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Low-mass current-carrying cosmic strings

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Cosmic-string properties are studied in the framework of the minimal extension of the standard electroweak theory with an extra U(1) symmetry. Provided certain critical minimum values are exceeded, currents can be trapped in a W-boson condensate within the vortex core, giving a behavior qualitatively similar to that in the simplified U(1)×U(1) Witten model.

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INTRODUCTION

There has been considerable interest in cosmic strings [1] which could have formed as topological vortex defects in the early Universe, since strings are almost the only possible defects that do not lead to a cosmological catastrophe [2] as happens for domain walls and monopoles. Various scenarios have been proposed to explain the observed large-scale structure of the Universe [3] and the formation of galaxies: models using both the gravitational wakes produced by strings together with dark matter succeeded in explaining the sheetlike structure [4], whereas superconducting string loops were the basis for an explosive model [5] that reproduced the large voids between sheets of galaxies.

The strings that were involved in these models had to be formed at a grand unified theory (GUT) energy scale in order to give correct orders of magnitude for the characteristic lengths of the large-scale structure. However, at such scales, the scenarios involving superconducting strings were more recently shown to be untenable [6–8] due to the survival of centrifugally supported relic loops that would be responsible for a premature collapse of the Universe. However, it has been argued [9] that low-mass (as opposed to GUT energy scale) current-carrying cosmic-string loops could be a dark-matter candidate, the cosmic closure condition $\Omega = 1$ yielding a mass per unit string length of order $(10\ \text{TeV})^2$, while GUT string formation would have yielded a remnant energy density corresponding to $\Omega \sim 10^{20}$ [7, 9] (Ω growing slightly more slowly than the square of the symmetry-breaking energy scale).

It is therefore of interest to consider the possibility of a field theory in which cosmic strings are generated at an energy scale which could be comparable to that of the standard electroweak symmetry breaking. Since the theory of Glashow, Salam, and Weinberg [10] is such that the vacuum manifold is simply connected, being isomorphic to the three-sphere S^3, topologically stable vortex defects are not present after the phase transition (even though stringlike solutions exist in this framework, as introduced by Nambu [11]). In order to satisfy the closure condition by means of current-supported string loops [7, 8], keeping in mind that electroweak-scale rings [12] would not be sufficient for this purpose, it is necessary to extend the electroweak model, the simplest possibility being the symmetry breaking of an extra U(1) invariance.

Such extensions have been considered in the literature for various motivations connected with supersymmetry or with GUT's. The supersymmetric extension of the standard model requires at least two Higgs doublets [13]; it is possible [14], in such a theory, to gauge an extra U(1) symmetry and to break it at an *a priori* arbitrary energy scale, thereby obtaining the formation of cosmic strings whose energy per unit length is not constrained. The conclusion is similar in the case of GUT's: many proposed Lie groups such as SO(10), E_6, or E_8 have been shown to yield cosmic string formation [15] in which fermionic as well as bosonic charged and neutral currents should be conserved [16]. [This is the case, in particular, within the schemes [17] SO(10)→SU(5)×U(1)→SU(3)×SU(2)×U(1)×U(1) or from the superstring-inspired model [18] E_6 →SU(3)×SU(2)×U(1)×U(1)×U(1) in which at least one new massive gauge boson has to be introduced.] Special emphasis on the phenomenological aspects [14, 19] revealed that experimental data [20] do not conflict [21] with these models provided the mass $M_{Z'}$ of the new gauge boson exceeds approximately 300–500 GeV, depending on the couplings [22]. These considerations, together with the cosmological requirement stated above, imply that the corresponding cosmic-string vortices, the subject of the present work, should not be more massive than roughly $1\ \text{g}\,\text{cm}^{-1}$. The purpose of this article is consequently to investigate the microscopic structure of the strings arising in these models, so as to provide a basis for further work concerned with macroscopic and cosmological effects. We show that the strings under consideration admit the formation of a W condensate (somehow similar to that examined in Ref. [23]) that will support a superconducting current of bosonic type, and we derive an effective action for describing their dynamics.

I. EXTENDED ELECTROWEAK MODEL

At a symmetry-breaking phase transition $G \to H$, a Higgs field Φ acquires a nonzero vacuum expectation value (VEV). In general, Φ belongs to an N-dimensional representation of the gauge group G, so that fixing its magnitude at the phase transition leaves $N-1$ variables arbitrary: the vacuum is degenerate, being invariant under the action of G/H (the quotient symmetry-group) transformations. Cosmic strings appear as a consequence of symmetry breaking whenever the vacuum degeneracy group is not simply connected, i.e., when its first homotopy group π_1 is nontrivial: for instance, in the simplest possibility $\mathrm{U}(1) \to \{0\}$ [with $\pi_1(G/H) \sim \pi_1$ [U(1)]$\sim \mathbf{Z}$], where the Higgs field VEV is $\langle\Phi\rangle_0 \propto e^{i\alpha(x_\mu)}$, it is only possible to remove the phase $\alpha(x_\mu)$ on scales smaller than the correlation length so that loops exceeding this size along which α varies by $2\pi n$, with n an integer winding number, might exist and are unshrinkable; these loops surround vortex tubes in the core of which $\langle\Phi\rangle_0 = 0$, i.e., cosmic strings. However, in the standard electroweak symmetry-breaking scheme, $\mathrm{SU}(2)\times\mathrm{U}(1)\to\mathrm{U}(1)$, this phenomenon does not occur since, in this case, $G/H \sim$SU(2), which is isomorphic to the three-sphere S^3 and therefore simply connected. In practice, this means that it is possible to perform a gauge transformation (the unitary gauge) which removes the Higgs field phases everywhere in space. (In this work, we shall disregard the possibility of the so-called "semilocal" strings of the standard model [24] which have been shown [25] to be unstable for the parameter values measured at LEP.)

In order to generate cosmic strings, the simplest way is to gauge an extra U(1) symmetry, thereby introducing a gauge vector C^μ, and to require this symmetry to be broken by means of a (string-forming) Higgs field hereafter denoted by Σ. The model we shall work with belongs to a more general family of models [14] which is now well understood phenomenologically, so that this section has been essentially included in order to fix the notation that is used throughout the rest of this work. The new hypercharge introduced will be noted F, so that we shall refer to the new invariance group as $\mathrm{U}(1)_F$. For simplicity, we shall consider that the standard Glashow-Salam-Weinberg fields, namely, the usual Higgs boson [the $\mathrm{SU}(2)_L$ doublet] φ, the $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ gauge vectors A_i^μ, B^μ, and the fermionic fields ψ, are coupled to the new fields Σ and C^μ by means of the "minimal extension" requirement: Σ is invariant under $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$, and symmetrically (contrary to most of the previous models), φ is invariant under the $\mathrm{U}(1)_F$ transformations (i.e., $F_\varphi = 0$). Because of this choice of vanishing F_φ, the new hypercharge has to be a linear combination of the baryonic and leptonic numbers B and L, respectively (this being necessary to gauge the Yukawa coupling terms between φ and the fermions [14]). The simplest possibility to cancel anomalies is to introduce a "right-handed" neutrino field ν_R which transforms trivially under $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$, so that, in general, a Dirac mass term for the neutrino is allowed (see, e.g., Ref. [26] for massive neutrino constraints and related dark-matter candidates).

Consequently, with a suitable normalization choice, it may be shown that the F hypercharge coincides with $B-L$ for the fermionic fields (although $B-L$ may be seen to be still conserved), and one can set the new hypercharge of the extra U(1) singlet to unity. The Lagrangian is thus chosen to be

$$\mathcal{L} = \mathcal{L}_W + \mathcal{L}_F + \mathcal{L}_{\text{fermions}}, \tag{1}$$

with $\mathcal{L}_W$ the bosonic part of the standard electroweak Lagrangian, given by

$$\mathcal{L}_W = -\frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - (D_\mu\varphi)^\dagger(D^\mu\varphi) - \lambda_\varphi(\varphi^\dagger\varphi - v_\varphi^2/2)^2, \tag{2}$$

the extra bosonic term given by

$$\mathcal{L}_F = -\frac{1}{4}H_{\mu\nu}H^{\mu\nu} - (D_\mu\Sigma)^\star(D^\mu\Sigma) - \lambda_\sigma(|\Sigma|^2 - v_\sigma^2/2)^2 - f(\varphi^\dagger\varphi - v_\varphi^2/2)(|\Sigma|^2 - v_\sigma^2/2) \tag{3}$$

and the fermionic Lagrangian being the usual standard model one with

$$D_\mu = \nabla_\mu - ig\mathbf{T}\cdot\mathbf{A}_\mu - ig'\frac{Y}{2}B_\mu - ig''\frac{F}{2}C_\mu, \tag{4}$$

$$\varphi = \begin{pmatrix}\varphi^+\\ \varphi^0\end{pmatrix}, \quad F^i_{\mu\nu} = \nabla_\mu A^i_\nu - \nabla_\nu A^i_\mu + g\varepsilon^{ijk}A^j_\mu A^k_\nu, \tag{5}$$

$$G_{\mu\nu} = \nabla_\mu B_\nu - \nabla_\nu B_\mu, \quad H_{\mu\nu} = \nabla_\mu C_\nu - \nabla_\nu C_\mu, \tag{6}$$

and $v_\varphi \simeq 246$ GeV.

We shall no longer be concerned with the fermionic sector of the theory and postpone its examination for future work, assuming here that the most important effects of currents generated in strings are those due to the $W^\pm$ and Z^0 bosons to which we now turn.

II. FIELD EQUATIONS IN THE UNITARY GAUGE

As already mentioned in the Introduction, the new gauge boson C^μ should be more massive than roughly 500 GeV, while the mass of the Higgs field Σ should be of order 10 TeV. Assuming the coupling constants to be at most of order unity, this leads to the requirement that the VEV of the Σ field v_σ is significantly larger than that of φ, v_φ. Therefore, we expect the phase transition where

Σ develops a nonzero VEV and may eventually condense into cosmic strings to occur prior to the usual electroweak symmetry breaking. For describing a vortex defect at a classical level, we need to solve the field equations that arise from variations of the Lagrangian (1); this requires that we fix the gauges.

Consider first the $U(1)_F$ fields. In general, we can write the Higgs field Σ as

$$\Sigma(x^\mu) = \sigma(x^\mu) e^{i\alpha(x^\mu)}, \tag{7}$$

where the phase α, being undefined on the vortex line, is impossible to remove by means of a smooth $U(1)_F$ transformation. Instead, we make use of the definition of the vortex as a topological defect: along any closed path surrounding the string, the phase α varies by $2\pi n$, n being an integral winding number. Therefore, we use a gauge in which α is identified with $n\theta$, with θ the polar angle in cylindrical coordinates.

Now, since Σ is invariant under $SU(2)_L \times U(1)_Y$ transformations, we have not lost any freedom in the choice of the latter gauge. Since the purpose here is to describe classical fields (i.e., VEV's), it is convenient (for subsequent interpretation) to work in the $SU(2)_L \times U(1)_Y$ unitary gauge in which only observable fields appear. This transforms the $SU(2)_L$ doublet φ into

$$\varphi(x) = \begin{pmatrix} 0 \\ \dfrac{v_\varphi + \eta(x)}{\sqrt{2}} \end{pmatrix}, \tag{8}$$

so that setting $D_\mu W^\pm_\nu = [\partial_\mu \pm ig(sA_\mu + cZ_\mu)] W^\pm_\nu$, we are left with the special form of the Lagrangian (1):

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{2}(D_\mu W^-_\nu - D_\nu W^-_\mu)(D^\mu W^{+\nu} - D^\nu W^{+\mu}) - \frac{1}{4}A_{\mu\nu}A^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}H_{\mu\nu}H^{\mu\nu} \\
& -(\partial_\mu\sigma)^2 - \sigma^2\left(\partial_\mu\alpha + \frac{g''}{2}C_\mu\right)^2 - \frac{1}{2}(\partial_\mu\eta)^2 - igW^{-\mu}W^{+\nu}(sA_{\mu\nu} + cZ_{\mu\nu}) \\
& -\frac{\lambda_\varphi}{4}(\eta^2 + 2v_\varphi\eta)^2 - \lambda_\sigma\left(\sigma^2 - \frac{v_\sigma^2}{2}\right)^2 - \frac{f}{2}\left(\sigma^2 - \frac{v_\sigma^2}{2}\right)(\eta^2 + 2v_\varphi\eta) \\
& -\frac{g^2}{4}(v_\varphi + \eta)^2 W^+ \cdot W^- - \frac{g^2}{4c^2}(v_\varphi + \eta)^2 Z_\mu^2 - \frac{g^2}{2}[(W^+ \cdot W^-)^2 - W^{+2}W^{-2}],
\end{aligned} \tag{9}$$

where

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(A_{1\mu} \mp iA_{2\mu}), \tag{10}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_{3\mu} \\ B_\mu \end{pmatrix}, \tag{11}$$

the weak angle θ_W is defined by $\tan\theta_W = g'/g$, and $s \equiv \sin\theta_W$, $c \equiv \cos\theta_W$.

The Euler-Lagrange equations arising from the variations of the Lagrangian (9) can then be derived to yield the equations of motion of the extra $U(1)_F$ fields:

$$\partial^\mu H_{\mu\nu} = 2\sigma^2\left(\partial_\nu\alpha + \frac{g''}{2}C_\nu\right), \tag{12}$$

$$\partial^\mu\left[\sigma^2\left(\partial_\mu\alpha + \frac{g''}{2}C_\mu\right)\right] = 0, \tag{13}$$

$$\Box\sigma = \sigma\left[\left(\partial_\mu\alpha + \frac{g''}{2}C_\mu\right)^2 + 2\lambda_\sigma(\sigma^2 - v_\sigma^2/2) + \frac{f}{2}(\eta^2 + 2\eta v_\varphi)\right], \tag{14}$$

and those of the "standard" fields

$$\Box\eta = (v_\varphi + \eta)\left[\frac{g^2}{2}W^+ \cdot W^- + \frac{g^2}{4c^2}Z_\mu^2 + \lambda_\varphi(\eta^2 + 2v_\varphi\eta) + f(\sigma^2 - v_\sigma^2/2)\right], \tag{15}$$

where the vectors A_μ, Z_μ, and $W^\pm_\mu$ satisfy

$$\partial_\mu A^{\mu\nu} + igs\partial_\mu(W^{-[\mu}W^{+\nu]}) = igs(W^+_\alpha W^{-\nu\alpha} - W^-_\alpha W^{+\nu\alpha}) + g^2 s\left(W^+_\alpha A_3^{[\nu}W^{-\alpha]} + W^-_\alpha A_3^{[\nu}W^{+\alpha]}\right), \tag{16}$$

$$\partial_\mu Z^{\mu\nu} + igc\partial_\mu(W^{-[\mu}W^{+\nu]}) = igc(W^+_\alpha W^{-\nu\alpha} - W^-_\alpha W^{+\nu\alpha}) + g^2 c\left(W^+_\alpha A_3^{[\nu}W^{-\alpha]} + W^-_\alpha A_3^{[\nu}W^{+\alpha]}\right) + \frac{g^2}{4c^2}(v_\varphi + \eta)^2 Z^\nu, \tag{17}$$

$$\partial^\mu W^-_{\mu\nu} - ig\partial^\mu(A_{3[\mu}W^-_{\nu]}) = igA_3^\mu(W^-_{\mu\nu} - igA_{3[\mu}W^-_{\nu]}) + igW^{-\mu}A_{\mu\nu} + g^2(W^+\cdot W^- W^-_\nu - W^{-2}W^+_\nu) + \frac{g^2}{4}(v_\varphi + \eta)^2 W^-_\nu. \tag{18}$$

We shall investigate the solutions of these equations in the next sections in order to show that the standard electroweak vacuum is unstable around the vortices under consideration: the Higgs boson η as well as the gauge vectors $W^\pm$ and Z^0 are trapped in the string core, generating, as we shall show, many nontrivial currents so as to yield a far more richer (and complicated) structure than in the original Witten [16] superconducting cosmic string model.

III. KIBBLE-TYPE VORTEX SOLUTION

In a vacuum where the $U(1)_F$ singlet has its nominal VEV v_σ, the φ-doublet VEV is measured as $\sqrt{2}\langle|\varphi|\rangle_0 = v_\varphi \simeq 246$ GeV. In a vortex line, however, where the singlet VEV vanishes, we shall see that there is a corresponding shift in $\langle|\varphi|\rangle_0$ which turns out to exceed its vacuum value, a result which can be interpreted as a Higgs-doublet trapping within the string. In particular, this implies that fermion masses, being directly related to this latter VEV, are increased in the vortex core. This is to be contrasted with the Witten model for which fermionic fields, being coupled with the string-forming Higgs field only, had vanishing masses in the vortex and were possibly trapped in the form of zero modes.

We now turn more specifically to vortexlike solutions. We shall consider a straight and stationary string generated by arbitrary choices of the phase of Σ: since we are looking at the microscopic structure of the string, it is possible to neglect its curvature and to postulate axisymmetry. In terms of the usual cylindrical coordinates, we therefore seek a solution having the special form for (7),

$$\Sigma = \sigma(r)e^{in\theta}, \tag{19}$$

which implies, according to Eqs. (12) and (13), that the only nonzero component of C^μ is C^θ. The effective vortex background of the theory can then be described by means of the functions $\sigma(r)$ and $Q(r)$ defined by

$$Q(r) \equiv n + \frac{g''}{2}C_\theta, \tag{20}$$

an expression which is seen to vanish for $r \to \infty$: integrating Eq. (13) over an arbitrary surface $\mathcal{S}$ crossing the string, and using the Stockes theorem, one finds the usual relation (with $\mathbf{C}$ the three-dimensional part of the vector C^μ)

$$\oint_{\partial S} d\ell\cdot\mathbf{C} = -4\pi n/g''$$

(i.e., the quantization of the "magnetic" flux) indicating that the magnitude of $\mathbf{C}$ behaves as $|\mathbf{C}| \sim -2n/g''r$. This is restated in terms of the field variables that are used here by $C_\theta \to -2n/g''$ and $C^\theta \to -2n/g''r^2$ because of the cylindrical metric.

Let us first ask whether the background vortex solution in which Σ and Q obey the field equations (12)–(14), and where the "standard" fields η, $W^\pm$, Z, and A all being zero represent a solution to the equations of motions. This is certainly the case for $W^\pm = Z = A = 0$ everywhere since these fields, according to the "minimal extension" postulate, are coupled to neither Σ nor C^μ. This statement is no longer true for the Higgs field η: the $SU(2)_L\times U(1)_Y$ vacuum solution $\eta = 0$ is consistent only with the $U(1)_F$ vacuum solution $\sigma = v_\sigma/\sqrt{2}$. However, in the vicinity of the vortex core, one has $\sigma = 0$. This implies a corresponding shift in the φ's VEV, or stated differently, a nontrivial solution for η as a function of the distance to the core. The discussion on the structure of the vortex in the presence of the real Higgs boson η proceeds in two steps: we exhibit a full solution of the field equations (still restricting ourselves to the case where $W^\pm = Z = A = 0$), and we show (numerically) that the corresponding configuration is stable, a conclusion which is necessary for the extension described by the Lagrangian (1) to be reliable but otherwise far from being obvious.

Consider a perturbation on the field η. This transforms Eq. (15) into

$$\Box\eta = v_\varphi f(\sigma^2 - v_\sigma^2/2) + \eta\left[f(\sigma^2 - v_\sigma^2/2) + 2\lambda_\varphi v_\varphi^2\right], \tag{21}$$

whose right-hand side contains a term independent of η: unlike most of the previous models [16], the configuration with $\eta(r) = 0$ everywhere is not a solution of the classical equations of motion. We are therefore led to conclude that the standard Higgs field φ is trapped in the core of the string. This fact, however, does not break the Lorentz invariance along the string because of the real nature of η (it cannot be responsible for a current along the string). Therefore, in a preferred frame in which the energy-momentum tensor is diagonal and expressible as [27]

$$T^{\mu\nu} = Uu^\mu u^\nu - Tv^\mu v^\nu, \tag{22}$$

with U the energy per unit length, T the tension, and u^μ and v^μ, respectively, a timelike and a spacelike unit vector lying on the string's world sheet, the boost invariance yields the equation of state relating U and T as $U = T =$ const: the effective action that describes the dynamics of the vortex is the Goto-Nambu action, i.e., proportional to the array spanned by the string. This is the reason why we call this configuration a Kibble-type solution [1, 2].

In order to investigate this solution, we have solved numerically the equations of the vortex, i.e., the system consisting of Eqs. (12), (14), and (15) with $W^\pm = Z = A = 0$. Denoting by a prime a derivative with respect to the radial distance, and rescaling the various fields and coupling constant by means of

$$\sigma = \frac{v_\sigma}{\sqrt{2}}X, \quad \eta = v_\varphi Y, \quad r = \frac{\rho}{\sqrt{\lambda_\sigma}v_\sigma}, \quad g'' = \sqrt{2\lambda_\sigma}\tilde{g}, \tag{23}$$

we have to solve the system

$$X'' + \rho^{-1}X' = \frac{XQ^2}{\rho^2} + X(X^2-1) + \alpha_2 XY(\tfrac{1}{2}Y+1),$$

$$Y'' + \rho^{-1}Y' = \frac{\alpha_2}{2\alpha_1}(X^2-1)(1+Y^2) + 2\frac{\alpha_3}{\alpha_1}Y(1+\tfrac{3}{2}Y), \tag{24}$$

$$Q'' - \rho^{-1}Q' = \tilde{g}^2X^2Q,$$

where the free parameters α_1, α_2, and α_3 are defined by

$$\alpha_1 = \left(\frac{v_\varphi}{v_\sigma}\right)^2, \quad \alpha_2 = \frac{f}{\lambda_\sigma}\alpha_1, \quad \alpha_3 = \frac{\lambda_\varphi}{\lambda_\sigma}\alpha_1^2, \tag{25}$$

and with the boundary conditions

$$X(0) = 0, \quad X(\infty) = 1,$$

$$Y'(0) = 0, \quad Y(\infty) = 0, \tag{26}$$

$$Q(0) = n, \quad Q'(0) = 0.$$

We have solved this system by means of successive over-relaxed iterations [28] on a discretized grid: Fig. 1 shows a typical solution for X as the solid curve and Y as the dashed curve.

It should be remarked that in the present model and even for this Kibble-type solution, one can actually define two characteristic radii, instead of one in the original model of Kibble [1]: noting the scalar masses $M_\sigma = \sqrt{2\lambda_\sigma}v_\sigma$ and $M_\varphi = \sqrt{2\lambda_\varphi}v_\varphi$, one finds the asymptotic behaviors of σ and η as given by

$$(v_\sigma/\sqrt{2} - \sigma)'' + r^{-1}(v_\sigma/\sqrt{2} - \sigma)' \sim M_\sigma^2\sigma,$$

$$\eta'' + r^{-1}\eta' \sim M_\varphi^2\eta, \tag{27}$$

so $(v_\sigma/\sqrt{2} - \sigma)$ and η behave like Bessel functions, decreasing exponentially with the distance to the core, yielding, respectively, the radii of order $1/M_\sigma$ and $1/M_\varphi$. This greatly complicates numerical investigations [29] since these masses (therefore the radii) can be quite different (a few orders of magnitude say).

It is not, however, obvious that the stationary solution of Eq. (24) is truly stable. This requires that the potential for the fields Σ and φ is minimized by the solution (24) in the string background, i.e.,

$$\frac{\delta^2 V(\sigma,\eta)}{\delta\eta^2} > 0. \tag{28}$$

To see that this is indeed the case, we write

$$\eta_{\rm tot} = \eta + f(t)h(r), \quad f(t)h(r) \ll \eta \tag{29}$$

with η the solution of Eq. (15) for $Z = W = 0$ that we have just obtained, and we insert this form into Eq. (15). This yields

$$\frac{\ddot{f}}{f} = \frac{\Delta h}{h} - f(\sigma^2 - v_\sigma^2/2) - 2\lambda_\varphi v_\varphi^2 - 6\eta\lambda_\varphi v_\varphi = -\omega^2, \tag{30}$$

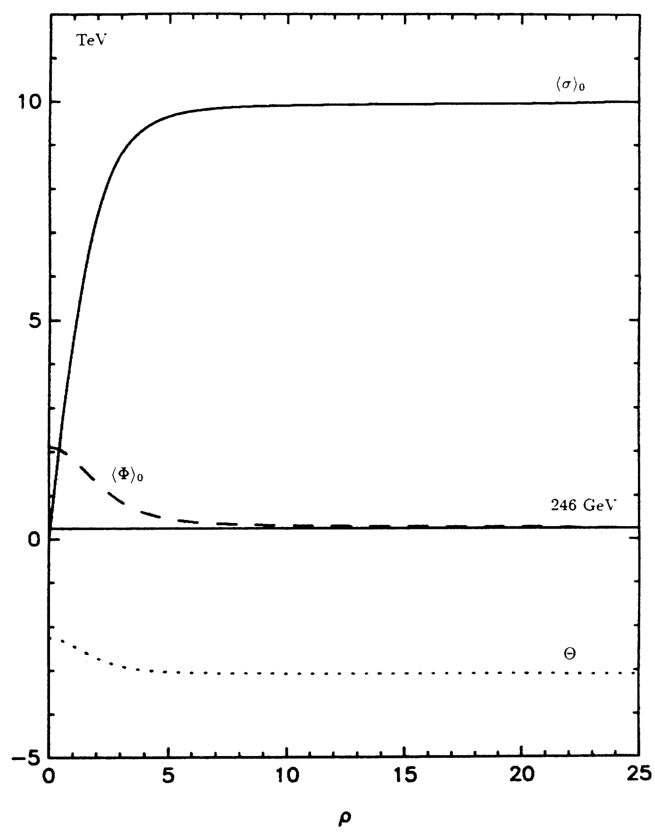

FIG. 1. Higgs fields as functions of the distance to the vortex core. The solid curve is the $U(1)_F$-singlet σ (undimensioned function X), the dashed curve is the usual doublet VEV $\langle|\varphi|\rangle_0$ (corresponding to Y) which goes asymptotically to its standard value 246 GeV, and the dotted curve represents the function Θ (see text); the fact that this latter function is found to always be less than $\langle|\varphi|\rangle$ proves that the corresponding configuration is stable.

where a dot represents a derivative with respect to time. This equation can be written in the form of a Schrödinger equation

$$-\Delta h + V(r)h = \omega^2 h, \tag{31}$$

with the potential $V(r)$ defined by

$$V(r) = f(\sigma^2 - v_\sigma^2/2) + 2\lambda_\varphi v_\varphi^2 + 6\lambda_\varphi v_\varphi \eta. \tag{32}$$

The solution for η can therefore only be stable provided Eq. (31) does not admit bound-state solutions; i.e., the eigenvalues should all be positive. This will certainly be the case if potential (32) is positive definite, so that a sufficient condition for the stability of this particular solution with respect to perturbations in the Higgs scalar η is

$$\eta(r) > \frac{f}{6\lambda_\varphi v_\varphi}(v_\sigma^2/2 - \sigma^2) - \frac{v_\varphi}{3}, \tag{33}$$

or equivalently, in terms of dimensionless functions,

$$Y > \Theta = \frac{\alpha_1^2\alpha_2}{12\alpha 3}(1 - X^2) - \frac{1}{3}. \tag{34}$$

The right-hand side of this inequality is shown in Fig. 1 as the dotted curve, and it is seen that the Y field (the dashed curve on the figure) actually exceeds the combination Θ as required. We performed an exploration over many possible parameter values (ranging between 10^{-4} up to 10^5) and we were always led to the same conclusion: the result presented in Fig. 1 represents a generic situation. Consequently, the potential $V(r)$ in (32) is numerically seen to be positive definite so that Eq. (28) is systematically satisfied: although we do not have a general proof, it turns out that the strings under consideration are effectively stable as far as the Higgs field η is concerned.

IV. POSSIBILITY OF A W CONDENSATE

Since we do not consider fermions, the only possible current is related with the W intermediate vector, and, as the squared mass of this vector is positive definite, the classically stable solution is $W = 0$: there is *a priori* no current along the cosmic strings of this model. However, in the presence of external fields, the situation changes: there exists a critical magnetic field above which this configuration becomes unstable, thereby leading to a W condensate (this phenomenon was originally studied in the framework of the standard model by Ambjørn and Olesen [23]). Here, we make this statement more precise, emphasizing the possibility that the string tunnels from the initial configuration (without current) to a new (less energetic) configuration in which charged as well as neutral vector fields are trapped: this section is essentially devoted to the demonstration that such tunneling processes can occur at least for some limited regions of space, while in Sec. V, we complete the proof by showing that the current-carrying strings are indeed globally less energetic than the ordinary ones.

The energy associated with a particular configuration is obtained by means of the energy-momentum tensor given by variations of the Lagrangian (9) with respect to the metric $g^{\mu\nu}$:

$$T^{\mu\sigma} = -2g^{\sigma\nu}g^{\mu\alpha}\frac{\delta\mathcal{L}}{\delta g^{\alpha\nu}} + g^{\mu\sigma}\mathcal{L}. \tag{35}$$

Explicitly, this gives

$$\begin{aligned} T^{\mu\nu} = &(D^\mu W^{-\gamma} - D^\gamma W^{-\mu})(D^\nu W_\gamma^+ - D_\gamma W_\nu^+) + (D^\nu W^{-\gamma} - D^\gamma W^{-\nu})(D^\mu W_\gamma^+ - D_\gamma W_\mu^+) \\ &+2ig(sA_\gamma^\nu + cZ_\gamma^\nu)(W^{-\mu}W^{+\gamma} - W^{+\mu}W^{-\gamma}) + \frac{g^2}{4}(v_\varphi+\eta)^2(W^{+\mu}W^{-\nu} + W^{+\nu}W^{-\nu}) \\ &+g^2[W^+\cdot W^-(W^{+\mu}W^{-\nu} + W^{+\nu}W^{-\nu}) - W^{+2}W^{-\mu}W^{-\nu} - W^{-2}W^{+\mu}W^{+\nu}] \\ &+A^{\mu\gamma}A_\gamma^\nu + Z^{\mu\gamma}Z_\gamma^\nu + H^{\mu\gamma}H_\gamma^\nu + 2(\partial^\mu\sigma)(\partial^\nu\sigma) + 2\sigma^2\left(\partial^\mu\alpha + \frac{g''}{2}C^\mu\right)\left(\partial^\nu\alpha + \frac{g''}{2}C^\nu\right) \\ &+(\partial^\mu\eta)(\partial^\nu\eta) + \frac{g^2}{4c^2}(v_\varphi+\eta)^2 Z^\mu Z^\nu + g^{\mu\nu}\mathcal{L}, \end{aligned} \tag{36}$$

with which we can obtain the effective potential V^{eff} for the W and Z fields as the time-and-gradients-independent part of the static energy T^{tt}:

$$\begin{aligned} V^{\text{eff}} = &\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2 + \mathbf{K}^2 + \mathbf{N}^2) + \frac{g^2}{4}(v_\varphi+\eta)^2[|W^t|^2 + |\mathbf{W}|^2] \\ &+\frac{g^2}{8c^2}(v_\varphi+\eta)^2[(Z^t)^2 + \mathbf{Z}^2] + ig(s\mathbf{E} + c\mathbf{K})\cdot(W^{-t}\mathbf{W}^+ - W^{+t}\mathbf{W}^-) \\ &+ig(s\mathbf{B} + c\mathbf{N})\cdot(\mathbf{W}^-\times\mathbf{W}^+) + |W^t|^2|\mathbf{W}|^2 + \frac{1}{2}(\mathbf{W}^+\times\mathbf{W}^-)^2, \end{aligned} \tag{37}$$

where boldface letters represent vectors in the ordinary (three-dimensionnal) space, $\mathbf{E}$ and $\mathbf{B}$ being the electric and the magnetic fields, with the symmetric definitions for the weak equivalent of the Maxwell fields, namely, $\mathbf{K}$ and $\mathbf{N}$, given by

$$\mathbf{K} \equiv -\nabla Z_t - \frac{\partial \mathbf{Z}}{\partial t}, \quad \mathbf{N} \equiv \nabla \times \mathbf{Z}. \tag{38}$$

To find the minimum of this effective potential, we look at a configuration having nonzero electric (and **K**) or magnetic (and **N**) fields. Since we seek a particular current-carrying string solution, we shall restrict our attention to electromagnetoweak fields having the cylindrical symmetry: **E** and **K** should be purely radial, and **B** and **N** should be orthoradial, i.e.,

$$E_\theta = K_\theta = 0, \quad E_z = K_z = 0, \quad B_r = N_r = 0, \quad B_z = N_z = 0. \tag{39}$$

Let us first consider the so-called "electric" situation for which $s\mathbf{E}^2 + c\mathbf{K}^2 > s\mathbf{B}^2 + c\mathbf{N}^2$. Then there exists a preferred frame in which the "magnetic" fields **B** and **N** can be set to zero everywhere. In this frame, we can write the potential given by Eq. (37) under assumptions (39) as

$$\begin{aligned} V^{\text{eff}}(W) = &\frac{1}{2}(E_r^2 + K_r^2) + \frac{g^2}{4}(v_\varphi + \eta)^2[|W^t|^2 + |W^r|^2 + |W^z|^2 + r^2|W^\theta|^2] \\ &+ \frac{g^2}{8c^2}(v_\varphi + \eta)^2[(Z^t)^2 + |Z^r|^2 + |Z^z|^2 + r^2|Z^\theta|^2] \\ &+ ig(sE_r + cK_r)(W^{-t}W^{+r} - W^{+t}W^{-r}) + |W^t|^2|\mathbf{W}|^2 + \frac{1}{2}(\mathbf{W}^+ \times \mathbf{W}^-)^2, \end{aligned} \tag{40}$$

which we minimize with respect to the various fields, assuming E_r and K_r to be fixed. As the coefficients of W^z and W^θ are positive definite, we immediately obtain a minimum potential requirement as

$$W^{+z} = W^{+\theta} = 0, \tag{41}$$

and we are left with the following system for $W^{\pm t}$ and $W^{\pm r}$:

$$\frac{g^2}{2}(v_\varphi + \eta)^2 W^{-t} - ig(sE_r + cK_r)W^{-r} + 2g^2 W^{-t}|W^r|^2 - g^2(W^{-r})^2 W^{+t} = 0, \tag{42}$$

$$\frac{g^2}{2}(v_\varphi + \eta)^2 W^{-r} + ig(sE_r + cK_r)W^{-t} + 2g^2 W^{-t}|W^t|^2 - g^2(W^{-t})^2 W^{+r} = 0. \tag{43}$$

Assuming W^{+r} and W^{+t} to both be nonzero, we can restate the previous system as [multiplying Eq. (42) by W^{+t} and Eq. (43) by W^{+r}]

$$\frac{g^2}{2}(v_\varphi + \eta)^2|W^t|^2 = \xi, \quad \frac{g^2}{2}(v_\varphi + \eta)^2|W^r|^2 = \xi^\star, \tag{44}$$

where

$$\begin{aligned} \xi = &-2g^2|W^r|^2|W^t|^2 + ig(sE_r + cK_r)W^{+r}W^{-t} \\ &+ g^2(W^{+r}W^{-t})^2. \end{aligned} \tag{45}$$

Since the right-hand side of Eq. (44) is real, we obtain $\xi = \xi^\star$, which yields $|W^r|^2 = |W^t|^2$ and

$$\begin{aligned} &g^2[(W^{-r})^2(W^{+t})^2 - (W^{+r})^2(W^{-t})^2] \\ &= -ig(sE_r + cK_r)(W^{-r}W^{+t} + W^{+r}W^{-t}), \end{aligned} \tag{46}$$

which is only consistent with $W^{-r}W^{+t} + W^{+r}W^{-t} = 0$, so we finally obtain

$$W^{+r} = \pm i W^{+t}, \tag{47}$$

which implies that the potential has the famous "Mexican hat" form

$$\begin{aligned} V^{\text{eff}}(W) = &\left[\frac{g^2}{2}(v_\varphi + \eta)^2 \pm 2g(sE_r + cK_r)\right]|W|^2 \\ &+ 2g^2|W|^4, \end{aligned} \tag{48}$$

whose minimum is nonzero provided

$$g(v_\varphi + \eta)^2 \pm 4(sE_r + cK_r) < 0. \tag{49}$$

Since the W field itself can be considered as the source of this electric field, there exists a nonzero probability for the vortex to tunnel locally from the ordinary configuration to a charged one.

It should be remarked that the + or − sign in Eq. (47) can be interpreted in terms of the electric field (without considering K) as follows (assuming the coupling constant g to be positive): for the choice $W^{+r} = +iW^{-r}$, it is necessary in order to have a W condensate, i.e., to satisfy condition (49), that E_r be positive, so the electric field should be directed outward. This corresponds to a positive electric charge per unit length. Conversely, for the solution $W^{+r} = -iW^{-r}$, the electric field should be directed inward, so that a negative electric charge per unit length is trapped along the string.

Consider now the so-called "magnetic" situation where $s\mathbf{E}^2 + c\mathbf{K}^2 < s\mathbf{B}^2 + c\mathbf{N}^2$. As in the previous situation, it is possible to select a preferred frame in which **E** and **K** are zero everywhere so that Eq. (37), with the cylindrical symmetry assumption (39), transforms to

$$V^{\rm eff}(W)=\frac{1}{2}(B_\theta^2+N_\theta^2)+\frac{g^2}{4}(v_\varphi+\eta)^2[|W^t|^2+|W^r|^2+|W^z|^2+r^2|W^\theta|^2]$$
$$+\frac{g^2}{8c^2}(v_\varphi+\eta)^2[(Z^t)^2+|Z^r|^2+|Z^z|^2+r^2|Z^\theta|^2]$$
$$+ig(sB_\theta+cN_\theta)(W^{-z}W^{+r}-W^{+z}W^{-r})+|W^t|^2|\mathbf{W}|^2+\frac{1}{2}(\mathbf{W}^+\times\mathbf{W}^-)^2. \quad (50)$$

The same arguments as in the previous "electric" situation show that, in this case, the minimum of the potential (50) is given by

$$W^{+t}=W^{+\theta}=0 \quad (51)$$

and

$$W^{+r}=\pm iW^{+z}. \quad (52)$$

The effective potential for a "magnetic" configuration can therefore be written as

$$V^{\rm eff}(W)=\left[\frac{g^2}{2}(v_\phi+\eta)^2\pm 2g(sB_\theta+cN_\theta)\right]|W|^2$$
$$+2g^2|W|^4, \quad (53)$$

and we find a nonzero minimum in this case for

$$g(v_\varphi+\eta)^2\pm 4(sB_\theta+cN_\theta)<0, \quad (54)$$

with the same interpretation for the $\pm$ sign as before.

The structure around a cosmic vortex is consequently seen to generate local instabilities of the electromagnetic vacuum. More precisely, since external electric or magnetic fields can be responsible for a change in the W mass term, and since the W boson itself can be the source of such electromagnetic fields, large fluctuations (i.e., having a low, but nonzero, probability to occur) can be stabilized around the string, leading to local condensates. Therefore, the $W=0$ configuration represents only a local, i.e., nonglobal, minimum of the potential, and thus a metastable state. We shall now discuss the global properties of the fluctuations to conclude that current-carrying strings are effectively stable, whereas ordinary vortices are only metastable.

V. EFFECTIVE ACTION, STABLE STATES

The dynamics of the vortices under consideration here are conveniently described by means of an effective action [27] which is, in principle, identified with the energy per unit length U or the tension T depending on whether the string is "magnetic" or "electric." In both of these situations, the W boson behaves as a scalar field in the transverse plane, so we can express its longitudinal components in the form

$$W^{\pm}_{t \text{ or } z}\equiv\Upsilon(r)e^{\pm i\psi(t \text{ or } z)}, \quad (55)$$

where the purely radial dependence of the amplitude is a consequence of Eq. (15). This form, together with assumptions (39), once inserted in the energy-momentum tensor, yield the desired effective action: it is shown that the corresponding field equations reproduce Eqs. (12)–(18) under the same assumptions. This effective action, in turn, is used to demonstrate that the stable string solution is that for which $W\neq 0$.

In order to describe the macroscopic dynamical behavior of the strings we have been considering thus far, we need to compute the equation of state, i.e., the energy per unit length U and the tension T defined by

$$U=2\pi\int r\,dr\,T^{tt},\quad T=-2\pi\int r\,dr\,T^{zz}, \quad (56)$$

in the preferred frame where $T^{\mu\nu}$ is diagonal, as well as the various conserved currents defined below (see next section).

In order to show that these quantities are suitable as effective actions for describing spacelike or timelike current-carrying vortices, we first consider an "electric" situation: one can set $W^+_t=\Upsilon(r)e^{i\psi(t)}$ [we shall now consider the + sign in Eqs. (47) and (52)] to obtain, still in the preferred frame in which the only nonzero components of $\mathbf{A}$ and $\mathbf{Z}$ are A_t and Z_t,

$$U=2\pi\int r\,dr\left(\Upsilon'^2+\sigma'^2+\frac{1}{2}\eta'^2+\frac{1}{2}A_t'^2+\frac{1}{2}Z_t'^2+\frac{Q'^2}{g''^2r^2}+\frac{\sigma^2Q^2}{r^2}+\frac{g^2}{8c^2}(v_\varphi+\eta)^2Z_t^2+V_{\rm e}^-(\Upsilon,A_t,Z_t)+\tilde V(\sigma,\eta)\right) \quad (57)$$

and

$$T=2\pi\int r\,dr\left(\Upsilon'^2+\sigma'^2+\frac{1}{2}\eta'^2-\frac{1}{2}A_t'^2-\frac{1}{2}Z_t'^2+\frac{Q'^2}{g''^2r^2}+\frac{\sigma^2Q^2}{r^2}-\frac{g^2}{8c^2}(v_\varphi+\eta)^2Z_t^2+V_{\rm e}^+(\Upsilon,A_t,Z_t)+\tilde V(\sigma,\eta)\right), \quad (58)$$

where the potentials are defined by

$$V_e^{\pm}(\Upsilon, A_t, Z_t) = 2g^2\Upsilon^4 + \Upsilon^2 \left[(\partial_t\psi + gsA_t + gcZ_t)^2 + \frac{g^2}{2}(v_\varphi + \eta)^2 \pm 2g(sA_t' + cZ_t')\right] \tag{59}$$

and

$$\tilde{V} = \frac{1}{4}\lambda_\varphi(\eta^2 + 2v_\varphi\eta)^2 + \lambda_\sigma(\sigma^2 - v_\sigma^2/2)^2 + \frac{f}{2}(\eta^2 + 2v_\varphi\eta)(\sigma^2 - v_\sigma^2/2). \tag{60}$$

It suffices to vary the tension T with respect to the various fields involved to find

$$\sigma'' + r^{-1}\sigma' = \sigma Q^2/r^2 + 2\lambda_\sigma\sigma(\sigma^2 - v_\sigma^2/2) + \frac{f}{2}\sigma(\eta^2 + 2v_\varphi\eta), \tag{61}$$

which is equivalent to Eq. (14),

$$Q'' - r^{-1}Q' = g''^2\sigma^2 Q, \tag{62}$$

which is Eq. (12),

$$\eta'' + r^{-1}\eta' = (v_\varphi + \eta)\left[g^2\Upsilon^2 - \frac{g^2}{4c^2}Z_t^2 + \lambda_\varphi(\eta^2 + 2v_\varphi\eta) + f(\sigma^2 - v_\sigma^2/2)\right], \tag{63}$$

i.e., Eq. (15) under assumption (47),

$$\Upsilon'' + r^{-1}\Upsilon' = 4g^2\Upsilon^3 + \left[\frac{g^2}{2}(v_\varphi + \eta)^2 + 2g(sA_t' + cZ_t') + (\partial_t\psi + gsA_t + gcZ_t)^2\right]\Upsilon, \tag{64}$$

which can be seen to be equivalent to Eq. (18) for $\nu \equiv t$, assuming (41) to hold, and finally the equations corresponding to Eqs. (16) and (17), also for $\nu \equiv t$,

$$A_t'' + r^{-1}A_t' = -\frac{2gs}{r}\frac{d}{dr}(r\Upsilon^2) + 2gs\Upsilon^2(\partial_t\psi + gsA_t + gcZ_t), \tag{65}$$

$$Z_t'' + r^{-1}Z_t' = -\frac{2gc}{r}\frac{d}{dr}(r\Upsilon^2) + 2gc\Upsilon^2(\partial_t\psi + gsA_t + gcZ_t) + \frac{g^2}{4c^2}(v_\varphi + \eta)^2 Z_t. \tag{66}$$

The corresponding expressions in the "magnetic" configuration are found, using in this case only the z component of the fields **A** and **Z** and $W_z^+ = \Upsilon(r)e^{i\psi(z)}$, as

$$U = 2\pi\int r\,dr\left(\Upsilon'^2 + \sigma'^2 + \frac{1}{2}\eta'^2 + \frac{1}{2}A_z'^2 + \frac{1}{2}Z_z'^2 + \frac{Q'^2}{g''^2r^2} + \frac{\sigma^2Q^2}{r^2} + \frac{g^2}{8c^2}(v_\varphi + \eta)^2Z_z^2 + V_m^+(\Upsilon, A_z, Z_z) + \tilde{V}(\sigma,\eta)\right) \tag{67}$$

and

$$T = 2\pi\int r\,dr\left(\Upsilon'^2 + \sigma'^2 + \frac{1}{2}\eta'^2 - \frac{1}{2}A_z'^2 - \frac{1}{2}Z_z'^2 + \frac{Q'^2}{g''^2r^2} + \frac{\sigma^2Q^2}{r^2} - \frac{g^2}{8c^2}(v_\varphi + \eta)^2Z_z^2 + V_m^-(\Upsilon, A_z, Z_z) + \tilde{V}(\sigma,\eta)\right), \tag{68}$$

with the potential $V_m^{\pm}$ expressible as

$$V_m^{\pm}(\Upsilon, A_z, Z_z) = 2g^2\Upsilon^4 + \Upsilon^2\left[(\partial_t\psi + gsA_t + gcZ_t)^2 + \frac{g^2}{2}(v_\varphi + \eta)^2 \pm 2g(sA_z' + cZ_z')\right]. \tag{69}$$

Variations of the energy per unit length yield the same equations as in the previous case, with the substitution $t \to z$, except for Eqs. (63) and (64) which transform according to the original equations (15) and (18), as

$$\eta'' + r^{-1}\eta' = (v_\varphi + \eta)\left[g^2\Upsilon^2 + \frac{g^2}{4c^2}Z_z^2 + \lambda_\varphi(\eta^2 + 2v_\varphi\eta) + f(\sigma^2 - v_\sigma^2/2)\right], \tag{70}$$

$$\Upsilon'' + r^{-1}\Upsilon' = 4g^2\Upsilon^3 + \left[\frac{g^2}{2}(v_\varphi + \eta)^2 - 2g(sA_z' + cZ_z') - (\partial_z\psi + gsA_z + gcZ_z)^2\right]\Upsilon. \tag{71}$$

Performing a Lorentz boost along the string for both "magnetic" and "electric" states, i.e., in the computation of the energy per unit length and the tension, yields equations for A_t and Z_t in a "magnetic" situation (A_z and Z_z in an "electric" one) that are equivalent to Eqs. (65) and (66) and linear in A_t, Z_t, A_z, and Z_z. Since regularity at $r = 0$ requires that the derivatives of these fields should vanish at the origin, we can define a new function $P_\star$ by means of

$$[\partial_t\psi + gsA_t + gcZ_t](r) = [\partial_t\psi + gsA_t + gcZ_t](0)P_\star(r), \quad (72)$$

$$[\partial_z\psi + gsA_z + gcZ_z](r) = [\partial_z\psi + gsA_z + gcZ_z](0)P_\star(r),$$

which allows one to define the fundamental parameter of the current-carrying vortex w as

$$[\partial_t\psi + gsA_t + gcZ_t]^2 - [\partial_z\psi + gsA_z + gcZ_z]^2 = wP_\star^2, \quad (73)$$

this parameter being the only entirely free parameter needed to describe any particular configuration [27, 30, 31], and whose positive (negative) sign reflects the "magnetic" ("electric") nature of the vortex since it determines the spacelike (timelike) character of the Nœther currents $j_\mu^{\rm em}$, j_μ^N, and $j_\mu^\pm$ defined in Sec. VI. It should be emphasized that, unlike what occurs in the original model of Witten, the knowledge of the function $P_\star$ is not sufficient to determine the complete structure of the vortex since it depends on both **A** and **Z** which are not subject to the same equations. However, fixing w allows, in principle, computation of this structure without ambiguity, so this parameter is indeed the fundamental one.

In order to know whether or not charged configurations actually exist as solutions of the equations of motion, we consider an "electric" situation (results being similar in case a "magnetic" configuration is studied) for which the effective tension, seen as a functional over the field functions Υ, A_t, Z_t, σ, η, and Q, should be minimized with respect to these fields. As we have seen in the previous section, there exists a solution to the equations of motion for which $\Upsilon = A_t = Z_t = 0$. This solution would be strictly stable if it led to an absolute minimum of the functional T. We now consider a perturbation of this solution, having nonzero electromagnetoweak fields, to argue that there exists a current-carrying configuration for which T is less than for an ordinary configuration; provided the action is bounded from below, this implies that $W \neq 0$ states are absolutely stable (at least in the short-wavelength limit to which the present analysis is restricted).

Indeed, the unperturbed solution yields a tension T_0 given by

$$T_0 = 2\pi \int r\,dr \left(\sigma'^2 + \frac{1}{2}\eta'^2 + \frac{'^2}{g''^2 r^2} + \frac{\sigma^2 Q^2}{r^2} + \tilde{V}\right), \quad (74)$$

whereas the perturbed one yields

$$T_1 = T_0 + 2\pi \int r\,dr \left(\Upsilon'^2 - \frac{1}{2}(A_t'^2 + Z_t'^2) - \frac{g^2}{8c^2}(v_\varphi + \eta)^2 Z_t^2 + V_e^+\right). \quad (75)$$

Current-carrying solutions exist if there exist electromagnetoweak field functions such that $T_1 < T_0$, and subject to the boundary conditions

$$\Upsilon'(0) = 0, \quad \Upsilon(\infty) = 0,$$
$$A_t'(0) = 0, \quad A_t(r) \propto \ln(r) \text{ for } r \to \infty, \quad (76)$$
$$Z_t'(0) = 0, \quad Z_t(\infty) = 0.$$

The existence of such solutions may be seen to be only dependent on the amplitude of A_t, i.e., on w, since this field gives the leading terms in the integral T_1 due to the logarithmic divergence. Therefore, the uncharged state is only locally stable, having a nonzero probability to tunnel to a current-carrying state having a fundamental parameter w exceeding in magnitude the critical one that leads to $T_1 = T_0$, smaller values leading to $T_1 > T_0$.

One important difference can already be noticed with the original model of Witten [16]: as shown in Fig. 2, the available range for the parameter w is much more restricted here than in the Witten situation. Indeed, there exists a gap between the possible electric and magnetic states which does not exist in the version of Witten: bosonic-current properties of the strings of the present model are restricted to those having large electric or magnetic fields. This is to be contrasted with the original model of Witten in which the coupling constant e, being arbitrary, could be set to zero, allowing a global limit [30,

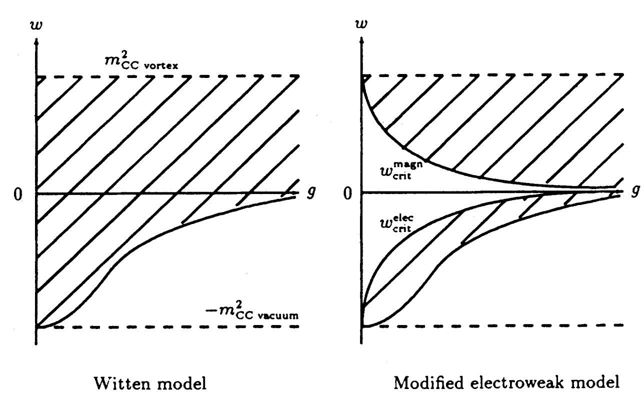

FIG. 2. Comparison between allowed ranges (filled arrays) of the variation of the phase gradient w of the charge carrier in the Witten model (left) and in the modified version of the electroweak theory (right) as functions of the coupling constant g (and g' in the latter case). This qualitative sketch shows that, contrary to the Witten model, the coupling constants g and g' cannot be set simultaneously to zero so that the global limit is trivial. In this figure, the straight dashed lines represent the limiting values $w = -m^2_{\rm CC\ vacuum}$, the mass of the current carrier in vacuum, and $w = m^2_{\rm CC\ vortex}$, the effective mass of the current carrier in the core of the vortex.

32]. In the present model, however, this limit does not exist (at least as far as bosonic currents are concerned) since the various currents, arising from a nonzero value of the field W, can only be nontrivially zero in case the coupling constants g and g' are both nonzero.

VI. STRING CURRENTS

We showed that local large-amplitude fluctuations yield instabilities so that the vacuum surrounding a cosmic string can be abolutely stabilized through the breaking of the electromagnetic symmetry: the W field acquires a nonzero VEV. Equations (16) and (17) [or, equivalently, (65) and (66)] consequently imply that A and Z are both nonzero. This corresponds to nontrivial electromagnetic, charged, and neutral currents [i.e., the four $SU(2)_L \times U(1)_Y$ phase invariances] which we shall now investigate.

To start with, let us consider form (9) of the Lagrangian. Since it is invariant under changes in the phase of W, there exists a conserved Nœther current,

$$j_\mu^{\rm em} \propto W^- \frac{\delta \mathcal{L}}{\delta(\partial^\mu W^-)} - W^+ \frac{\delta \mathcal{L}}{\delta(\partial^\mu W^+)}, \qquad (77)$$

whose internal components (i.e., lying along the axis of the string) are, with a Latin a index to denote z or t,

$$j_a^{\rm em} = e\Upsilon[\partial_a \psi + g(sA_a + cZ_a)], \qquad (78)$$

which define the electromagnetic part of the total current ($e = gs$ is the usual electromagnetic coupling constant) since it is directly (i.e., at the tree order) coupled with the photon A.

We shall now consider the Higgs-doublet kinetic term $\mathcal{K}$ in the Lagrangian (1) given by

$$\mathcal{K} = -\left| \left(\partial_\mu - i\frac{g}{2}\boldsymbol{\tau}\cdot\mathbf{A}_\mu - i\frac{g'}{2}B_\mu \right) \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \right|^2, \qquad (79)$$

which we write as

$$\mathcal{K} = -\left| \left[\partial_\mu - i\frac{g}{2}\left(\frac{\cos 2\theta_W}{c} Z_\mu + 2sA_\mu \right) \right] \varphi^+ - i\frac{g}{\sqrt{2}} W_\mu^+ \varphi^0 \right|^2 - \left| \left(\partial_\mu + i\frac{g}{2c} Z_\mu \right) \varphi^0 - i\frac{g}{\sqrt{2}} W_\mu^- \varphi^+ \right|^2 . \qquad (80)$$

This term allows us to compute a charge-coupled and a weak neutral current associated (via the Nœther theorem) with the required invariance of the initial Lagrangian (1) under whatever changes in the $SU(2)_L$ phases of φ^+ and φ^0: this yields a current coupled with the W boson $j_\mu^\pm$ expressible in the unitary gauge as

$$j_a^\pm = \frac{g}{2\sqrt{2}} W_a^\pm (v_\varphi + \eta)^2, \qquad (81)$$

as well as a Z-coupled current j_μ^N, again in the unitary gauge

$$j_a^N = \frac{g}{\cos\theta_W} Z_a (v_\varphi + \eta)^2. \qquad (82)$$

As stated before, these currents are both nonzero according to Eqs. (18) and (17) if $W \neq 0$.

One can define a total current $\mathcal{J}_a$ as the set $\{j_a^{\rm em}, j_a^\pm, j_a^N\}$, the four components of which are related, as mentioned above, in such a way that the knowledge of the fundamental parameter w is sufficient to their overall determination. Therefore, in the simplest possible extension of the standard electroweak model, we find that the cosmic strings which form at the symmetry-breaking phase transition can be either of the (metastable) Kibble kind, or (stable) Witten kind. In the latter case, however, the total current flowing along the string always has not only one electromagnetic-coupled component, but also two charged and one neutral components. The vortex structure is therefore describable by means of four different (though related) currents which cannot vanish separately.

CONCLUSION

The microscopic structure of cosmic strings arising at the symmetry-breaking phase transition of the minimal extension of the standard electroweak theory is investigated in detail. This reveals that such strings are endowed with current-carrying properties that produce a much more complicated internal structure than in the original superconducting cosmic-string model proposed by Witten [16]. However, they can conveniently be described by this latter model as an approximation in which neutral currents as well as (more generally) any currents not coupled to the photon at the tree order have been neglected.

It is found that a "minimally coupled" (i.e., only through scalar-scalar interaction) topological vortex defect modifies locally the standard electroweak vacuum structure in such a way that the usual Higgs boson gets trapped in the core of the strings, the corresponding field configuration being shown to be stable against perturbation modes of the Higgs-field VEV. Owing to the fact that the baryon and lepton masses are related to this latter VEV, we believe that an examination of the potential cosmological influence of such strings deserves theoretical attention in the near future.

An important characteristic feature of the strings that are produced by the model of Witten is that electromagnetic gauge symmetry is spontaneously broken in their core by means of a charged scalar field acquiring a nonzero VEV due to its coupling with the string-forming field. This phenomenon is independent of the actual value of the electromagnetic coupling constant e, so that the latter is arbitrary, allowing for a global limit $e = 0$. In the present model, however, the symmetry is not broken by means of a hypothetical scalar field but by the W vectorial field. It is shown that this implies that such a global limit cannot be defined here: electromagnetic symmetry breaking on the vortex core achieved by a charged vector requires the effective squared mass of this vector to become negative. As this mass is calculated via the Higgs-boson VEV (a quantity which is positive definite) and the electromagnetic fields through the coupling constant, the latter should not vanish for any local instability to be present, a condition which is necessary for current possibilities.

Furthermore, the electromagnetic symmetry breaking is not induced by the Higgs mechanism; i.e., it is not an instantaneous process, but rather by a quantum tunneling process: it is only locally that the energy per unit length is minimized for vanishing currents, so the corresponding state is found to be metastable. Globally, however, a new (absolute) minimum is found for current-carrying configurations. Thus, it is only through the fluctuations of the W field that the string may reach a stable state, a process which requires much more time (due to the low probability) than the usual Higgs mechanism. An interesting cosmological consequence of this conclusion is that electromagnetic effects of such strings should probably be disregarded during the very early epoch of the Universe (e.g., before the quark-hadron phase transition).

The present analysis is restricted to the bosonic part of the model but its conclusions may not be drastically changed by the inclusion of fermionic fields. Indeed, if fermions are trapped along the vortex (presumably not in the form of the zero modes used by Witten [16] since their masses do not vanish), the most important effect (regarding the bosonic configuration) should be in an enhancement of the tunneling probability: if fermions were responsible for the generation of an electromagnetic current, they would increase the background amplitude of the vector potential A, and finally the tunneling probability discussed above. Further work is, however, needed to conclude as to the existence of fermionic condensate in this model.

It should again be remarked that even though the strings described here are gravitationally insignificant, having a negligible mass (per unit length) compared with the characteristic GUT energy scale, they might still be cosmologically relevant for various applications such as primordial inhomogeneous nucleosynthesis or dark matter.

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