PHYSICAL REVIEW D VOLUME 46, NUMBER 7 1 OCTOBER 1992

Periodic orbits of the Skyrmion breathing mode: Classical and quantal analysis

Abdellatif Abada and Dominique Vautherin
Division de Physique Théorique, Institut de Physique Nucléaire, F-91406, Orsay CEDEX, France
(Received 8 May 1992)

We construct the periodic classical orbits of the Skyrmion breathing mode by a perturbation expansion in the amplitude of the vibration. We first examine the lowest-order construction associated with linear response theory. We find that the monopole response function exhibits a sharp unbound peak which we identify as the Roper resonance $N(1440)$. A calculation of second-order terms provides an evaluation of the anharmonic corrections. In a second part, we construct a collective Bohr-type Hamiltonian using the knowledge of periodic trajectories. This provides a natural requantization scheme with which we calculate the spectrum of monopole excitations of the Skyrmion. Finally we apply our results to the calculation of color transparency effects. We find that anharmonicities decrease significantly the time taken by a nucleon of small radius to regain its normal size. This effect should diminish the importance of the color transparency phenomenon.

PACS number(s): 11.10.Lm, 11.10.Ef, 14.20.Gk

INTRODUCTION

The Skyrme model [1] is an effective Lagrangian for QCD at low energies and large number of colors N_c [2]. At the classical level, the model produces stable localized objects, solitons, which are identified with baryons. It has been used to predict the static properties of baryons [3] (nucleon, $\Delta, \ldots$). The model is reasonably successful: energy differences are rather well reproduced [4–7], the most visible discrepancy being only an overall shift which can be explained as a Casimir energy of the Skyrmion [8].

The model has also been used to investigate the simplest vibrational excitation of the Skyrmion: the breathing mode. Some authors use simple scaling approximation [9–12]. More elaborate methods have been investigated by Zahed, Meissner, and Kaulfuss [13] as well as Breit and Nappi [14] who expand the Lagrangian to leading order in the semiclassical approximation to find the phase shifts of the vibrational modes. The energy at which the phase shift passes through $\frac{\pi}{2}$ is then identified with the mass of a resonance.

In this work, we explore an alternative approach to describe this low-lying resonance. It exploits a perturbative construction of finite amplitude periodic orbits which we have developed recently [15, 16] in the context of giant collective resonances in nuclear physics. It allows one to build solutions of the nonlinear time-dependent mean-field equations with a given period T. Semiclassical quantization of these solutions yields collective nuclear spectra. The method is similar in spirit to the work of Dashen, Hasslacher, and Neveu who performed a semiclassical quantization of periodic orbits [17] to compute the particle spectrum of the ϕ^4 theory in 1+1 dimensions.

An advantage of our expansion method is that it is simple enough to deal with realistic problems in 3+1 dimensions. It thus appears well suited to discuss collective vibrations of the Skyrmion. We have focused in the present paper on the special case of monopole oscillations. Quadrupole modes can be treated as well without further complications by the same technique.

We wish to emphasize that the perturbation expansion we use is not an exact scheme for building periodic orbits of arbitrary nonlinear systems. It is indeed limited by the possible appearance of well-known resonant terms in the construction of higher-order terms (as is already the case for two-dimensional systems with commensurate frequencies) [18–20]. It is however a useful approximation to describe systems with well-developed collective motions (e.g., nuclei), decoupled from other modes, and well described by effective one-dimensional collective Hamiltonians. In this case, the method is useful to identify the collective variables and to build perturbatively the first few anharmonic terms in the collective Hamiltonian.

It is also possible to deal with resonant terms at the cost of more cumbersome analytical work. This was illustrated in the case of coupled nuclear quadrupole and monopole oscillations in Ref. [16].

The present article is organized as follows. Section I presents a brief summary of Skyrme's model and defines our notation. In Sec. II we review the expansion method of Refs. [15, 16] to build periodic orbits and specialize to the case of the time-dependent Skyrme model. In Sec. III we discuss the first-order equations which are equivalent to the linearized evolution equations and therefore to linear response theory. We investigate in detail the case of an external time-dependent monopole field with a frequency Ω and determine the response function by looking at the evolution of the Skyrmion root mean square radius. This function is found to have a pronounced peak which we identify to the Roper resonance. In Sec. IV we construct a collective Bohr-type Hamiltonian by identifying periodic orbits in this Hamiltonian to the ones built in Sec. II. From the knowledge of periodic orbits up to second order in the elongation we are able to construct cubic terms in the collective potential. Quantizing this Hamiltonian by means of the Pauli prescription provides the spectrum of collective states and allows one to evaluate the importance of anharmonic terms. Section V contains an application to the calculation of color transparency

 ©1992 The American Physical Society

while Sec. VI contains a discussion of our main results and a critical comparison with earlier approaches.

I. THE SKYRME MODEL

The basis of Skyrme's model [1] is a chiral-invariant nonlinear theory involving only mesonic degrees of freedom and from which baryons emerge as topologically stable solitons (Skyrmions). In this work, we use the simplest Lagrangian density as introduced by Skyrme [1]:

$$\mathcal{L} = \left[\frac{F_\pi^2}{16}\right] \text{Tr}\,(\partial_\mu U \partial^\mu U^\dagger) + \left[\frac{1}{32e^2}\right] \text{Tr}\,\{\,[(\partial_\mu U)U^\dagger, (\partial_\nu U)U^\dagger]^2\,\}\,. \quad (1.1)$$

Here U is an SU(2) matrix; $F_\pi = 186$ MeV is the pion decay constant. The last term, which contains the dimensionless parameter e, was introduced by Skyrme to stabilize the soliton. In order to reproduce the axial-vector coupling constant g_A, e is taken to be 4.76 [21]. Following Skyrme, we make the hedgehog ansatz

$$U = \exp[\, iF(r,t)\, \boldsymbol{\tau}\cdot\hat{\mathbf{r}}]\,, \quad (1.2)$$

τ_i being the usual Pauli matrices. With this parametrization of the field U, the Lagrangian density (1.1) becomes

$$\mathcal{L} = \left[\frac{F_\pi^2}{8} + \frac{\sin^2(F)}{e^2r^2}\right]\partial_\mu F \partial^\mu F - \frac{\sin^2(F)}{4r^2}\left[F_\pi^2 + 2\frac{\sin^2(F)}{e^2r^2}\right]\,. \quad (1.3)$$

The corresponding classical Euler-Lagrange equation is

$$\left[\frac{(eF_\pi)^2}{4}r^2 + 2\sin^2 F\right]\ddot{F} + \sin(2F)\,(\dot{F})^2 = \left[\frac{(eF_\pi)^2}{4}r^2 + 2\sin^2 F\right]F'' + \frac{(eF_\pi)^2}{2}rF' + \sin(2F)\,F'^2 - \left[\frac{(eF_\pi)^2}{4}r^2 + \sin^2 F\right]\frac{\sin(2F)}{r^2}\,. \quad (1.4)$$

Primes and dots indicate radial coordinate differentiations and time differentiations respectively.

The energy of the Skyrmion is defined as $M = \int d^3r\, \mathcal{H}$ where $\mathcal{H}$ is the Hamiltonian density. Explicitly M reads

$$M = 4\pi F_\pi^2 \int_0^\infty \mathrm{d}r\, \left[\left(\frac{r^2}{8} + \frac{\sin^2(F)}{e^2F_\pi^2}\right)(\dot{F}^2 + F'^2) + \left(r^2 + 2\frac{\sin^2(F)}{e^2F_\pi^2}\right)\frac{\sin^2(F)}{4r^2}\right]\,. \quad (1.5)$$

II. PERTURBATIVE CONSTRUCTION OF PERIODIC ORBITS

We look for periodic solutions of the time-dependent equation (1.4) in the form [15, 16]

$$F(r,t) = F_0(r) + \epsilon F_1\left(r, \frac{\omega}{\omega_0}t\right) + \epsilon^2 F_2\left(r, \frac{\omega}{\omega_0}t\right) + \cdots\,, \quad (2.1)$$

where $F_0(r)$ is the static Skyrme solution with the boundary conditions $F_0(0) = \pi$ and $F_0(\infty) = 0$. The frequency ω is also expanded into a power series in the amplitude of the vibration ϵ according to the standard procedure [22]:

$$\omega = \omega_0 + \epsilon\omega_1 + \epsilon^2\omega_2 + \cdots.$$

In order to ensure that the baryon number is unchanged we impose the conditions

$$F_i(0,t) = F_i(\infty,t) = 0 \quad \text{for all} \quad i = 1,2,\ldots\,. \quad (2.2)$$

To first order we obtain the following evolution equation for F_1:

$$(\partial_0^2 + \mathcal{A}_0)(g_0 F_1) = 0, \quad (2.3)$$

where the function g_0 and the operator $\mathcal{A}_0$ are, respectively,

$$g_0 = \left[\frac{1}{4}(eF_\pi r)^2 + 2\sin^2 F_0\right]^{\frac{1}{2}}\,,$$
$$\mathcal{A}_0 \equiv -\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{g_0''}{g_0} - \frac{1}{g_0^2}\left[2\sin(2F_0)F_0'' + \frac{2}{r^2}\cos(2F_0)\,(r^2F_0'^2 + \tfrac{1}{2} - g_0^2\,) - \frac{1}{r^2}\right]. \quad (2.4)$$

The solution of Eq. (2.3) with $F_1(r,0) = 0$ is

$$F_1(r,t) = R_1(r)\sin(\omega_0 t) \quad (2.5)$$

with R_1 such that

$$(\omega_0^2 - \mathcal{A}_0)\,(g_0 R_1) = 0\ . \tag{2.6}$$

As in earlier studies [13], we find that the operator $\mathcal{A}_0$ has no bound states. Note that $R_1 \to \alpha j_1(\omega_0 r) + \beta n_1(\omega_0 r)$ in the limit $r \to \infty$, where j_1 and n_1 are the usual spherical Bessel functions of order 1.

An expansion of Eq. (1.4) to second order in ϵ gives

$$(\partial_0^2 + \mathcal{A}_0)(g_0 F_2) = \frac{2}{g_0}\sin 2F_0\left[-F_1\ddot{F}_1 - \frac{1}{2}\dot{F}_1^2 + \frac{1}{2}F_1^{'2} + F_1 F_1^{''} + F_1^2 f_0\right] + \frac{2}{g_0}\cos 2F_0\,(F_1^2 F_0')' - 2\frac{\omega_1}{\omega_0}\,g_0\ddot{F}_1, \tag{2.7}$$

where the function $f_0(r)$ is $[(eF_\pi)^2/4 - F_0^{'2} + (4\sin^2 F_0 - \frac{3}{2})/r^2]$ and g_0 and $\mathcal{A}_0$ are defined in Eq. (2.4). Since the operator $(\partial_0^2 + \mathcal{A}_0)$ admits $R_1 \sin\omega_0 t$ as a zero mode, the solution of Eq. (2.7) exists only if the source term in this equation has no component along this vector. This is the case for all terms of the type $F_1 \times F_1$ (whose time dependences are $\cos 2\omega_0 t$ or 1) but not for the last term. Therefore the coefficient of this term must vanish, which yields

$$\omega_1 = 0,$$

and consequently the time dependence of $F_2(r,t)$ becomes

$$F_2(r,t) = R_2(r)\cos(2\omega_0 t) + T_2(r)\ . \tag{2.8}$$

Actually the condition $\omega_1 = 0$ is not sufficient to eliminate all the resonant terms in the right-hand side of Eq. (2.7). Indeed, since the operator $\mathcal{A}_0$ has a continuous spectrum, there is also a zero mode with frequency $2\omega_0$. This mode should in principle be included in our first-order solution (2.5) as described in Ref. [16]. We have found, however, that its coupling is weak and can be neglected.

We now apply these equations to a description of the Roper resonance. In the work of Breit and Nappi [14] the location of this resonance was determined by looking at the phase shift of the solution of the first-order radial equation (2.6). In the next section, we propose an alternative procedure inspired from the standard methods developed for giant resonances in nuclear physics.

III. LINEAR RESPONSE

As a starting point, we consider the response function of the Skyrmion to an external monopole field with a frequency Ω and examine the resonant structures of this function. Within our approximation scheme, this is the most natural procedure since the first-order equation (2.3) corresponds to the linearized equations of motion which are at the basis of linear response theory [23].

An external time-dependent monopole field corresponds to the addition of the following term to the Lagrangian density (1.3):

$$\mathcal{L}_{\rm int} = eF_\pi^3 r^2 B^0(r,t)\ \epsilon\ \sin(\Omega t)\exp(\eta t), \tag{3.1}$$

where $B^0(r,t)$ is baryonic current [3],

$$B^0(r,t) = -\frac{1}{2\pi^2}\frac{\sin^2(F)}{r^2}\frac{\partial F}{\partial r},$$

and η a vanishingly small positive number. Adding this interaction term (3.1) to the Lagrangian density (1.3), the new corresponding Euler-Lagrange equation reads

$$\left[\frac{(eF_\pi)^2}{4}r^2 + 2\sin^2 F\right]\ddot{F} + \sin(2F)\,(\dot{F})^2 = \left[\frac{(eF_\pi)^2}{4}r^2 + 2\sin^2 F\right]F^{''} + \frac{(eF_\pi)^2}{2}rF' + \sin(2F)\,F^{'2} - \left[\frac{(eF_\pi)^2}{4}r^2 + \sin^2 F\right]\frac{\sin(2F)}{r^2} + \epsilon\frac{(eF_\pi)^3 r}{\pi^2}\sin^2(F)\sin(\Omega t)\exp(\eta t)\ . \tag{3.2}$$

This last equation is to be solved with the boundary conditions $F(t=-\infty, r) = F_0(r)$ and $\dot{F}(t=-\infty, r) = 0$ where $F_0(r)$ is the static Skyrme solution.

Because the field (3.1) is weak (in the domain $[-\infty, 0]$), it introduces small changes of the Skyrme solution which we can treat in a linear approximation. To first order in ϵ, $F(r,t)$ has the form

$$F(r,t) = F_0(r) + \delta F(r,t) + \delta F^*(r,t)\ ,$$

where

$$\delta F(r,t) = -i\frac{\epsilon}{2}R_1(r)\exp[i(\Omega - i\eta)t]$$

is linear in the field strength ϵ. Thus, Eq. (3.2) becomes

$$[(\Omega - i\eta)^2 - \mathcal{A}_0](g_0 R_1) = -\frac{(eF_\pi)^3}{\pi^2}\frac{r\sin^2(F_0)}{g_0}, \tag{3.3}$$

where g_0 and $\mathcal{A}_0$ have been defined in the preceding section. The isoscalar mean square radius [3] is given by

$$\langle r^2\rangle = -\frac{2}{\pi}\int_0^\infty r^2\sin^2(F)F'\,dr.$$

Up to first order it reads

$$\langle r^2\rangle = \langle r^2\rangle_0 - i\frac{\epsilon}{2}\{ f_1(\Omega)\exp[i(\Omega - i\eta)t] - f_1^*(\Omega)\exp[-i(\Omega + i\eta)t]\}$$

where $\langle r^2\rangle_0$ is the static mean square radius [3] and f_1 the response function

$$f_1(\Omega) = \frac{4}{\pi}\int_0^\infty r\frac{\sin^2(F_0)}{g_0}(g_0R_1)\,dr.$$

Let us introduce the state ϕ defined by

$$\langle r|\phi\rangle = \frac{2eF_\pi}{\pi}\frac{r\sin^2(F_0)}{g_0}, \tag{3.4}$$

and the eigenstates ϕ_n of the operator $\mathcal{A}_0$,

$$\langle r|\mathcal{A}_0|\phi_n\rangle = \omega_n^2\langle r|\phi_n\rangle, \tag{3.5}$$

normalized according to $\int_0^\infty \phi_n(r)\phi_m(r)eF_\pi dr = \delta_{nm}$. Using Eq. (3.3), we obtain the following spectral representation of the response function:

$$f_1(\Omega) = -\frac{1}{\pi}\sum_n \frac{|\langle\phi|\phi_n\rangle|^2}{(\Omega - i\eta)^2 - \omega_n^2}\ , \tag{3.6}$$

where, as usual [23], the limit $\eta \to 0^+$ is implicit and corresponds to the boundary condition specified above.

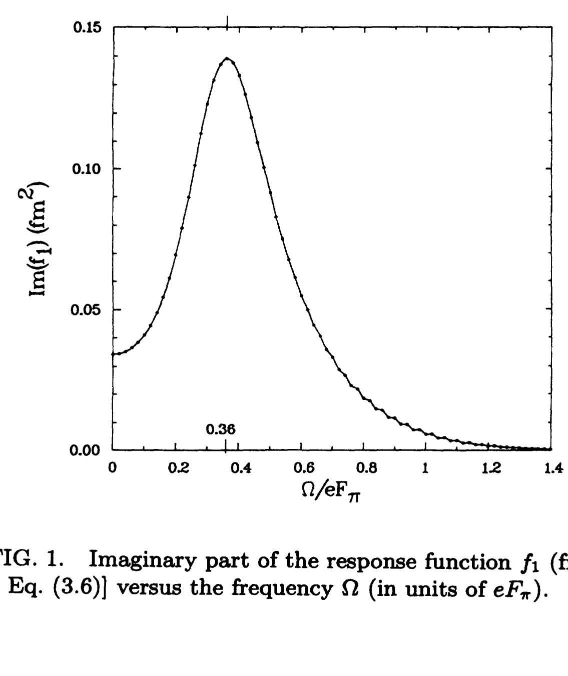

FIG. 1. Imaginary part of the response function f_1 (fm^2) [see Eq. (3.6)] versus the frequency Ω (in units of eF_π).

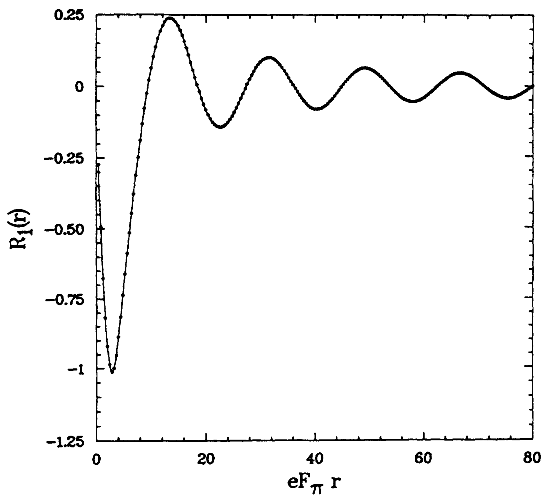

FIG. 2. First-order correction $R_1(r)$ to the static Skyrme solution corresponding to the resonance energy $0.36eF_\pi$ versus the dimensionless radius $eF_\pi r$.

The quantity of interest for our purpose is the imaginary part of the response function, which is directly related to the distribution of collective strength (see, e.g., [23]). By using the identity $1/(x+i\eta) = P(1/x) - i\pi\delta(x)$ one finds that

$$\mathrm{Im}f_1(\Omega) = \sum_n \delta(\Omega^2 - \omega_n^2)|\langle\phi|\phi_n\rangle|^2\ . \tag{3.7}$$

This function is shown in Fig. 1. We find a resonance at $\Omega = 0.36\ eF_\pi$ (318.7 MeV) with a width $\Gamma = 0.35\ eF_\pi$ (310 MeV) which we identify with the Roper resonance.

The functions R_1, T_2, and R_2 defined in the preceding section [see Eqs. (2.5) and (2.8)], at the value $\omega_0 = 0.36\ eF_\pi$, have been plotted in Figs. 2 and 3, re-

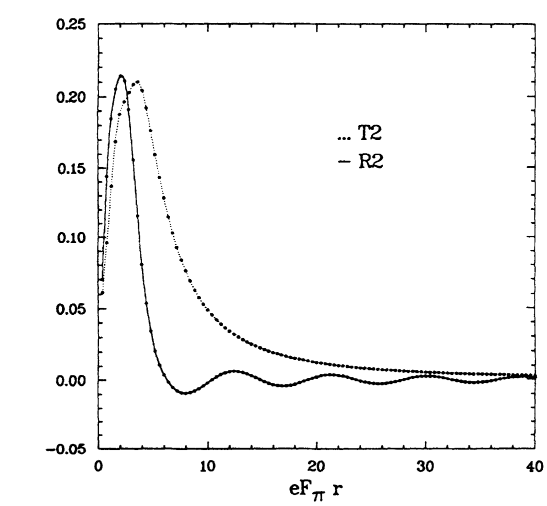

FIG. 3. Second-order corrections $R_2(r)$ (full line) and $T_2(r)$ (dashed line) to the static Skyrme solution corresponding to the resonance energy $0.36eF_\pi$ versus the dimensionless radius $eF_\pi r$.

spectively. It can be seen that the function R_2 has the same asymptotic behavior as R_1, but with an argument $2\omega_0 r$ while T_2 decays algebraically at large distance.

At this stage, it is worthwhile introducing a word of caution about the above procedure of identification of the Roper resonance. Our Skyrmion is in fact a superposition of the nucleon and delta resonance whereas the Roper resonance refers to the nucleon only. However, as emphasized by Zahed, Meissner, and Kaulfuss [13], there also exists a delta-Roper resonance at 1670 MeV. It is therefore plausible that a proper projection of our breathing mode would generate both the Roper and the delta-Roper resonance. This statement is further supported by the fact that the coupling between rotations of the Skyrmion and its vibrations is of order e^2, i.e., $1/N_c$ [11], so that it appears possible to calculate vibrations first and perform a projection afterwards on states with proper spin and isospin.

IV. COLLECTIVE HAMILTONIAN

From the knowledge of periodic orbits the prescription used frequently to construct energy spectra is to perform a semiclassical quantization of these orbits [13,15,16,24]. In the present case however this procedure is not adequate because we are dealing with resonances embedded in a continuum. For this reason we have used a different approach: We construct a collective Hamiltonian by requiring that periodic orbits of this Hamiltonian reproduce those obtained in Sec. II. First we must choose an adequate collective variable, i.e., an observable that suitably describes the collective motion. In the present case, the mean square radius already encountered in the preceding section appears as the most natural choice. Let X,

$$X(t) = \langle r^2\rangle - \langle r^2\rangle_0 = -\frac{2}{\pi}\int_0^\infty r^2\left[\,\sin^2(F)F' - \sin^2(F_0)F_0'\,\right]dr, \quad (4.1)$$

be the collective variable. Inserting Eq. (2.1), truncated at second order, into Eq. (4.1) we find that X has the time dependence

$$X(t) = \epsilon x_1 \sin(\omega_0 t) + \epsilon^2\left[x_{2c}\cos(2\omega_0 t) + x_{2f}\right] . \quad (4.2)$$

The constants x_1, x_{2c} and x_{2f} appearing in the last equation are

$$x_1 = \frac{4}{\pi}\int_0^\infty r\sin^2(F_0)R_1\,dr,$$

$$x_{2c} = \frac{4}{\pi}\int_0^\infty r\left[\sin^2(F_0)R_2 - \frac{1}{4}R_1^2\sin(2F_0)\right]dr,$$

$$x_{2f} = \frac{4}{\pi}\int_0^\infty r\left[\sin^2(F_0)T_2 + \frac{1}{4}R_1^2\sin(2F_0)\right]dr, \quad (4.3)$$

where the functions F_0, R_1, R_2, and T_2 are defined in Sec. II.

In order to build a collective Bohr-type Hamiltonian

$$H = \frac{1}{2}M(X)\dot{X}^2 + V(X)\ , \quad (4.4)$$

we proceed as follows. We first calculate the periodic orbits in this Hamiltonian up to second order in the elongation and require the resulting expression to be identical to Eq. (4.2). The identification is possible only if H parametrizes as

$$H = M_{\rm cl} + \frac{1}{2}(m + m'X)\dot{X}^2 + \frac{1}{2}m\omega_0^2 X^2 + \lambda X^3, \quad (4.5)$$

where $M_{\rm cl}$ is the classical Skyrmion mass [3]. The periodic orbits in this Hamiltonian up to second order in the amplitude η are given by [22]

$$X(t) = \eta\sin(\omega_0 t) + \eta^2\left[\left(-\frac{\lambda}{2m\omega_0^2} + \frac{m'}{4m}\right)\cos(2\omega_0 t) - \frac{3\lambda}{2m\omega_0^2} + \frac{m'}{4m}\right]. \quad (4.6)$$

By identifying Eqs. (4.6) and (4.2) we obtain the values of the constants m'/m and λ/m in terms of the x's:

$$m'/m = (6x_{2c} - 2x_{2f})/x_1^2,$$
$$\lambda/m = \ \omega_0^2\ (x_{2c} - x_{2f})/x_1^2. \quad (4.7)$$

The value of the mass parameter m can be easily found by expanding the kinetic (or potential) energy up to second order in the elongation in Eq. (1.5). The result is

$$m = \frac{4\pi}{e^2 x_1^2}\int_0^\infty dr\ (g_0 R_1)^2. \quad (4.8)$$

One can see that the quantities m, m', and λ appearing in Eqs. (4.7) and (4.8) are independent of the normalization of R_1.

Let us now perform a numerical calculation of the parameters in the collective Hamiltonian. For the Roper resonance the appropriate value of ω_0 is $0.36eF_\pi$ as was found in the preceding section. Using the above formulas we obtain for the dimensionless parameters $\bar{m}$, $\bar{m}'$, and $\bar{\lambda}$ defined as

$$m = \ eF_\pi^3\ \bar{m}, \quad m' = \ (eF_\pi)^2 m\ \bar{m}', \quad \lambda = \ (eF_\pi)^2 m\omega_0^2\ \bar{\lambda} \quad (4.9)$$

the values

$$\bar{m} = 29, \quad \bar{m}' = -0.4, \quad \bar{\lambda} = -0.13.$$

The graph of the collective potential is given in Fig. 4. It can be seen that the potential exhibits a barrier whose maximum is at about 0.18 in units of eF_π while the unperturbed value of the Roper resonance is 0.36. The unbound state character of the Roper resonance is thus manifest.

The procedure we have used has allowed the unambiguous determination of the cubic terms in the collective potential only. The determination of quartic and higher-order terms can be achieved by considering further terms in the expansion (2.1).

Having obtained a Bohr Hamiltonian, we now proceed in the usual way by performing a standard quantization of this Hamiltonian via the Pauli prescription [25], with the result

$$\hat{H} = \frac{1}{2}\frac{1}{\sqrt{m(\hat{X})}}\hat{P}\frac{1}{\sqrt{m(\hat{X})}}\hat{P} + \frac{1}{2}m\omega_0^2\hat{X}^2 + \lambda\hat{X}^3 , \tag{4.10}$$

where $\hat{P} = -id/dX$. Note that this Hamiltonian is Hermitian with respect to the measure $\sqrt{m(X)}$.

Let us now calculate the eigenvalues of $\hat{H}$ by treating the anharmonic terms in $\bar{m}'$ and $\bar{\lambda}$ in second-order perturbation theory. The energy of the state with n quanta is

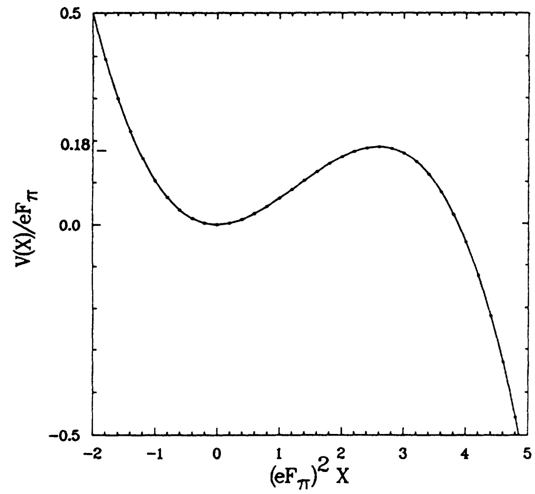

FIG. 4. The collective potential V (in units of eF_π) versus the collective variable $X = \langle r^2\rangle - \langle r^2\rangle_0$ [in units of $1/(eF_\pi)^2$].

$$M_n - M_{\rm cl} = \omega_0\left(n+\frac{1}{2}\right) + \frac{e^3F_\pi}{16\bar{m}}\left\{\bar{m}'^2(n^2+n+1) - 2\bar{\lambda}^2(11+30n+30n^2) + 2\bar{m}'\bar{\lambda}(1+6n+6n^2)\right\} . \tag{4.11}$$

The energy of the ground state is equal to

$$M_0 = M_{\rm cl} + \frac{\omega_0}{2} + \frac{e^3F_\pi}{8\bar{m}}\left[\frac{\bar{m}'^2}{2} - 11\bar{\lambda}^2 + \bar{m}'\bar{\lambda}\right] . \tag{4.12}$$

Using our numerical results we obtain

$$M_0 = (36.44)\left[\frac{F_\pi}{e}\right] + (0.18)\left[eF_\pi\right] - (2.18\times10^{-4})\left[e^3F_\pi\right] = 1578.9 \text{ MeV} . \tag{4.13}$$

This value represents the mass of the Skyrmion incorporating the contribution of the quantum zero-point monopole fluctuations.

It is worthwhile examining the relative importance of the various terms in the previous formula. In the limit of a large number of colors N_c, F_π increases as $N_c^{\frac{1}{2}}$ and e decreases as $N_c^{-\frac{1}{2}}$ so that the leading contribution to the quantized Skyrmion mass M_0 is $M_{\rm cl}$, namely, $36.44F_\pi/e$ which grows as N_c. The next-to-leading contribution is $(0.18)eF_\pi$ which is independent of N_c in the high-N_c limit. Note that Biedenharn, Dothan, and Tarlini [11] find this last term to be $(0.144)eF_\pi$. The last contribution to M_0 is $2.18\times10^{-4}e^3F_\pi$ and decreases as N_c^{-1}. We thus expect the next correction terms in Eq. (4.13) to be negligible.

The excitation energies of the Skyrmion are

$$\begin{aligned} E_n^* &= M_n - M_0 \\ &= n\omega_0 + n(n+1)e^3F_\pi\left[\frac{\bar{m}'^2}{4} - 15\bar{\lambda}^2 + 3\bar{m}'\bar{\lambda}\right]\Big/4\bar{m} . \end{aligned} \tag{4.14}$$

For $n=1$, the excitation energy of the first excited state which we interpret as the Roper resonance is $E_1^* = 300.4$ MeV compared to an unperturbed value of 318.7 MeV. This small difference justifies the perturbative treatment of the anharmonicities we have adopted. The difference between the mass of the Roper defined as the one-phonon state and the classical mass of the Skyrmion is found to be 455.4 MeV.

Let us mention another equivalent derivation of the collective Hamiltonian. Starting from Eq. (4.2) one can express the quantities $\epsilon\sin(\omega_0 t)$ and ϵ^2 in terms of X, $\dot{X}^2$, and X^2. Returning to the expression (1.5) of the Skyrmion mass provides the desired collective Hamiltonian. This procedure yields exactly the same formulas (4.7) and (4.8).

V. APPLICATION TO COLOR TRANSPARENCY

In this section we use the collective Hamiltonian calculated in the preceding section to discuss the phenomenon of color transparency. We adopt the model description presented in Refs. [26, 27], in which the reaction

$$e + A \rightarrow e' + p + (A-1)$$

is pictured as follows. The transfer of a momentum Q to the nucleon selects out of its wave function a component with a small radius $R(Q)$ which, in Ref. [26], was chosen to be

$$R^2(Q) = 1/(Q^2 + q_0^2) \tag{5.1}$$

with $q_0 = 1/R_0 = 0.42$ GeV, R_0 being the isoscalar root mean square radius [3]. The probability P for the struck nucleon to leave the nucleus without further interactions was calculated in [26, 27] by means of the formulas

$$P(Q) = \exp\left[-\int_0^\infty \frac{p}{m} dt\, \sigma(Q,t)\, \rho\left(z = \frac{p}{m}t\right) \right]$$

where m is the nucleon mass, ρ the nuclear density, and

$$\sigma = \begin{cases} \sigma_0 \left[\dfrac{R(t)}{R_0}\right]^2, & R(t) < R_0 \,, \\ \sigma_0, & R(t) > R_0 \,. \end{cases}$$

The quantity $\sigma_0 = 4$ fm^2 is the total nucleon-nucleon cross section and $R(t)$ is the radius at time t of a nucleon created at time $t = 0$ with a radius $R(Q)$.

From this simplified picture, one expects the nucleus to become transparent at large momentum transfer since a small object is more likely to cross the nucleus without interacting. As a result the ratio of the cross sections $\sigma(A,Q)/A\sigma(A=1,Q)$ should increase and tend to one when Q increases as supported by some experimental data (see the references quoted in [27]).

We thus see that an important input in the preceding discussion of color transparency is the characteristic time it takes a nucleon created with a small size $R(Q)$ to return to its normal size R_0. From the collective Hamiltonian we have built in the preceding section we can extract the information needed. For a nucleon with an initial radius $R_i = R(Q)$ the time τ needed to reach the equilibrium value is

$$\tau = \int_{X_i}^0 dX \left[\frac{m(X)}{2[V(X_i) - V(X)]}\right]^{\frac{1}{2}}, \tag{5.2}$$

where the collective variable $X = R^2 - R_0^2$ runs from $X_i = -Q_i^2/q_0^2(Q_i^2 + q_0^2)$ to zero. When the collective potential is assumed to be a harmonic oscillator this time is just equal to the quarter of the period $t_0 = 2\pi/\omega_0 = 1.3 \times 10^{-23}$ sec. It is interesting to see how much this value is affected by anharmonic terms. In terms of the dimensionless variables defined in (4.9), τ reads

$$\tau = \frac{t_0}{2\pi} \int_0^1 \mathrm{d}u \left[\frac{1 + \bar{m}'\tilde{X}_i u}{1 - u^2 + 2\bar{\lambda}\tilde{X}_i(1-u^3)}\right]^{\frac{1}{2}}, \tag{5.3}$$

where $\tilde{X}_i$ is the dimensionless term $(eF_\pi)^2 X_i$. In Fig. 5 we show the ratio τ/t_0 as a function of the momentum transfer Q. From this figure it can be seen that the characteristic time τ is strongly reduced by anharmonic terms up to momenta of the order of 1 GeV which makes color transparency less efficient. Beyond this value the decrease appears to be weaker. However, in this region the second-order truncation we have made is presumably no longer sufficient.

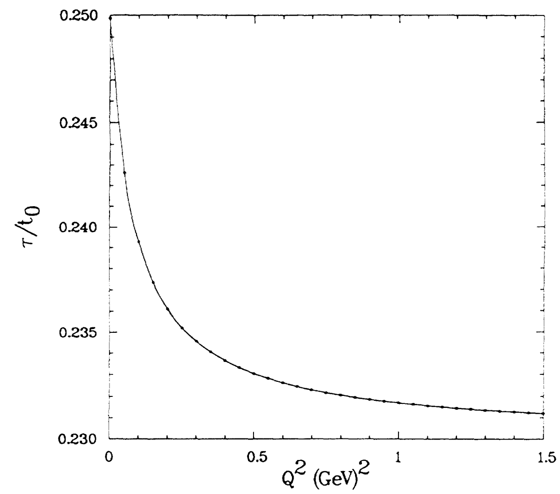

FIG. 5. The characteristic time τ (in units of the period $t_0 = 1.3 \times 10^{-23}$ sec), it takes a nucleon created with a small size $R(Q)$ [see Eq. (5.1)] to return to its normal size, as a function of the square of the transferred momentum Q^2.

Let us briefly discuss the special case Q^2=1 GeV^2 for which our calculation may still be adequate. In this case the time needed for a nucleon to return to its normal size is 0.3×10^{-23} sec (to be corrected by a Lorentz factor of the order of 1.3). In contrast the traversal time of the nucleus is approximately 3×10^{-23} sec (in ^{208}Pb). Unfortunately our model cannot be extented to the region of interest (~ 10 GeV^2) where color transparency has been experimentaly tested. Indeed the effective character of the Skyrme model, which is a low energy approximation, makes it no longer suitable in this domain.

VI. CONCLUSION

In this article we have used a perturbative method to build the classical periodic orbits of a Skyrmion as a power series in the amplitude of the oscillations. The method used up to second order has been applied to the Roper resonance described in terms of monopole vibrations. In first order already the method provides a convenient prescription to identify the location of the resonance. To this order the method is equivalent to linear response theory and we find that the response function displays a well-developed peak.

In the second part of this article we have also presented a powerful method which uses the knowledge of periodic orbits to construct a collective Bohr-type Hamiltonian. We have applied it to the case of monopole vibrations and derived the corresponding first anharmonic terms in the collective Hamiltonian. Although the cubic terms do not produce large shifts in the unperturbed harmonic spectrum, they have important qualitative effects. Indeed due to the finite height of the potential barrier, the spectrum of the collective states becomes unbound. Such detailed information was not available in the simplified approaches using the scaling assumption.

In the last part of the paper we have also shown that anharmonic terms affect significantly conclusions regarding color transparency effects associated with electroproduction of protons from nuclei. We find that anharmonicities significantly decrease the time needed by a nucleon of small radius to regain its normal size. This phenomenon should reduce the magnitude of color transparency effects.

Further improvements and applications of the present work are now being considered [28]. They include a calculation of quadrupole resonances (already considered by Mattis and Karliner [29]) and a calculation of the distribution of quadrupole strength. Moreover, it is our opinion that projection on states of good spin and isospin should also be performed including a determination of the coupling between rotations and vibrations in order to achieve an accurate description of the nucleon and Δ resonances within the Skyrme model.

ACKNOWLEDGMENTS

We wish to thank E. Bogomolny for a discussion concerning the appearance of resonant terms in the construction of higher order terms in Eq. (2.1). We are grateful to J. F. Mathiot, and R. Mendez-Galain for suggestions about color transparency. We acknowledge stimulating discussions with D. Kalafatis and B. Moussalam and express our appreciation to H. Flocard, J. F. Mathiot, and B. Moussalam for a critical reading of the manuscript. Division de Physique Théorique is Unité de Recherche des Universités Paris XI et Paris VI associée au CNRS.

[1] T. H. R. Skyrme, Proc. R. Soc. London **A260**, 127 (1961).
[2] E. Witten, Nucl. Phys. **B223**, 422 (1983); **B223**, 433 (1983).
[3] G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983).
[4] R. Vinh Mau, Prog. Part. Nucl. Phys. **20**, 221 (1988).
[5] B. Schwesinger, H. Weigel, G. Holzwarth, and A. Hayashi, Phys. Rep. **173**, 175 (1989).
[6] I. Klebanov, in *Hadrons and Hadronic Matter*, Proceedings of the NATO Advanced Study Institute, Cargèse, France, 1989, edited by D. Vautherin, F. Lenz, and J. Negele (Plenum, New York, 1991).
[7] U. G. Meissner, Phys. Rep. **161**, 213 (1988).
[8] D. Kalafatis and B. Moussalam, Phys. Lett. B **272**, 196 (1991).
[9] Ch. Hajduk and B. Schwesinger, Phys. Lett. **140B**, 172 (1984).
[10] H. Walliser and G. Eckart, Nucl. Phys. **A429**, 514 (1984).
[11] L. C. Biedenharn, Y. Dothan, and M. Tarlini, Phys. Rev. D **31**, 649 (1985).
[12] A. Hayashi and G. Holzwarth, Phys. Lett. **140B**, 175 (1984).
[13] I. Zahed, U. G. Meissner, and U. B. Kaulfuss, Nucl. Phys. **A426**, 525 (1984).
[14] J. D. Breit and C. R. Nappi, Phys. Rev. Lett. **53**, 889 (1984).
[15] A. Abada and D. Vautherin, Phys. Lett. B **258**, 1 (1991).
[16] A. Abada and D. Vautherin, Phys. Rev. C **45**, 2205 (1992).
[17] R. F. Dashen, B. Hasslacher, and A. Neveu, Phys. Rev. D **11**, 3424 (1975).
[18] J. Moser, Mem. Am. Math. Soc **81**, 1 (1968).
[19] F. Gustavson, Astron. J. **71**, 670 (1966).
[20] A. J. Lichtenberg and M. A. Lieberman, *Regular and Stochastic Motion* (Springer, Berlin, 1984).
[21] A. D. Jackson and M. Rho, Phys. Rev. Lett. **51**, 751 (1983).
[22] L. D. Landau and E. L. Lifschitz, *Mécanique Classique* (Mir, Moscow, 1969), Sec. 28.
[23] P. Ring and P. Schuck, *The Nuclear Many Body Problem* (Springer, Berlin, 1980).
[24] J. Schechter and H. Weigel, Phys. Lett. B **261**, 235 (1991).
[25] W. Pauli, *Handbuch der Physik* (Springer-Verlag, Berlin, 1933), Vol. XXIV, p. 120.
[26] P. Jain, J. Schechter, and H. Weigel, Phys. Rev. D **45**, 1470 (1992).
[27] G. R. Farrar, H. Liu, L. L. Frankfurt, and M. I. Strikman, Phys. Rev. Lett. **61**, 686 (1988).
[28] A. Abada, Ph.D. thesis, Université Paris XI.
[29] M. P. Mattis and M. Karliner, Phys. Rev. D **31**, 2833 (1985); **32**, 58 (1985).