Quark diagrams and the $\Delta I = \frac{1}{2}$ rule

Yoji Kohara
Nihon University at Fujisawa, Fujisawa, Kanagawa 252, Japan
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Nonleptonic decays of hyperons are analyzed on the basis of the quark-diagram scheme. It is shown that the external W-emission amplitudes and the internal W-emission amplitudes lead to the breaking of the $\Delta I = \frac{1}{2}$ rule. A sum rule among the decay amplitudes which include the $\Delta I = \frac{3}{2}$ effects is derived.

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Much theoretical effort has been made in order to understand the $\Delta I = \frac{1}{2}$ rule for $\Delta S = 1$, $\Delta C = 0$ weak nonleptonic decays [1]. It was shown that the Fierz transformation symmetry of the $V-A$ interaction and color antisymmetry of baryons lead to the $\Delta I = \frac{1}{2}$ rule [2]. As there remain some diagrams in which the $\Delta I = \frac{1}{2}$ rule cannot be concluded, other assumptions such as current algebra and partial conservation of axial-vector current are needed. Shifman *et al.* [3] asserted that the W-loop diagrams dominate these decays and lead to the $\Delta I = \frac{1}{2}$ rule. However, the magnitudes of W-loop amplitudes are not sufficient to explain the experimental data. The quark-diagram scheme gave a successful model-independent framework to analyze decays of charm and bottom particles [4,5], and it was pointed out that the W-exchange diagrams are as important as the W-loop diagrams for the $\Delta I = \frac{1}{2}$ dominance in $K \to 2\pi$ decays [6]. In this Brief Report we investigate the relation between quark diagrams and the breaking of the $\Delta I = \frac{1}{2}$ rule in nonleptonic decays of hyperons on the basis of the quark-diagram scheme.

Octet baryons can be expressed in terms of the wave functions in which two of the quarks are symmetric or antisymmetric in flavor. Whichever wave functions we use, the same physical results are obtained. It can be verified by actual calculations. However, the antisymmetric representation is more convenient for our purpose, so we use it here. The octet baryon wave function $B_{i[jk]}$ satisfies the Jacobi identity

$$B_{i[jk]} + B_{j[ki]} + B_{k[ij]} = 0 \ .$$

Therefore there are 15 independent diagrams in hyperon decays, as depicted in Fig. 1. They are classified into seven types. The brackets in the diagrams denote antisymmetric pairs of constituent quarks. In (c)-, (d)-, and (e)-type diagrams it was required that the quark pair participating in weak interaction is antisymmetric in one baryon. This is a result of the fact that the $(V-A)\times(V-A)$ structure of weak interactions is invariant under Fierz transformations and that the quarks in baryons are color antisymmetric [2].

The decay amplitudes obtained from these diagrams are given in Table I. The parameter a_1 corresponds to the diagram (a1) and so on. The final-state interactions were neglected here because they do not have a significant effect on these processes. These forms of the amplitudes hold for both S- and P-wave decays. The terms of c, d, and e satisfy the $\Delta I = \frac{1}{2}$ rule because of the antisymmetric property mentioned above. The terms of f and g also satisfy the $\Delta I = \frac{1}{2}$ rule, as they are from W-loop diagrams. The $\Delta I = \frac{3}{2}$ amplitudes arise from the external W-emission diagrams (a) and the internal W-emission diagrams (b). From Table I it is easily seen that the $\Delta I = \frac{1}{2}$ rule holds when the relations

$$\frac{a_i}{b_i} = -1 \quad (i=1,2)$$

are satisfied. The parton model which does not include perturbative QCD effects gives

$$\frac{a_i}{b_i} = 3 \quad (i=1,2)$$

because of color mismatching of the diagrams (b). When we write the effective Hamiltonian as

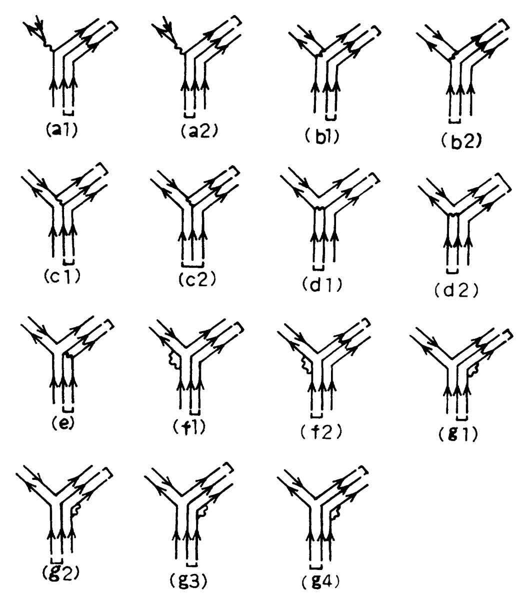

FIG. 1. 15 independent quark diagrams of nonleptonic decays of hyperons.

TABLE I. Nonleptonic decay amplitudes of hyperons in quark diagram scheme.

Decays	Amplitudes
Λ_0^0	$\frac{1}{2\sqrt{3}}(-2b_1-b_2-c_1-2c_2+d_1+d_2-e+4f_1+f_2+g_1+2g_2)$
Λ_-^0	$\frac{1}{\sqrt{6}}(-2a_1-a_2+c_1+2c_2-d_1-d_2+e-4f_1-f_2-g_1-2g_2)$
Ξ_0^0	$\frac{1}{2\sqrt{3}}(b_1+2b_2+2c_2-2f_1-2f_2-g_2+g_4)$
Ξ_-^-	$\frac{1}{\sqrt{6}}(a_1+2a_2-2c_2+2f_1+2f_2+g_2-g_4)$
Σ_+^+	$-d_1-d_2+e-g_1-g_3$
Σ_0^+	$\frac{1}{\sqrt{2}}(-b_2+c_1-d_1-d_2+e+f_2-g_1)$
Σ_-^-	$-a_2-c_1-f_2-g_3$

$$H_{\text{eff}}=\frac{G_F}{\sqrt{2}}V_{ud}^*V_{us}\left[\frac{c_++c_-}{2}(\bar{u}s)_L(\bar{d}u)_L+\frac{c_+-c_-}{2}(\bar{d}s)_L(\bar{u}u)_L\right],$$

the value of coefficients with QCD corrections are $c_-\approx 2.4$ and $c_+\approx 0.65$ by renormalization equations [7]. The factorization approximation leads to

$$\frac{a_i}{b_i}=\frac{2c_++c_-}{2c_+-c_-}\approx -3.4 \quad (i=1,2) .$$

Furthermore, the new factorization scheme with a large-N_c approximation gives

$$\frac{a_i}{b_i}=\frac{c_++c_-}{c_+-c_-}\approx -1.74 \quad (i=1,2) .$$

The discrepancies of these ratios from -1 cause the breaking of the $\Delta I=\frac{1}{2}$ rule.

The $\Delta I=\frac{3}{2}$ contributions are represented in terms of two parameters of a_1+b_1 and a_2+b_2. Experimental decay amplitudes are given in Table II [7]. From this table, the magnitudes of the $\Delta I=\frac{3}{2}$ amplitudes are

$$\Delta\Lambda\equiv\Lambda_-^0+\sqrt{2}\Lambda_0^0=-0.09\pm0.05 ,$$

$$\Delta\Xi\equiv\Xi_-^-+\sqrt{2}\Xi_0^0=0.36\pm0.09 ,$$

$$\Delta\Sigma\equiv\Sigma_+^+-\Sigma_-^--\sqrt{2}\Sigma_0^+=0.48\pm0.16 ,$$

TABLE II. Experimental decay amplitudes ($\times 10^{-7}$).

Decays	S wave	P wave
Λ_0^0	-2.36 ± 0.03	-15.61 ± 1.40
Λ_-^0	3.25 ± 0.02	22.40 ± 0.54
Ξ_0^0	3.43 ± 0.06	-12.13 ± 0.71
Ξ_-^-	-4.49 ± 0.02	17.45 ± 0.58
Σ_+^+	0.14 ± 0.03	41.83 ± 0.17
Σ_0^+	-3.26 ± 0.11	26.74 ± 1.32
Σ_-^-	4.27 ± 0.01	-1.44 ± 0.17

(in units of 10^{-7}) for the S-wave amplitudes and

$$\Delta\Lambda=0.33\pm2.05 ,$$

$$\Delta\Xi=0.30\pm1.16 ,$$

$$\Delta\Sigma=5.46\pm1.88$$

(in units of 10^{-7}) for the P-wave amplitudes. We can derive the following relation from the theoretical amplitudes of Table I:

$$\sqrt{\tfrac{2}{3}}(\Delta\Lambda+2\Delta\Xi)=\Delta\Sigma . \tag{1}$$

Using the above experimental values, both sides of Eq. (1) are

$$0.52\pm0.15=0.48\pm0.16 \text{ for } S \text{ wave} ,$$

$$0.75\pm2.52=5.46\pm1.88 \text{ for } P \text{ wave} .$$

For S-wave decays very good agreement is obtained. On the other hand agreement is poor for P-wave decays. However, as the experimental errors are rather large, we cannot judge now whether or not this equation is correct. The relation between the $\Delta I=\frac{3}{2}$ amplitudes and the factorization approximation was argued by Pakvasa and Trampetic [8].

Similarly, in $K\to 2\pi$ decays and $\Omega^-\to\Xi\pi$ decays the breaking of the $\Delta I=\frac{1}{2}$ rule is due to the external and the internal W-emission terms, and the $\Delta I=\frac{3}{2}$ amplitudes are represented in terms of one parameter $a+b$. This is already known in $K\to 2\pi$ decays [9], and no new results can be obtained. The final-state interactions and other effects play an important role in K decays [10].

In conclusion, the $\Delta I=\frac{3}{2}$ amplitudes in hyperon nonleptonic decays are caused by the external W-emission amplitudes and the internal W-emission amplitudes. They are represented in terms of two parameters, and lead to Eq. (1), which is consistent with present experimental data for both S- and P-wave decays.

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