Thermal bath and decoherence of Rindler spacetimes

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The Minkowski vacuum state is a thermal state as far as any uniformly accelerated observer is concerned. However, if we allow a thermal bath, or any system, to come to equilibrium with that thermal state, the state is not left as the Minkowski vacuum. The thermal bath, or the scatterer, destroys crucial coherences in the state across the horizon which destroy the character of the state.

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I. ACCELERATED OSCILLATOR AND THERMAL BATH

It is now well known that an accelerated observer in flat spacetime sees the vacuum state of some quantum field as a thermal bath with a temperature equal to [1]

$$T = \frac{\hbar a}{2\pi ck}. \tag{1}$$

In particular, an accelerated body will come to equilibrium with this temperature. Furthermore, an accelerated thermal bath at this temperature will be in equilibrium with the field. However, this does not mean that the body in equilibrium or the thermal bath does not affect the state. In particular, the vacuum state of the field is characterized by coherences which extend across the horizon of the accelerated observer. These coherences are altered (or destroyed) by the thermal bath of the body. This paper will calculate these effects for a simple harmonic oscillator in equilibrium with the accelerated temperature, and for a simple model of a heat bath in equilibrium with the accelerated temperature in a two-dimensional spacetime. The field of interest will be assumed to be a massless scalar field.

Recall that the response of the simplest type of "monopole" detector to a field ϕ to which it is coupled is given by the expression

$$P_{nm} = \int_{-\infty}^{\infty} e^{-i(E_n-E_m)(t-t')}\langle m|O(t)|n\rangle\langle\phi(y(t))\phi(y(t'))\rangle\langle n|O(t')|m\rangle dt\, dt' \tag{2}$$

to lowest order in the coupling between the detector and the field. Here $y(t)$ is the path of the detector in the spacetime, $O(t)$ is the coupling operator coupling the detector to the field ϕ, which may be time dependent, E_m is the energy of the of the initial state of the detector, E_n the energy of the final state of the detector, and P_{nm} is the probability that the detector will make a transition from energy eigenstate $|m\rangle$ to $|n\rangle$ due to the presence of the interaction with the field. The important point is that the response of the detector depends on the two-point function $\langle\phi(y)\phi(y')\rangle$. By an appropriate choice of $O(t)$ and of the path of the detector, the probability of transition is thus a measure of this two-point function. Any changes in this two-point function can therefore be detected by their change in the transition probabilities. We will concentrate on calculating this two-point function when we couple the field to a heat bath via an accelerated harmonic oscillator. This thus is a generalization of the situation investigated by Raine *et al.* [2].

The spacetime metric will be given by

$$ds^2 = dt^2 - dz^2 = \frac{\rho^2}{\rho_0^2}d\tau^2 - d\rho^2 = dT^2 - \frac{T^2}{\rho_0^2}dR^2, \tag{3}$$

where $\tau,\ \rho$ are coordinates related to $t,\ z$ by

$$\rho = \mathrm{sgn}(z)\,[z^2 - t^2]^{\frac{1}{2}}, \tag{4}$$

$$\tau = \frac{\rho_0}{2}\ln\frac{t+z}{z-t} \tag{5}$$

in the region where $|z| > |t|$, and $T,\ R$ are given by

$$T = \mathrm{sgn}(t)\,[t^2 - z^2]^{\frac{1}{2}}, \tag{6}$$

$$R = \frac{\rho_0}{2}\ln\frac{z+t}{t-z} \tag{7}$$

in the region where $|t| > |z|$. Note that τ, ρ coordinates cover only half of the full Minkowski spacetime, namely those regions which have a spacelike separation from $t = z = 0$, while $T,\ R$ cover those regions with a timelike separation from 0, 0.

The quantum scalar field ϕ obeys

$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial z^2} = \rho_0\frac{\partial^2\phi}{\partial\tau^2} - \frac{\partial^2\phi}{\partial(\ln\rho)^2} = \frac{\partial^2\phi}{\partial\ln^2 T} - \rho_0^2\frac{\partial^2\phi}{\partial R^2} = 0 \tag{8}$$

with the usual expansion into normal modes of

$$\begin{aligned}\phi &= \int \left(a_{\Omega k}e^{-i(\Omega t-z)} + a^\dagger_{\Omega k}e^{i(\Omega t-kz)}\right)\frac{dk}{\sqrt{2\pi\Omega}}\\ &= \int \Theta(\rho)\left(b_{\omega\kappa+}e^{-i(\omega\tau-\kappa\rho_0\ln\rho)} + b^\dagger_{\omega\kappa+}e^{i(\omega\tau-\kappa\rho_0\ln\rho)}\right)\frac{d\kappa}{\sqrt{2\pi\omega}}\\ &\quad + \int \Theta(-\rho)\left(b^\dagger_{\omega\kappa-}e^{-i(\omega\tau-\kappa\rho_0\ln|\rho|)} + b_{\omega\kappa-}e^{i(\omega\tau-\kappa\rho_0\ln|\rho|)}\right)\frac{d\kappa}{\sqrt{2\pi\omega}}\end{aligned} \tag{9}$$

or

$$\begin{aligned}\phi &= \int_{\kappa<0} \Theta(T)\left(b_{\omega\kappa+}e^{-i(\omega R-\kappa\rho_0\ln T)} + b^\dagger_{\omega\kappa+}e^{i(\omega R-\kappa\rho_0\ln T)}\right)\frac{d\kappa}{\sqrt{2\pi|\kappa|}}\\ &\quad + \int_{\kappa>0} \Theta(T)\left(b^\dagger_{\omega\kappa-}e^{-i(\omega R-\rho_0\kappa\ln T)} + b_{\omega\kappa-}e^{i(\omega R-\kappa\rho_0\ln T)}\right)\frac{d\kappa}{\sqrt{2\pi|\kappa|}}\\ &\quad + \int_{\kappa>0} \Theta(-T)\left(b_{\omega\kappa+}e^{-i(\omega R-\kappa\rho_0\ln|T|)} + b^\dagger_{\omega\kappa+}e^{i(\omega R-\kappa\rho_0\ln|T|)}\right)\frac{d\kappa}{\sqrt{2\pi\omega}}\\ &\quad + \int_{\kappa<0} \Theta(-T)\left(b^\dagger_{\omega\kappa-}e^{-i(\omega R-\kappa\rho_0\ln|T|)} + b_{\omega\kappa-}e^{i(\omega R-\kappa\rho_0\ln|T|)}\right)\frac{d\kappa}{\sqrt{2\pi|\kappa|}},\end{aligned} \tag{10}$$

where the solutions in terms of the two sets of Rindler coordinates are assumed to be valid only in the coordinate patches which the coordinates cover. Here we have

$$\Omega = |k|, \qquad \omega = |\kappa|. \tag{11}$$

Note that in these expressions, we have to take into account the fact that the times τ and $\ln|T|$ run backwards in time for $\rho < 0$ and $T < 0$, respectively.

The Minkowski vacuum is now defined by

$$a_{\omega k}|0\rangle = 0. \tag{12}$$

This condition is equivalent to the conditions

$$\begin{aligned}\left[e^{\pi\Omega\rho_0/2}b_{\Omega\kappa+} + e^{-\pi\Omega\rho_0/2}b^\dagger_{\Omega\kappa-}\right]|0\rangle &= 0,\\ \left[e^{-\pi\Omega\rho_0/2}b^\dagger_{\Omega\kappa+} + e^{\pi\Omega\rho_0/2}b_{\Omega\kappa-}\right]|0\rangle &= 0\end{aligned} \tag{13}$$

because those particular combinations of $b, b^\dagger$ can be written solely in terms of the annihilation operators a.

Now let us introduce an accelerated harmonic oscillator into the spacetime, located at the position, $\rho = \rho_0$. This harmonic oscillator is defined to have an internal configuration variable q, and is coupled to the field ϕ via a coupling

$$I_{\rm int} = \epsilon\int q\frac{d\phi(\tau,\rho_0)}{d\tau}d\tau, \tag{14}$$

where I is the action. The action for the oscillator is assumed to be given by

$$I_{\rm osc} = \int \frac{1}{2}\left\{\left(\frac{dq}{d\tau}\right)^2 - q^2\right\}d\tau, \tag{15}$$

where I have chosen the units so that the frequency of the oscillator is unity. Note that I have also chosen the coordinate τ so that it corresponds to the proper time along the path $\rho = \rho_0$.

In addition, we will couple the oscillator to a heat bath of temperature $\mathbf{T} = \frac{1}{2\pi\rho_0}$, which is the temperature corresponding to the acceleration of the oscillator located at position ρ_0. The heat bath will be taken to be a massless one-dimensional scalar field ψ traveling in the internal space with coordinates τ,ζ with action

$$I_{hb} = \int\left\{\int\frac{1}{2}\left(\frac{d\psi}{d\tau}^2 - \frac{d\psi}{d\zeta}^2\right)\right\}d\zeta. \tag{16}$$

We expand ψ in terms of creation and annihilation operators by

$$\psi(\tau,\zeta) = \int\left\{c_{\nu\lambda}e^{-i(\nu\tau-\lambda\zeta)} + c^\dagger_{\nu\lambda}e^{i(\nu\tau-\lambda\zeta)}\right\}\frac{d\lambda}{\sqrt{2\pi\nu}}, \tag{17}$$

where

$$\nu = |\lambda|. \tag{18}$$

The state of the ψ field will be taken to be a thermal state, with temperature $\mathbf{T} = 1/2\pi\rho_0$, so that

$$\begin{aligned}\langle c_{\nu\lambda}c_{\nu'\lambda'}\rangle &= \langle c^\dagger_{\nu\lambda}c^\dagger_{\nu'\lambda'}\rangle = 0,\\ \langle c^\dagger_{\nu\lambda}c_{\nu'\lambda'}\rangle &= \coth(-2\pi\rho_0\nu)\delta(\lambda-\lambda').\end{aligned} \tag{19}$$

Finally, we couple the oscillator to the heat bath as well via the coupling

$$I_{\rm hbint} = \int \mu q(\tau)\frac{d\psi(\tau,0)}{d\tau}d\tau. \tag{20}$$

We can now solve the equations of motion for these fields completely. Let Φ, Ψ be the solutions to the coupled equations, and ϕ, ψ be the free field solutions given above. Then we have

$$\Psi(\tau,\zeta) = \psi(\tau,\zeta) - \frac{\mu}{2} q(\tau - |\zeta|), \tag{21}$$

$$\Phi(\tau,\rho) = \phi(\tau,\rho) - \frac{\epsilon}{2}\Theta(\rho) q\left(\tau - \rho_0 \left|\ln\frac{\rho}{\rho_0}\right|\right), \tag{22}$$

$$\Phi(T,R) = \phi(T,R) + \Theta(T)\,\frac{\epsilon}{2} q\left(\rho_0 \ln\frac{T}{\rho_0} + R\right), \tag{23}$$

$$\frac{d^2q}{d\tau^2} + \frac{\epsilon^2+\mu^2}{2}\frac{dq}{d\tau} + q = \frac{d}{d\tau}\left[\epsilon\phi(\tau,\rho_0) + \mu\psi(\tau,0)\right]. \tag{24}$$

This last equation can be solved easily. I will assume that the coupling has been in place forever, so that the q of the oscillator is completely determined by the external fields

$$\begin{aligned} q(\tau) &= \int_{-\infty}^{\tau} \frac{\sin[W(\tau-\tilde\tau)]}{W} e^{-\gamma(\tau-\tilde\tau)}\left(\epsilon\frac{d\phi(\tilde\tau,\rho_0)}{d\tilde\tau} + \mu\frac{d\psi(\tilde\tau,0)}{d\tilde\tau}\right) \\ &= \int_{-\infty}^{\infty}\left(\frac{i\omega}{-\omega^2+2i\gamma\omega+1}\left(\epsilon b^\dagger_{\omega\kappa+} + \mu c^\dagger_{\omega\kappa}\right)\right) e^{i\omega\tau}\frac{d\kappa}{\sqrt{2\pi\omega}} \quad + \quad \text{H.c.}, \end{aligned} \tag{25}$$

where $W = \sqrt{1-\gamma^2}$ and $\gamma = \epsilon^2 + \mu^2/4$.

Now let us examine the two-point function $\langle\Phi(y)\Phi(y')\rangle$. The claim made by Raine, Sciama, and Grove [2] (see also Grove [5]) is that the presence of the thermal bath (or of the oscillator on its own, which is just the $\mu \to 0$ limit) will not change this two-point function, i.e., that there are no observable effects of the presence of an accelerated detector on the system if it is in equilibrium with the thermal radiation. To assist in this calculation, let us first look at $\langle q(\eta)q(\eta')\rangle$ and at $\langle\phi(\tau,\rho), q(\eta)\rangle$. We have

$$\begin{aligned} \langle q(\eta)q(\eta')\rangle &= \int_{-\infty}^{\eta}\frac{\sin[W(\eta-\tilde\tau)]}{W}e^{-\gamma(\eta-\tilde\tau)}\int_{-\infty}^{\eta'}\frac{\sin[W(\eta'-\tilde\tau')]}{W}e^{-\gamma(\eta'-\tilde\tau')} \\ &\qquad\times\left(\epsilon^2\langle\dot\phi(\tilde\tau,\rho_0)\dot\phi(\tilde\tau',\rho_0)\rangle + \mu^2\langle\dot\psi(\tilde\tau,0)\dot\psi(\tilde\tau',0)\rangle\right)d\tilde\tau\, d\tilde\tau' \\ &= \int_{-\infty}^{\infty}\left|\frac{i\omega}{-\omega^2-2i\gamma\omega+1}\right|^2 e^{i\omega(\eta-\eta')}2(\epsilon^2+\mu^2)\left(\frac{1}{e^{2\pi|\omega|\rho_0}-1}+\Theta(-\omega)\right)\frac{d\omega}{2\pi|\omega|}. \end{aligned} \tag{26}$$

Writing

$$\Sigma(\omega) = \frac{i\omega}{-\omega^2+2i\gamma\omega+1}, \tag{27}$$

$$\Lambda(\omega) = \left|\frac{1}{e^{\pi\omega\rho_0}-e^{-\pi\omega\rho_0}}\right|, \tag{28}$$

this can be written as

$$\langle q(\eta)q(\eta')\rangle = 8\gamma\int_{-\infty}^{\infty}|\Sigma(\omega)|^2 e^{i\omega(\eta-\eta')}e^{-\pi\omega\rho_0}\Lambda(\omega)\frac{d\omega}{2\pi|\omega|}. \tag{29}$$

The other expectation values we need are

$$\langle\phi(\tau,\rho)q(\tau')\rangle = \epsilon\int_{-\infty}^{\infty}\Sigma(-\omega)\left(e^{i\omega(\tau-\tau'+\rho_0\ln\frac{\rho}{\rho_0})} + e^{i\omega(\tau-\tau'-\rho_0\ln\frac{\rho}{\rho_0})}\right)e^{-\pi\omega\rho_0}\Lambda(\omega)\frac{d\omega}{2\pi|\omega|} \tag{30}$$

for $\rho > 0$,

$$\langle\phi(\tau,\rho)q(\tau')\rangle = \epsilon\int_{-\infty}^{\infty}\Sigma(-\omega)\left(e^{i\omega(\tau-\tau'+\rho_0\ln\frac{\rho}{\rho_0})} + e^{i\omega(\tau-\tau'-\rho_0\ln\frac{\rho}{\rho_0})}\right)\Lambda(\omega)\frac{d\omega}{2\pi|\omega|} \tag{31}$$

for $\rho < 0$ and

$$\langle\phi(T,R)q(\tau')\rangle = \epsilon\int_{-\infty}^{\infty}\Sigma(-\omega)\left(e^{i\omega[R-\tau'+\sigma(T)\rho_0\ln\frac{|T|}{\rho_0}]}e^{-\pi\omega\rho_0} + e^{i\omega[R-\tau'-\sigma(T)\rho_0\ln\frac{|T|}{\rho_0}]}\right)\Lambda(\omega)\frac{d\omega}{2\pi|\omega|} \tag{32}$$

for the T, R region of spacetime, where $\sigma(x) = \Theta(x) - \Theta(-x)$.

We thus are left to calculate the expectation value of the two-point function $\langle\Phi(y),\Phi(y')\rangle$ for y, y' in different regions of the Rindler spacetime. For both y, y' in the region of either $T < 0$ or of $\rho < 0$, the expectation value will be unchanged from the one with no interaction with the heat bath. In the remaining regions, we will again calculate the relevant expectation values. We will however calculate them, not in the τ, ρ or the T, R coordinates, but rather in the U, V coordinates, a set of Minkowski null coordinates, where

$$U = -\mathrm{sgn}(\rho)\exp\left[-\frac{\tau}{\rho_0} + \ln\left(\frac{\rho}{\rho_0}\right)\right] = \mathrm{sgn}(T)\exp\left(\ln\frac{T}{\rho_0} - \frac{R}{\rho_0}\right),$$

$$V = \mathrm{sgn}(\rho)\exp\left[\frac{\tau}{\rho_0} + \ln\left(\frac{\rho}{\rho_0}\right)\right] = \mathrm{sgn}(T)\exp\left(\ln\frac{T}{\rho_0} + \frac{R}{\rho_0}\right). \tag{33}$$

Note that the metric in these coordinates is given by

$$ds^2 = \rho_0^2 dU\, dV. \tag{34}$$

I will furthermore restrict my attention to the case in which both of the points y, y' lie to the left of the path of the oscillator, i.e., $\rho < \rho_0$. In that case we have

$$\langle\Phi(y)\Phi(y')\rangle = \langle\phi(y)\phi(y')\rangle - \frac{\epsilon}{2}\{\Theta(V')\langle\phi(y)q[\rho_0\ln(V')]\rangle - \Theta(V)\langle q[\rho_0\ln(V)]\phi(y')\rangle\}$$
$$+\frac{\epsilon}{2}^2\Theta(V)\Theta(V')\langle q[\rho_0\ln(V)]q[\rho_0 ln(V')]\rangle. \tag{35}$$

Expressing these various terms, we have

$$\Theta(V)\Theta(V')\langle q(\rho_0\ln V)q(\rho_0\ln V')\rangle$$
$$= \Theta(V)\Theta(V')8\gamma\int_{-\infty}^{\infty}|\Sigma(\omega)|^2\left(\frac{V}{V'}\right)^{i\omega\rho_0}e^{-\pi\omega\rho_0}\Lambda(\omega)\frac{d\omega}{2\pi|\omega|}$$
$$= \lim_{\delta,\delta'\to 0} 8\gamma\int_{-\infty}^{\infty}|\Sigma(\omega)|^2\left[e^{\pi\omega\rho_0}(V+i\delta)^{i\omega\rho_0} - e^{-\pi\omega\rho_0}(V-i\delta)^{i\omega\rho_0}\right]$$
$$\times\left[e^{-\pi\omega\rho_0}(V'+i\delta')^{-i\omega\rho_0} - e^{\pi\omega\rho_0}(V'-i\delta')^{-i\omega\rho_0}\right]e^{-\pi\rho_0\omega}\Lambda(\omega)^3\frac{d\omega}{2\pi|\omega|} \tag{36}$$

and

$$\Theta(V')\langle\phi(U,V)q(\rho_0\ln V')\rangle = \Theta(V')\langle q(\rho_0\ln V')\phi(U,V)\rangle^*$$
$$= \lim_{\delta,\delta'\to 0}\epsilon\int_{-\infty}^{\infty}\Sigma(-\omega)e^{-\pi\omega\rho_0}\left[(V-i\delta)^{i\omega\rho_0} + (-U+i\delta)^{-i\omega\rho_0}\right]$$
$$\times\left[-e^{-\pi\omega\rho_0}(V'+i\delta')^{-i\omega\rho_0} + e^{\pi\omega\rho_0}(V'-i\delta')^{-i\omega\rho_0}\right]e^{-\pi\omega\rho_0}\Lambda(\omega)^2\frac{d\omega}{2\pi|\omega|}. \tag{37}$$

The important point is that the function $\lim_{\delta\to 0}(V - i\delta)^{i\omega\rho_0}$ is a positive-frequency Minkowski function (analytic in the lower half complex V plane) and $\lim_{\delta\to 0}(V + i\delta)^{i\omega\rho_0}$ is negative frequency for all values of ω. Thus we see that $\langle\phi(U,V)q(\rho_0\ln V')\rangle$ is purely positive frequency in U, V and a mixture in U', V' while $\Theta(V)\Theta(V')\langle q[\rho_0\ln(V)]q[\rho_0 ln(V')]\rangle$ is a mixture of positive and negative frequencies in both U, V and U', V'. Thus, an inertial detector, whose response is determined to lowest order by the expression

$$\int\int e^{-iE(t-t')}\langle\Phi[y(t)]\Phi[y(t')]\rangle dt\, dt', \tag{38}$$

where E is the energy difference between the lower and the upper state of the detector, will respond if $\langle\Phi[y(t)]\Phi[y(t')]\rangle$ contains positive-frequency components in y and negative-frequency components in y'. It is the $\langle qq\rangle$ term in $\langle\Phi[y(t)]\Phi[y(t')]\rangle$ which contains such terms. Thus an inertial particle detector will respond nontrivially to the presence of an accelerated heat bath, or to the presence of an accelerated harmonic scatterer. This conclusion is not limited to the two-dimensional model, but is also true in the full four-dimensional situation as well.

It is of interest to calculate the energy-momentum expectation value for the scalar field in this situation as well. The energy-momentum tensor can be calculated by using the above two-point function $\langle\Phi(y)\Phi(y')\rangle$. In particular we have

$$T_{UU} = \lim_{y\to y'}\langle\Phi(y)_{,U}\,\Phi(y')_{,U}\rangle - \lim_{y\to y'}\langle\phi(y)_{,U}\,\phi(y')_{,U}\rangle, \tag{39}$$

$$T_{VV} = \lim_{y\to y'}\langle\Phi(y)_{,V}\,\Phi(y')_{,V}\rangle - \lim_{y\to y'}\langle\phi(y)_{,V}\,\phi(y')_{,V}\rangle, \tag{40}$$

$$T_{UV} = 0, \tag{41}$$

where I have renormalized the expression so that the expectation value in the Minkowski vacuum state is zero.

Let us first calculate $\langle\Phi(y)\Phi(y')\rangle$ where the points y, y' are close to each other. We have

$$\begin{aligned}
&\langle\Phi(U,V)\Phi(U',V')\rangle - \langle\phi(U,V)\phi(U',V')\rangle \\
&= -\frac{\epsilon}{2}\left[\Theta(V')\left\langle\phi(U,V)q\left(\rho_0\ln\frac{V'}{\rho_0}\right)\right\rangle + \Theta(V)\left\langle q\left(\rho_0\ln\frac{V}{\rho_0}\right)\phi(U',V')\right\rangle\right] \\
&\quad\times\frac{\epsilon^2}{4}\Theta(V')\Theta(V)\left\langle q\left(\rho_0\ln\frac{V}{\rho_0}\right)q\left(\rho_0\ln\frac{V'}{\rho_0}\right)\right\rangle \\
&= -\frac{\epsilon^2}{2}\lim_{\delta,\delta'\to 0}\int_{-\infty}^{\infty}\Sigma(-\omega)\left[(V-i\delta)^{i\omega\rho_0}+(-U+i\delta)^{-i\omega\rho_0}\right] \\
&\qquad\times\left[-e^{-\pi\omega\rho_0}(V'+i\delta')^{-i\omega\rho_0}+e^{\pi\omega\rho_0}(V'-i\delta')^{-i\omega\rho_0}\right]e^{-\pi\omega\rho_0}\Lambda(\omega)^2\frac{d\omega}{2\pi|\omega|} \\
&\quad-\frac{\epsilon^2}{2}\lim_{\delta,\delta'\to 0}\int_{-\infty}^{\infty}\Sigma(\omega)\left[(V'+i\delta)^{-i\omega\rho_0}+(-U'-i\delta)^{i\omega\rho_0}\right] \\
&\qquad\times\left[-e^{-\pi\omega\rho_0}(V-i\delta')^{i\omega\rho_0}+e^{\pi\omega\rho_0}(V+i\delta')^{i\omega\rho_0}\right]e^{-\pi\omega\rho_0}\Lambda(\omega)^2\frac{d\omega}{2\pi|\omega|} \\
&\quad+\epsilon^2 2\gamma\lim_{\delta,\delta'\to 0}\int_{-\infty}^{\infty}|\Sigma(\omega)|^2\left[e^{\pi\omega\rho_0}(V+i\delta)^{i\omega\rho_0}-e^{-\pi\omega\rho_0}(V-i\delta)^{i\omega\rho_0}\right] \\
&\qquad\times\left[e^{-\pi\omega\rho_0}(V'+i\delta')^{-i\omega\rho_0}-e^{\pi\omega\rho_0}(V'-i\delta')^{-i\omega\rho_0}\right]e^{-\pi\rho_0\omega}\Lambda(\omega)^3\frac{d\omega}{2\pi|\omega|}.
\end{aligned} \tag{42}$$

If V, V' are both greater than zero (i.e., we are interested only in the region in causal contact with the oscillator), this becomes

$$\begin{aligned}
&\langle\Phi(U,V)\Phi(U',V')\rangle - \langle\phi(U,V)\phi(U',V')\rangle \\
&= -\frac{\epsilon^2}{2}\int_{-\infty}^{\infty}\Sigma(-\omega)\left[(V)^{i\omega\rho_0}+(-U)^{-i\omega\rho_0}\right]\left[(V')^{-i\omega\rho_0}\right]e^{-\pi\omega\rho_0}\Lambda(\omega)\frac{d\omega}{2\pi|\omega|} \\
&\quad-\frac{\epsilon^2}{2}\int_{-\infty}^{\infty}\Sigma(\omega)\left[(V')^{-i\omega\rho_0}+(-U')^{i\omega\rho_0}\right]\left[(V)^{i\omega\rho_0}\right]e^{-\pi\omega\rho_0}\Lambda(\omega)\frac{d\omega}{2\pi|\omega|} \\
&\quad+\epsilon^2 2\gamma\int_{-\infty}^{\infty}|\Sigma(\omega)|^2\left[(V)^{i\omega\rho_0}\right]\left[(V')^{-i\omega\rho_0}\right]e^{-\pi\rho_0\omega}\Lambda(\omega)\frac{d\omega}{2\pi|\omega|} \\
&= \epsilon^2\int_{-\infty}^{\infty}\left(-\frac{\Sigma(\omega)+\Sigma(-\omega)}{2}+2\gamma|\Sigma(\omega)|^2\right)\left(\frac{V}{V'}\right)^{i\omega\rho_0}e^{-\pi\omega\rho_0}\Lambda(\omega)\frac{d\omega}{2\pi|\omega|} \\
&\quad-\frac{\epsilon^2}{2}\int_{-\infty}^{\infty}\left(\Sigma(-\omega)(-UV')^{-i\omega\rho}+\Sigma(\omega)(-VU')^{i\omega\rho}\right)e^{-\pi\omega\rho_0}\Lambda(\omega)\frac{d\omega}{2\pi|\omega|} \\
&= -\frac{\epsilon^2}{2}\int_{-\infty}^{\infty}\left(\Sigma(-\omega)(-UV')^{-i\omega\rho}+\Sigma(\omega)(-VU')^{i\omega\rho}\right)e^{-\pi\omega\rho_0}\Lambda(\omega)\frac{d\omega}{2\pi|\omega|}.
\end{aligned} \tag{43}$$

In the region where $V, V' < 0$, we have

$$\langle\Phi(U,V)\Phi(U',V')\rangle - \langle\phi(U,V)\phi(U',V')\rangle = 0. \tag{44}$$

Thus in both of these regions

$$T_{UU} = T_{VV} = 0. \tag{45}$$

However, on the boundary, $V = 0$, one must be more careful. Again we have that $T_{UU} = 0$. However, T_{VV} diverges as $1/\delta\delta'$ for nonzero δ's. On the other hand, T_{VV} is nonzero over a region ΔV of width roughly the minimum of δ or δ'. Thus the divergence is stronger than that of a δ function. Thus in this problem there is an infinite energy density of radiation along the past horizon $V = 0$. This arises because the thermal bath and the acceleration radiation have been in equilibrium for all times. Had the detector been switched on at some finite time in the past, one would have had a finite band in spacetime in which the energy-momentum tensor was nonzero, corresponding to the time during which the coupling between the oscillator and the radiation field ϕ was switched on and the oscillator came into equilibrium with the field.

However, even though $T_{\mu\nu}$ is trivial over most of the spacetime, the same is not true of other quadratic functions. In particular, the expectation value of the field squared, again renormalized so that its value is zero in the Minkowski vacuum state, is not trivial even within the region near the oscillator. We have, in the region $V > 0$,

$$\begin{aligned}\langle\Phi(U,V)^2\rangle-\langle\phi(U,V)^2\rangle&=-\frac{\epsilon^2}{2}\int_{-\infty}^{\infty}\left(\Sigma(\omega)\,(-UV)^{-i\omega\rho_0}+\Sigma(-\omega)\,(-UV)^{i\omega\rho_0}\right)e^{-\pi\omega\rho_0}\Lambda(\omega)\frac{d\omega}{2\pi|\omega|}\\&=\frac{\epsilon^2}{2}\int_{-\infty}^{\infty}\Sigma(\omega)\,(-UV)^{-i\omega\rho_0}\left(e^{\pi\omega\rho_0}+e^{-\pi\omega\rho_0}\right)\Lambda(\omega)\frac{d\omega}{2\pi|\omega|}\\&=\frac{\epsilon^2}{2}\int_{-\infty}^{\infty}\Sigma(\omega)\,(-UV)^{-i\omega\rho_0}\,\sigma(\omega)\coth(\pi\omega\rho_0)\frac{d\omega}{2\pi\omega}\\&\neq 0.\end{aligned} \tag{46}$$

Recall that $m^2\langle\Phi(U,V)^2\rangle$ is the trace of the energy-momentum tensor for a scalar field with mass m, and that this expression would be expected to be a good approximation for the trace of the energy momentum tensor if the field had a very small mass.

We thus see that the presence of the thermal bath, or of a scatterer ($\mu=0$), does affect the field even if the thermal bath or the scatterer is in complete thermal equilibrium with the acceleration radiation.

Where has the disagreement with Raine *et al.* [3] come from? The answer is subtle. If we switch on the inertial detector for all times, the detector will respond to terms in $\langle\Phi(y)\Phi(y')\rangle$ which are positive frequency in y and negative in y' as mentioned above. In this case the response arises from the behavior of the $\langle\Theta(V)\Theta(V')<q(\rho_0\ln V)q(\rho_0\ln V')\rangle$ term. If on the other hand the detector is switched on only while it is in the region $V>0$, then the terms in $\langle\Phi(y)\Phi(y')\rangle$ which depend on V,V' in fact cancel, as we saw in the calculation of $T_{VV'}$ above. In that case it is the terms which depend on U,V' or V,U' in the cross terms between ϕ and q which excite the detector. The detector response is then proportional to

$$\int\int\epsilon(t)\epsilon(t')e^{-iE(t-t')}\langle\Phi(t)\Phi(t')\rangle dt\,dt'. \tag{47}$$

Now because of the presence of the nontrivial time dependence of the ϵ the detector will respond not only to the positive, but also the negative-frequency parts of Φ. Thus if one wishes the inertial detector to operate only in the $V>0$ region (for example, to escape from the burst of radiation along the $V=0$ surface expected because of the radiation along this line expected from the oscillator/heat bath coming into equilibrium with the thermal acceleration field in the far past), it will still respond differently from the way it would in the Minkowski vacuum state. Any inertial detector will see the system with a heat bath as different from the Minkowski unless it is entirely confined to the region out of causal contact with the heat bath ($V<0$). I would note the similarity of this line of argument to that of Grove [5] in the context of the radiation from moving mirrors.

II. QUANTUM EQUIVALENCE PRINCIPLE

As an extension of these results, we can also ask a question which was asked by Candelas and Sciama [3] and Grove [4]. Can an inertial observer who looks at the scalar field ϕ only within the Rindler wedge, $\rho>0$, tell the difference between the Minkowski or the Rindler vacuum state? The Rindler vacuum is defined by the condition that

$$b_{\omega\kappa+}|0\rangle_R=b_{\omega\kappa-}|0\rangle_R=0. \tag{48}$$

To answer this question we calculate the expectation value of the two-point function $\phi(U,V)\phi(U',V')$ in the two states. The two-point function for a massless scalar field in two dimensions in the Minkowski vacuum is well known to be given by

$$\langle\phi(U,V)\phi(U',V')\rangle \tag{49}$$

$$=i[\Theta(U-U')\Theta(V-V')-\Theta(U'-U)\Theta(V'-V)]+[\ln(U-U')+\ln(V-V')+2\ln\rho_0]. \tag{50}$$

The expectation value in the Rindler vacuum on the other hand is given by

$$\begin{aligned}\langle\phi(U,V)\phi(U',V')\rangle_R=\int_0^{\infty}\Bigg[&\Theta(U)\Theta(U')\left(\frac{U}{U'}\right)^{-i\omega\rho_0}+\Theta(-U)\Theta(-U')\left(\frac{U}{U'}\right)^{-i\omega\rho_0}\\&+\Theta(V)\Theta(V')\left(\frac{V}{V'}\right)^{i\omega\rho_0}+\Theta(-V)\Theta(-V')\left(\frac{V}{V'}\right)^{i\omega\rho_0}\Bigg]\frac{d\omega}{2\pi|\omega|},\end{aligned} \tag{51}$$

where I have used the relations that

$$e^{i\omega(\tau-\rho_0\ln|\rho|)}=[-\sigma(\rho)\rho_0U]^{-i\omega\rho_0}, \tag{52}$$

$$e^{i\omega(\tau+\rho_0\ln|\rho|)}=\left(\frac{\sigma(\rho)V}{\rho_0}\right)^{i\omega\rho_0}. \tag{53}$$

Integrating, we get

$$\begin{aligned}&\langle\phi(U,V)\phi(U',V')\rangle_R\\&=i[\Theta(U-U')\Theta(V-V')-\Theta(U'-U)\Theta(V'-V)]\\&\quad+\Theta\left(\frac{U}{U'}\right)\ln\left(-\ln\frac{U}{U'}\right)+\Theta\left(\frac{V}{V'}\right)\ln\left(\ln\frac{V}{V'}\right),\end{aligned} \tag{54}$$

which certainly differs from the Minkowski expression everywhere. An inertial detector will respond differently in the two states, even if the detector is switched on only in the region $\rho > 0$. This conclusion differs from that of Candelas and Sciama [3] it seems because of the limits which they take and the terms they throw away as "transients" which are really expected time-dependent terms in the response of the detectors. These results agree with those of Grove [4], who analyzed this problem earlier from a slightly different point of view.

As might be expected from the boost invariance of the problem, the extra terms depend on Rindler coordinates only through ρ and not through τ. Thus, one might with some justification claim that these terms are tied to the oscillator and do not represent radiation from the oscillator, just as, for an electron in uniform acceleration, there is in conventional terms no radiation emitted from an electron in uniform acceleration, even though the electromagnetic field around the electron differs from that of a stationary electron. In two dimensions, the difference between a distortion of the field tied to the particle, and one which is in some sense independent of the particle (free radiation) is difficult to make. Even in three dimensions, what one means by radiation is difficult. One is on much safer ground asking not, does the oscillator radiate, but does it alter measurable properties of the field. This, as the above analysis shows, it does. Ascribing those changes to "radiation" or to "vacuum polarization" is terminology rather than physics, although the connotations of one or the other term may convey the physics of one or another physical situation more accurately.

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