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Bulk viscosity of strange quark matter, damping of quark star vibration, and the maximum rotation rate of pulsars

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The bulk viscosity of strange quark matter is calculated in the light of new rate expressions for the reaction $u+s \leftrightarrow d+u$. The results are orders of magnitude larger than previously assumed, in particular, in the limit of high amplitude perturbations. This has important consequences for the vibration and rotation properties of strange stars and neutron stars with quark cores, implying rapid damping of stellar vibrations and suppression of nonaxisymmetric instabilities for rapidly spinning pulsars, probably decreasing the minimum rotation period to the Keplerian limit of 0.6 ms.

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I. INTRODUCTION

Quark matter composed of comparable numbers of u, d, and s quarks may be stable at zero pressure and temperature [1–3], in which case some or all neutron stars can turn out to be so-called strange stars [1, 4, 5, 3]. If strong interaction parameters are such that strange matter is only metastable, the high pressure in the central regions of neutron stars may lead to formation of hybrid stars, having strange matter cores.

Observationally it is not easy to distinguish strange, hybrid, and ordinary neutron stars (see [3] for reviews and references). In the observed mass region near $1.4M_\odot$ the stars have rather similar radii, since at these masses they are bound mainly by gravity, not by strong interactions. Neutrino cooling may be more efficient in stars with quark cores, but recent revival of the ordinary URCA process in neutron stars [6] has reduced the importance of this potential "smoking gun." Another discriminant could be the seismic "glitch events" in pulsars, but it is not yet clear whether strange stars may be able to explain glitches under certain circumstances.

Perhaps the best test for the existence of hybrid or strange stars will turn out to be studies of phenomena related to stellar vibration or rotation. A decisive issue here is the time scale for damping of the vibrations, and of the gravitational radiation reaction instability limiting the maximum rotation rate—the former perhaps related to γ-burst events, the latter to millisecond pulsars.

Both of these mechanisms are strongly influenced by the bulk viscosity of strange quark matter, which again depends on the rate of the nonleptonic interaction:

$$u+d \leftrightarrow s+u. \qquad (1)$$

This reaction changes the concentrations of down and strange quarks in response to the density changes involved in vibration or rotational instabilities, thereby causing dissipation. This dissipation is most efficient if the rate of reaction (1) is comparable to the frequency of the density change. If the weak rate is very small, the quark concentrations keep their original values in spite of a periodic density fluctuation, whereas a very high weak rate means that the matter immediately adjusts to follow the true equilibrium values reversibly. But in the intermediate range dissipation is important.

The importance of dissipation due to Eq. (1) was first stressed by Wang and Lu [7] in the case of neutron stars with quark cores. These authors made a numerical study of the evolution of the vibrational energy of a neutron star with an $0.2M_\odot$ quark core, governed by the energy dissipation due to Eq. (1). Sawyer [8] expressed the damping in terms of the bulk viscosity, a function of temperature and oscillation frequency, which he tabulated for a range of densities and strange quark masses. Sawyer's tabulation was later used in studies of quark star vibration [9] and of the gravitational radiation reaction instability determining the maximum rotation rate of pulsars [10]. The latter study concluded that the bulk viscosity is large enough to be important for temperatures exceeding 0.01 MeV, but that it should be a few orders of magnitude larger to generally dominate the stability properties.

However, as has been pointed out in [11], the bulk viscosities calculated by Sawyer depend on the assumption that the rate of Eq. (1) can be expanded to first order in $\delta\mu = \mu_s - \mu_d$, where μ_i are the quark chemical potentials.[1] This assumption is not correct at low temperatures ($2\pi T \ll \delta\mu$), where the dominating term in the rate is proportional to $\delta\mu^3$. Furthermore, Sawyer's rate is too small by an overall factor of 3, and, as discussed below, a discrepancy of 2–3 orders of magnitude, perhaps due to unit conversions, appears as well. Taken together, these effects lead to an upward correction of the bulk viscosity by several orders of magnitude. The nonlinearity of the rate also means that the bulk viscosity is no longer independent of the amplitude of the density variations. But all effects increase the viscosity, and thereby the importance for the astrophysical applications.

[1] In equilibrium $\mu_i \approx 235\ \mathrm{MeV}(\rho/\rho_{\mathrm{nuc}})^{1/3}$, with nuclear matter density $\rho_{\mathrm{nuc}} = 2.8\times10^{14}\ \mathrm{g\,cm^{-3}}$.

II. DERIVATION OF THE BULK VISCOSITY

In this section we derive the bulk viscosity of strange quark matter using a strategy similar to that taken by Wang and Lu [7] and by Sawyer [8]; in spite of quite different notation, those two treatments lead to almost identical expressions for the bulk viscosity. We shall adopt a notation close to that of Ref. [7].

Assume that the volume per unit mass, v, changes periodically in time according to the relation

$$v(t) = v_0 + \Delta v \sin\left(\frac{2\pi t}{\tau}\right), \tag{2}$$

where v_0 is the equilibrium volume, Δv is the perturbation amplitude, and τ is the period. The mean dissipation rate of energy per unit mass can be expressed as

$$\left(\frac{dw}{dt}\right)_{\rm av} = -\frac{1}{\tau}\int_0^\tau P(t)\frac{dv}{dt}dt. \tag{3}$$

Here $P(t)$ is the pressure, which can be expanded near the equilibrium value P_0 according to

$$P(t) = P_0 + \left(\frac{\partial P}{\partial v}\right)_0 \delta v + \left(\frac{\partial P}{\partial n_d}\right)_0 \delta n_d + \left(\frac{\partial P}{\partial n_s}\right)_0 \delta n_s. \tag{4}$$

n_i are quark numbers per unit mass, and $\delta v = v - v_0$.

The changes in d- and s-quark numbers per unit mass are given by the rate of reaction (1), so that

$$\delta n_d = -\delta n_s = \int_0^t \frac{dn_d}{dt}dt. \tag{5}$$

The net rate of $u+s \leftrightarrow d+u$ can be well approximated at low temperatures ($T \ll \mu_i$) and s-quark mass ($m_s \ll \mu_s$), and for comparable quark chemical potentials by [11, 12]

$$\frac{dn_d}{dt} \approx \frac{16}{5\pi^5}G_F^2 \sin^2\theta_C \cos^2\theta_C \mu_d^5 \delta\mu[\delta\mu^2 + 4\pi^2 T^2]v_0, \tag{6}$$

with

$$\frac{16}{5\pi^5}G_F^2 \sin^2\theta_C \cos^2\theta_C = 6.76 \times 10^{-26}\ \mathrm{MeV}^{-4}. \tag{7}$$

When calculating the energy dissipation according to Eq. (3), only the third and fourth terms in Eq. (4) contribute. Using the thermodynamical relations

$$\frac{\partial P}{\partial n_i} = -\frac{\partial \mu_i}{\partial v}, \tag{8}$$

one gets the formula for the contributing terms:

$$\delta P(t) = \frac{m_s^2}{3\mu_d v}\int_0^t \frac{dn_d}{dt}dt \tag{9}$$

[cf. Eqs. (6) and (7) in [7]].

The chemical potential difference can be expanded like the pressure in Eq. (4), giving

$$\delta\mu(t) = \left(\frac{\partial \delta\mu}{\partial v}\right)_0 \delta v + \left(\frac{\partial \delta\mu}{\partial n_d}\right)_0 \delta n_d + \left(\frac{\partial \delta\mu}{\partial n_s}\right)_0 \delta n_s. \tag{10}$$

(Note that $\delta\mu = 0$ in equilibrium.) This can be evaluated using the zero-temperature relations

$$n_d = \frac{\mu_d^3 v}{\pi^2}, \quad n_s = \frac{(\mu_s^2 - m_s^2)^{3/2} v}{\pi^2}, \tag{11}$$

giving

$$\delta\mu = \frac{m_s^2}{3\mu_d}\frac{\delta v}{v} - \frac{2\mu_d^2 + \frac{1}{2}m_s^2}{3\mu_d n_d}\int_0^t \frac{dn_d}{dt}dt. \tag{12}$$

This is the same as Eq. (12) in [7], except for the factor $\frac{1}{2}$ rather than -1 on m_s^2, which comes from expanding $(\mu_s^2 - m_s^2)^{-1/2}$ for $m_s \ll \mu_s$. Equation (12) is equivalent to

$$\frac{\partial \delta\mu}{\partial t} = \frac{m_s^2}{3\mu_d}\frac{2\pi}{\tau}\frac{\Delta v}{v_0}\cos\left(\frac{2\pi t}{\tau}\right) - \frac{2\mu_d^2 + \frac{1}{2}m_s^2}{3\mu_d n_d}\frac{dn_d}{dt}. \tag{13}$$

Defining the bulk viscosity like [8]

$$\zeta \equiv 2\frac{(dw/dt)_{\rm av}}{v_0}\left(\frac{v_0}{\Delta v}\right)^2\left(\frac{\tau}{2\pi}\right)^2 \tag{14}$$

we finally get

$$\zeta = -2\left(\frac{v_0}{\Delta v}\right)\left(\frac{\tau}{2\pi}\right)\frac{m_s^2}{3\mu_d v}\frac{1}{\tau} \times \int_0^\tau dt\left[\int_0^t \frac{dn_d}{dt}dt\right]\cos\left(\frac{2\pi t}{\tau}\right). \tag{15}$$

A. The linear regime ($2\pi T \gg \delta\mu$)

Simultaneously solving Eqs. (6), (13), and (15) is not possible analytically. In the high-temperature limit ($2\pi T \gg \delta\mu$), where the term in the weak rate proportional to $\delta\mu^3$ can be neglected, the bulk viscosity can be found analytically, as

$$\zeta = \frac{\alpha T^2}{\omega^2 + \beta T^4}\left[1 - [1 - \exp(-\beta^{1/2}T^2\tau)]\frac{2\beta^{1/2}T^2/\tau}{\omega^2 + \beta T^4}\right]. \tag{16}$$

In this expression $\omega = 2\pi/\tau$, and

$$\alpha = \frac{64}{45\pi^3}G_F^2 \sin^2\theta_C \cos^2\theta_C m_s^4 \mu_d^3 = 9.39 \times 10^{22} m_s^4 \mu_d^3\ \ \mathrm{g\,cm^{-1}s^{-1}}, \tag{17}$$

$$\beta = \frac{36}{\pi^2}\left(\frac{64}{45}\right)^2 G_F^4 \sin^4\theta_C \cos^4\theta_C \mu_d^6 (1 + m_s^2/4\mu_d^2)^2 = 7.11 \times 10^{-4}\ \mathrm{s}^{-2}\mu_d^6(1 + m_s^2/4\mu_d^2)^2. \tag{18}$$

(The numerical values here and below assume T, m_s, and μ_d in MeV, ω^2 and β in s^{-2}.)

The bulk viscosity as given by Eq. (16) never deviates by more than 16% from the prefactor

$$\zeta = \alpha T^2(\omega^2 + \beta T^4)^{-1}. \tag{19}$$

This has the same form as Eq. (16) in [8], except that $\alpha = 3\alpha_S$ and $\beta = 81\beta_S$, where α_S and β_S are the expressions derived by Sawyer. The factor 3 on α and a factor of 9 on β can be traced back to the factor of 3 discrepancy in the weak rate. It has not been possible to trace the last factor of 9 on β. Nor has it been possible to explain why the *numerical* values for α quoted in [8] and later used in [9, 10] are too low by roughly 3 orders of magnitude, rather than a factor of 3.

B. The general solution

In general Eqs. (6), (13), and (15) must be solved numerically. Results of such calculations are shown in Figs. 1 and 2. One notices that the behavior of ζ as a function of $\Delta v/v_0$ is divided into three regimes. For low values of $\Delta v/v_0$, the bulk viscosity is constant (other parameters fixed). Then there is a branch where $\zeta \propto (\Delta v/v_0)^2$, followed by a bendover or even decline at high $\Delta v/v_0$. These trends can be understood by exploring Eq. (13).

Assume for the moment that the first term in Eq. (13) dominates the second (it will be shown below that this is the case for most of an oscillation cycle in certain parameter ranges). Then an integration gives

$$\delta\mu(t) \approx \frac{m_s^2}{3\mu_d}\frac{\Delta v}{v_0}\sin\left(\frac{2\pi t}{\tau}\right). \tag{20}$$

This admits an analytical integration of Eqs. (6) and (15), leading to

$$\zeta \approx \frac{16}{45\pi^5}G_F^2\sin^2\theta_C\cos^2\theta_C m_s^4\mu_d^3\omega^{-2} \times\left[\frac{3}{4}\left(\frac{m_s^2}{3\mu_d}\frac{\Delta v}{v_0}\right)^2 + 4\pi^2T^2\right]. \tag{21}$$

This is simply

$$\zeta \approx \alpha T^2/\omega^2 \tag{22}$$

when the T^2 term dominates, or

$$\zeta \approx \gamma m_s^8\mu_d(\Delta v/v_0)^2/\omega^2, \tag{23}$$

with $\gamma = 1.98 \times 10^{20}\ \mathrm{g\,cm^{-1}s^{-1}}$ when the perturbation amplitude term dominates. The division between the two regimes is at $\Delta v/v_0 \approx 6\pi T\mu_d/m_s^2$.

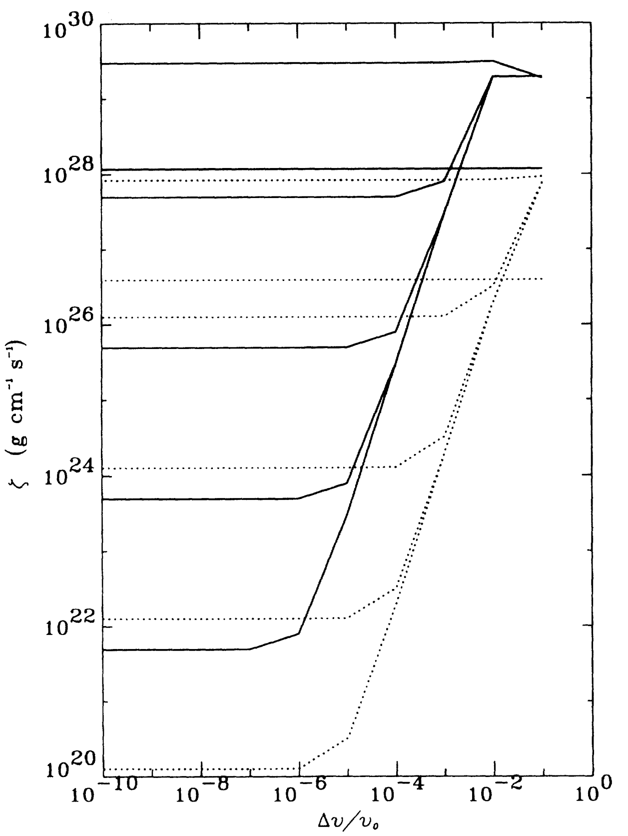

FIG. 1. Bulk viscosity as function of relative volume perturbation amplitude for $\mu_d = 235$ MeV, $m_s = 80$ MeV (dotted curves), $m_s = 200$ MeV (full curves). For both sets of curves $\tau = 10^{-3}$ s, and the temperatures are 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 1, and 10^{-1} MeV, respectively, from bottom to top on the flat part of the curves.

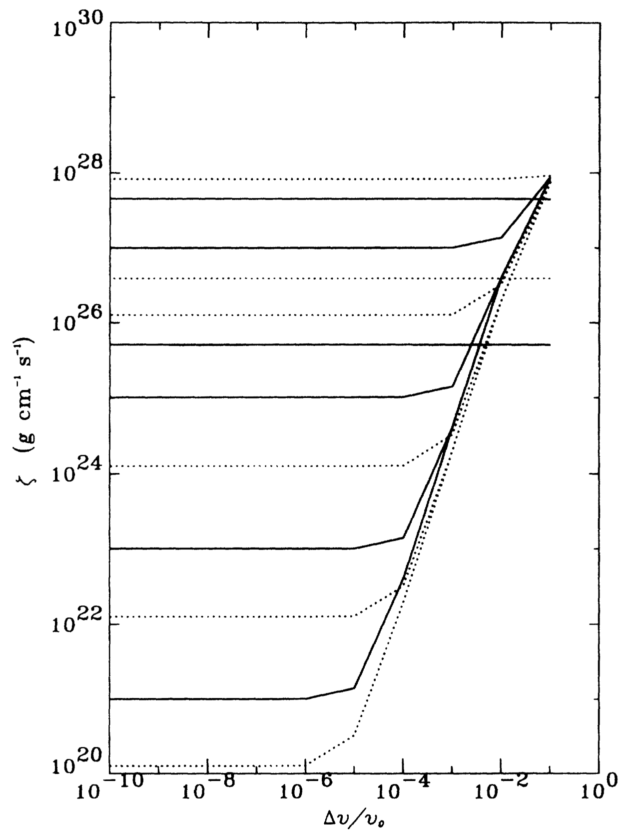

FIG. 2. Bulk viscosity as function of relative volume perturbation amplitude for $\tau = 10^{-3}$ s, $m_s = 80$ MeV, $\mu_d = 235$ MeV (dotted curves), $\mu_d = 470$ MeV (full curves). For $\mu_d = 235$ MeV the order of the curves are as in Fig. 1. For $\mu_d = 470$ MeV curves from bottom to top (flat part) correspond to temperatures of 10^{-5}, 10^{-4}, 10^{-3}, 1, 10^{-2}, and 10^{-1} MeV.

These approximations fit very well with the flat and rising branches of ζ shown in Figs. 1 and 2. The validity depended on the approximation in Eq. (20), which is self-consistent provided that

$$\left[\frac{m_s^2}{3\mu_d}\frac{\Delta v}{v_0}\sin\left(\frac{2\pi t}{\tau}\right)\right]^2 + 4\pi^2T^2 \ll \frac{15\pi^4|\cot(2\pi t/\tau)|}{16G_F^2\sin^2\theta_C\cos^2\theta_C\tau\mu_d(\mu_d^2+m_s^2/4)}. \quad (24)$$

This condition is obeyed over a significant fraction of an oscillation period when $T \leq 15\mu_d^{-3/2}\tau^{-1/2}$ and $\Delta v/v_0 \leq 289\mu_d^{-1/2}m_s^{-2}\tau^{-1/2}$ (cf. Fig. 3). These limits fit the bendovers occurring in Figs. 1 and 2.

III. ASTROPHYSICAL APPLICATIONS OF THE BULK VISCOSITY

A. Damping time for vibrating strange stars

A main motivation for the studies in Refs. [7, 8] was to calculate the damping time for neutron star and/or strange star vibrations. For a star of constant density (an excellent approximation for a strange star, except very close to the gravitational instability limit) Sawyer [8] estimates the kinetic energy per unit volume of the star as

$$E_K = 60^{-1}(\Delta v/v_0)^2\rho\omega^2R^2, \quad (25)$$

where ρ is the mass density and R the radius. A typical damping time is thus $\tau_D = E_K/(dw/dt)_{\text{av}}$, or

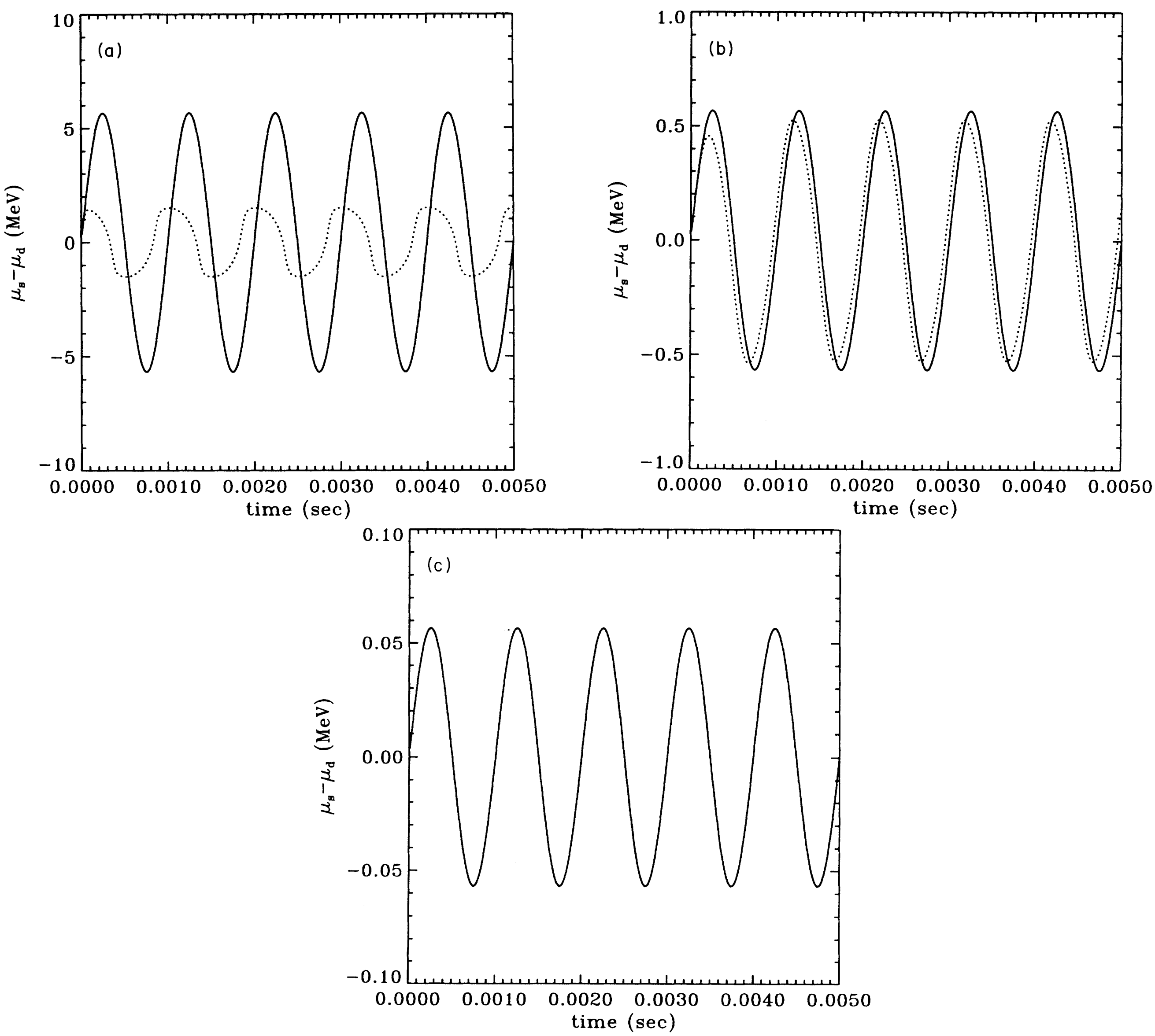

FIG. 3. $\delta\mu(t)$ for five cycles for parameters m_s = 200 MeV, μ_d = 235 MeV, $T = 10^{-4}$ MeV, $\tau = 10^{-3}$ s, and $\Delta v/v_0 = 10^{-1}, 10^{-2}$, and 10^{-3}, respectively. Dotted curves are based on the numerical integrations. Full curves are based on Eq. (20). $\delta\mu$ is indistinguishable from the approximative expression for $\Delta v/v_0 \leq 10^{-3}$, i.e., on the flat and rising parts of the ζ curves in Fig. 1.

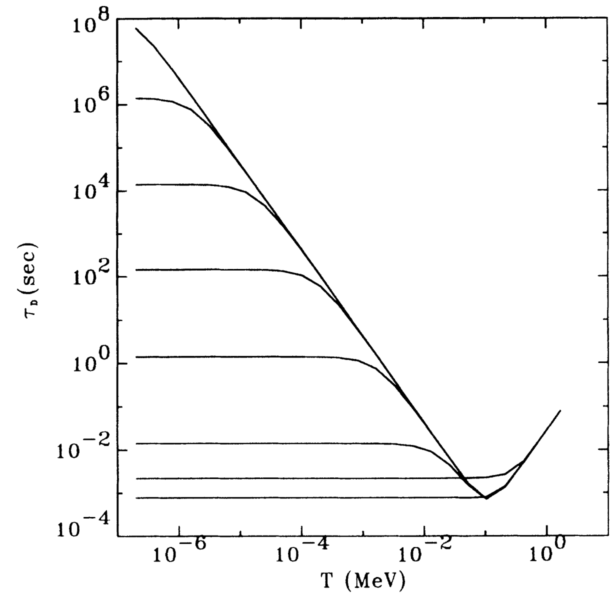

FIG. 4. Damping time for a $1.4M_\odot$ strange star of density 4×10^{14} g cm^{-3}, assuming $m_s = 100$ MeV, $\tau = 10^{-3}$ s. Curves from bottom to top correspond to perturbation amplitudes 10^{-1}, 1, 10^{-2}, 10^{-3}, etc., ending at 10^{-7}.

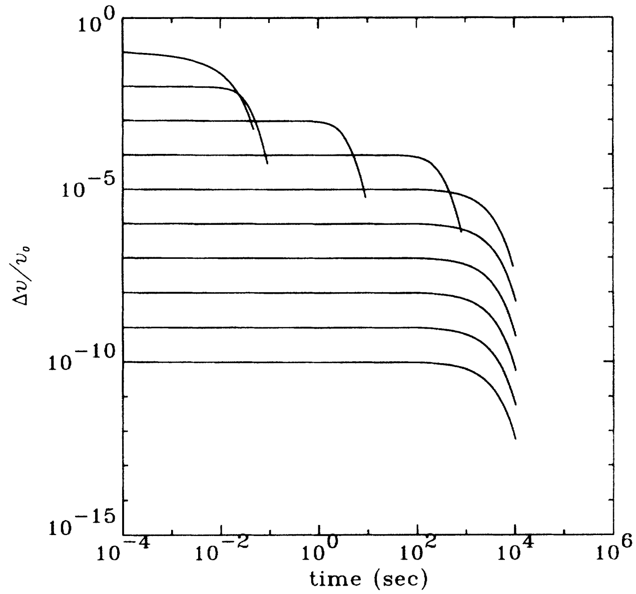

FIG. 5. Evolution of amplitude for damped strange stars with initial amplitudes ranging from 10^{-10}–10^{-1}. The oscillation period was fixed at $\tau = 10^{-3}$ s, and the initial temperature set to 10^{-4} MeV.

$$\tau_D = 30^{-1}\rho R^2\zeta^{-1}. \quad (26)$$

Figure 4 shows the damping time for a typical strange star with $M = 1.4M_\odot$, $\rho = 4\times10^{14}$ g cm^{-3}, and $R = 1.06\times10^6$ cm, which has $\tau_D \approx 1.5\times10^{25}\zeta^{-1}$ s, with ζ in cgs units. The oscillation time is taken to be 10^{-3} s, which is typical for the fundamental mode. Even at very low temperatures, high-amplitude oscillations are damped in fractions of a second. This flat part of the curves are given by the limit of Eq. (23), $\tau_D \approx 286$ s $m_s^{-8}\omega^2(\Delta v/v_0)^{-2}$. At higher temperatures ζ is roughly given by Eq. (22), with $\tau_D \approx 142$ s $m_s^{-4}\mu_d^{-3}\omega^2T^{-2}$. And finally in the high-T limit $[\zeta \approx \alpha/(\beta T^2)]$, $\tau_D \approx 1.8\times10^6$ s $m_s^{-4}T^2$.[2]

Compared to Ref. [8] the new feature is the flat, amplitude-dependent part in Fig. 4, in addition to an overall decrease in the damping time scales of some 3 orders of magnitude in the amplitude-independent regime.

The damping time as such is not the whole story, however. As pointed out by Wang and Lu [7] the temperature of the star increases due to the heat released by viscous dissipation. Except for $T > 0.1$ MeV this can speed up the damping of vibrations. Crudely assuming that the decrease in oscillation energy goes to a uniform increase in thermal energy[3] E_T the star heats up to a temperature given from

$$E_T \approx 2.5\times10^{31}\ \mathrm{erg\,cm^{-3}}T^2. \quad (27)$$

If the heating takes the star from the flat to the declining part of the damping time curves in Fig. 4, the heating can significantly speed up the damping. This is only a marginal effect in the example of Fig. 4, but for lower frequencies the effect can be sizable. Figure 5 shows the damping in terms of the amplitude evolution for a range of initial values. Significant dissipative heating occurred for the highest amplitudes, but as τ_D is insensitive to T for high amplitudes (cf. Fig. 4) this had no major effect on the damping.

The discussion above was based on rather crude estimates. A detailed, general relativistic, numerical treatment along the lines of Ref. [9] is clearly needed.

B. Maximum rotation rate of pulsars

Viscosity plays an important role in setting the maximum rotation rate of pulsars. Gravitational radiation reaction instabilities (as opposed to "Keplerian mass shedding") is supposed to set the rotation rate limit, but the larger the damping by shear and bulk viscosity is, the closer the rate can get to the Keplerian limit.

The relative importance of bulk viscosity depends on the ratio of bulk-to-shear viscosity, where the latter is normally taken to be [13]

$$\eta \approx 5.2\times10^{15}\left(\frac{0.1}{\alpha_S}\right)^{3/2}\left(\frac{\rho}{\rho_{\rm nuc}}\right)^{5/3}T^{-2}\ \mathrm{g\,cm^{-1}s^{-1}}. \quad (28)$$

Here α_S is the strong fine structure constant.[4] Investi-

[2]When $\tau_D < \tau$ the use of ζ for determining the damping is questionable—a detailed treatment at the microscopic level seems required.

[3]Neutrino cooling is not efficient on the damping time scales of interest here.

[4]Recent calculations [14] lead to a slightly different expression, $\eta \propto \alpha_S^{-5/3}\rho^{14/9}T^{-5/3}$.

gations by Colpi and Miller [10] indicated that the bulk viscosity dominates over shear viscosity for temperatures exceeding 0.01 MeV, when it comes to calculate the maximum rotation frequency of a strange star (or neutron star with a large quark core), but the new bulk viscosities derived above are decisive at much lower temperatures.

Choosing the parameters of Fig. 1, the bulk viscosity exceeds the shear viscosity for low-amplitude perturbations when $T > 10^{-4}$ MeV, whereas for high amplitudes bulk viscosity dominates even for very low T. Thus bulk viscosity is decisive for most parameters. Furthermore the value of the viscosity is several orders of magnitude larger than assumed in [10]. It therefore seems reasonable to expect that nonaxisymmetric instabilities will be suppressed, and the maximum rotation frequency of strange stars will be given by the Keplerian limit (see [15] for a review). Detailed numerical calculations like those in [10], including the new viscosities and effects of dissipative heating, are required to settle the issue.

IV. CONCLUSIONS

The bulk viscosity of strange quark matter has been calculated for a range of parameters. Three different regimes were recognized as a function of the perturbation amplitude, and analytical approximations were shown to fit this behavior. The viscosities derived are orders of magnitude larger than previously assumed. Consequences hereof are rapid damping of strange star pulsations, and suppression of nonaxisymmetric instabilities in rapidly rotating strange stars. The latter probably changes the minimum rotation period of pulsars from 1 to 0.6 ms (the Keplerian limit). More detailed, general relativistic calculations should be performed.

ACKNOWLEDGMENTS

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