Leptonic CP-violating asymmetry in e^+e^- annihilation due to Z width effects

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CP-violating rate asymmetry can be generated in a process only if its amplitude possesses an absorptive part. Such an absorptive part can be provided in e^+e^- annihilation by the presence of a Z resonance of nonzero width. The lepton-flavor-violating CP asymmetry in the process $e^+e^- \to l_i\overline{l_j}\,(\overline{l_i}l_j)$, where l_i are charged leptons, is discussed in several models. In a specific two-Z model, with an extra U(1) gauge group corresponding to $L_e - L_\tau$ (L_e,L_τ being the lepton numbers associated with the electron and the τ lepton), large and observable CP asymmetry in $e^+e^- \to \tau^+e^-(\tau^-e^+)$ is shown to be possible at energies reached at the CERN e^+e^- collider and SLAC Linear Collider. Various constraints coming from present experimental data have been taken into account while making this prediction.

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I. INTRODUCTION

There have been several proposals in recent times to look for CP violation outside the neutral kaon system, especially in Z decays [1–3], in view of the currently operational CERN e^+e^- collider (LEP) and SLAC Linear Collider (SLC) which have the capability of producing a large number of Z's. The suggestions include investigating the rate asymmetry between two processes related to each other by CP and looking for CP-odd correlations in various processes. Of these, rate asymmetries are easier to search for experimentally. But they can arise theoretically only if the relevant amplitude possesses an absorptive part. This requires either the knowledge of strong interaction phases (final-state interactions) or calculation of loop diagrams. The need for an absorptive part may be avoided in case of CP-odd correlations; processes in which a nonzero signal can occur are, however, quite complicated [4].

It is indeed possible to have a simplification in the calculation of a CP-violating rate asymmetry in the neighborhood of a resonance such as the Z. The Breit-Wigner form for the Z propagator has an imaginary part proportional to the width of the Z, and hence the resulting CP-violating asymmetry would be proportional to this width. Since in this case the need for nonperturbative phase shifts or loop diagrams is avoided, the calculation of the asymmetry involves a simple evaluation of tree diagrams.

In a recent short paper [5] we had examined the possibility that the absorptive part required for CP-violating asymmetry comes from the propagator of the Z. In this paper we give a more detailed discussion of various models and include details of calculations omitted in [5].

The Z width effect was considered earlier by Hoogeveen and Stodolsky [6] in the context of an electric dipole type of CP-violating interaction. Nowakowski and Pilaftsis [7] and Eilam *et al.* [8] have recently considered width effects of the top quark and the W boson, respectively.

Although it is possible to think of quark processes in the standard model (SM) with CP violation coming from complex quark-antiquark couplings to unstable $W^\pm$, they would need at least two W's and also involve flavor tagging to distinguish between charge-conjugate processes to be of practical relevance. We consider here, instead, extensions of SM, where the new mechanism can be illustrated in a somewhat simpler leptonic process, viz.,

$$e^+e^- \to l_i\overline{l_j}\,(\overline{l_i}l_j) \quad (l_1 = e, l_2 = \mu, l_3 = \tau). \tag{1}$$

Since lepton flavor detection is simpler, this would be experimentally easier to observe. In SU(2)×U(1) models with exotic charged leptons, Z couplings can violate leptonic flavor and make this process possible. The difference in the rates for $e^+e^- \to l_i\overline{l_j}$ and $e^+e^- \to \overline{l_i}l_j$ would then be a measure of CP violation.

As we shall see, to get an observable asymmetry consistent with known constraints on lepton flavor violations, it is not enough to stay within SU(2)×U(1) with an extended fermionic sector. We need to extend the model further to include at least one extra neutral gauge boson. We have examined several extensions here, and analyzed the CP-violating asymmetry in them. In most popular models, such as, e.g., those arising from E_6 grand unification, the asymmetry is found to be unobservably small. Most remarkably, in a certain SU(2)×U(1)×U(1)′ model, where the extra U(1)′ couples to $L_e - L_\tau$, L_e and L_τ being the lepton numbers associated with e and τ, respectively, we find in the region of the Z resonance rather large asymmetries, nearing 100%, for a choice of parameters allowed by all known data. This effect, if present, could be easily observable at current e^+e^- colliders. This is to be contrasted with rather small and unobservable rate asymmetries found in the SM [1] and its extensions [2, 3].

Though we restrict ourselves in this work to leptonic final states for reasons mentioned above, many of the considerations below, including some of the expressions, can be trivially modified to the case when the final states are hadronic.

The plan of the rest of the paper is as follows. Section II contains a discussion of general conditions for a CP-violating rate asymmetry in the process (1) and the possibility of finding a nonzero asymmetry in various models. Section III deals with a specific model with an extra U(1) corresponding to gauged $L_e - L_\tau$ and the rate asymmetry therein. Section IV contains our conclusions.

II. RATE ASYMMETRIES IN VARIOUS MODELS

Before we look at any specific model, let us consider the necessary conditions for a CP-violating rate asymmetry to occur. The amplitude for a process can be written as

$$M = \sum_m g_m f_m A_m, \tag{2}$$

where g_m are complex products of coupling constants (including mixing angles and phases), f_m are invariant functions of kinematic variables (including propagators and momentum integrals over propagators), and A_m are tensors (which are assumed real). The sum is over the distinct Feynman diagrams in perturbation theory. The amplitude for the CP-conjugate process is then

$$\overline{M} = \sum_m g_m^* f_m A_m. \tag{3}$$

The difference in the rates for the two processes is proportional to

$$|M|^2 - |\overline{M}|^2 = -4 \sum_{m>n} \mathrm{Im}(g_m g_n^*) \mathrm{Im}(f_m f_n^*) A_m A_n. \tag{4}$$

It immediately follows that, for a nonzero rate asymmetry, (i) at least two terms must contribute, (ii) of which at least one must have complex couplings and complex f_m, and (iii) if two or more terms with complex g_m and f_m contribute, then not all g_m and not all f_m should have identical phases.

We will now consider various extensions of the standard models in which lepton flavor violation can occur in the Z couplings in the lowest order and investigate to what extent the requirements of a nonzero CP-violating rate asymmetry in the process (1) can be met in these models.

A. $\mathrm{SU(2)}_L \times \mathrm{U(1)}$ models with exotic charged leptons

The simplest extensions of SM satisfying conditions (i) and (ii) above are $\mathrm{SU(2)}_L \times \mathrm{U(1)}$ models which include at least one generation of charged leptons, transforming under $\mathrm{SU(2)}_L$ as (A) left-handed (LH) and right-handed (RH) singlets (vector singlets), or (B) LH and RH doublets (vector doublets), or (C) LH singlets and RH doublets (mirrors). One can of course think of more complicated models with combinations of the three possibilities of vector singlets, vector doublets, and mirrors occurring simultaneously. However, we will only study the simpler models A, B, and C described above.

In all these models, the mixing of ordinary with exotic leptons violates the conditions for the leptonic Glashow-Iliopoulos-Maiani (GIM) mechanism, and therefore introduces flavor-changing Z couplings which are complex in general. The corresponding terms in the Lagrangian can be parametrized in general as

$$\mathcal{L}_Z = -\left(G_{ij}^L \overline{f_{iL}} \gamma_\mu f_{jL} Z^\mu + L \leftrightarrow R\right), \tag{5}$$

where f_i refer to the ordinary fermions. The constants $G_{ij}^{L,R}$ are given by

$$G_{ij}^L = \delta_{ij} z_L + (z_{\alpha L} - z_L) U_{\alpha j}^L U_{\alpha i}^{L*}, \tag{6}$$

$$G_{ij}^R = \delta_{ij} z_R + (z_{\alpha R} - z_R) U_{\alpha j}^R U_{\alpha i}^{R*}, \tag{7}$$

where $z_{L,R}$ are the couplings of the weak eigenstates of the ordinary fermions $f'_{iL,R}$ to the Z, $z_{\alpha L,R}$ are those of the weak eigenstates of the exotic fermions $F'_{\alpha L,R}$, and the unitary matrices $U^{L,R}$ relate these states to the mass eigenstates $f_{iL,R}$ and $F_{\alpha L,R}$, respectively. The couplings $z_{L,R}$ are given in all the models by

$$z_L = \frac{g}{\cos\theta_W}\left(-\frac{1}{2} + \sin^2\theta_W\right), \quad z_R = \frac{g}{\cos\theta_W}\sin^2\theta_W. \tag{8}$$

The couplings $z_{\alpha L,R}$ of the exotic leptons in a general model would be α dependent. However, in the models A, B, and C, these couplings are independent of α and are given by

$$z_{\alpha L} = z_{\alpha R} = \frac{g}{\cos\theta_W}\sin^2\theta_W \quad \text{(model A)}, \tag{9}$$

$$z_{\alpha L} = z_{\alpha R} = \frac{g}{\cos\theta_W}\left(-\frac{1}{2} + \sin^2\theta_W\right) \quad \text{(model B)}, \tag{10}$$

$$z_{\alpha L} = \frac{g}{\cos\theta_W}\sin^2\theta_W, \qquad z_{\alpha R} = \frac{g}{\cos\theta_W}\left(-\frac{1}{2} + \sin^2\theta_W\right) \quad \text{(model C)}. \tag{11}$$

Restricting to only tree diagrams, the two terms needed for a nonzero CP asymmetry [condition (i) above] can either be the terms with LH and RH couplings to the Z, or, in the case of $e^+e^- \to l^+e^-, l^-e^+$ ($l = \mu, \tau$), those involving s- and t-channel exchanges of the Z. In either case the width of the Z can provide an imaginary part to the propagator to make the amplitudes complex. However, condition (iii) is satisfied only when s- as well as t-channel exchanges are present, and even then, only in model C. This is because with only s-channel exchange all terms have the same factor of f_m, and hence condition (iii) is not satisfied. Even when both s- and t-channel exchanges are present, the phases of the product of couplings to Z are identical in models A and B. This is because, as can be seen from Eqs. (6) and (7), together with Eqs. (9) and (10), model A has only LH flavor-changing couplings and model B only RH ones. In model A, the same complex phase corresponding to the LH coupling occurs in both s- and t-channel exchanges, making them relatively real. Similarly, in model B, there is a single phase from RH couplings. Only model C has both LH and RH flavor-violating couplings. These couplings occur with different phases, and their interference can give rise to a rate asymmetry.

It turns out, however, that this asymmetry is constrained to be extremely small for two reasons. Since in model C both LH and RH couplings enter, there have to be two helicity flips, suppressing the effect by a factor of at least $m_e m_l/M_Z^2$. Moreover, the mixing angles in model C are severely constrained by the experimental limit on the electric dipole moment of the electron [9]. As a result, the *CP* asymmetry turns out to be unobservably small. We do not discuss this case any further.

B. Models with extra Z

Another class of popular models in which the conditions (i) and (ii) can be satisfied is based on E_6 as a grand unification group [10]. In these models, when E_6 is broken to a low energy group SU(2)$\times$[U(1)]^2 or SU(2)$\times$[U(1)]^3, there is at least one extra neutral gauge boson (Z'), and vector doublets of exotic leptons, which mix with the ordinary leptons. The presence of Z' provides the additional amplitude needed to satisfy condition (i). This has the advantage that one no longer needs to invoke the interference between the LH and the RH fermionic couplings as in the previous case. The consequent mass suppression can be avoided as a result and the asymmetries could be potentially large. Moreover, one could now obtain asymmetries in the processes $e^+e^- \to f_i\overline{f_j}$ (f=any quark or lepton) as t-channel exchanges are not required. The presence of exotic leptons also generates the flavor changing, in general complex, couplings to Z and Z'. Relative phases between the Z and Z' couplings depend upon the way the fermions transform under the gauge group. However, these phases are zero unless the Z (Z') distinguishes between various generations of the exotic (ordinary or exotic) fermions [11]. To see this, let us look at the explicit expressions for the Z and Z' couplings in such models.

The couplings of the ordinary fermions to Z bosons Z_m ($m = 1, 2$) can be parametrized in general as

$$\mathcal{L}_Z = -\left[(G_m^L)_{ij}\,\overline{f_{iL}}\gamma_\mu f_{jL}\,Z_m^\mu + L \leftrightarrow R\right], \qquad (12)$$

where Z_m are mass eigenstates related to Z and Z' by a 2x2 orthogonal matrix with mixing angle ϕ. f_{ia} ($a = L, R$) denote the chiral projections of the mass eigenstates of the ordinary fermions. They are related to the weak eigenstates of the ordinary (f'_{ia}) and exotic ($F'_{\alpha a}$) fermions through unitary matrices U^a. If the couplings of $f'_{ia}(F'_{\alpha a})$ to Z and Z' are denoted by $z_a(z_{a\alpha})$ and $z'_a(z'_{a\alpha})$, respectively, then

$$\begin{aligned}(G_1^a)_{ij} = {}& \cos(\phi)\left[\delta_{ij}z_a + (z_{a\alpha} - z_a)U^a_{\alpha j}U^{a*}_{\alpha i}\right] \\ & -\sin(\phi)\left(z'_{ak}U^a_{kj}U^{a*}_{ki} + z'_{a\alpha}U^a_{\alpha j}U^{a*}_{\alpha i}\right)\end{aligned}$$
$$(a = L, R). \qquad (13)$$

$(G_2^a)_{ij}$ are obtained from the above by the interchange $\cos(\phi) \to \sin(\phi)$, $-\sin(\phi) \to \cos(\phi)$. Here the index $k(\alpha)$ refers to the ordinary (exotic) fermions.

If the Z and Z' couplings of the weak eigenstates are independent of the flavor (index k or α above) then unitarity of U^a can be used to show that the Z and Z' couplings have the same phase. Hence unless Z or Z' distinguishes between the fermionic generations, one cannot obtain a *CP*-violating rate asymmetry for massless leptons. E_6-based models do not contain any horizontal subgroups and hence cannot lead to the desired asymmetry.

The above considerations lead us to study SU(2)$\times$U(1)$\times$U(1)$'$ models with the U(1)$'$ acting horizontally on the ordinary fermions. The requirement of anomaly cancellation restricts the choice of U(1)$'$ considerably. In particular, if U(1)$'$ acts only on leptons, then only three choices are allowed in the absence of exotic fermions [12]. While other choices of U(1)$'$ would be possible once one introduces exotic fermions or allows quarks to transform nontrivially under U(1)$'$, we shall concentrate for illustrative purposes on a specific SU(2)$\times$U(1)$\times$U(1)$'$ model with the U(1)$'$ hypercharge Y' identified with $L_e - L_\tau$. L_e, L_τ represent here the lepton numbers of e and τ, respectively. The phenomenology of this and two other related models is quite interesting and has been extensively discussed [12]. We shall consider here a specific extension which allows large flavor violation in the e-τ sector. It is possible to satisfy all three constraints in this model and obtain a large *CP*-violating rate asymmetry in the process $e^+e^- \to \tau^+e^-(\tau^-e^+)$.

III. SU(2)$\times$U(1)$\times$U(1)$'$ MODEL WITH $Y' = L_e - L_\tau$

A. The model

We now turn to a specific model with SU(2)$\times$U(1)$\times$U(1)$'$ symmetry. If U(1)$'$ acts only on leptons then the three possible choices for the U(1)$'$ hypercharge Y' are $L_\mu - L_\tau$, $L_e - L_\mu$, and $L_e - L_\tau$. In the first case, electrons do not couple to Z' as long as $L_\mu - L_\tau$ symmetry remains exact. The second choice would be significantly constrained from processes such as $\mu \to eee$ and $\mu \to e\gamma$. We therefore turn to the choice $Y' = L_e - L_\tau$. The minimal model with this choice was considered in [12]. In that model, mass terms for the charged leptons also respect the symmetry. As a consequence, the $L_e - L_\tau$ symmetry is respected by the mixing among leptons and this results in the absence of any flavor-violating couplings of Z' to leptons in spite of the fact that Z' distinguishes among flavors. Our main interest is in the flavor-changing transitions. These can be induced if we add exotic fermions to the model. We therefore enlarge the model of [12] by adding a vectorial charge--1 SU(2)$\times$U(1)-singlet lepton E' with Y'=1. This does not introduce any new anomalies. The $L_e - L_\tau$ symmetry is broken by introducing an SU(2)$\times$U(1)-singlet scalar field η with $Y' = 2$. Since η does not couple to Z, the Z and Z' do not mix and coincide, respectively, with Z_1 and Z_2. Consequently, the data from LEP do not put any restrictions on $M_{Z'}$.

Denoting the weak basis state of the singlet charged lepton E' by e'_4, we have the following

$SU(2)\times U(1)\times U(1)'$-invariant couplings involving the charged leptons e'_a $(a=1,\ldots,4)$:

$$-\mathcal{L}_Y = \frac{1}{\langle\phi^0\rangle}\left(m_{ii}\overline{e'_{iL}}e'_{iR} + m_{14}\overline{e'_{1L}}e'_{4R}\right)\phi^0 + m_{41}\overline{e'_{4L}}e'_{1R} + \frac{1}{\langle\eta\rangle}m_{43}\overline{e'_{4L}}e'_{3R}\eta + m_{44}\overline{e'_{4L}}e'_{4R} + \text{H.c.} \tag{14}$$

$(i=1,2,3)$. This results in the following mass matrix for the charged leptons:

$$M = \begin{pmatrix} m_{11} & 0 & 0 & m_{14} \\ 0 & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & 0 \\ m_{41} & 0 & m_{43} & m_{44} \end{pmatrix}. \tag{15}$$

The second generation does not mix with the remaining ones. This is a consequence of an unbroken L_μ. Moreover, if m_{43} were set to zero, e'_3 also does not mix with the rest. In that situation, $L_e - L_\tau$ would remain an exact symmetry of M, and the flavor-violating coupling of Z' to $e\tau$ would not arise. The presence of η simultaneously breaks the $L_e - L_\tau$ spontaneously and generates the required flavor-violating interactions.

M can be diagonalized by a biunitary transformation:

$$U^L M U^{R\dagger} = M_{\text{diagonal}}. \tag{16}$$

The $U^{L,R}$ are 4×4 matrices relating the weak and mass eigenstates:

$$e'_{aL,R} = U^{L,R\dagger}_{ab} e_{bL,R}. \tag{17}$$

Given the form of M as in Eq. (15), the elements $U^{L,R}_{ab}$ $(a,b\neq2)$ are nonzero and one gets the required flavor violation.

The fermionic couplings [Eq. (5)] to $Z\equiv Z_1$ and $Z'\equiv Z_2$ are now given by

$$(G_1^L)_{ij} = \frac{g}{\cos\theta_W}[(-\tfrac{1}{2}+\sin^2\theta_W)\delta_{ij} + \tfrac{1}{2}U^{L*}_{4i}U^L_{4j}], \tag{18}$$

$$(G_1^R)_{ij} = \frac{g}{\cos\theta_W}\left[\sin^2\theta_W\delta_{ij}\right], \tag{19}$$

$$(G_2^a)_{ij} = g'\left[\delta_{ij} - 2U^{a*}_{3i}U^a_{3j}\right] \quad (i,j\neq2). \tag{20}$$

$i,j=1,3$ label the e and τ, respectively, while the index 4 corresponds to E. $(G_2^a)_{ij}$ is zero when i or j is 2. g, g' are, respectively, $SU(2)$,$U(1)'$ coupling constants and θ_W is the SM weak mixing angle. The observed leptonic universality of the Z decay requires $|U_{41}|$ and $|U_{43}|$ to be small compared to 1. In this case, it is convenient to adopt the following parametrization for the elements of U^a:

$$\begin{aligned} U^a_{11} &= U^a_{33} = \cos\theta_a, \\ U^a_{13} &= -U^a_{31} = \sin\theta_a, \\ U^a_{43} &= |U^a_{43}|e^{i\delta_a}, \end{aligned} \tag{21}$$

with U^a_{41} chosen real.

B. Calculation of the asymmetry

The cross section for $e^+e^-\to l_i\overline{l_j}$ $(i,j=1,3)$ including s- and t-channel Z_1 and Z_2 exchanges is given by

$$\sigma(e^+e^-\to l_i\overline{l_j}) = \int\frac{dt}{16\pi s^2}[\,(G_n^L)^*_{ij}(G_m^L)_{ij}(G_m^L)_{11}(G_n^L)_{11}(s+t)^2(f_m(s)+f_m(t))(f_n(s)+f_n(t))^* + (G_n^L)^*_{ij}(G_m^L)_{ij}(G_m^R)_{11}(G_n^R)_{11}(s^2f_m(t)f_n^*(t)+t^2f_m(s)f_n^*(s)) + L\longleftrightarrow R]. \tag{22}$$

Lepton masses have been neglected and s,t are the usual Mandelstam variables. f_m are the Breit-Wigner functions occurring in the $Z_{1,2}$ propagators [13]:

$$f_m(s) = \frac{1}{s-M_m^2+iM_m\Gamma_m}, \tag{23}$$

and likewise for $f_m(t)$. M_m (Γ_m) denotes the mass (total width) of Z_m.

The difference and the sum of the cross sections for $e^+e^-\to e^-\tau^+, e^+\tau^-$ follow from Eqs. (6) and (9)–(12). The difference is given by

$$\begin{aligned}\sigma(e^+e^-\to e^-\tau^+) - \sigma(e^+e^-\to e^+\tau^-) = \int\frac{dt}{8\pi s^2}\frac{-g^2g'^2}{\cos^2\theta_W}&|U^L_{41}U^L_{43}|\sin2\theta_L\sin\delta_L \\ &\times\{(-\tfrac{1}{2}+\sin^2\theta_W+\tfrac{1}{2}|U^L_{41}|^2)\cos2\theta_L\,(s+t)^2 \\ &\times\text{Im}[(f_1(s)+f_1(t))(f_2(s)+f_2(t))^*] \\ &+\sin^2\theta_W\cos2\theta_R\,\text{Im}[t^2f_1(s)f_2(s)^*+s^2f_1(t)f_2(t)^*]\}.\end{aligned} \tag{24}$$

The sum is given by

$$\sigma(e^+e^- \to e^-\tau^+) + \sigma(e^+e^- \to e^+\tau^-)$$

$$\begin{aligned}
= \int \frac{dt}{8\pi s^2}\Bigg(& \frac{g^4}{4\cos^4\theta_W}[(s+t)^2(|f_1(s)+f_1(t)|^2)(-\tfrac{1}{2}+\sin^2\theta_W+\tfrac{1}{2}|U^L_{41}|^2)^2 \\
& \qquad +(s^2|f_1(t)|^2+t^2|f_1(s)|^2)\sin^4\theta_W]|U^L_{41}U^L_{43}|^2 \\
& +\frac{g^2g'^2}{\cos^2\theta_W}\{(s+t)^2\mathrm{Re}[(f_1(s)+f_1(t))(f_2(s)+f_2(t))^*](-\tfrac{1}{2}+\sin^2\theta_W+\tfrac{1}{2}|U^L_{41}|^2)\cos 2\theta_L \\
& \qquad +\mathrm{Re}[s^2f_1(t)f_2(t)^*+t^2f_1(s)f_2(s)^*]\sin^2\theta_W\cos 2\theta_R\}|U^L_{41}U^L_{43}|\cos\delta\sin 2\theta_L \\
& +g'^4\{(s+t)^2|f_2(s)+f_2(t)|^2(\sin^2 2\theta_L\cos^2 2\theta_L+\sin^2 2\theta_R\cos^2 2\theta_R) \\
& \qquad +[s^2|f_2(t)|^2+t^2|f_2(s)|^2](\sin^2 2\theta_L\cos^2 2\theta_R+\sin^2 2\theta_R\cos^2 2\theta_L)\}\Bigg). \qquad (25)
\end{aligned}$$

The ratio of the above equations gives the asymmetry

$$A = \frac{\sigma(e^+e^- \to e^-\tau^+) - \sigma(e^+e^- \to e^+\tau^-)}{\sigma(e^+e^- \to e^-\tau^+) + \sigma(e^+e^- \to e^+\tau^-)}\,. \qquad (26)$$

C. Experimental constraints

Before presenting the results, a discussion of constraints on various parameters in the model is in order. The constraints on the model from various experiments involving flavor-diagonal neutral currents have been discussed in great detail by He *et al.* [12], and those continue to apply. However, since the model considered here has an additional sector with flavor violation, there are further constraints coming from nonobservation of leptonic flavor-violating processes, which put limits on the mixing of ordinary with heavy leptons.

Flavor-violating Z decays into leptons have been looked for at LEP [14], and their nonobservation limits the couplings $(G^a)_{ij} (i \neq j)$ in our model. However, more stringent bounds [3, 15] on the mixing come from the flavor-diagonal Z decays into leptons measured at LEP, together with the Schwarz inequality for $(U^a)_{\alpha i}$. These can be shown [15] to imply the upper limit $|U^L_{41}U^L_{43}| \leq 0.0106$. We have assumed the maximum value $|U^L_{41}U^L_{43}| = 0.0106$ for our numerical estimates. This corresponds [15] to a branching ratio for $Z \to \tau e$ a factor of about 10 less than the bound put by the direct search [14].

A further constraint comes from limits on the branching ratio for $\tau^- \to e^-e^-e^+$, which can occur in the model via Z and Z' exchange. The branching ratio is given by [16]

$$\begin{aligned}
B(\tau \to 3e) = 2B(\tau \to e\nu\overline{\nu})\Bigg[& |U^L_{41}U^{L*}_{43}|^2[(-\tfrac{1}{2}+\sin^2\theta_W+\tfrac{1}{2}|U^L_{41}|^2)^2+\tfrac{1}{2}\sin^4\theta_W] \\
& -4\frac{M_Z^2}{M_{Z'}^2}\left(\frac{g'\cos\theta_W}{g}\right)^2(-\tfrac{1}{2}+\sin^2\theta_W+\tfrac{1}{2}|U^L_{41}|^2)\,|U^L_{41}U^L_{43}|\sin 2\theta_L\cos 2\theta_L\cos\delta \\
& +\frac{M_Z^4}{M_{Z'}^4}\left(\frac{g'\cos\theta_W}{g}\right)^4[3(\sin^2 2\theta_L+\sin^2 2\theta_R)(\cos^2 2\theta_L+\cos^2 2\theta_R) \\
& \qquad +(\sin^2 2\theta_L-\sin^2 2\theta_R)(\cos^2 2\theta_L-\cos^2 2\theta_R)]\Bigg]. \qquad (27)
\end{aligned}$$

Using the experimental limit $B(\tau \to eee) \leq 3.8 \times 10^{-5}$ we have calculated the lower limit on $M_{Z'}$ as a function of g'/g, which is shown in Fig. 1 [17].

IV. RESULTS AND DISCUSSION

Using the constraints on the parameters coming from the previous discussion still leaves the parameters θ_L, θ_R, and δ free. For a suitable choice of these parameters, we can hope to get a large asymmetry as well as the flavor-violating total cross sections.

In Fig. 1 is shown, together with the minimum value of $M_{Z'}$ coming from the previous section, the asymmetry A for this minimum $M_{Z'}$ at $\sqrt{s} = M_Z$ for different values of g'/g. The continuous and dashed lines correspond, respectively, to $\theta_L = 80°$ and $\theta_L = 45°$. We have taken $\theta_R = 0$, $\delta_L = 90°$, $\sin^2\theta_W = 0.23$ and the fine structure constant $\alpha(M_Z^2) = \frac{1}{128}$. The Z resonance parameters chosen are M_Z=91.16 GeV and Γ_Z=2.55 GeV. The Z' total width in the model is given by

$$\Gamma_{Z'} = \frac{g'^2}{4\pi}M_{Z'}. \qquad (28)$$

It is seen from Fig. 1 that the asymmetry is generally quite large (about 20–25 %) near $\sqrt{s} = M_Z$, and can even approach 100% for $\theta_L = 80°$ and small g'/g.

In Fig. 2, the asymmetry is plotted as a function of $\sqrt{s}$ for somewhat larger values of $M_{Z'}$ compared to the

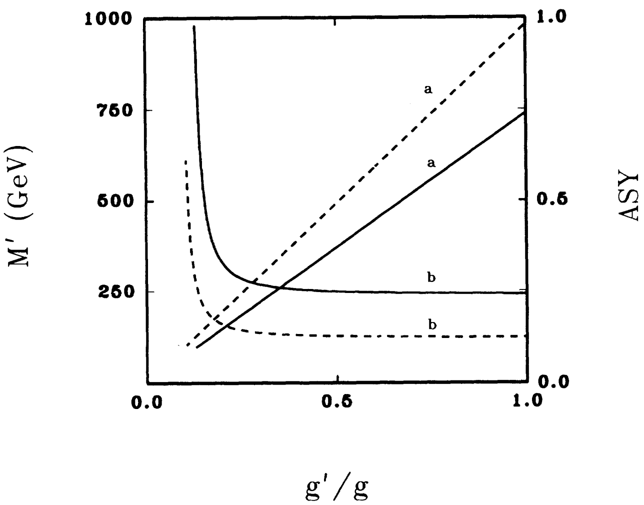

FIG. 1. (a) The lower limit M' on $M_{Z'}$ coming from the experimental limit on $\tau \rightarrow eee$, and (b) the value of the asymmetry A (labeled ASY) corresponding to this minimum value of $M_{Z'}$ plotted against g'/g. The solid and dashed curves correspond, respectively, to $\theta_L = 80°$ and $\theta_L = 45°$.

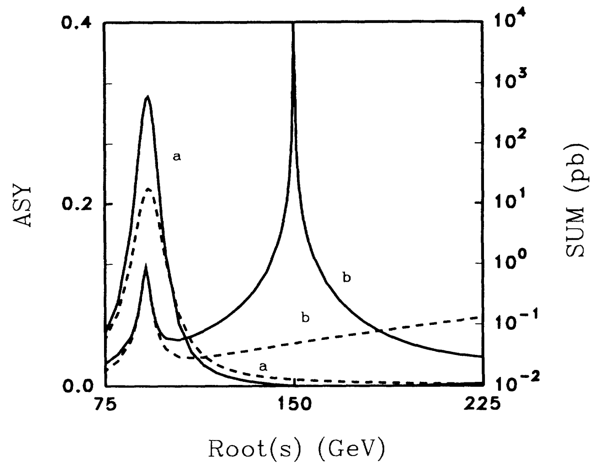

FIG. 2. (a) The asymmetry and (b) the sum of $\tau^- e^+$ and $\tau^+ e^-$ cross sections in e^+e^- annihilation plotted against $\sqrt{s}$ for $M_{Z'}$ = 150 GeV and g'/g = 0.2 (solid curve) and for $M_{Z'}$ = 400 GeV and $g'/g = 0.5$ (dashed curve). Both correspond to $\theta_L = 80°$ and $\theta_R = 0$.

minimum allowed, and for $\theta_L = 80°$. These are more conservative cases. The asymmetry peaks at $\sqrt{s} = M_Z$, where it is fairly large. To get an idea of the number of events expected, the sum of $\tau^- e^+$ and $\tau^+ e^-$ production is also plotted in Fig. 2 as a function of $\sqrt{s}$. Though the cross section is considerably higher at the Z' peak than at the Z peak, and can provide large flavor violation, the asymmetry is large only at $\sqrt{s} = M_Z$.

The cross section for τe production at the Z peak is in the picobarn range, which should be compared to the total peak cross section, which is about 50 nb. Assuming an overall τe detection efficiency of 0.1, observation of a τe cross section at this level would be possible with $\geq 5 \times 10^5$ Z events. The CP asymmetry A would then be observable with a minimum number N_Z of Z events given by $N_Z = [A^2/5 \times 10^5]^{-1}$. For $A \approx 0.2$, for example, $N_Z \approx 1.25 \times 10^7$. We know, however, from Figs. 1 and 2, that much higher asymmetries are possible. These would easily be observable in future runs of LEP.

Though we have included the effect of the width of the Z', it does not play a significant role in determining asymmetries near Z. More interestingly, as displayed in Fig. 2, the asymmetry can be large even if $M_{Z'}$ is significantly higher than M_Z [18]. Hence, even if Z' is not directly observable at present energies, its effect in the CP asymmetry could be observed.

In conclusion, we have explored a novel type of contribution to the CP-violating rate asymmetry in $e^+e^- \rightarrow l_i \bar{l}_j$ due to the nonzero width of Z, Z' bosons. In models with large leptonic flavor violation, observable asymmetries can arise. This effect, if present, should be easily observable at LEP. It is worthwhile emphasizing that similar effects would be present in other processes and models, and should be investigated.

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[13] Strictly speaking, the Breit-Wigner form gives the correct absorptive part of the amplitude only for s-channel exchange (and even then, Γ should be a function of s). However, our numerical results do not change significantly by dropping the imaginary part of $f_m(t)$.

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[17] We consider only $M_{Z'} > M_Z$, since for $M_{Z'} \approx M_Z$ the large Z' contribution to the cross section for $e^+e^- \to e^+e^-$ for $\sqrt{s} \approx M_Z$ is in conflict with experiment.

[18] Large asymmetries can result for large $M_{Z'}$ by choosing a correspondingly large g'. We do not need a g' larger than 1 even for $M_{Z'}$ of order TeV in order to get asymmetries at a few percent level.