Bag constant and deconfinement phase transition in a nontopological soliton model

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By calculating the effective potential of the nontopological soliton model at finite temperature and density, the temperature and the chemical potential dependence of the bag constant is obtained. The feature of deconfinement phase transition is analyzed. It turns out that, in this model, the deconfinement phase transition is of first order; the analysis of the phase transition feature suggests the coexistence of a deconfinement phase and a confinement phase.

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Because of the complications of the QCD vacuum, when investigating the thermal properties of QCD a common treatment is to assume that the thermodynamic potential of QCD is the summation of the thermodynamic potential resulting from perturbative QCD and the bag constant (which is independent of the temperature and chemical potential) representing the nonperturbative effects of QCD. Under the above assumption, some authors obtained many valuable results [1]. It must, however, be pointed out that there exists an obvious defect to consider: the bag constant is independent of the temperature and chemical potential. For example, when we calculate some important thermodynamic quantities such as entropy, the contribution of the nonperturbative effects (bag constant) is not contained in the calculation because of the derivation with respect to the temperature or chemical potential. The thermal influence of the nonperturbative effects is mainly embodied in the temperature and the chemical-potential dependence of the bag constant $B=B(T,\mu)$.

Recently, at finite temperature and density the research of the nontopological soliton model, initiated by Friedberg and Lee (FL) [2], has received a great deal of attention [3]. We notice that in the FL model the bag constant is connected with the difference between the effective potential at the perturbative vacuum state and the effective potential at the physical vacuum state, so it is more convenient to investigate the behavior of the bag constant at finite temperature and density in detail. This may lead to some new results.

The Lagrangian of the FL model has the form [2]

$$\mathcal{L}=\bar{\Psi}(i\gamma_\mu\partial^\mu-g\sigma)\Psi+\tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma-U(\sigma)\ , \tag{1a}$$

$$U(\sigma)=\frac{a}{2!}\sigma^2+\frac{b}{3!}\sigma^3+\frac{c}{4!}\sigma^4+B\ , \tag{1b}$$

where Ψ is the quark field, σ is the phenomenological scalar field, $U(\sigma)$ is the potential function, and g,a,b,c are four model parameters.

$U(\sigma)$ has the two minima and a local maximum. The local minimum corresponding to a perturbative vacuum state and the absolute minimum corresponding to a physical vacuum state are located at

$$\sigma_0=0\ , \tag{2a}$$

$$\sigma_v=\frac{3|b|}{2c}\left[1+\left[1-\frac{8ac}{3b^2}\right]^{1/2}\right]\ , \tag{2b}$$

respectively. The difference in potential of the two vacuum states is the bag constant B, which measures the pressure of the physical vacuum on the perturbative vacuum. If we take $U(\sigma_v)=0$, then the bag constant can be expressed as

$$-B=\frac{a}{2!}\sigma_v^2+\frac{b}{3!}\sigma_v^3+\frac{c}{4!}\sigma_v^4\ . \tag{3}$$

Table I lists the values of B for eight sets of parameters.

By solving the equation of motion, the numerical calculation indicates that the σ field has a soliton solution which is of a spherical cavitylike structure, $\sigma\to\sigma_0=0$ inside the cavity and $\sigma\to\sigma_v$ outside, and the quarks are confined inside the cavity.

In order to investigate the temperature and the chemical-potential dependence of the bag constant and the features of the deconfinement phase transition, we first calculate the effective potential at finite temperature and density. The effective potential of the FL model at finite temperature and density may be expressed as

$$V(\sigma,T,\mu)=U(\sigma)+V_S(\sigma,T)+V_F(\sigma,T,\mu)\ , \tag{4}$$

where $U(\sigma)$ is the classical potential at zero temperature, $V_S(\sigma,T)$ is the effective potential of the scalar σ field at finite temperature, and $V_F(\sigma,T,\mu)$ is the contribution of the fermion field on the effective potential at finite temperature and density. As pointed out by Lee in Ref. [2], since σ is only a phenomenological field describing the long-range collective effects of QCD, its short-wave com-

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TABLE I. Values of T_{c1}, T_{c2} and μ_{c1}, μ_{c2} for eight sets of model parameters.

a (fm^{-2})	$-b$ (fm^{-1})	c	g	B (MeV/fm^3)	T_{c1} (MeV)	T_{c2} (MeV)	μ_{c1} (MeV)	μ_{c2} (MeV)
69.73	2112.6	20 000	12.416	21.7	83.7	135.4	216.2	540
188.86	7774.0	100 000	15.333	31.8	92.8	144.8	237.6	496
11.60	834.4	10 000	10.957	44.2	102.2	126.0	258.4	450
17.70	1457.4	20 000	12.16	51.4	106.9	130.0	268.3	467
45.21	5208.5	100 000	16.379	67.1	116.2	135.9	286.7	442
6.734	778.48	10 000	10.963	48.0	104.8	126.0	263.9	462
10.25	1358.4	20 000	12.211	55.7	109.6	129.8	273.7	439
26.12	4848.4	100 000	16.537	72.3	119.0	138.4	292.2	437

ponents do not exist in reality; as an approximation, all σ quantum loop diagrams may be ignored, so it is unnecessary to consider the quantum correction of the σ field in Eq. (4) for such a phenomenological model.

With the aid of the method [4] for calculating loop corrections in imaginary-time temperature field theory, the effective potential to one-loop correction may be expressed as

$$V(\sigma,T,\mu)=\frac{a}{2!}(\sigma^2-\sigma_v^2)+\frac{b}{3!}(\sigma^3-\sigma_v^3)+\frac{c}{4!}(\sigma^4-\sigma_v^4)-\frac{1}{6\pi^2\beta^4}\int_0^\infty \frac{x^4dx}{\sqrt{x^2+\beta^2m_\sigma^2}\,[\exp(\sqrt{x^2+\beta^2m_\sigma^2})-1]}$$
$$-\frac{\gamma}{6\pi^2\beta^4}\left[\int_0^\infty \frac{x^4dx}{\sqrt{x^2+\beta^2m_q^2}[\exp(\sqrt{x^2+\beta^2m_q^2}+\beta\mu)+1]}+\int_0^\infty \frac{x^4dx}{\sqrt{x^2+\beta^2m_q^2}[\exp(\sqrt{x^2+\beta^2m_q^2}-\beta\mu)+1]}\right], \tag{5}$$

where $\beta=1/T$ and $\gamma=2(\text{spin})\times2(\text{flavor})\times3(\text{color})$ is the total degeneracy factor for a quark. m_q and m_σ are the effective mass of the quark field and the σ field, respectively:

$$m_q=g\sigma(T)\ , \tag{6a}$$

$$m_\sigma^2=a+b\sigma(T)+\frac{c}{2}\sigma^2(T)\ . \tag{6b}$$

With the help of the method proposed in Ref. [5], we fix m_σ^2 by taking its value at the physical vacuum state.

When the chemical potential is fixed, the effective potential at different temperatures is illustrated in Fig. 1. Figure 2 shows the effective potential at different chemical potentials when fixing temperature. From these figures, we can see clearly that the difference between the effective potential at the perturbative vacuum state and the effective potential at the physical vacuum state decreases continuously with increasing temperature (or chemical potential). At a certain temperature T_{c1} (or chemical potential μ_{c1}), the effective potentials at the two vacuum states are equal; i.e., the bag constant becomes zero, which is the limiting case in which the soliton solu-

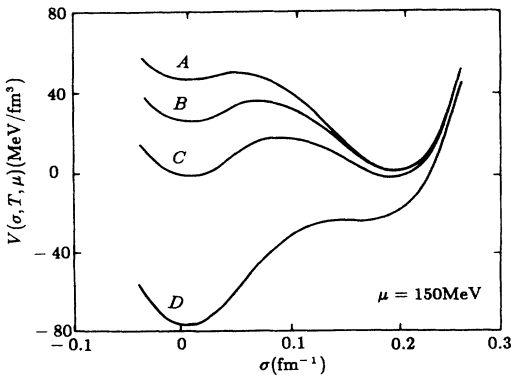

FIG. 1. Effective potential at different temperatures when fixing the chemical potential μ at 150 MeV. A: $T=10$ MeV, B: $T=60$ MeV, C: $T=T_{c1}=82$ MeV, D: $T=T_{c2}=112$ MeV. The following set of model parameters are taken: $a=17.70$ fm^{-2}, $b=-1457.4$ fm^{-1}, $c=2\times10^4$, $g=12.16$.

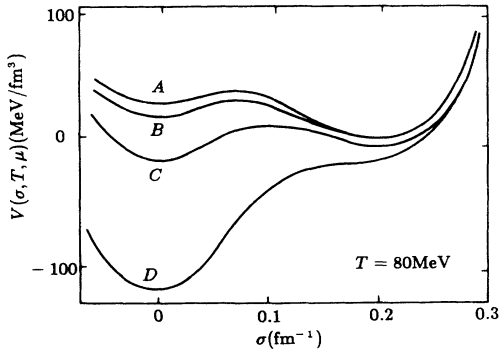

FIG. 2. Effective potential at different chemical potentials when fixing temperature T at 80 MeV. A: $\mu=50$ MeV, B: $\mu=100$ MeV, C: $\mu=\mu_{c1}=180$ MeV, D: $\mu=\mu_{c2}=280$ MeV. The same model parameters as in Fig. 1 are taken.

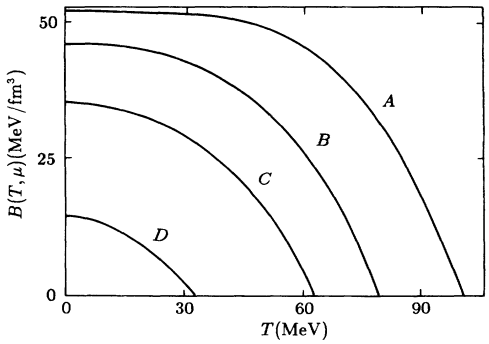

FIG. 3. Bag constant vs temperature at different chemical potentials. A: $\mu=0$, B: $\mu=150$ MeV, C: $\mu=200$ MeV, D: $\mu=250$ MeV.

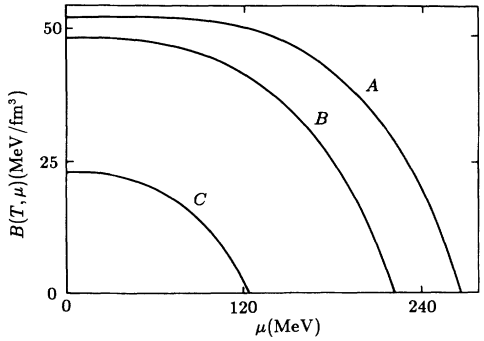

FIG. 4. Bag constant vs chemical potential at different temperatures. A: $T=2$ MeV, B: $T=50$ MeV, C: $T=90$ MeV.

tion of the original nontopological type still exists [3]. When $T>T_{c1}$ (or $\mu>\mu_{c1}$), the soliton solution above disappears; the deconfinement phase transition begins to occur. As the temperature (or chemical potential) approaches another higher temperature T_{c2} (or chemical potential μ_{c2}), the original physical vacuum state $\sigma_v(T)$ disappears, and the effective potential has a unique minimum which corresponds to the stable perturbative vacuum state σ_0; such a potential no longer ensures the existence of the soliton bag, and the confinement of the quarks is removed completely. When $T_{c1}<T<T_{c2}$ (or $\mu_{c1}<\mu<\mu_{c2}$), the original perturbative vacuum state σ_0 becomes a stable state and the original physical vacuum state σ_v becomes a metastable one. If the quark is at the stable state above, from Eq. (6a) its effective mass is zero, which indicates that it is situated in the deconfinement state. Otherwise, if the quark is at the metastable state, its effective mass is very large, so that it is in the confinement state. The coexistence of the stable state as a perturbative vacuum state and the metastable state as an original physical vacuum state shows the coexistence of the deconfinement phase and the confinement phase, which also tells us that the phase transition above is of first order.

For eight sets of parameters fitting the static properties of the hadrons, Table I lists the values of T_{c1},T_{c2} when the chemical potential is taken as zero and the values of μ_{c1},μ_{c2} at zero temperature.

From the effective potential above, it is convenient to investigate the temperature and the chemical-potential dependence of the bag constant. The bag constant in the FL model is defined as the difference between the effective potential at the perturbative vacuum state and the effective potential at the physical vacuum state; i.e.,

$$B(T,\mu)=V(\sigma_0,T,\mu)-V(\sigma_v,T,\mu)\ . \tag{7}$$

From effective potential (5) the temperature dependence of the bag constant is illustrated in Fig. 3 when the chemical potential is fixed. We can see clearly that the bag constant decreases continuously with increasing temperature. At temperature T_{c1}, $B(T_{c1})=0$ and the deconfinement phase transition begins to occur. According to the curve for $\mu=0$ in Fig. 3, the analytical formula of the bag constant as a function of temperature can be simulated by

$$B(T)=B-\alpha T^4-\delta T^2\ , \tag{8}$$

where B is the bag constant at zero temperature, $\alpha=3.016$, and $\delta=46.039$ MeV^2. Obviously, the second term in Eq. (8) corresponds to the contribution of the perfect gas, and the third term corresponds to the correction of the imperfect gas.

The bag constant versus chemical potential at different temperatures is shown in Fig. 4. Similarly to Fig. 3, we can see the deconfinement phase transition begin to take place at the chemical potential μ_{c1}. In the limiting case for low temperatures and high density ($\mu/T\gg1$) the thermal excitation of the σ field can be ignored, the effective potential $V(\sigma_v)=0$ at the physical vacuum state, and the bag constant, from Eqs. (5) and (7), can be deduced as

$$\begin{aligned}B(\mu,T)&=U(\sigma_0)+V_F(\sigma_0,T,\mu)\\&=B-\left[\frac{3}{4\pi^2}\mu^4+\frac{3}{2}T^2\mu^2+\frac{7\pi^2}{20}T^4\right]\ .\end{aligned} \tag{9}$$

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