$T=\pi\int r\,dr\left[\varphi'^2+\sigma'^2+\frac{Q'^2}{4\pi q^2r^2}-\frac{wP'^{2*}}{4\pi e^2}+\frac{\varphi^2Q^2}{r^2}-wP_*^2\sigma^2+2V\right]=-\tilde{L}\ , \tag{53}$$

and for an electric state, with $w<0$, we obtain

$$U=\pi\int r\,dr\left[\varphi'^2+\sigma'^2+\frac{Q'^2}{4\pi q^2r^2}+\frac{wP_*'^2}{4\pi e^2}+\frac{\varphi^2Q^2}{r^2}+wP_*^2\sigma^2+2V\right]=-\tilde{L}\ . \tag{54}$$

$$T=\pi\int r\,dr\left[\varphi'^2+\sigma'^2+\frac{Q'^2}{4\pi q^2r^2}+\frac{wP_*'^2}{4\pi e^2}+\frac{\varphi^2Q^2}{r^2}+wP_*^2\sigma^2+2V\right]=-L\ . \tag{55}$$

All these macroscopic quantities have been computed numerically (effectively including all orders in $4\pi\tilde{e}^2$) by means of the previously defined cutoffs Λ and $\rho_{\rm num}$, and are shown in Fig. 4 (with U and T normalized to their value in the Kibble limit) and Fig. 5 (with $\mathcal{C}$ and $\tilde{K}$ defined in the next section) as functions of $\tilde{v}$, both for the neutral limit (26) (solid curves) and the fully coupled system (24) (dashed). Differences, plotted in Figs. 6 and 7, are found to be of order $4\pi\tilde{e}^2$, so from now on we are going to concentrate on the first-order expansion and its relations with the neutral limit.

Definitions (52)–(55), together with (49), allow us to compute

$$U-T=2\pi w\int r\,dr\left[\frac{P_*'^2}{4\pi e^2}+P_*^2\sigma^2\right]\ , \tag{56}$$

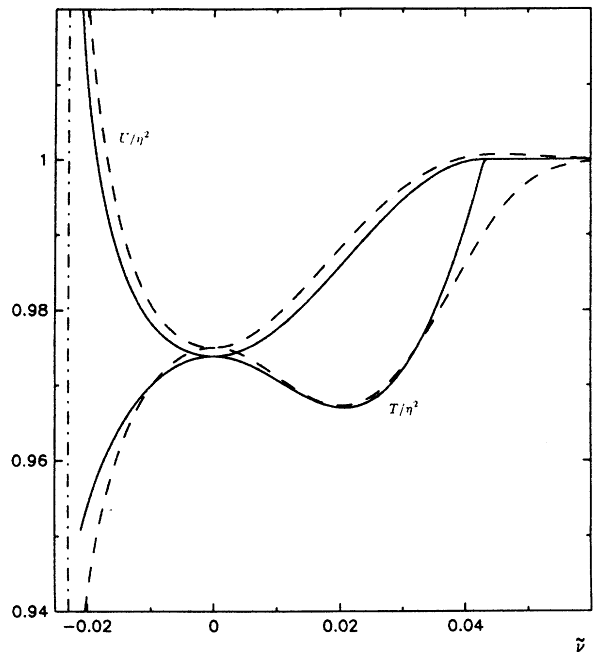

FIG. 4. U and T as functions of $\tilde{v}$. Solid curves, neutral limit; dashed curves, $4\pi\tilde{e}^2=0.1$.

and, due to the fact that the system (15) arises from the variations of U (T) with respect to ν in the magnetic (electric) regime, one has

$$\mu\equiv\frac{dL}{d\nu}=2\pi\nu\int r\,dr\left[\frac{P_*'^2}{4\pi e^2}+P_*^2\sigma^2\right]+\frac{\nu^2}{2}\lim_{r\to\infty}\left[r\frac{dP_*}{dr}\frac{dP_*}{d\nu}\right]\ , \tag{57}$$

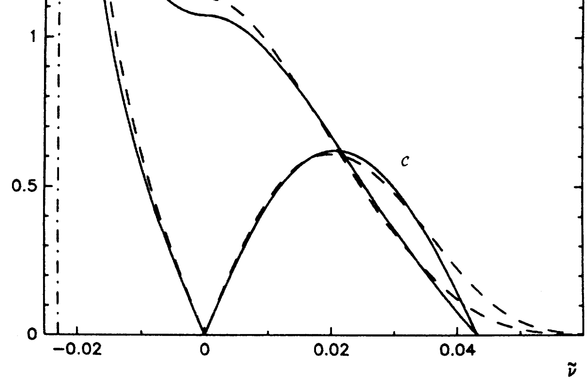

FIG. 5. $\mathcal{C}$ and $\tilde{K}/6$ vs $\tilde{v}$. The first two derivatives of the latter function are found to be zero at the origin.

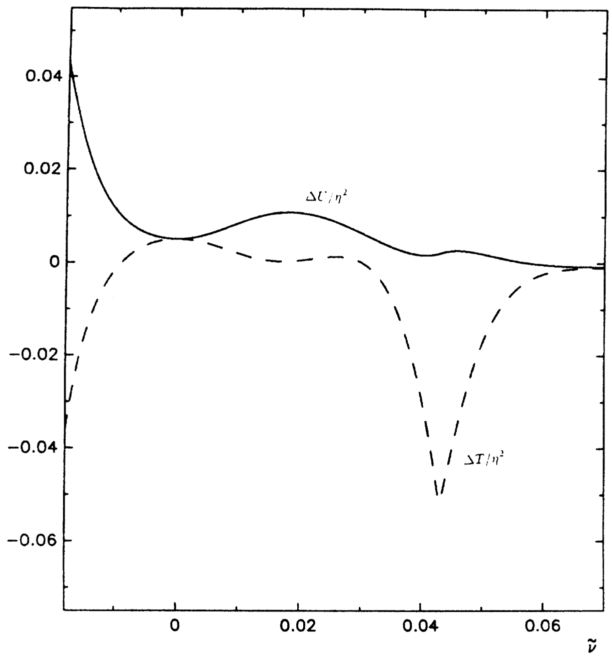

FIG. 6. $U(e=0)-U(e\neq 0)$ (solid) and $T(e=0)-T(e\neq 0)$ (dashed) vs $\tilde{\nu}$. It may be remarked that in magnetic situations maximum differences are found for ν such that the energy exceeds that of the Kibble case.

taking into account the possible variations of P_* with ν as r approaches infinity (due to the nonlocality of the vortex, the boundary conditions cannot be held fixed as ν varies).

Expanding this relation in powers of e^2, and using Eq. (32), one finds, after some algebra,

$$\mu=\frac{U-T}{\nu}-2\frac{e^2}{\nu}\mathcal{C}_0\int\frac{dr}{r}\mathcal{F}_0(r)+2e^2\mathcal{C}_0\frac{d}{d\nu}\int\frac{dr}{r}\mathcal{F}_0(r)+O(e^4) \tag{58}$$

(the subscript 0 on a quantity still has the meaning that its value is computed in the neutral limit), where we define the incomplete charge function $\mathcal{F}$ by

$$\mathcal{F}=2\pi\nu\int_0^r R\,dR\,\sigma^2(R)P_*(R)\ . \tag{59}$$

We are then able to compute the difference between the true value of the charge number density $\mathcal{C}$ and the estimate μ obtained from the macroscopic theory [4,6] as

$$\mu-\mathcal{C}=e^2\Upsilon+O(e^4)\ , \tag{60}$$

where

$$\Upsilon=2\mathcal{C}_0\frac{d}{d\nu}\int\frac{dr}{r}\mathcal{F}_0(r)\ , \tag{61}$$

which shows explicitly the correction to the zeroth order due to the nonlocality of the vortex under consideration here. The variables $\mathcal{C}$, μ, and Υ are represented in Figs. 8 and 9, computed for the same parameter values as before. The function $\Upsilon(\nu)$ is seen to be of order unity or so, giving a justification a posteriori for the e^2 expansion, at least as far as the "magnetic" regime is concerned. However, due to the phase-frequency-threshold shift as implied by Eq. (45), the coupling constant can give rise to important effects in the "electric" regime: the plots of μ and $\mathcal{C}$ in Fig. 8 as functions of ν reveal large deviations for timelike currents since $\Upsilon(\nu)$ is a divergent quantity for $\nu\rightarrow\nu_c$. This contrasts with the spacelike case for which the difference is really of order $\tilde{e}^2$. Furthermore,

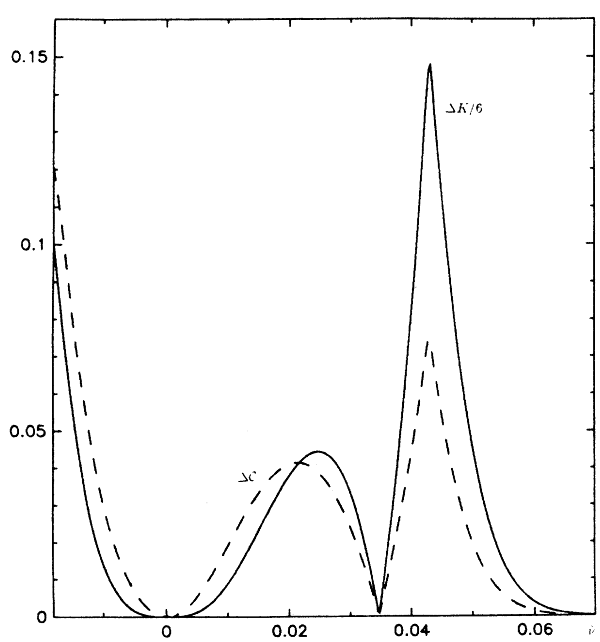

FIG. 7. $\mathcal{C}(e=0)-\mathcal{C}(e\neq 0)$ (solid) and $\tilde{K}(e=0)-\tilde{K}(e\neq 0)$ (dashed) as functions of $\tilde{\nu}$.

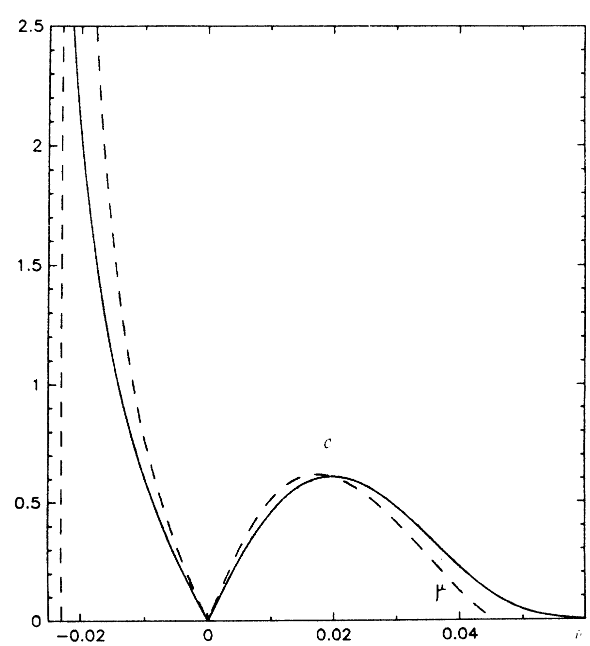

FIG. 8. $\mathcal{C}$ (solid line) and μ (dashed) vs ν.

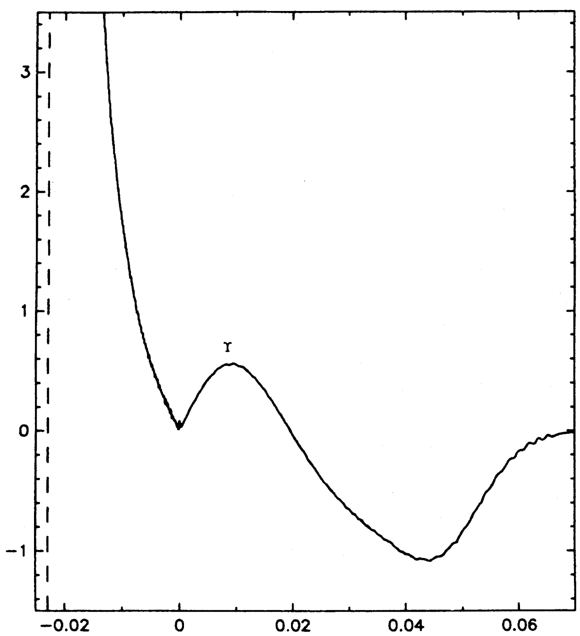

FIG. 9. First-order difference $\Upsilon(\nu)$ between $\mathcal{C}$ and μ.

it has to be remarked that the maximum of μ does not correspond exactly to that of $\mathcal{C}$ since

$$\frac{d\mu}{d\nu}=\frac{d\mathcal{C}}{d\nu}+2e^2\left[\frac{d\mathcal{C}_0}{d\nu}\frac{d}{d\nu}\int\frac{dr}{r}\mathcal{F}_0(r)+\mathcal{C}_0\frac{d^2}{d\nu^2}\int\frac{dr}{r}\mathcal{F}_0(r)\right], \tag{62}$$

which implies that the value of k for which μ is maximum is shifted to the right or to the left (depending on the sign of the quantity inside the square brackets) of the value of k which corresponds to the maximum of $\mathcal{C}$. It may also be seen that Υ passes through zero precisely (up to numerical errors) where $\mathcal{C}$ is maximum; that is, $\mu=\mathcal{C}$ for k such that $d\mathcal{C}/d\nu=0$. We conjecture this phenomenon to be of exact nature (i.e., valid at all orders in e^2) since it has been generically found for various values of the α parameters, even if its interpretation seems up to now rather unclear.

There is another interesting feature to be noticed in Fig. 8, namely, the fact that μ is found to become negative [25] where the tension (Fig. 4) is still increasing. This means that there exists another region in which the string is stable against longitudinal perturbations (see next section), the existence of this region being entirely due to e being nonzero. Indeed, for $e=0$, one has $\mathcal{C}=\mu$ with $\mathcal{C}$ defined positive and therefore the energy (in the magnetic regime, whereas for the electric regime it is the tension) must always be an increasing function of ν. This the reason why the energy so abruptly reaches the value it has in the decoupled version of Kibble. Here, however, the energy can exceed this latter value by an amount of order e^2. It is not clear whether this kind of string state will be stable anyway because the system is on a local maximum and it might well be possible to find a mechanism by which the charged bosons flow out of the string so as to minimize the energy.

A last modification that is imposed by the nonzero value of the electromagnetic constant in the "magnetic" regime concerns the minimum of the tension. It turns out that in the neutral limit, as in the macroscopic formalism [7], this minimum coincides with the maximum of the current. This relation is modified at the first order in e^2 by

$$\frac{dT}{d\nu}=-\nu\frac{d\mathcal{C}}{d\nu}-2e^2\frac{d\mathcal{C}_0}{d\nu}\int\frac{dr}{r}\mathcal{F}_0+O(e^4), \tag{63}$$

so that, assuming the integral to be positive, one finds that the minimum of the tension is shifted to the right or to the left of that of $\mathcal{C}$ depending on whether $\mathcal{C}_0$ is found to be decreasing or increasing for the corresponding value of ν. All the previous considerations have been used as tests of the numerical code developed to solve (24) and it has been found that discrepancies between the predicted values (at the first order) and the "observed" ones were always roughly of order e^4 as required.

The effective equation of state is shown in Fig. 10 where the tension is plotted as a function of the energy per unit length. It may be remarked in this figure that the charge coupling does not affect qualitatively the equation of state so that a macroscopic local treatment is in any case appropriate (as far as global effects are negligible). Also in Fig. 10 is plotted the special (integrable) equation of state

$$UT=\text{const} \tag{64}$$

(represented as the dot-dashed curve), which results in particular from models using dimensional reduction [7].

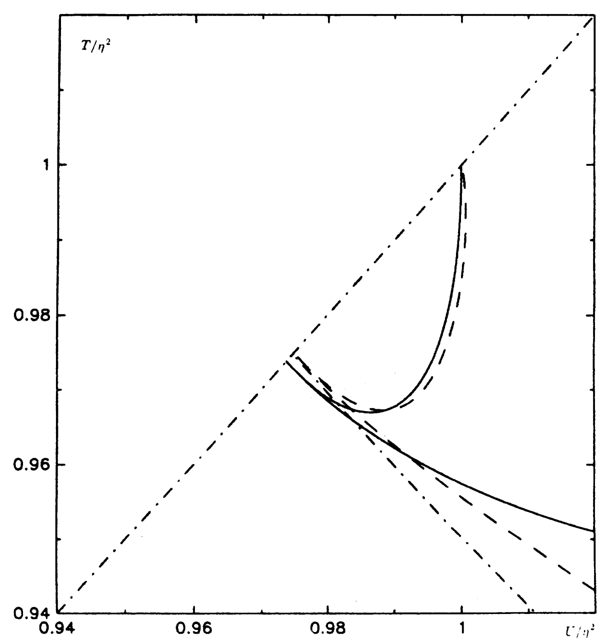

FIG. 10. The effective equation of state: the tension T is plotted vs the energy per unit length U, both of them being normalized to the Kibble case.

It is found that, contrary to previous claims [26], the dynamical properties of the Kaluza-Klein string models are not consistent with those of the Witten-like vortices whose equations of state are systematically above the integrable one (64).

Another widely used form of the equation of state is shown in Fig. 11 where U and T are plotted as functions of the total current. Discrepancies with the standard approximation $U=U_0+\text{const}\mathcal{C}^2$ and $T=U_0-\text{const}\mathcal{C}^2$ are illustrated in this figure, this approximation being plotted as a dot-dashed curve. This indicates that an expansion in powers of $\mathcal{C}$ is likely not to converge quickly so that a complete numerical solution seems to be necessary in order to get a correct description of a current-carrying vortex.

To conclude this section, we wish to emphasize that the effects of an electromagnetic coupling in electrically charged timelike current-carrying cosmic strings are, for most of the "physically reasonable" configurations, far more important than those found in spacelike current-carrying cosmic strings. The discrepancy between the neutral limit and the fully coupled version of the model would be mathematically divergent as soon as $\nu<0$ and physically divergent for a finite value of the cutoff Λ when $\nu\to\nu_c$. The actual definition of the phase-frequency threshold is moved as a function of the coupling strength and the local-vortex approximation ceases to be valid beyond this threshold so that the parameter space available is considerably reduced. However, the qualitative behavior for small negative ν is much the same as for the neutral limit, mainly due to the fact that Taylor expansion around $\nu=0$ is still physically defined as long as a finite regularization parameter Λ is used. It turns out that although differences due to charge coupling never exceed a few percent on the magnetic side, on the other hand, the electric side shows increasing differences which are not limited.

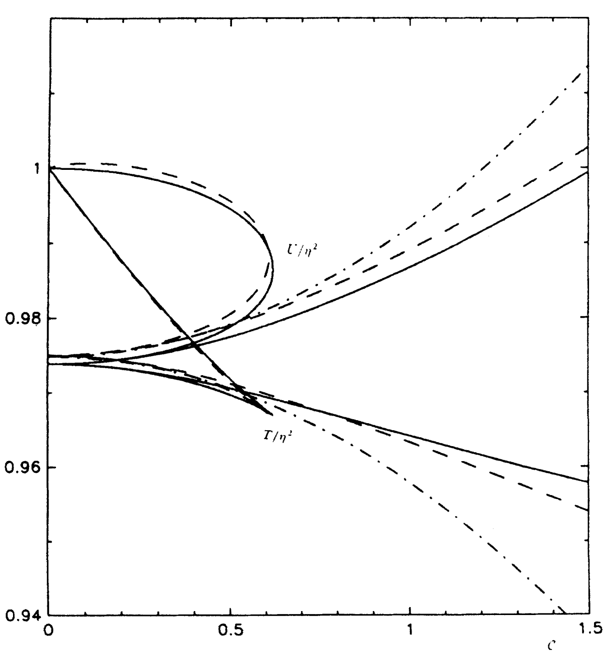

FIG. 11. U and T as functions of the current (for magnetic configurations) or the charge (electric). Dot-dashed curves represent the parabolic approximation.

B. Stability

In order to evaluate the cosmological importance of superconducting cosmic strings of the kind considered here, it has to be shown that the solutions described above may yield stable macroscopic configurations. In the following paragraphs, we list what we consider to be the two most important problems in this respect, the first one involving electric as well as magnetic strings, whereas the second problem is specific to charged strings.

1. Mechanical stability and radiative instability

Using the equation of state that we obtained numerically above, it is possible to compute the characteristic-perturbation velocities given by [4]

$$c_T^2=\frac{U}{T}\quad(\text{transverse})$$

and (65)

$$c_L^2=-\frac{dT}{dU}\quad(\text{longitudinal})\ ,$$

which are shown in Figs. 12 and 13, respectively, as functions of $\tilde{\nu}$. As discussed above, we obtain the largest discrepancies with respect to the neutral limit in the electric regime but the behaviors are seen to be qualitatively similar. In particular, the ratio c_L/c_T (shown in Fig. 14) is found to be less than unity, this effect being unchanged by the introduction of the charge coupling constant. This provides another difference with the Nielsen-Olesen [26] vortex for which this ratio is strictly found to be uni-

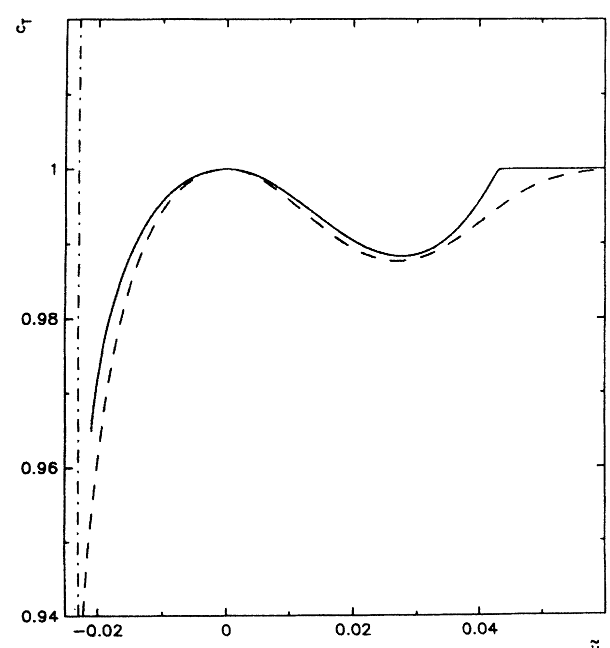

FIG. 12. Transverse-perturbation propagation speed $c_T(\nu)$.

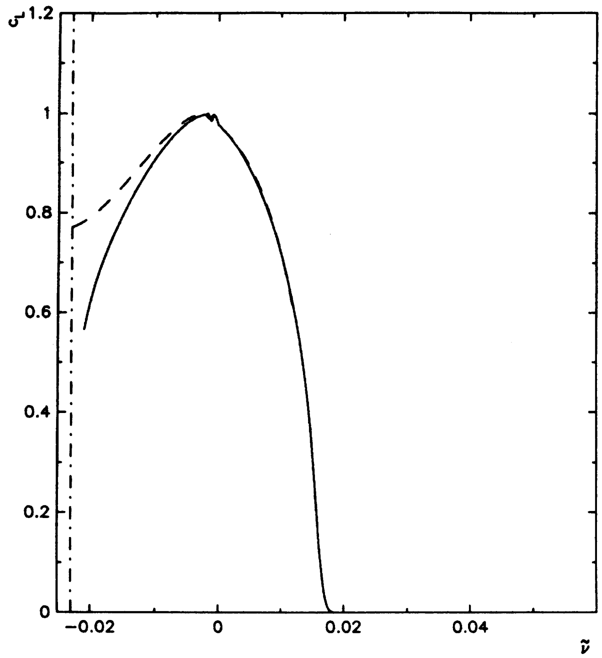

FIG. 13. Longitudinal-perturbation propagation speed $c_L(\nu)$.

ty [7]. One is therefore led to conclude that Witten superconducting cosmic strings generically yield rigidly rotating configurations [23,27] that are, in principle, unstable due to the fact that a nonaxisymmetric perturbation mode of such a configuration is amplified by radiation if it has a prograde but relatively retrograde rotation [28], which is possible for longitudinal modes here since mechanical equilibrium of a loop requires [7] the rotation

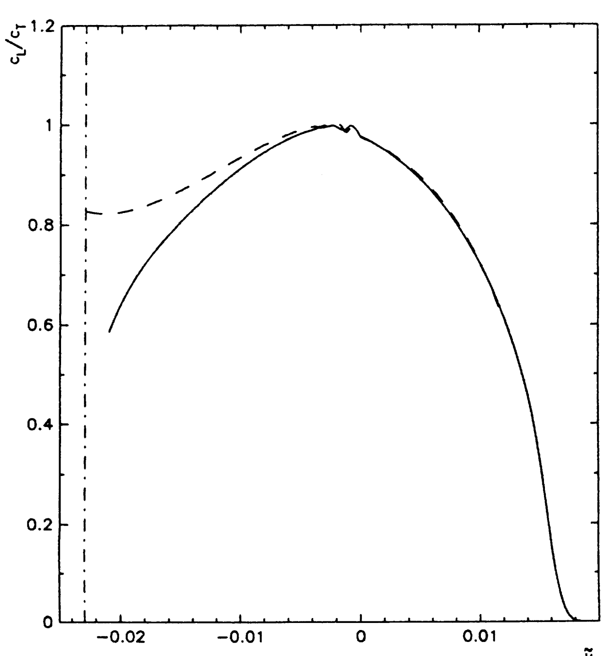

FIG. 14. The ratio c_L/c_T which is always seen to be less than unity.

velocity to coincide with c_T. Further work will be required to determine whether this kind of instability is cosmologically important (e.g., in restoring the possibility of a current-carrying-string formation at the GUT energy scale that is incompatible with effectively stable ring formation [23,18,27]) or whether it is, in practice, irrelevant due either to the extreme length of the decay time or to the dominance of some stabilizing effect such as electrostatic repulsion.

Another more obvious essential kind of mechanical requirement for stability concerns the total mass of a string loop as a function of its equilibrium radius, which has to be a minimum. This requires that the quantity $\mathcal{S}$ defined by

$$\mathcal{S}=\left[\frac{c_T}{c_L}\right]^2-\frac{3c_T^2-1}{3-c_T^2} \tag{66}$$

has to be positive [2]. That this condition is generally fulfilled is illustrated in Fig. 15 for both the neutral limit and the charge coupled cases. The problem of radiative instability in the preceeding paragraph is then seen to be a consequence of the result of Fig. 12 which shows that transverse perturbations propagate nearly at the speed of light: setting $c_T^2=1-\delta$ with $\delta \ll 1$, one finds that the positivity of $\mathcal{S}$ requires

$$\left[\frac{c_T}{c_L}\right] > 1-2\delta \ , \tag{67}$$

so that the corresponding states could only marginally satisfy the radiative stability requirement

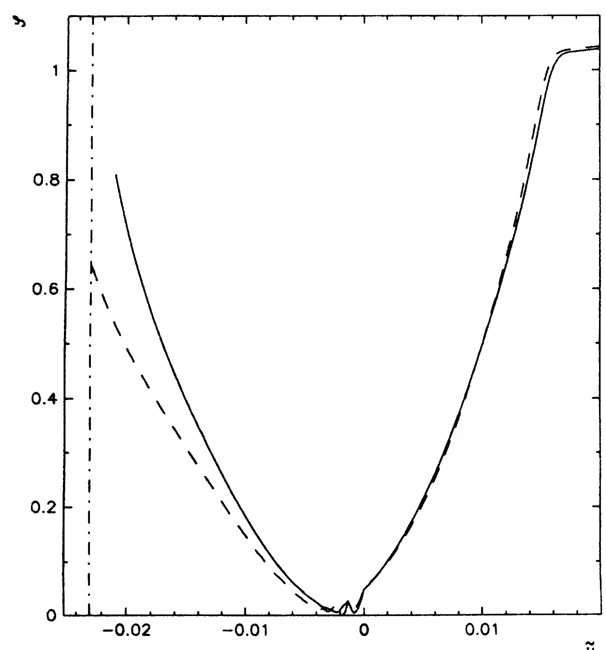

FIG. 15. The global mechanical-stability function $\mathcal{S}$: the requirement that $\mathcal{S}$ be positive is seen to be fulfilled throughout the domain of the local stability of the string.

2. Charge-loss time scale

Let us now consider current-carrier-particle emission by the string itself. The basic argument that charged strings emit particles for any $w<0$ is, of course, related to the Coulomb repulsive force between the trapped charged bosons, but the emission itself can also be considered even in the limiting case of zero coupling if the phase frequency is beyond the threshold, as we shall see at the end of this section, but for now, let us first investigate the modification of solution (29) for $w>-m_\sigma^2$ in the form

$$Y \sim CK_0(\kappa\rho)\zeta(\rho) \tag{68}$$

with the same κ as before (i.e., with $\tilde{w}_c=-\alpha_3$). Inserting this form into Eq. (15), we obtain the dominant behavior for ζ as given by

$$\zeta'' \sim -\tilde{\kappa}^2\zeta \ , \tag{69}$$

where the nearly constant $\tilde{\kappa}$ is defined as

$$\tilde{\kappa}^2=\frac{-8\pi\tilde{e}^2\tilde{w}}{\alpha_1}(B\ln\rho+A)>0 \ , \tag{70}$$

so that the full solution for Σ may be written as

$$\Sigma=\Sigma_{\rm in}+\Sigma_{\rm out} \ , \tag{71}$$

where

$$\Sigma_{\rm in}=\frac{m_\sigma C}{\sqrt{\lambda_\sigma}}K_0(\kappa\rho)e^{i(\omega t+\tilde{\kappa}\rho+\delta)} \ ,$$
$$\Sigma_{\rm out}=\frac{m_\sigma C}{\sqrt{\lambda_\sigma}}K_0(\kappa\rho)e^{i(-\omega t+\tilde{\kappa}\rho+\delta)} \ , \tag{72}$$

with δ an arbitrary constant phase.

The two terms in this equation can be interpreted as ingoing and outgoing waves, respectively, so that in order for such a configuration to be stationary there should exist a Σ source at infinity to take into account the ingoing wave. Since no such extra source would be expected to occur in natural circumstances, physically realistic solutions would not be strictly stationary but would be characterizable by a finite lifetime $\tau \equiv -\mathcal{C}/(d\mathcal{C}/dt)$, where $d\mathcal{C}/dt$ is the charge-loss rate computed from the outgoing contribution $\Sigma_{\rm out}$. (It should be remarked that, for spacelike currents for which w is positive, $\tilde{\kappa}^2$ would be negative so no emission process could take place.)

In order to compute this characteristic decay time, we first look at the solution (71) at a distance sufficiently large so that the charge carrier Σ behaves like an ordinary free Klein-Gordon field and the four-current (12) may be written as

$$\mathcal{T}_\mu=i[\Sigma^*(\nabla_\mu\Sigma)-(\nabla_\mu\Sigma^*)\Sigma] \ , \tag{73}$$

and it is conserved, i.e., $\nabla^\mu\mathcal{T}_\mu=0$. We transform this local conservation law into a global one by integration: the flux of charge per unit length crossing a cylindrical surface surrounding the string is equal to minus the derivative of the total charge per unit length in the cylinder with respect to time. More specifically, we compute the radial component of $\mathcal{T}$ by $\mathcal{T}_r=i[\Sigma(\Sigma_0^*)'-(\Sigma)'\Sigma^*]$ and we use

$$\mathcal{J}=\frac{d\mathcal{C}}{dt}=-\oint \mathcal{T}\cdot d\mathbf{s} \ , \tag{74}$$

where we can replace Σ by the asymptotic solution (72) if the surface s is far enough from the core of the string. Supposing that only the outgoing part of the flux is physically meaningful, we see that the effective lifetime is finite, but the flux of charge is exponentially damped with the distance to the core of the string. To obtain a characteristic time corresponding to the emission of charged particles, we need to use the effective cutoff length Λ as given by (38) since it is at this distance that we have to compute the flux of charge $\mathcal{J}$.

Considering the charge which crosses a cylinder of unit length located at a distance Λ, we obtain this flux, according to Eqs. (29) and (69), by

$$\mathcal{J}=4i\pi\frac{m_\sigma^2C^2}{\lambda_\sigma}e^{-2\kappa\Lambda}\left[1+i\frac{\tilde{\kappa}}{\kappa}\right] \ , \tag{75}$$

and since the charge per unit length is

$$\mathcal{C}=2\pi\sqrt{-w}\,\frac{m_\sigma^2}{\lambda_\sigma M_\varphi^2}B_\Lambda \ , \tag{76}$$

we obtain the characteristic lifetime of the timelike current-carrying configuration τ as

$$\tau=\frac{B_\Lambda}{4M_\varphi C^2}e^{2\kappa\Lambda}\left[\left[1-\frac{\omega^2}{m_\sigma^2}\right]\frac{\lambda_\sigma}{2\pi e^2(B\ln\rho_\varepsilon+A)}\right]^{1/2} \ , \tag{77}$$

which turns out to be of the same order of magnitude as ρ_∞ according to the specification (38) of the cutoff Λ. This expression may be used to plot τ^{-1} for $v<0$ (Fig. 3).

As a final remark before concluding, let us notice that in the neutral-limit situation the previously obtained lifetime is infinite for $w>-m_\sigma^2$. However, for $w<-m_\sigma^2$, although $\tilde{\kappa}=0$, Eq. (29) indicates that the field Σ may also be written as a sum (71), with

$$\Sigma_{\rm in\ out}=\frac{m_\sigma C}{\sqrt{\lambda_\sigma\kappa\rho}}[e^{i(\mp\omega t+\kappa\rho+\delta)}] \ , \tag{78}$$

the corresponding lifetime expressible as $\tau_0 \equiv -\mathcal{C}_0/(d\mathcal{C}_0/dt)$ with $\mathcal{C}_0$ computed by Eq. (27).

To compute this neutral-limit time, we again look at solution (29) and $\tilde{w}<-\alpha_3$ far away from the core of the string, and perform the same operations as in the charge coupled case. This yields the outgoing current

$$\mathcal{T}_{0r}=2\frac{\sqrt{\lambda_\varphi}}{\lambda_\sigma}\eta m_\sigma^2C^2\frac{1}{\rho} \ , \tag{79}$$

so that the flux of charge can be estimated as

$$\frac{d\mathcal{C}_0}{dt}=-4\pi m_\sigma^2\frac{C^2}{\lambda_\sigma} \ , \tag{80}$$

and finally the characteristic decay time becomes

$$\tau_0 = \frac{\sqrt{-w}}{2M_\varphi^2 C^2} B \ . \tag{81}$$

This lifetime essentially depends on the mass of the Higgs field M_φ so that, assuming the integral B and the constant C to be of order unity, which we verified numerically, and since for practical situations the parameter $\sqrt{-w}$ is of order the mass of the charge carrier m_σ, then it is possible to give a rough estimate of the lifetime of such configurations as

$$\tau_0 \approx m_\sigma / M_\varphi^2 \ . \tag{82}$$

Consequently, if the Higgs particle becomes massive at the GUT symmetry breakdown, i.e., $m_\varphi \sim 10^{15}$ GeV, and if the charge carrier is a "standard" particle ($m_\sigma \sim 100$ GeV), then the lifetime which is obtained ($\sim 10^{-52}$ s) turns out to be much less than even the Planck time ($t_P \sim 10^{-43}$ s) and so it may be conjectured that quantum gravitational effects may act upon these configurations in an unpredictable way. This time is greatly enhanced in case the symmetry breakdown occurs at a much smaller energy scale, but then, experimental constraints (e.g., neutrino counting [29] at the CERN e^+e^- collider LEP) yield a limit on this scale, i.e., $M_\varphi \gtrsim 300$ GeV and therefore a lifetime already less than 10^{-30} s so these string states remain irrelevant for cosmological or astrophysical purposes.

It may be observed that the finite value that is obtained from Eq. (77) for $\omega = -m_\sigma$ is a direct consequence of the behavior of the zeroth-order charge density (30), so that this behavior turns out to be essential for matching τ and τ_0 in a monotonic way. This gives another justification for having the constant C in (29) independent of κ.

CONCLUSIONS AND DISCUSSION

The most radically new result that is obtained in this paper can be summarized in a few words: the transverse-perturbation speed generically exceeds the longitudinal-perturbation speed so that superconducting cosmic-string loops are, in principle, radiatively unstable. The result holds for the parameter values chosen here, but it seems to be a quite generic result. Investigation in the parameter space is in progress and is postponed for future work [22].

The conclusions concerning the influence of the electromagnetic coupling constant may be summarized as follows.

As far as the "magnetic" regime is concerned, the differences between the neutral limit and the fully coupled version of the model is found to be of the order $\sim 10^{-2}$ only for a quite large value of the coupling constant $\tilde{e}$ and for reasonable (with a meaning which would still need to be defined) values of the α parameters. This is very interesting since it implies that the macroscopic formalism [4,6] (which has been found very suitable for the neutral limit beforehand) should yield the correct description of "real" superconducting cosmic strings if they exist.

As far as the "electric" regime is concerned, it is possible to neglect charge loss because the lifetime due to the Coulomb tunneling effect, being proportional to the exponential of the inverse of the coupling constant, is found to be very large (in practical situations in which the Higgs-boson mass is much larger than the current-carrier mass). Nevertheless, the effect of charge coupling can be rather important in the "electric" regime: discrepancies with the neutral limit can reach 10% or so already for small ν. For large ν, the conclusion is even more drastic: the phase-frequency threshold is moved in such a way that, depending on an effective cutoff (which can eventually be the horizon of the universe if no matter is present), configurations that were stable in the neutral-limit case find themselves to have finite lifetime with a numerical density of charge, an energy per unit length, and a tension which are all divergent (therefore undefined). The inverse-square asymptotic behavior (for $\nu \to \nu_c$) for these quantities is no longer valid, but conclusions concerning perturbation speed as well as all other macroscopic quantities are (qualitatively) not affected at all by the effect of charge coupling.

It once again has to be emphasized that the value used for $\tilde{e}$ has been taken to be much higher than would be realistic so that discrepancies between the neutral limit and the fully coupled version, as large as they may be, are actually not quantitatively significant: the neutral-limit model compared with a realistic coupling yield results in (nearly) perfect agreement, and the physics of both systems is very well described by the Carter formalism.

Let us finish by discussing the extent to which the "toy-model" results presented here concerning charged strings may be valid when one considers a more realistic theory describing, among the various fields that might be

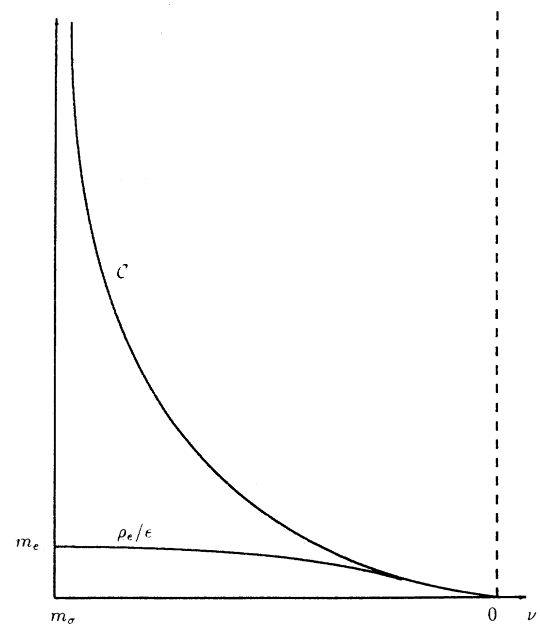

FIG. 16. Qualitative estimated behavior of the mechanical charge density $\mathcal{C}$ and the effective charge number density ρ_e/e with ν (in arbitrary units). This representation takes pair creation into account to yield a charge saturation for ν approaching the mass of the electron.

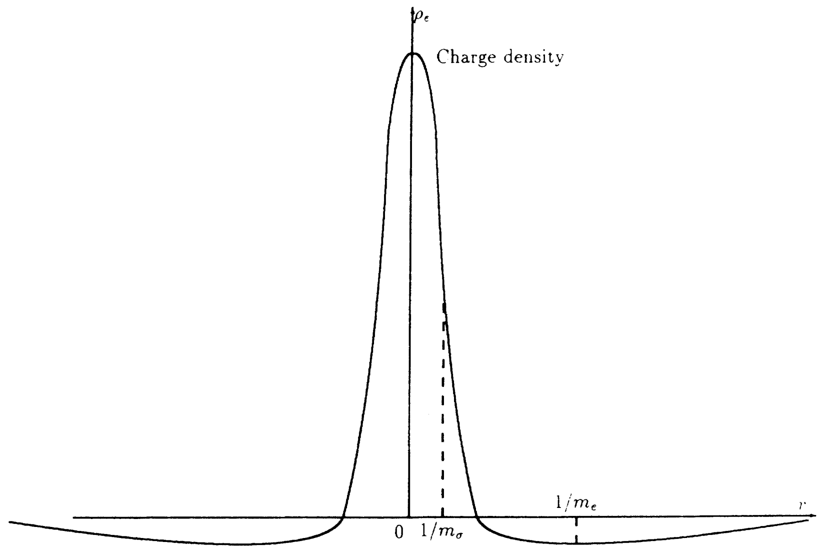

FIG. 17. Qualitative form of the charge density profile around a partly screened charged cosmic string (scales are arbitrary).

conceivable, the usual particles that are known to exist. Indeed, although there is no possibility to neutralize identically the internal conserved current $\mathcal{C}$ because of the phenomenon of localized absence [30] (due to the conical nature of space-time a cylindrical mode of any particle other than Φ and Σ goes to zero at the vortex), most of the microscopic electric configurations we have been studying here need corrections in order to be exactly realistic as far as the corresponding electric field is concerned since the existence of light charged particles (specifically the electrons) tends to screen the total effective charge: for an electric string state, the electric field surrounding the vortex is generally sufficient for pair creation so that the structure around a charged cosmic string is that of a cylindrical capacitor consisting of a hollow cylinder made of light charged particles, and a central straight core. This yields a maximum value for the effective total charge per unit length depending only on the mass of the electron and (in principle) on the age of the universe.

Indeed, if $\mathbf{E}$ is the electric field surrounding the string, then it is well known [31] that the probability of creating an electron-positron pair goes like

$$\mathcal{P}\sim(e\mathbf{E})^2 e^{-2\pi m_e^2/e|\mathbf{E}|} \tag{83}$$

per unit volume and unit time, so that the typical time τ_s characterizing the screening of one unit of charge, assuming the characteristic distance between the string and the bound electron to be of the order of the Compton wavelength of the electron $\sim 1/m_e$, is given by [32]

$$\tau_s \sim \frac{m_e^4}{e^4\mathcal{C}^3} e^{2\pi m_e/e\mathcal{C}} . \tag{84}$$

Thus, stationarity is recovered over a cosmological time scale if $\tau\sim 1/H$, where $1/H$ is the age of the Universe. This gives an order of magnitude of the maximum amount of the effective charge (and the most probable since the initial distribution gives values of order m_σ) as

$$\rho_e \sim e m_e , \tag{85}$$

as illustrated in Fig. 16. This shows that the actual configuration would be that of a cylindrical shell of electrons (or positrons, depending on the initial sign of the charge density inside the core of the string) surrounding an oppositely charged core. As presented in Fig. 17, it may be argued [33–35] that the typical distance at which electrons or positrons may remain in bounded annular states is their Compton wavelength, namely, $1/m_e$. This well-defined picture of the electromagnetic structure around a cosmic string has the advantage of being quite universal since it depends only on the mass of the electron.

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